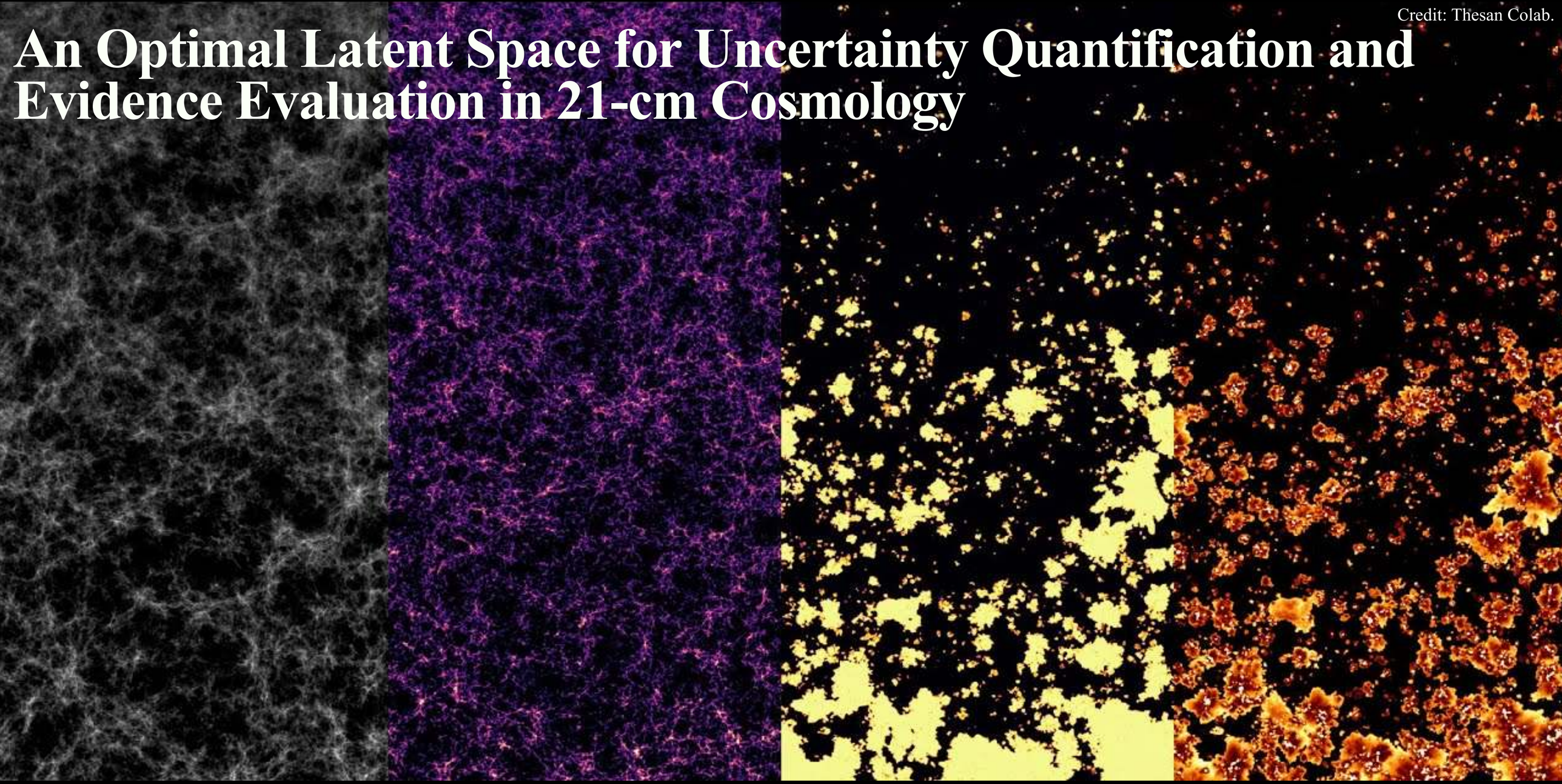
Credit: Thesan Colab.

An Optimal Latent Space for Uncertainty Quantification and Evidence Evaluation in 21-cm Cosmology

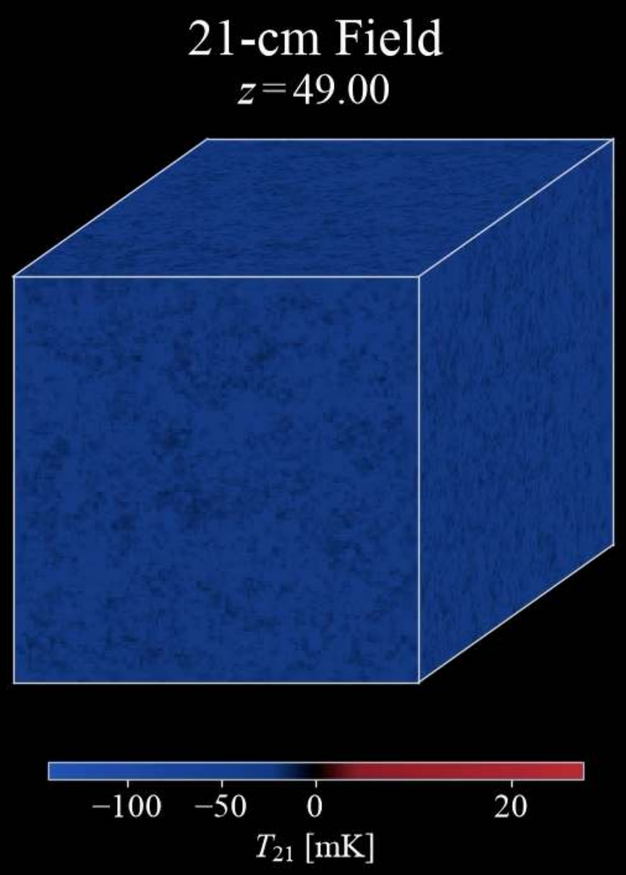


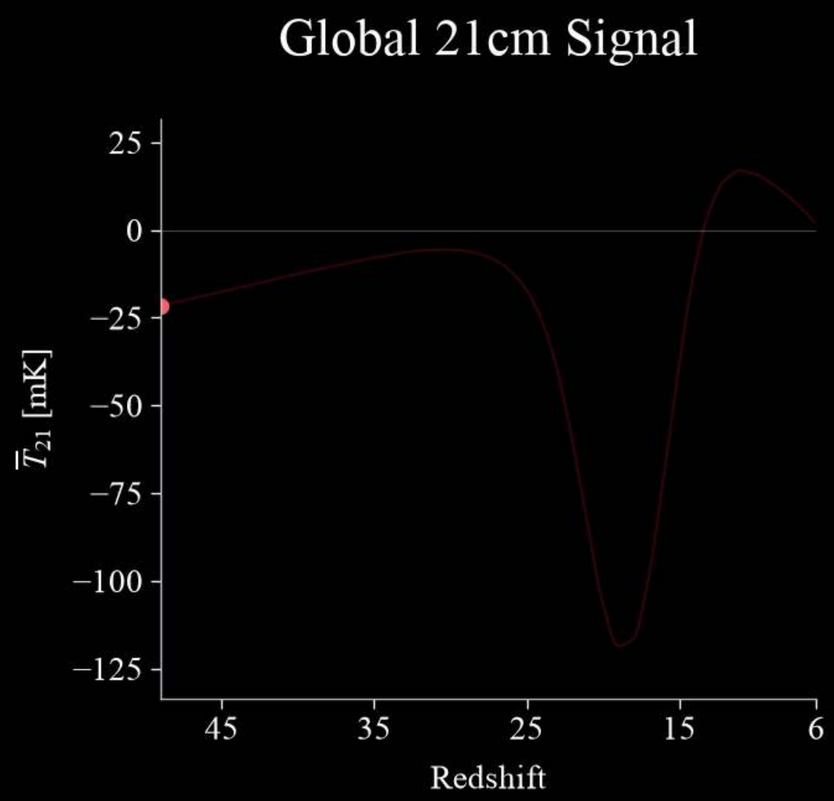
Jacob Tutt (with Peter Sims and Eloy de Lera Acedo)

