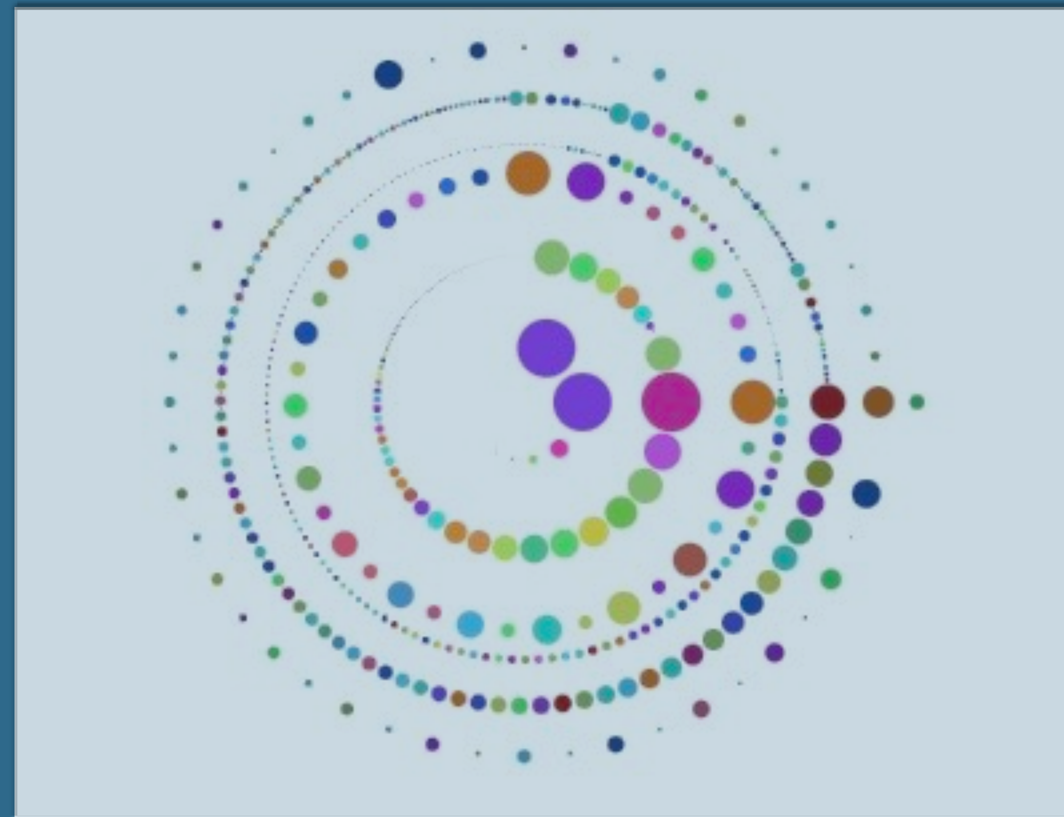


Managing data in the LMFDB

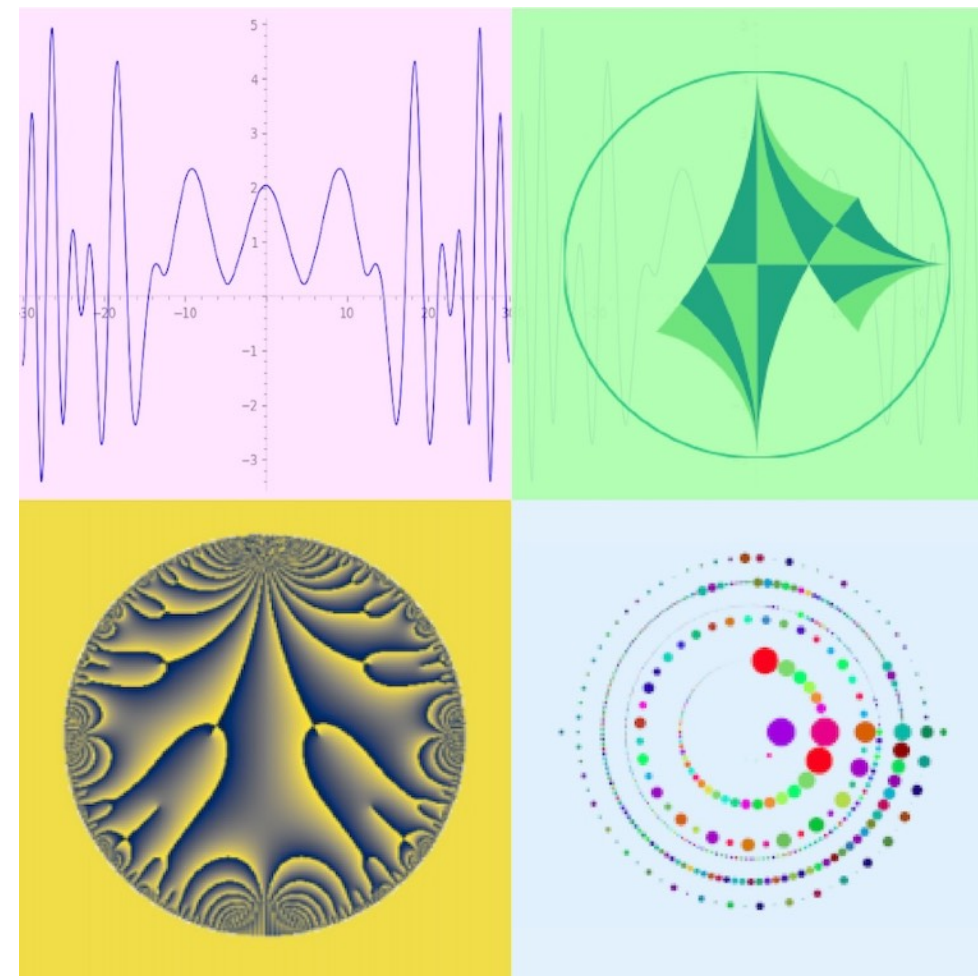
Jen Paulhus
now at Mount Holyoke College

LMFDB, Computation and Number Theory (LuCaNT) 2025

7-11 July 2025
ICERM (Providence, RI)

https://lucant.org

- LMFDB, ***mathematical databases***, computation, number theory, and related topics
- A peer-reviewed proceedings volume will be published for accepted papers, including “short communications”: submission deadline in January 2025.



Organizers: John Jones, Jen Paulhus, Drew Sutherland, and John Voight

www.lmfdb.org

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The L-functions and modular forms database (LMFDB)

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Overview · Random
Universe · Knowledge

L-functions
Rational · All

Modular forms
Classical · Maass
Hilbert · Bianchi

Varieties
Elliptic curves over $\mathbb{Q}$
Elliptic curves over $\mathbb{Q}(\alpha)$
Genus 2 curves over $\mathbb{Q}$
Higher genus families
Abelian varieties over $\mathbb{F}_q$

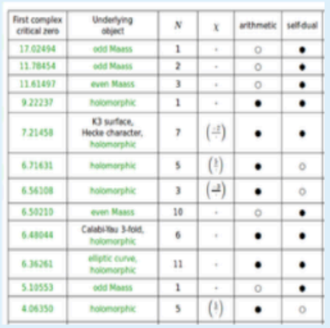
Fields
Number fields
p-adic fields

Representations
Dirichlet characters
Artin representations

Groups
Galois groups
Sato-Tate groups

Database

A database



The LMFDB is a database of mathematical objects arising in number theory and arithmetic geometry that illustrates some of the mathematical connections predicted by the Langlands program.

Click a heading on the left to browse, or go to a random page.

Announcements

The first LuCaNT conference took place July 10-14, 2023 at ICERM. Thanks to everyone who attended! Conference proceedings will be published soon.

Check out the recently updated abstract groups database [beta].

Check out the new modular curves database [beta].

Learn more

Information is available regarding the source, reliability, and completeness of the database.

Knowls provide explanations when you need them.

Overview · LMFDB universe · Knowledge · Data

Citations and acknowledgments

- How to cite the LMFDB
- Source code repository
- Editorial board
- Acknowledgments

Plan of Attack

Use a newer database of abstract groups in the LMFDB to talk about a few things we learned, decisions we made, and challenges we faced in creating and managing such a project.

What is the LMFDB?

- Initial goal to *connect* objects in Langland Program.
- Supporting cast of characters came along. Still about c*onnecting* objects.

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- Initial goal to *connect* objects in Langland Program.
- Supporting cast of characters came along. Still about c*onnecting* objects.
- Massive collaboration (over 100 contributors).
- Journal structure: managing and associate editors.
- Code is on GitHub. Anyone can contribute!
- Webpages hosted on Google Cloud.

Usage Statistics

	2020	2021	2022	2023
Users	32,301	48,664	47,667	48,396
Pageviews	438,095	514,350	522,373	580,161
Session length	4:48	4:06	4:32	4:25

In 2023 we had users in 175 countries and all 50 US states.

In the first half of 2024 there were over 340,000 pageviews, versus 220,000 in the same period in 2023.

www.lmfdb.org/Groups/Abstract/

LMFDB BETA

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Abstract groups

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The database currently contains 544,831 groups from many different sources, the largest of which is S_{47} of order $47!$. In addition, it contains 275,379,753 of their subgroups and 39,933,457 of their irreducible complex characters. You can browse further statistics.

Learn more
Source and acknowledgements
Completeness of the data
Reliability of the data
Abstract group labeling

Browse

By order: 1-64 65-127 128 129-255 256 257-383 384 385-511 513-1000 1001-1500 1501-2000 2001-

By nilpotency class: 1 2 3 4 5 6 7 8 9 (and not nilpotent)

By property: abelian nonabelian solvable nonsolvable simple perfect rational

Some interesting groups or a random group

Search for subgroups or complex characters

Search

Advanced search options

Order	3	e.g. 4, or a range like 3..5	Exponent	2, 3, 7	e.g. 2, or list of integers like 2, 3, 7
Automorphism group	4.2	e.g. 4.2	Nilpotency class	3	e.g. 4, or a range like 3..5
Automorphism group order	3	e.g. 4, or a range like 3..5	Commutator	4.2, 8	e.g. 4 or 4.2 (order or label)
Center	4.2, 8	e.g. 4 or 4.2 (order or label)	Abelianization	4.2, 8	e.g. 4 or 4.2 (order or label)
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Label or name 8.3 Find

e.g. 8.3, GL(2,3), 8T34, C3:C4, C2*A5, C16.D4, or 12.4.2.b1.a1

Why Another Groups Database?

- Abstract groups appear throughout the LMFDB: automorphism groups of curves, Sato-Tate groups, Galois groups, subgroups of $\mathrm{GL}(2,\mathbb{Z}/N)$ corresponding to modular curves.

- We wanted to add pages for each of these groups with core information, and then connect those pages with the number theoretic objects' pages.

- Much of our initial work was motivated by groupnames.org.

Group pages co-conspirators:

Lewis Combes, John Jones, David Roberts, David Roe, Manami Roy, Sam Schiavone, Drew Sutherland

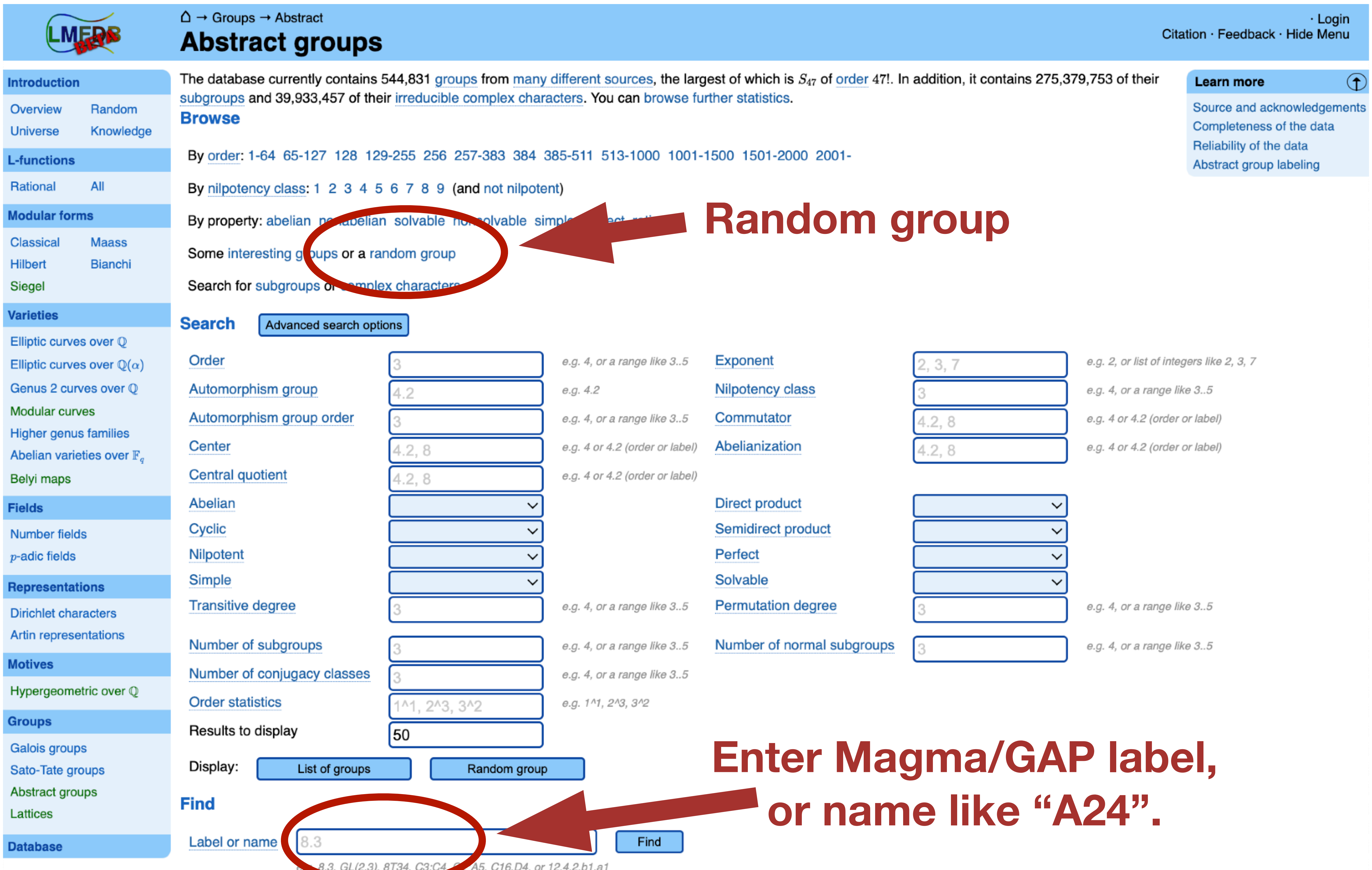
Acknowledgements:

All the computational group theorists who have contributed in any way to the many algorithms in Magma and GAP.

John Cannon and others at Magma who responded quickly to bugs.

Tim Dokchitser generously shared code and database expertise with us.

www.lmfdb.org/Groups/Abstract/



www.lmfdb.org/Groups/Abstract/

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Abstract group A_{24}

Group information

Description:	A_{24}
Order:	$310\cdots000 = 2^{21} \cdot 3^{10} \cdot 5^4 \cdot 7^3 \cdot 11^2 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
Exponent:	$5354228880 = 2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
Automorphism group:	Group of order $620\cdots000 = 2^{22} \cdot 3^{10} \cdot 5^4 \cdot 7^3 \cdot 11^2 \cdot 13 \cdot 17 \cdot 19 \cdot 23$ (generators)
Outer automorphisms:	C_2, of order 2
Composition factors:	A_{24}
Derived length:	0

This group is nonabelian and simple (hence nonsolvable, perfect, quasisimple, and almost simple).

Group statistics

Order	1	2	3	4	5
Elements	1	8792390355903	13428028220072048	25429245463784131120	1725747644222610624
Conjugacy classes	1	6	8	18	4
Divisions	1	6	8	18	4
Autjugacy classes	1	6	8	18	4

Minimal Presentations

Permutation degree:	24
Transitive degree:	24
Rank:	not computed
Inequivalent generating tuples:	not computed

Properties

Label	310224200866619719680000.a
Order	$2^{21} \cdot 3^{10} \cdot 5^4 \cdot 7^3 \cdot 11^2 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
Exponent	$2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$

Simple	yes
$\#G^{ab}$	1
$\#Z(G)$	1
$\#\mathrm{Aut}(G)$	$2^{22} \cdot 3^{10} \cdot 5^4 \cdot 7^3 \cdot 11^2 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
$\#\mathrm{Out}(G)$	2
Perm deg.	24
Trans deg.	24
Rank	not computed