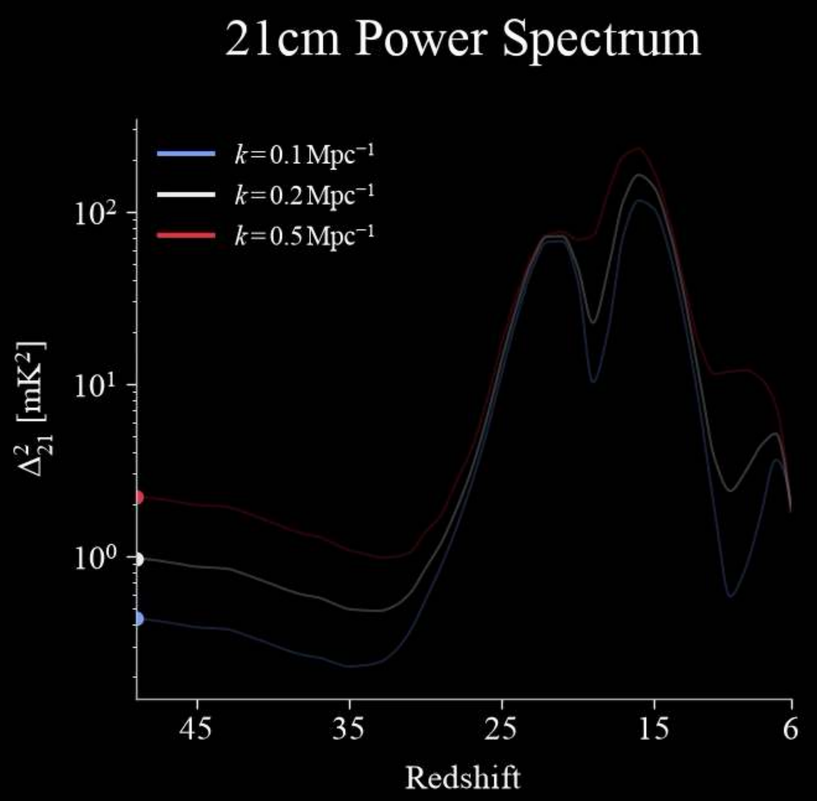
21cm Meeting Series 2026

CAVENDISH ASTROPHYSICS

Insights to the High-Z Universe
The 21cm Brightness Field





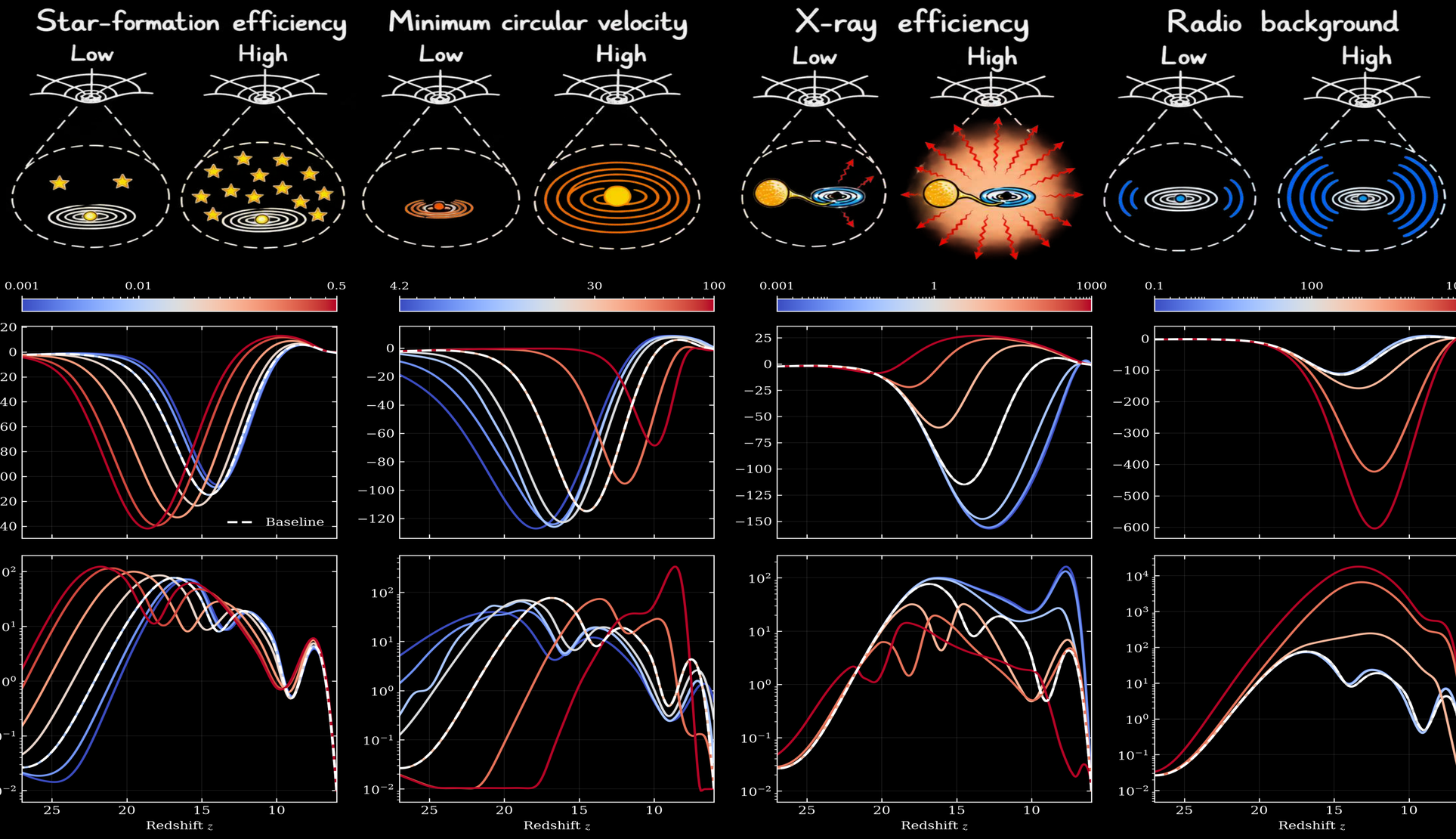


Credit: 21cmSPACE Simulation (Cosmic Dawn Lab, Cambridge)

Insights to the High-Z Universe
What can we learn?