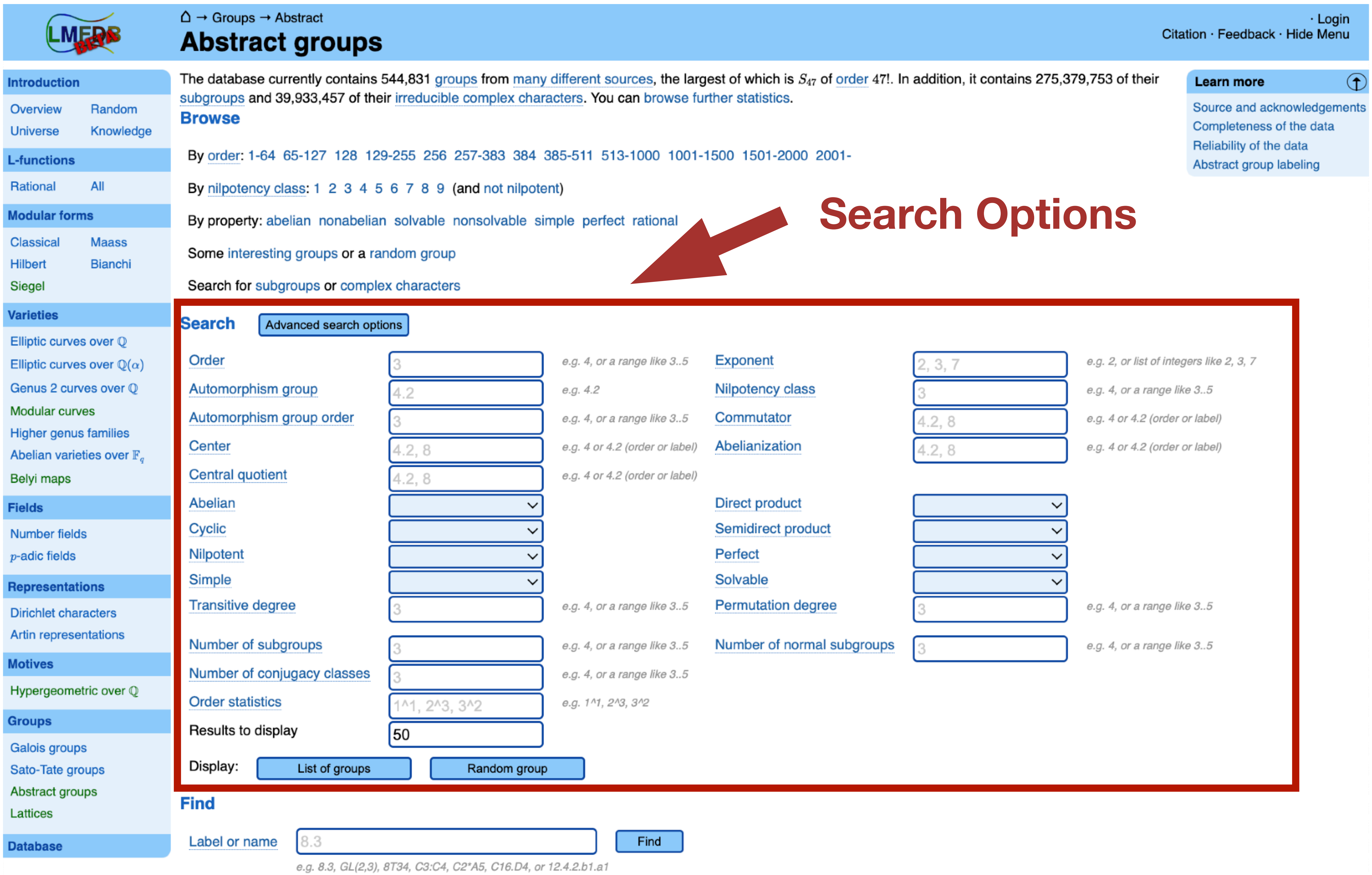
Related objects

Subgroups
Extensions
Supergroups
As a transitive group

Downloads

Group to Gap
Group to Magma

www.lmfdb.org/Groups/Abstract/



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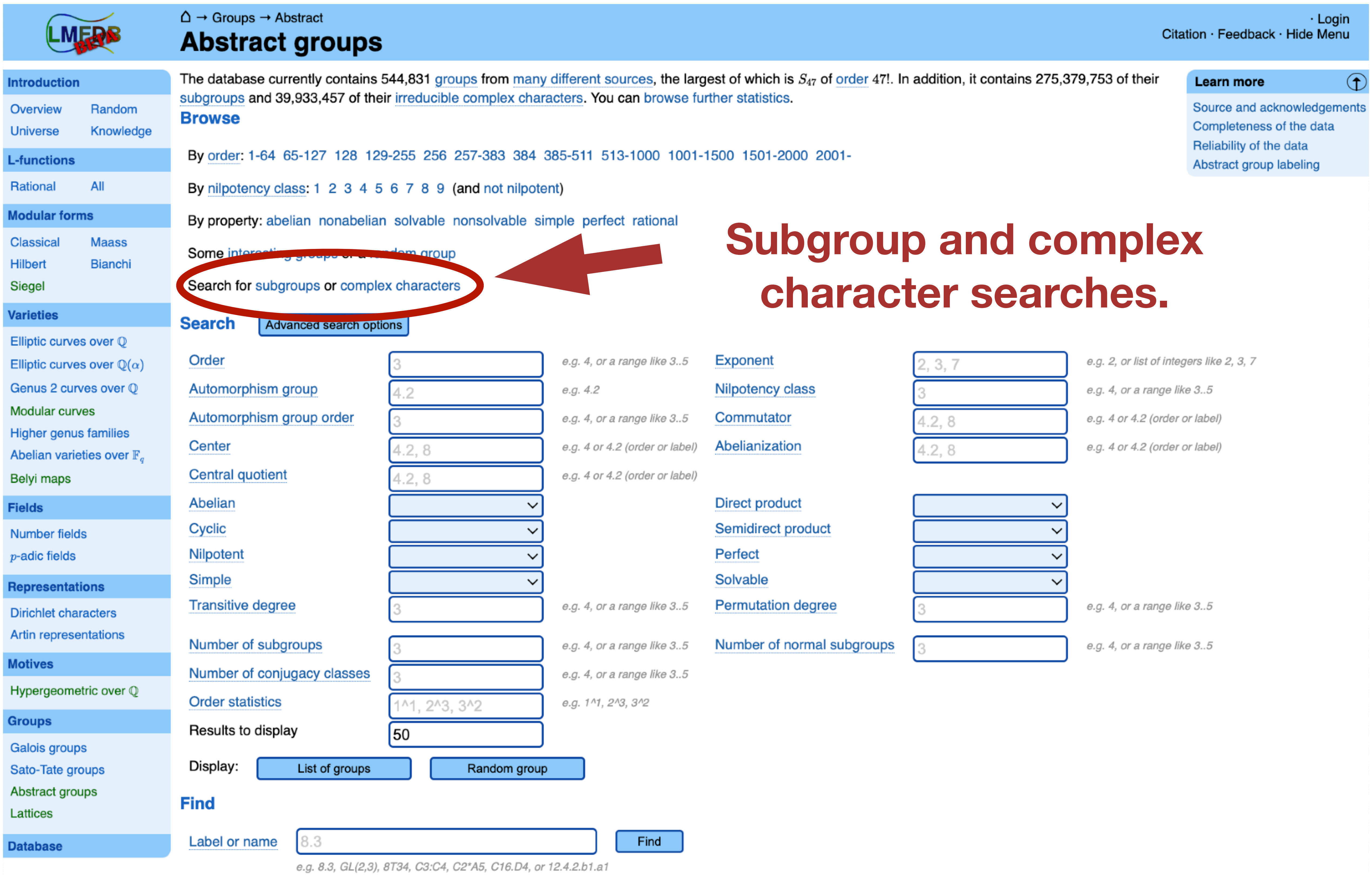
Search Options

Features

- In a browser
- Easily searchable
- GitHub code makes it easy to keep track of issues and feature requests
- 544,831 groups from multiple sources (see "Completeness of the data")
- Over 275 million subgroups, almost 40 million irreducible complex characters

www.lmfdb.org/Groups/Abstract/



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Subgroup and complex character searches.

Unique Searches

We can search for all groups which satisfy the following short exact sequence:

$$1 \to D_6 \to G \to D_{10} \to 1$$

These types of searches are possible because of several different database tables: groups, subgroups, conjugacy classes, both rational and complex characters

Click on "Underlying data" to see the different tables.

Backend Basics

Postgres is our database management system.

The code to create the group database is on GitHub:
https://github.com/roed314/FiniteGroups

We used hundreds of years of virtual CPU time on Google Compute Engine machines of type n2d-standard-2.

Making Choices

- Chose to store groups connected to LMFDB first (so not all groups of order 512, but much larger groups that connect to one of the other databases in LMFDB).

- Sometimes only stored subgroups up to automorphisms instead of up to conjugation (C_2^{11} has 11 subgroups up to automorphism but over 229 million up to conjugation).

Runtime Issues

The computational time is CPU centuries. What happens if there are bugs?

Runtimes are unpredictable and not based on, say, the order of the group.

- How many conjugacy classes.
- How many subgroups.
- Is it trivial to determine that a group does or does not satisfy certain booleans?

(partial) Solutions

- Parallelize as much as we can.
- Structure code to recover from one computation taking too long or having a bug.
- Store partial data to reload into future versions if our current version times out.

(partial) Solutions

- Parallelize as much as we can.
- Structure code to recover from one computation taking too long or having a bug.
- Store partial data to reload into future versions if our current version times out.
- Verification (internal and external) in progress.

Adding New Groups

Hash for Isomorphisms

- If order is identifiable by Magma or GAP, use their functions to identify group.
- If the group is abelian, use abelian invariants.
- Else create 64 bit integer that combines list of pairs *(order, size)* for conjugacy classes of maximal subgroups of the group.

On 408,641,062 groups of order 1536, there were 408,597,690 distinct values. Maximum cluster was 72 groups.

Groups of Lie Type

SL(2,16)

Constructions

Show commands: Gap / Magma

Groups of Lie type: $\mathrm{SL}(2,16)$, $\mathrm{SO}(3,16)$, $\mathrm{SU}(2,16)$, $\Omega(3,16)$, $\Omega^-(4,4)$, $\mathrm{PGL}(2,16)$

Multiple Presentations

SmallGroup(48,10)

Constructions

Show commands: Gap / Magma

Presentation:	$\langle a,b \mid b^{12}=1, a^4=b^6, b^a=b^{11} \rangle$	
Permutation group:	Degree 11 $\langle(1,2,3,6,4,5,7,8)(10,11),(2,5)(6,8),(1,3,4,7)(2,6,5,8),(1,4)(2,5)(3,7)(6,8),(9,10,11)\rangle$	
Matrix group:	$\left\langle \begin{pmatrix} -1 & 0 & 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} \right\rangle \subseteq \mathrm{GL}_6(\mathbb{Z})$ $\left\langle \begin{pmatrix} 1 & 0 \\ 0 & 12 \end{pmatrix}, \begin{pmatrix} 0 & 8 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 7 \end{pmatrix} \right\rangle \subseteq \mathrm{GL}_2(\mathbb{F}_{13})$	
Transitive group:	24T20	more information
Direct product:	not isomorphic to a non-trivial direct product	
Semidirect product:	$(C_3 : C_8) \rtimes C_2$ (2)	more information
Trans. wreath product:	not isomorphic to a non-trivial transitive wreath product	
Non-split product:	$C_4 \,.\, D_6$ $C_{12} \,.\, C_4$ $C_3 \,.\, \mathrm{OD}_{16}$ $C_{12} \,.\, C_2^2$	all 11

Code Snippets

Constructions

Show commands: Gap / Magma

Presentation: $\langle a, b \mid b^{12} = 1, a^4 = b^6, b^a = b^{11} \rangle$

```
magma: G := PCGroup([5, -2, -2, -2, -2, -3, 10, 126, 662, 42, 803, 58, 804]); a,b := Explode([G.1, G.3]); AssignNames(~G, ["a", "a2", "b", "b2
```

Permutation group: Degree 11 $\langle (1,2,3,6,4,5,7,8)(10,11), (2,5)(6,8), (1,3,4,7)(2,6,5,8), (1,4)(2,5)(3,7)(6,8), (9,10,11) \rangle$

```
magma: G := PermutationGroup< 11 | (1,2,3,6,4,5,7,8)(10,11), (2,5)(6,8), (1,3,4,7)(2,6,5,8), (1,4)(2,5)(3,7)(6,8), (9,10,11) >;
```

Matrix group:

$$\left\langle \begin{pmatrix} -1 & 0 & 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix} \right\rangle \subseteq \mathrm{GL}_6(\mathbb{Z})$$