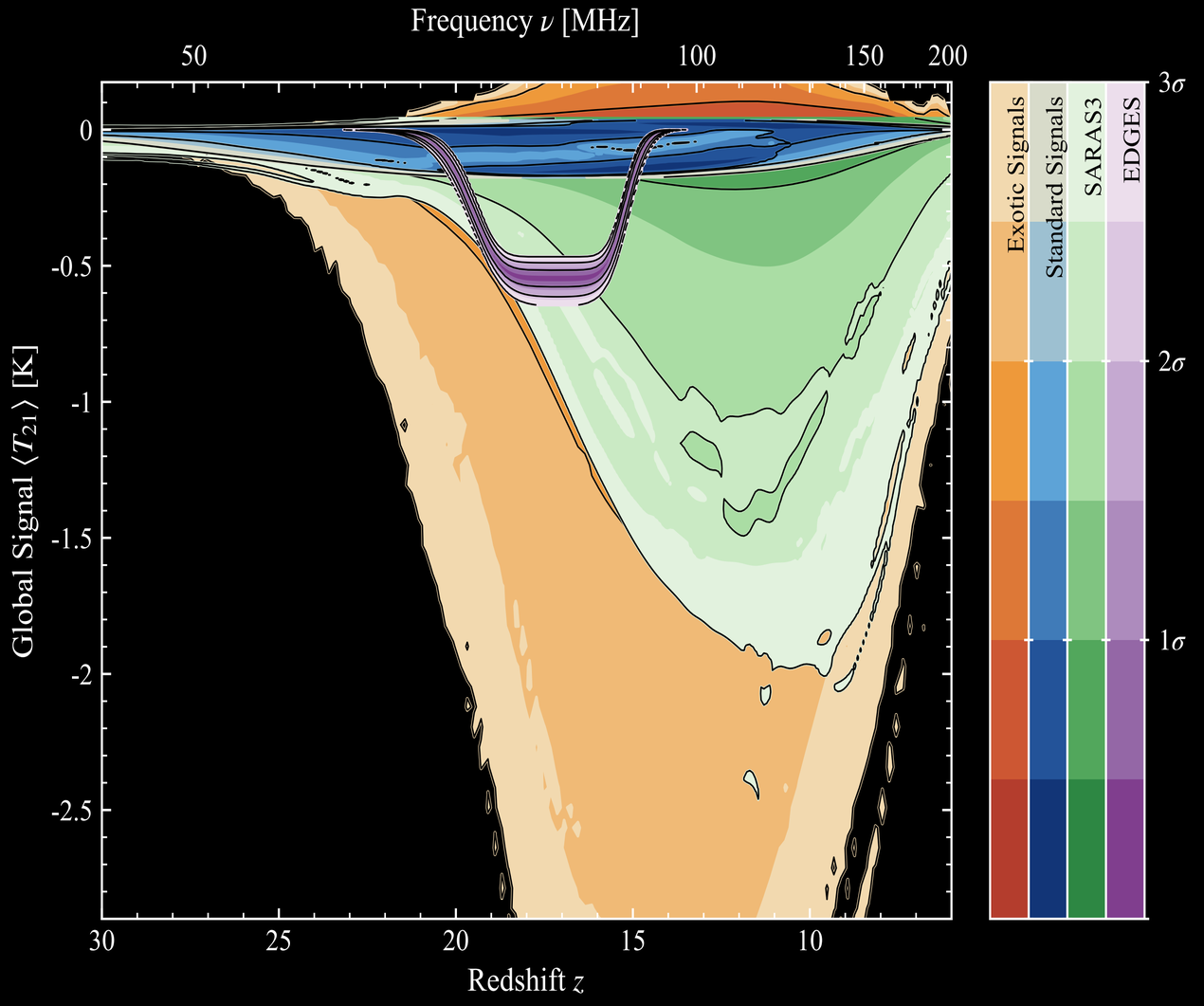


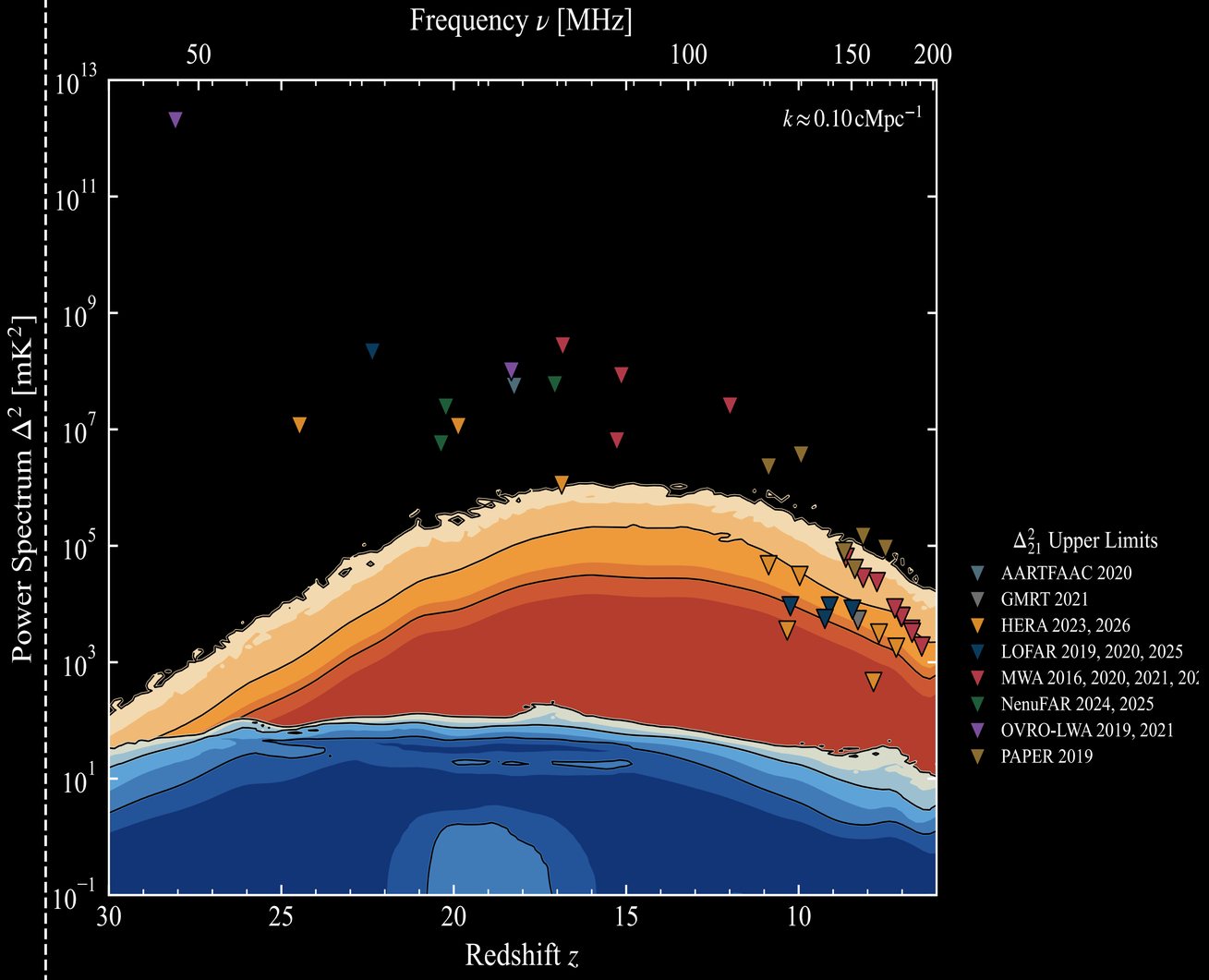




Credit: Reis et al. 2020

Insights to the High-Z Universe
Current Observational Status

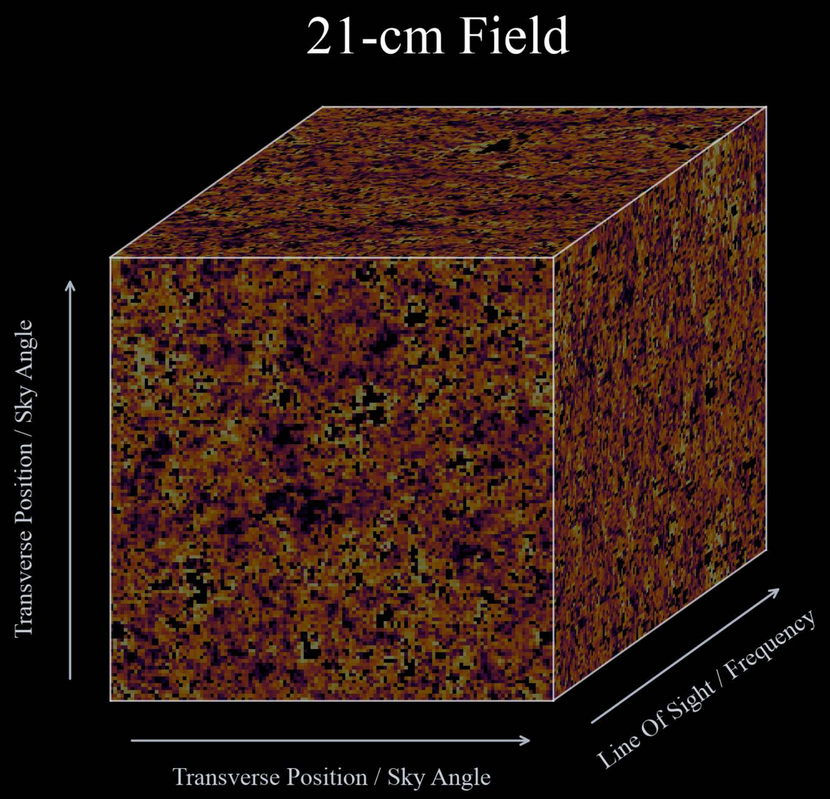




Credit: Inspired by Gessey-Jones 2024

Why is Detection so Challenging ?

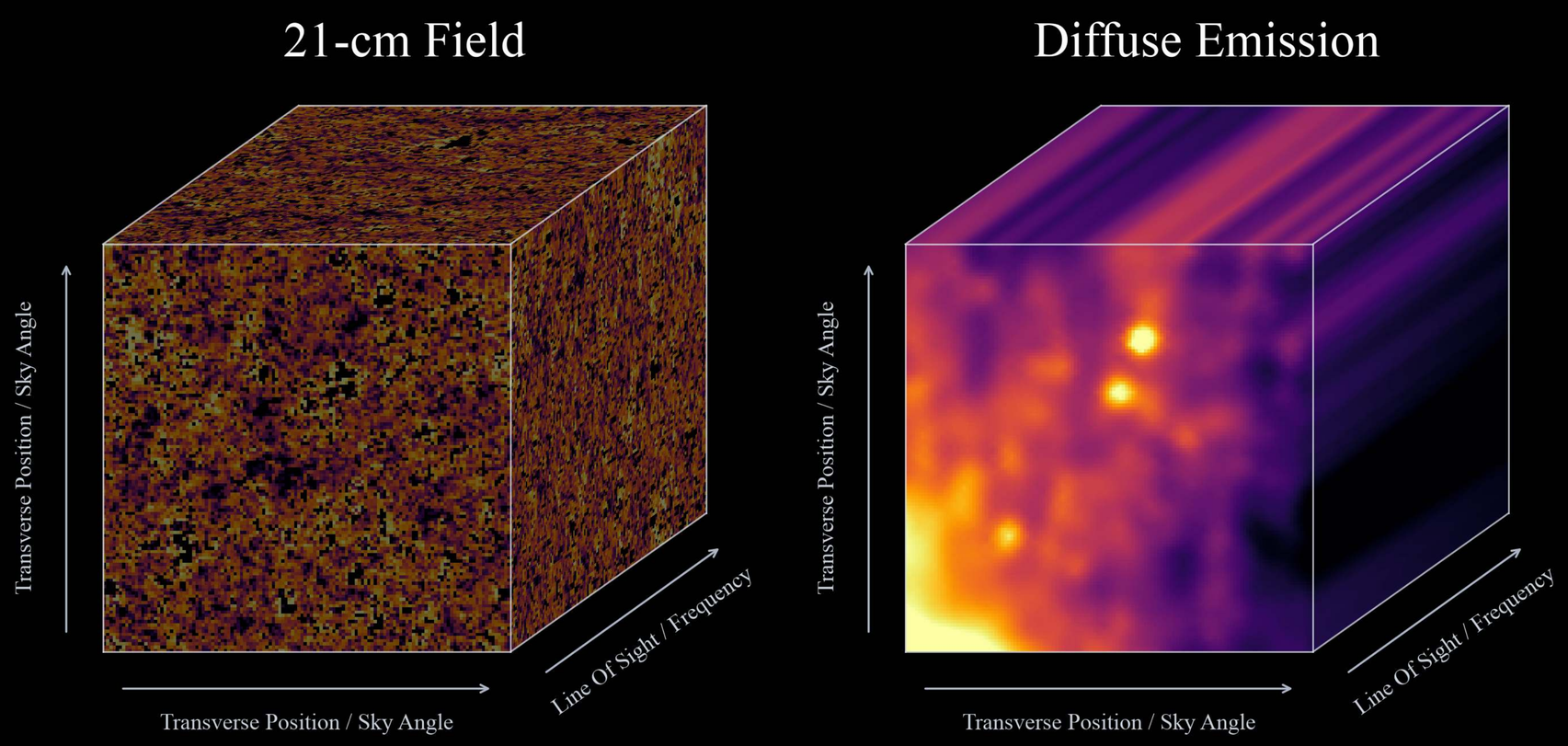
Low Signal to ~~Noise~~ Nuisance

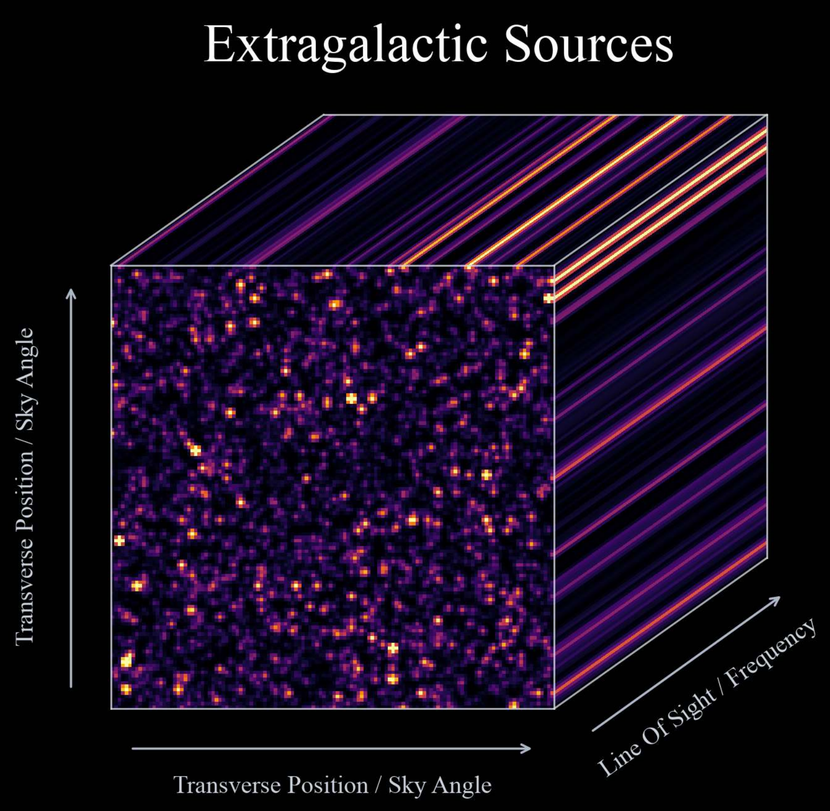


Credit: 21cmSPACE Simulation (Cosmic Dawn Lab, Cambridge)

Why is Detection so Challenging ?

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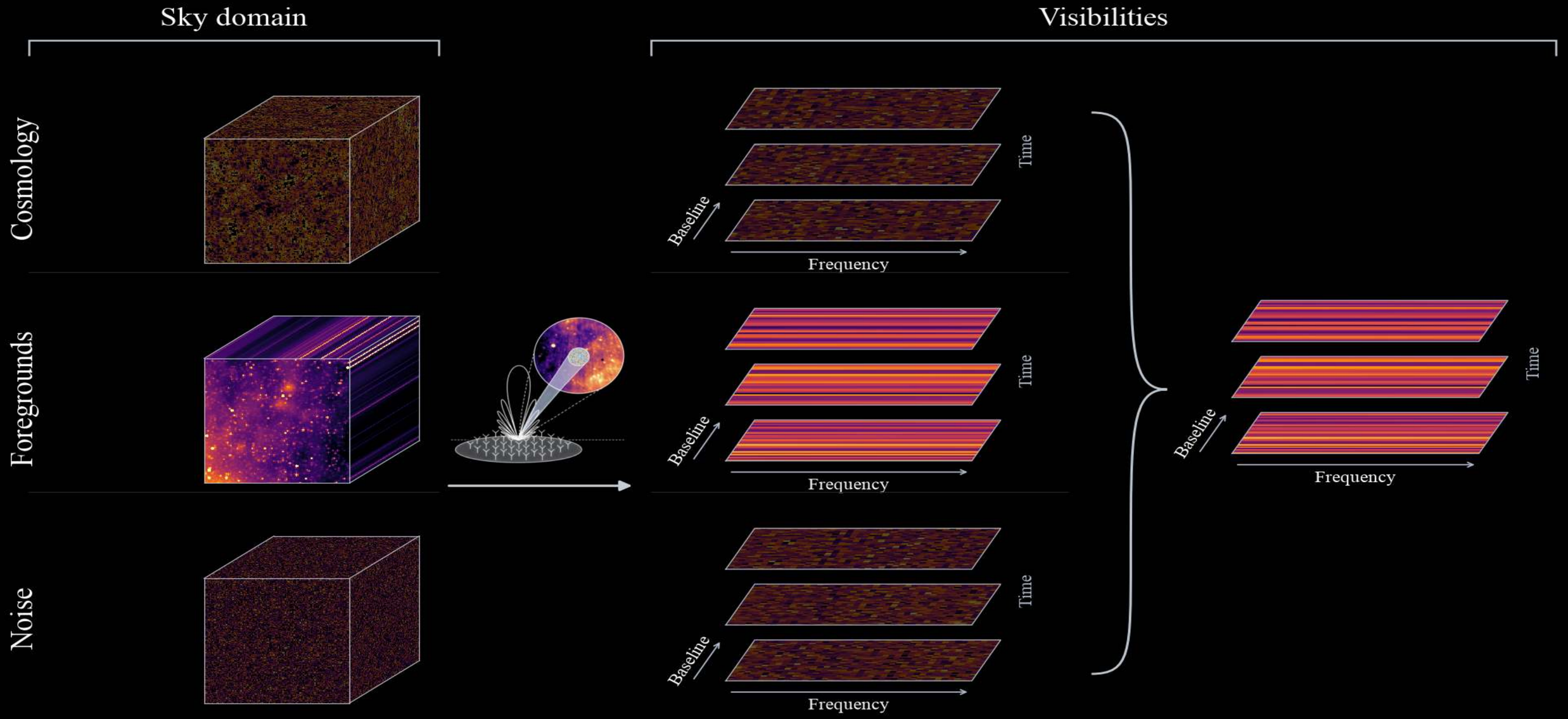




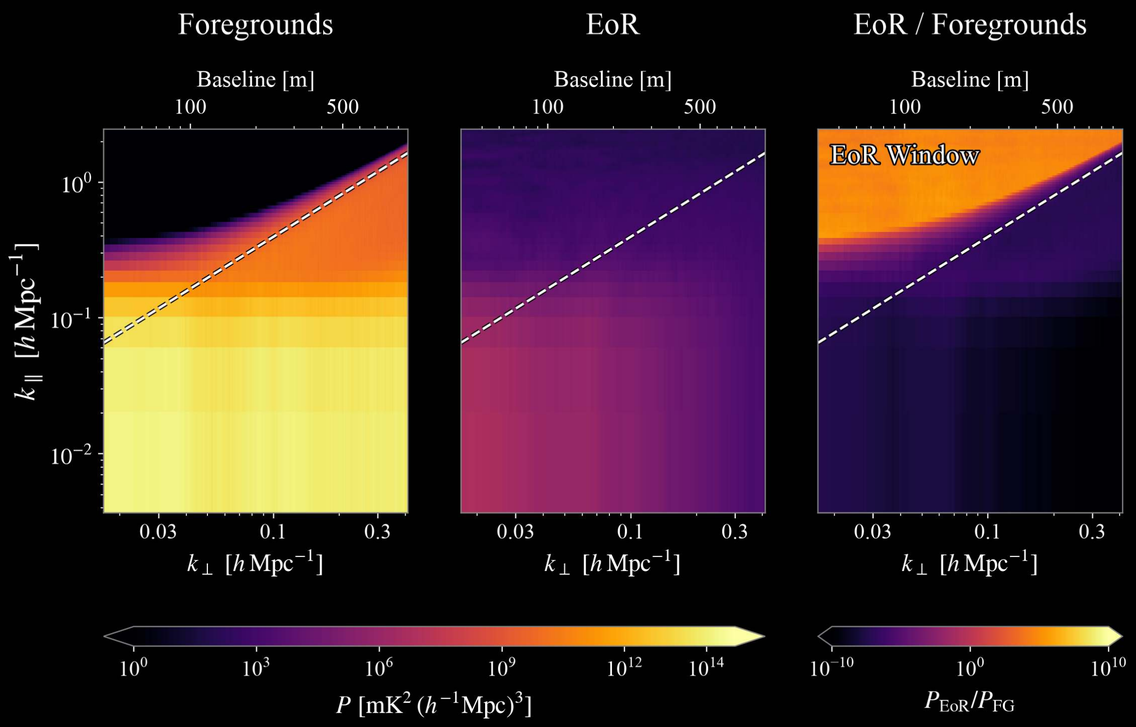
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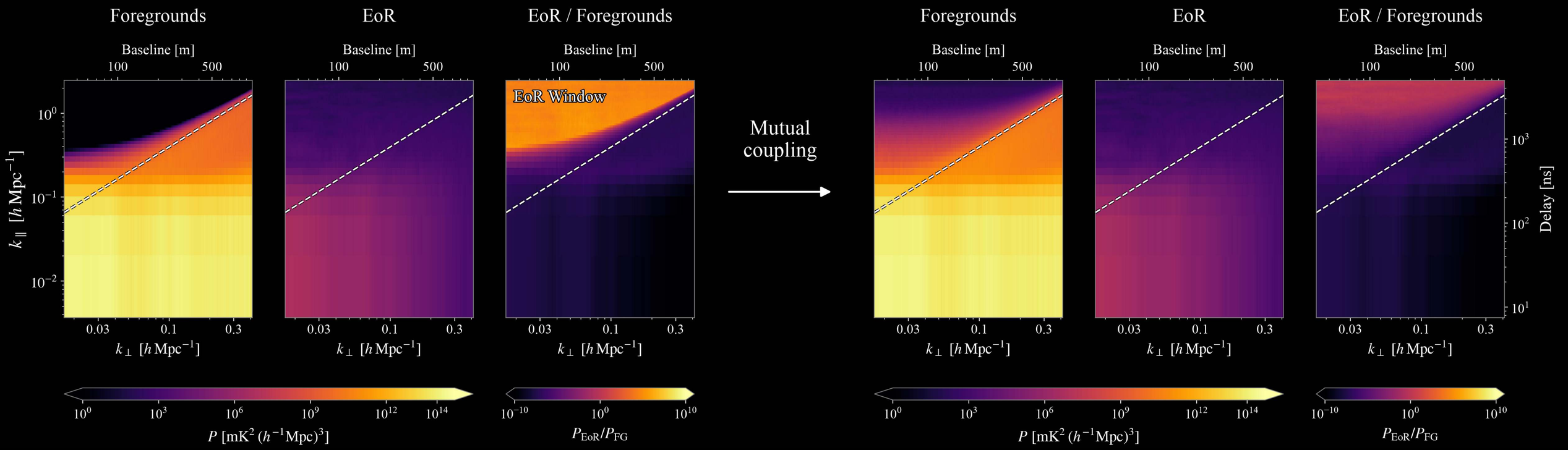


Traditional Approaches
Foreground Avoidance



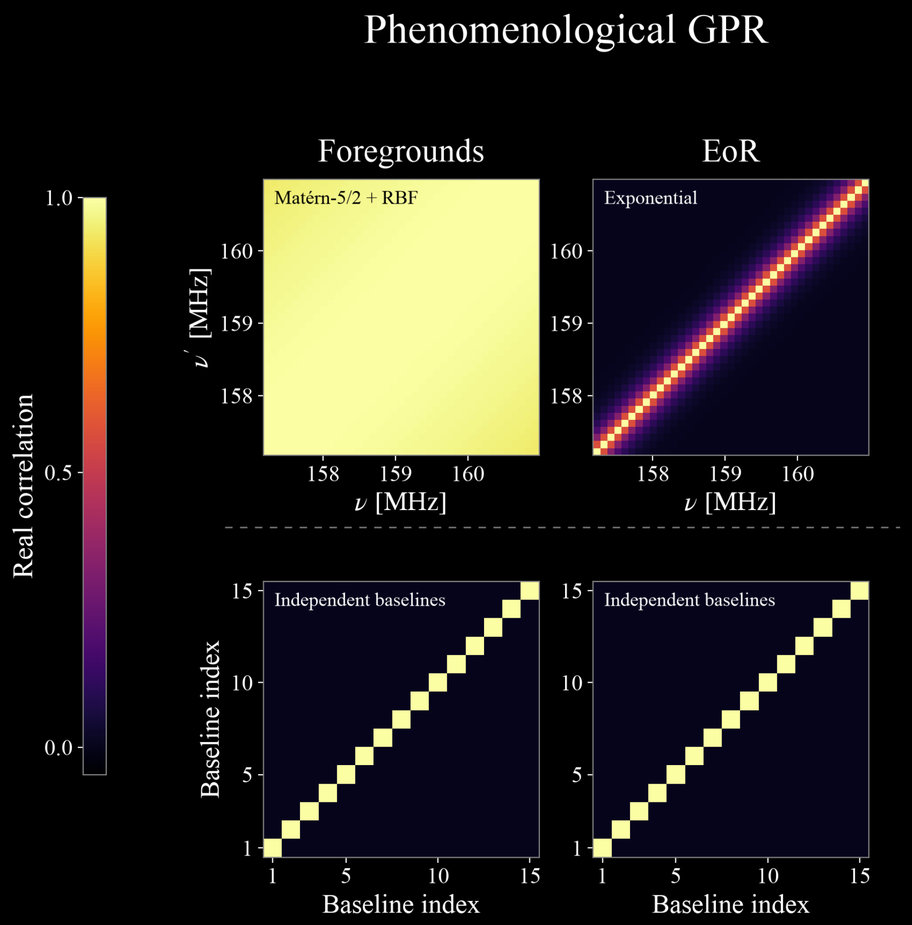
Credit: Oscar O'Hara (private communication), from https://arxiv.org/pdf/2607.11438