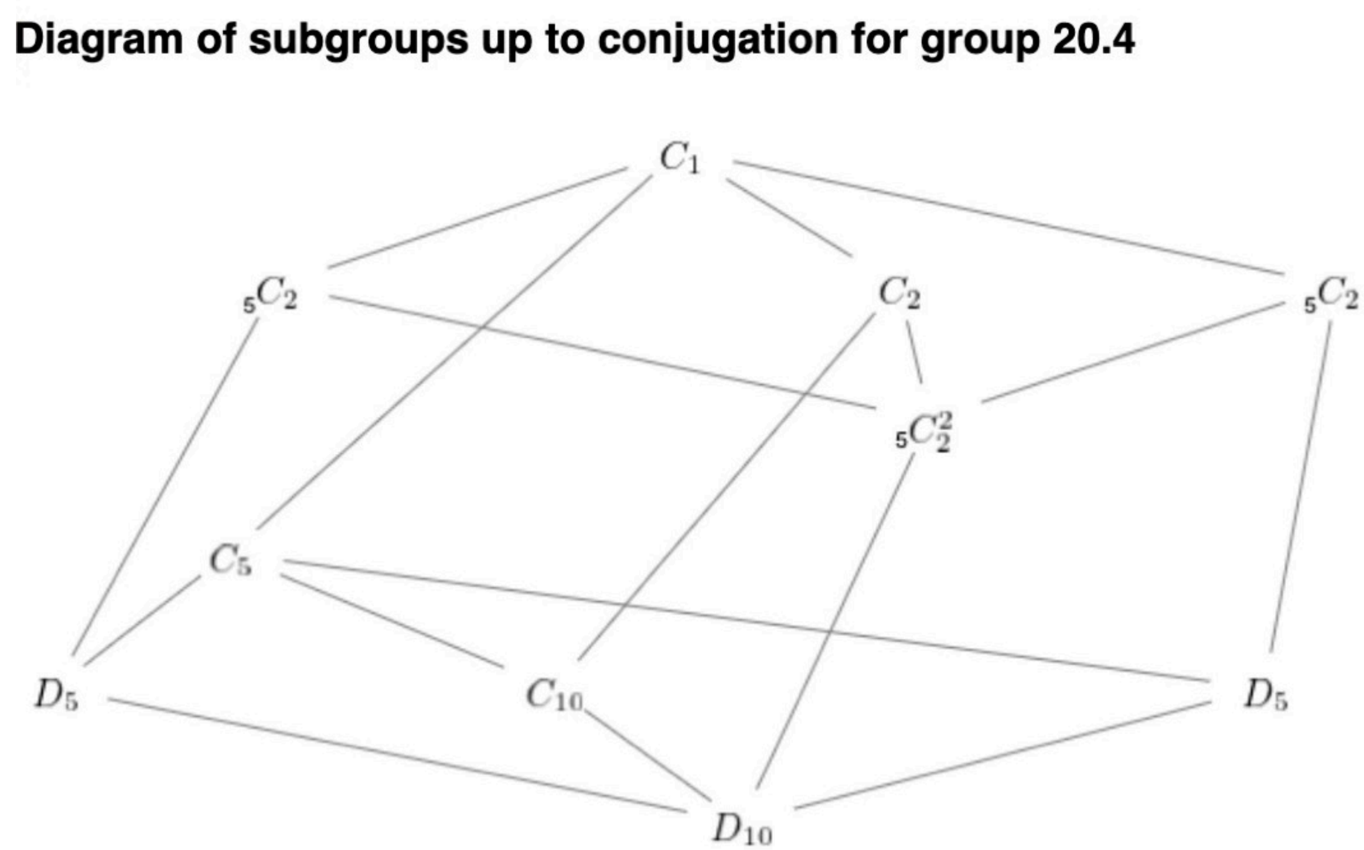
```
magma: G := MatrixGroup< 6, Integers() | [[-1, 0, 1, 1, 0, 0, -1, 0, 1, 0, 0, 0, 0, -1, 1, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
```

$$\left\langle \begin{pmatrix} 1 & 0 \\ 0 & 12 \end{pmatrix}, \begin{pmatrix} 0 & 8 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 7 \end{pmatrix} \right\rangle \subseteq \mathrm{GL}_2(\mathbb{F}_{13})$$

```
magma: G := MatrixGroup< 2, GF(13) | [[1, 0, 0, 12], [0, 8, 1, 0], [2, 0, 0, 7]] >;
```

Galois Correspondence in Action

Number field 20.0.99282070616165289306357 76.1 and group D_{10}



Intermediate fields

$\mathbb{Q}(\sqrt{-79})$, $\mathbb{Q}(\sqrt{-1})$, $\mathbb{Q}(\sqrt{79})$, $\mathbb{Q}(i, \sqrt{79})$, 5.1.6241.1 x5, 10.0.3077056399.1, 10.0.39884882944.4 x5, 10.2.3150905752576.3 x5

The End