Traditional Approaches
Foreground Avoidance

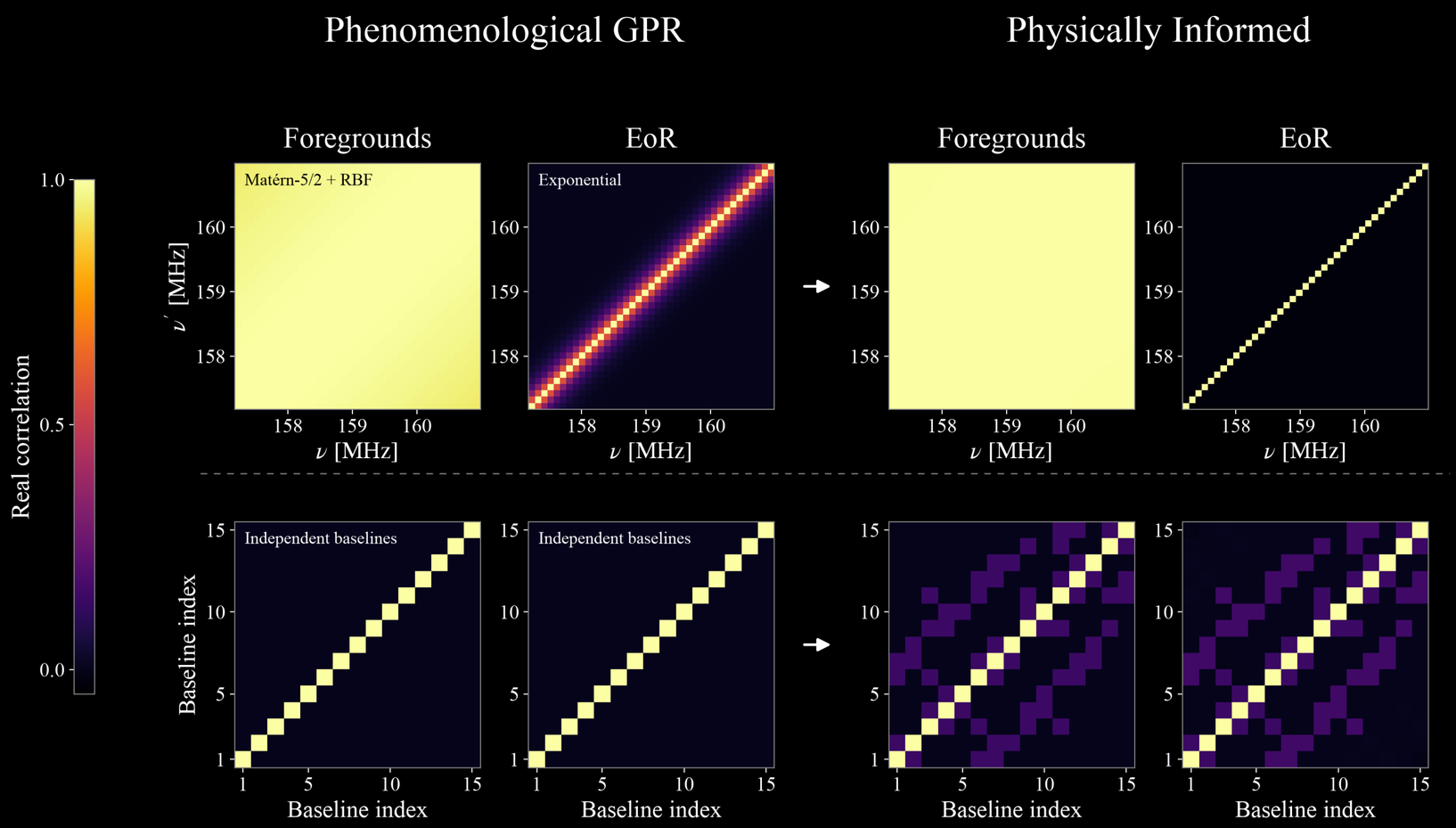


Credit: Oscar O'Hara (private communication), from https://arxiv.org/pdf/2607.11438

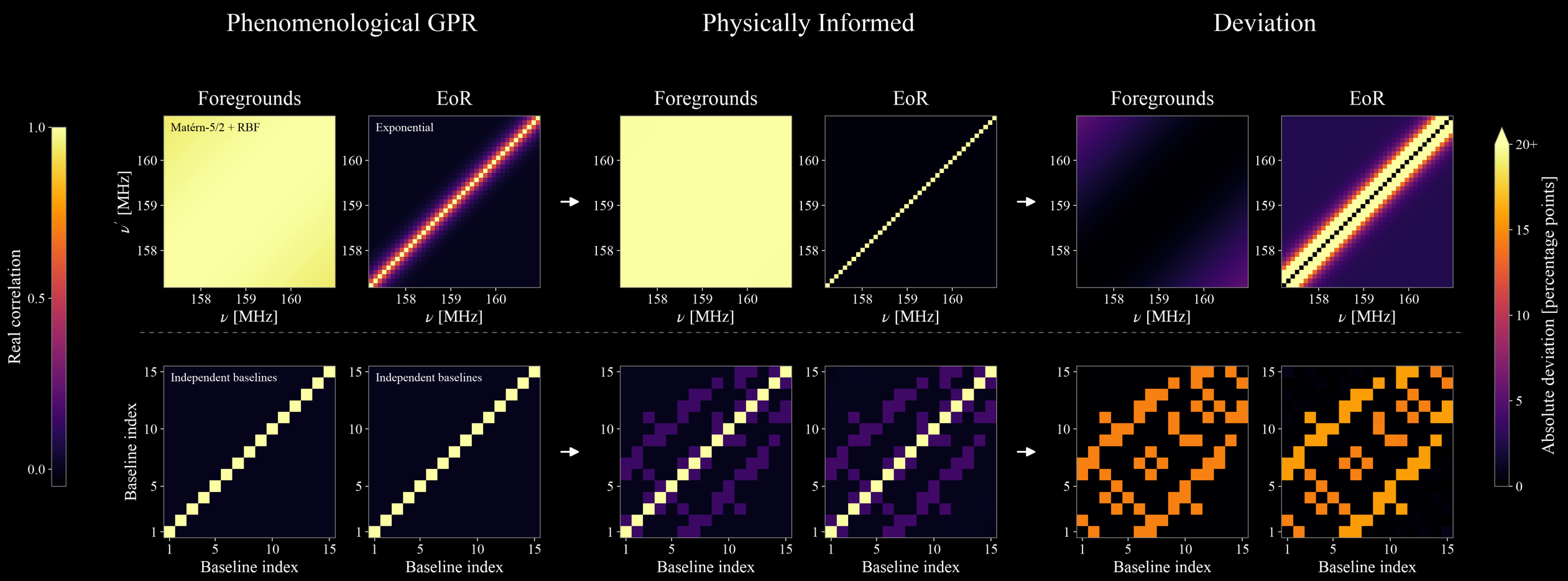
Traditional Approaches
Foreground Removal



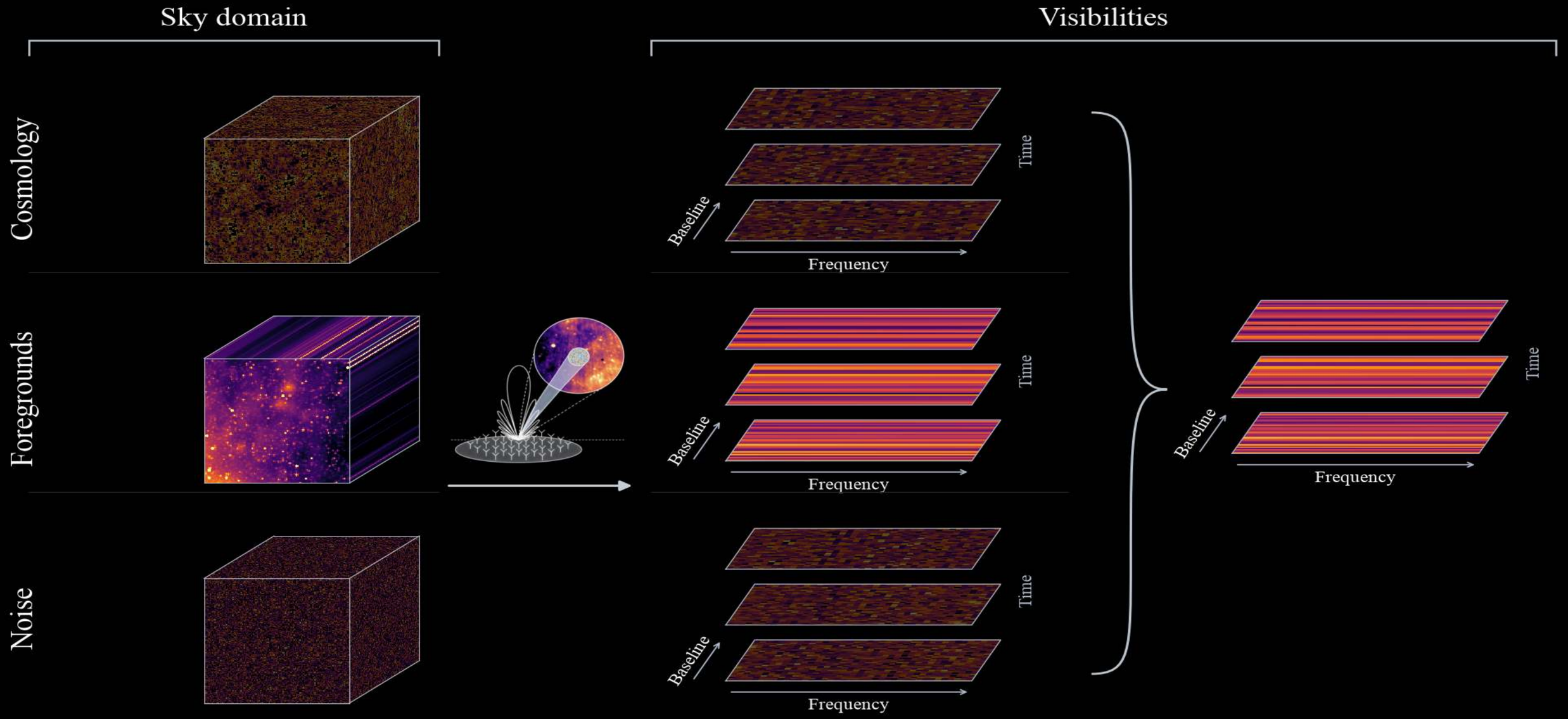
Traditional Approaches
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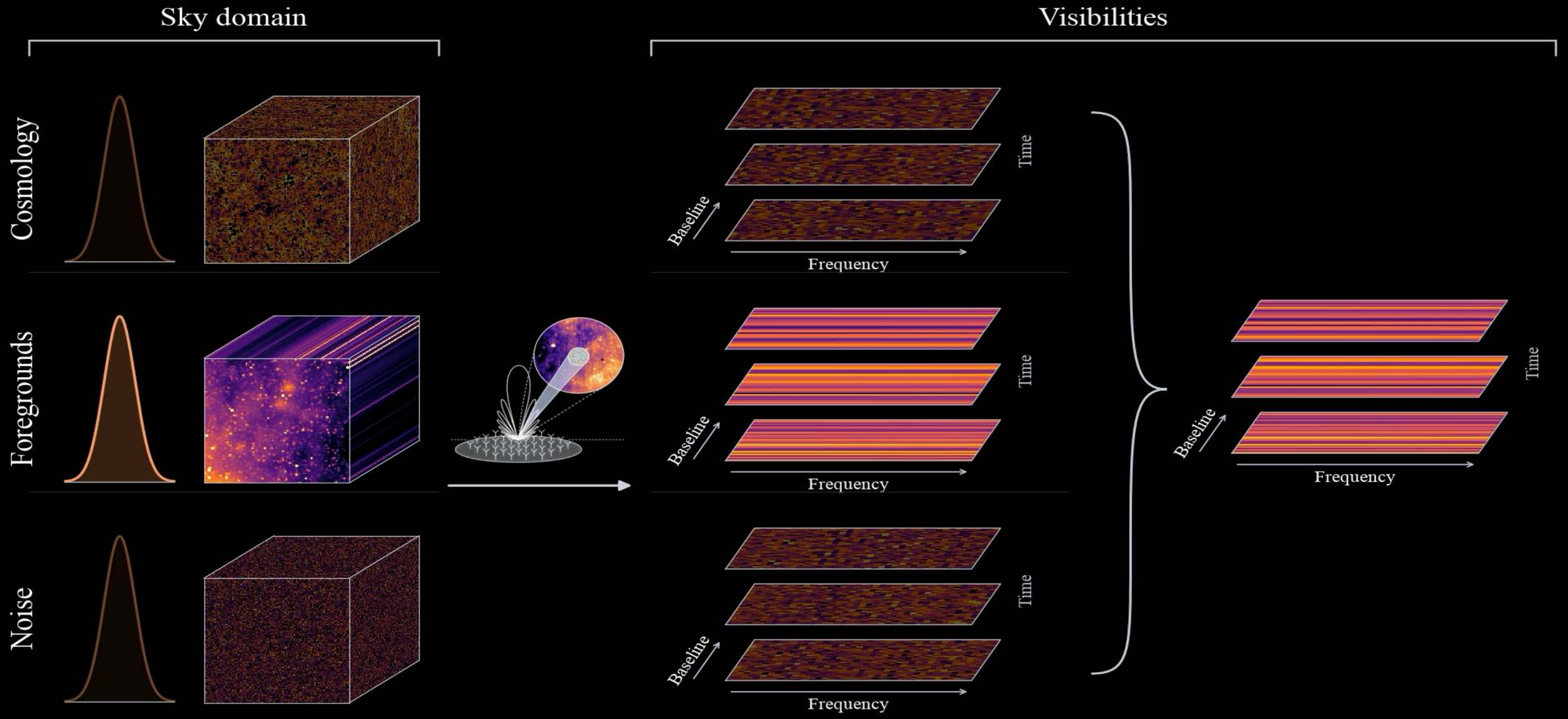
Traditional Approaches
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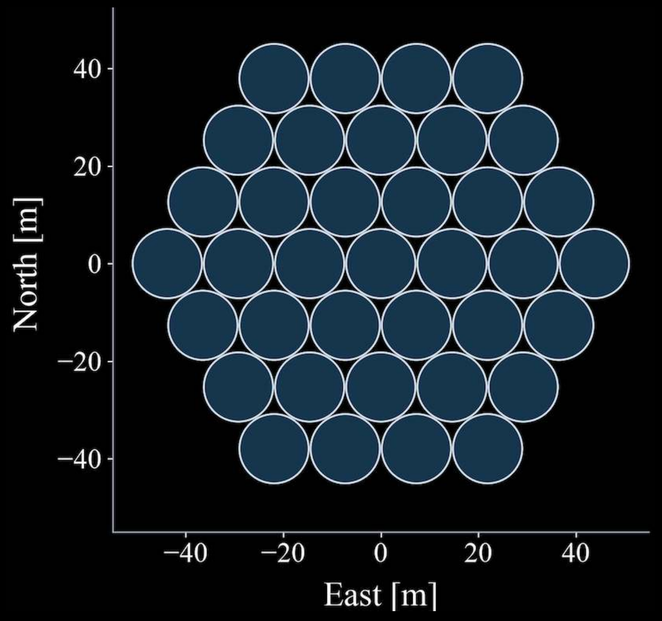
Traditional Approaches
The risks



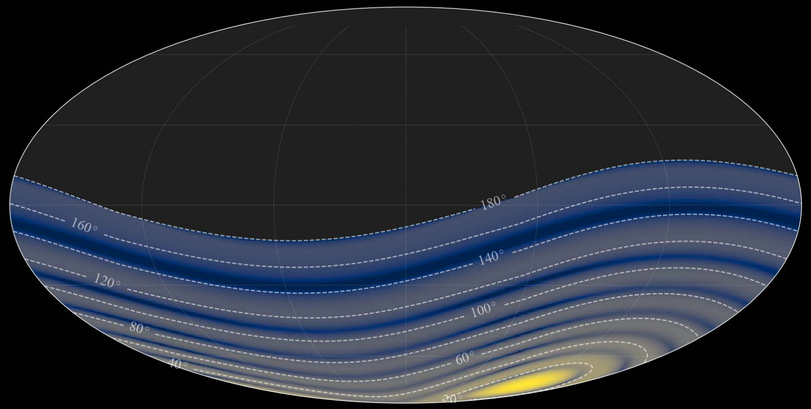
Traditional Approaches
The risks



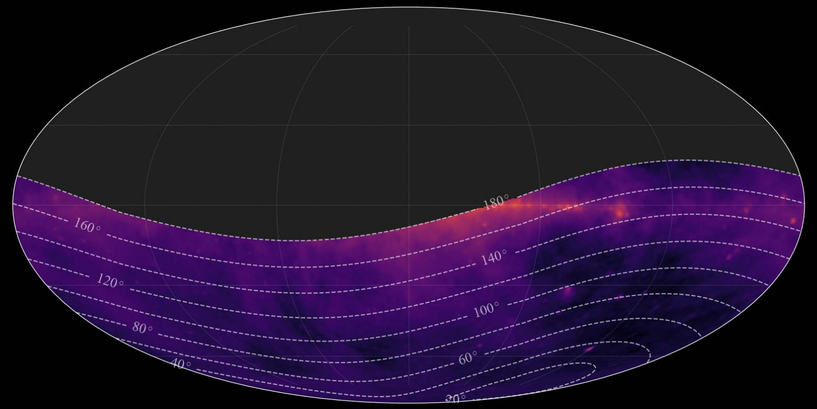
Signal Recovery
Hera-Like Full Fidelity Data Generation



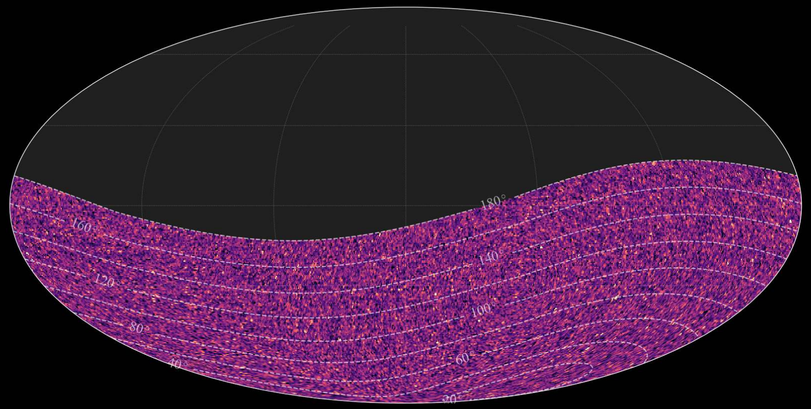
(a) Time-Integrated Beam



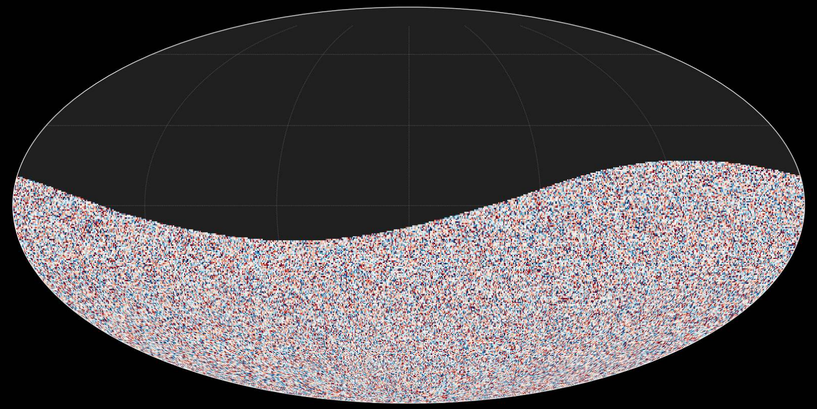
(b) Diffuse Foregrounds



(c) Extragalactic and A-team Sources



(d) Cosmological 21-cm Signal



Configuration:

- **37 Antennas**
- **Baselines < 40m**
- **14.2m Airy Beam**
- **1000h Integration**
- **157.2-160.9 MHz**

Credit: Tutt et al. (In Prep)

Bayesian Forward Modelling (BayesEoR)

For a confident detection:

- Understand the physics behind the structure in our data
  - **Physically Informed Forward Model (Digital Twin)**
- Understand their contributions down to noise
  - **Noise-Like Residuals**

Credit: Tutt et al. (In Prep)

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- Understand the physics behind the structure in our data
  - **Physically Informed Forward Model (Digital Twin)**
- Understand their contributions down to noise
  - **Noise-Like Residuals**
- Statistically favour a signal model
  - **Bayes Factor**
- Ensure not susceptible to Spurious Detections
  - **Bayesian Validation (Null Test)**

Bayes Theorem

$$p(\boldsymbol{\theta} \mid \boldsymbol{d}, M) = \frac{p(\boldsymbol{d} \mid \boldsymbol{\theta}, M)\, p(\boldsymbol{\theta} \mid M)}{Z_M}$$

Bayesian Evidence

$$Z_M \equiv p(\boldsymbol{d} \mid M) = \int p(\boldsymbol{d} \mid \boldsymbol{\theta}, M)\, p(\boldsymbol{\theta} \mid M)\, d\boldsymbol{\theta}$$

Bayes Factor

$$B_{12} = \frac{Z_1}{Z_2}$$

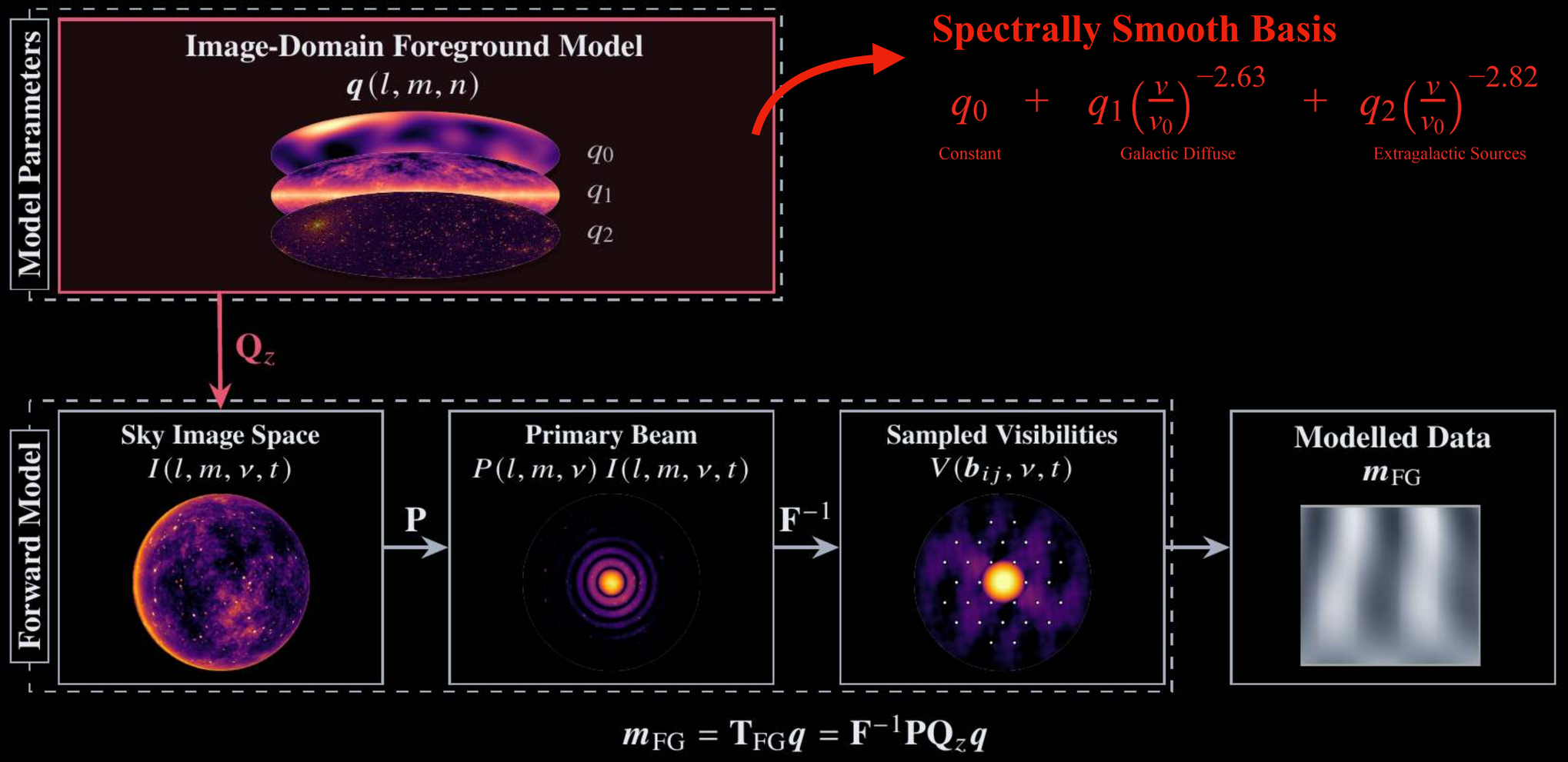
Credit: Tutt et al. (In Prep)

Bayesian Forward Modelling (BayesEoR)
Challenges

- The Physics:
  - **Formulating the Forward Model**
- Parameter Formulation
  - **Natural Component Separation**
- Dimensionality
  - **Memory/ Compute Requirements**
  - **Statistical Sampling**
- Evidence Evaluation
  - **Order(N^2) ——> Order(N^3)**

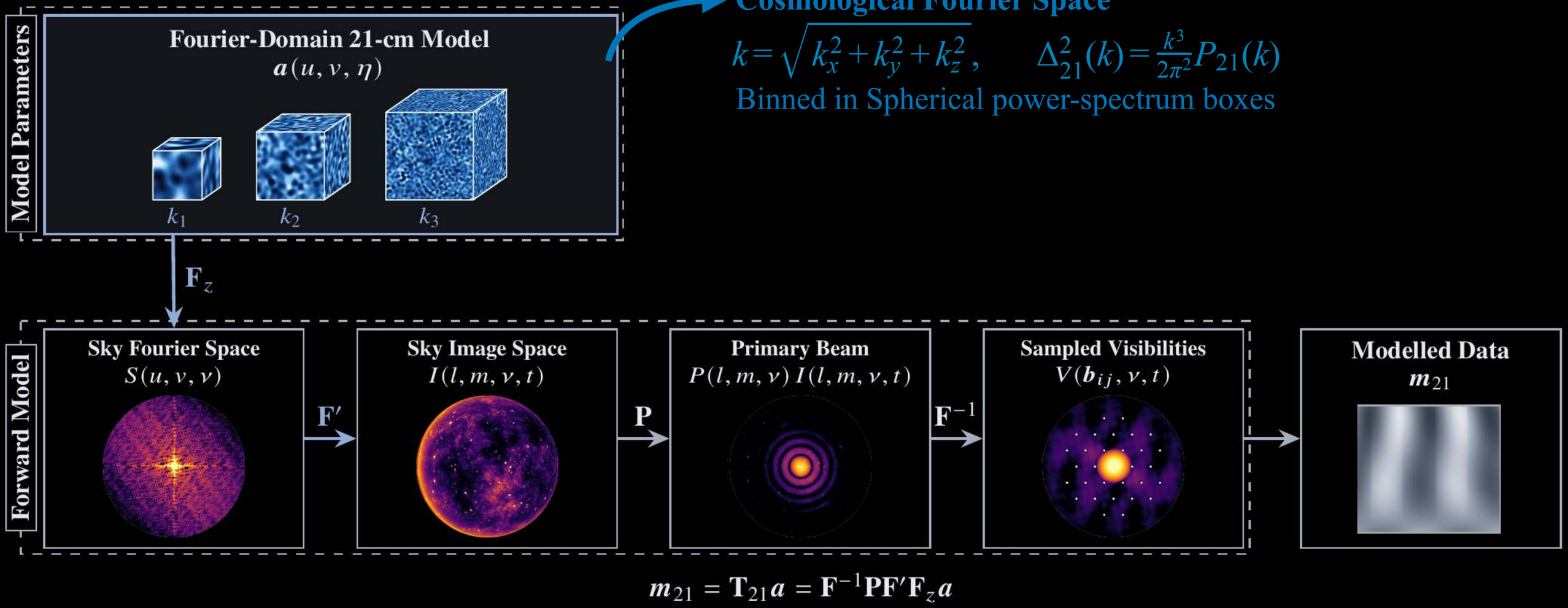
Credit: Tutt et al. (In Prep)

Bayesian Forward Modelling (BayesEoR)
Foreground Contributions



Credit: Tutt et al. (In Prep)

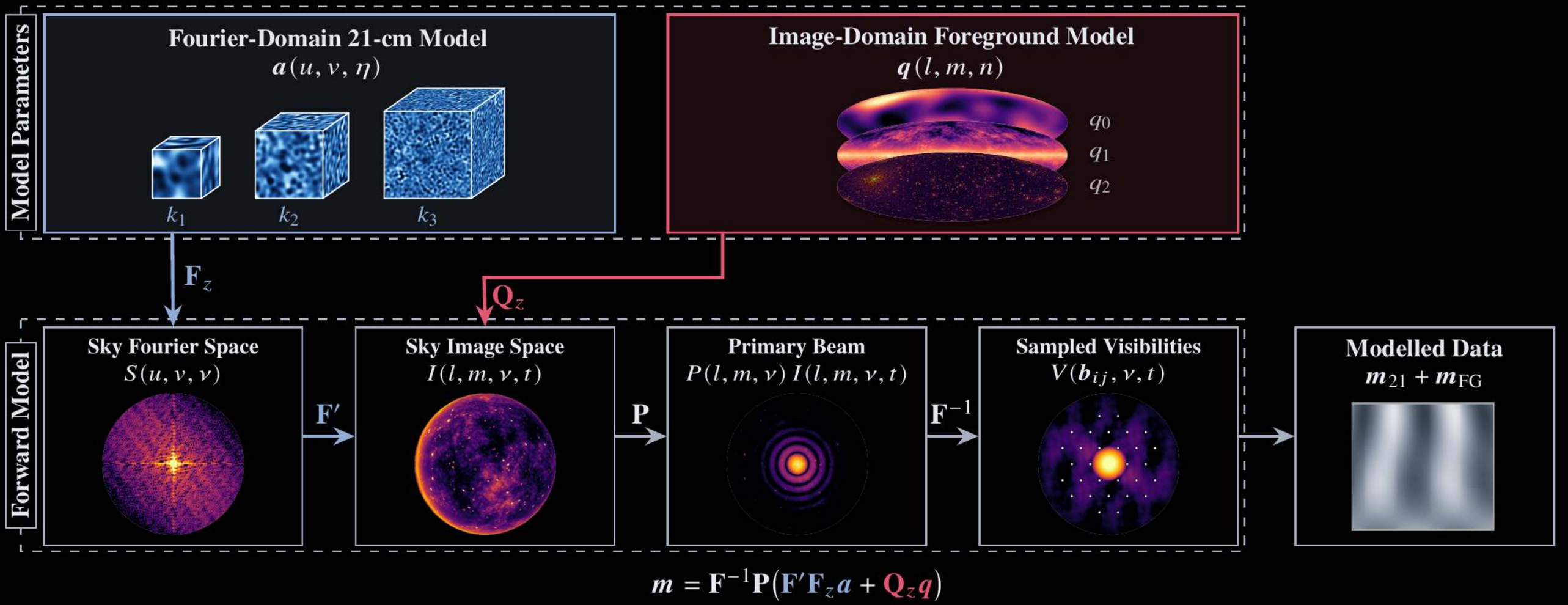
Bayesian Forward Modelling (BayesEoR)
Cosmological Contributions



$$\boldsymbol{m}_{21} = \mathbf{T}_{21}\boldsymbol{a} = \mathbf{F}^{-1}\mathbf{P}\mathbf{F}'\mathbf{F}_z\boldsymbol{a}$$

Credit: Tutt et al. (In Prep)

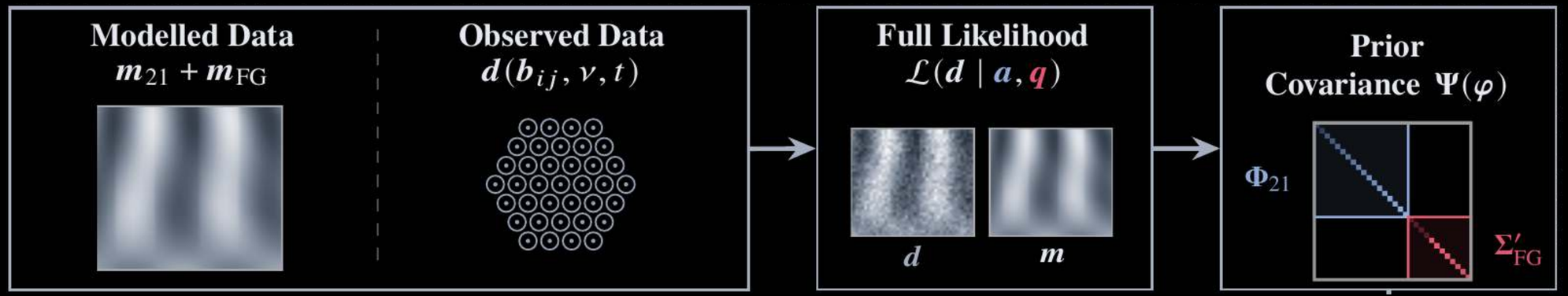
Bayesian Forward Modelling (BayesEoR)
Joint Forward Modelling



$$\boldsymbol{m} = \mathbf{F}^{-1}\mathbf{P}\left(\mathbf{F}'\mathbf{F}_z\boldsymbol{a} + \mathbf{Q}_z\boldsymbol{q}\right)$$

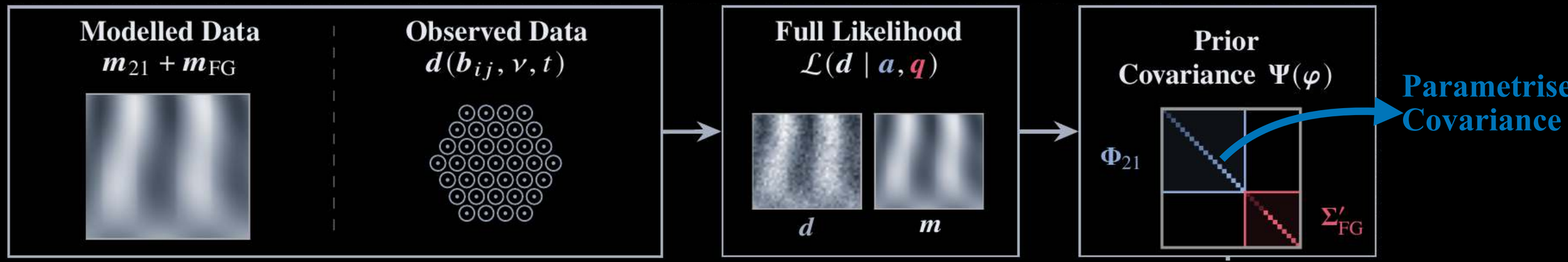
Credit: Tutt et al. (In Prep)

Bayesian Forward Modelling (BayesEoR)
Joint Inference



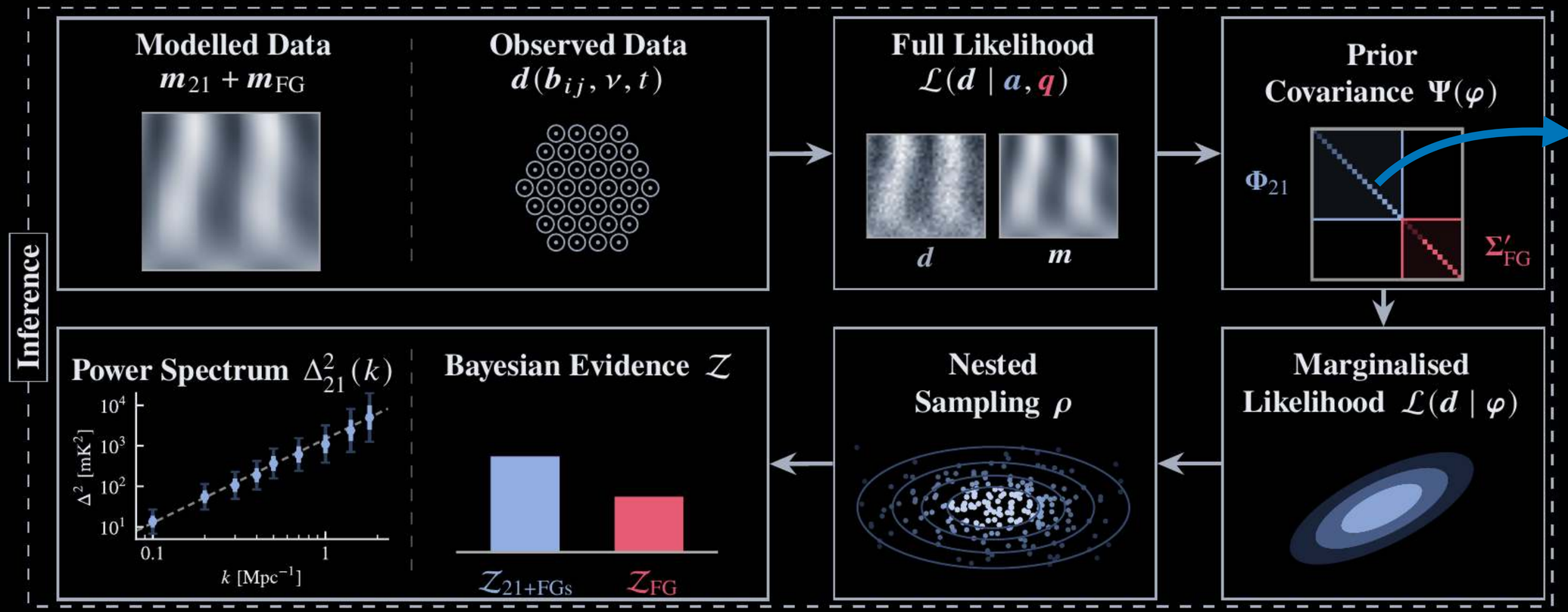
Credit: Tutt et al. (In Prep)

Bayesian Forward Modelling (BayesEoR)
Joint Inference



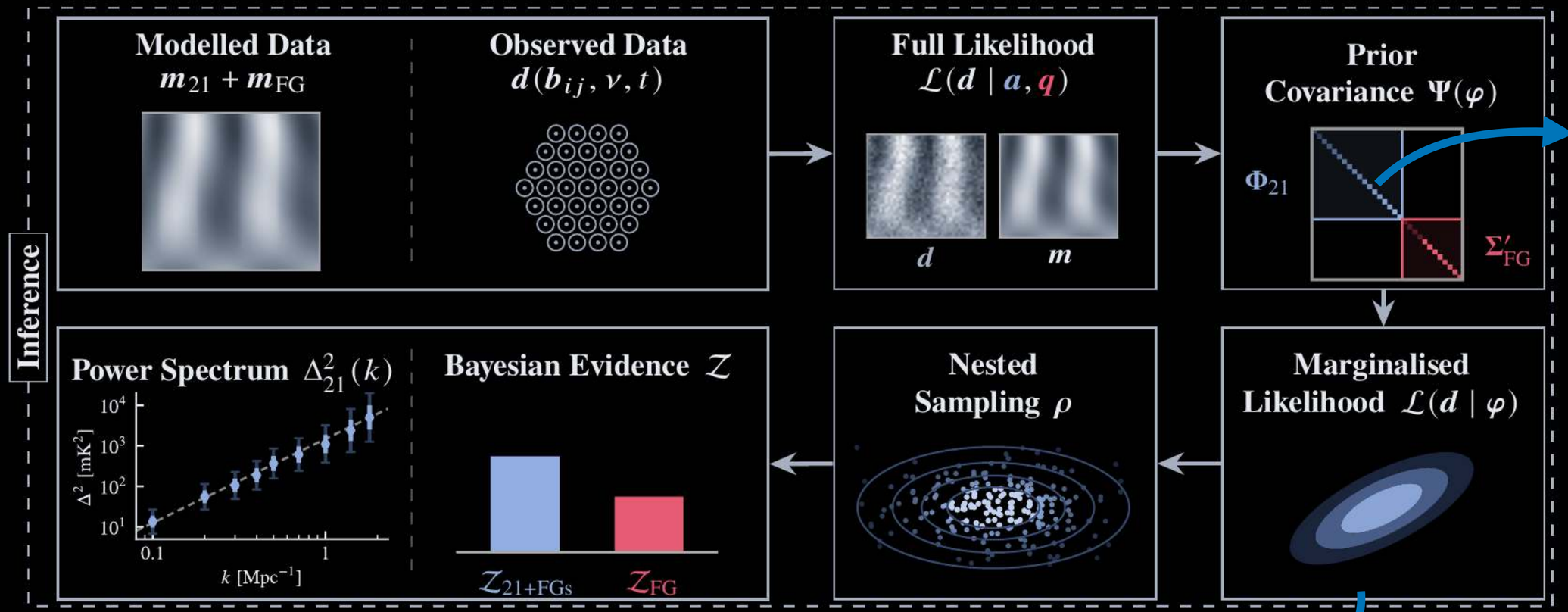
Credit: Tutt et al. (In Prep)

Bayesian Forward Modelling (BayesEoR)
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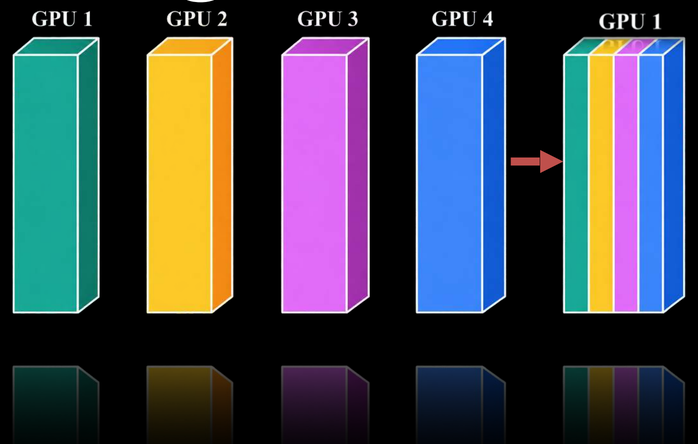
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Bayesian Forward Modelling (BayesEoR)
Joint Inference

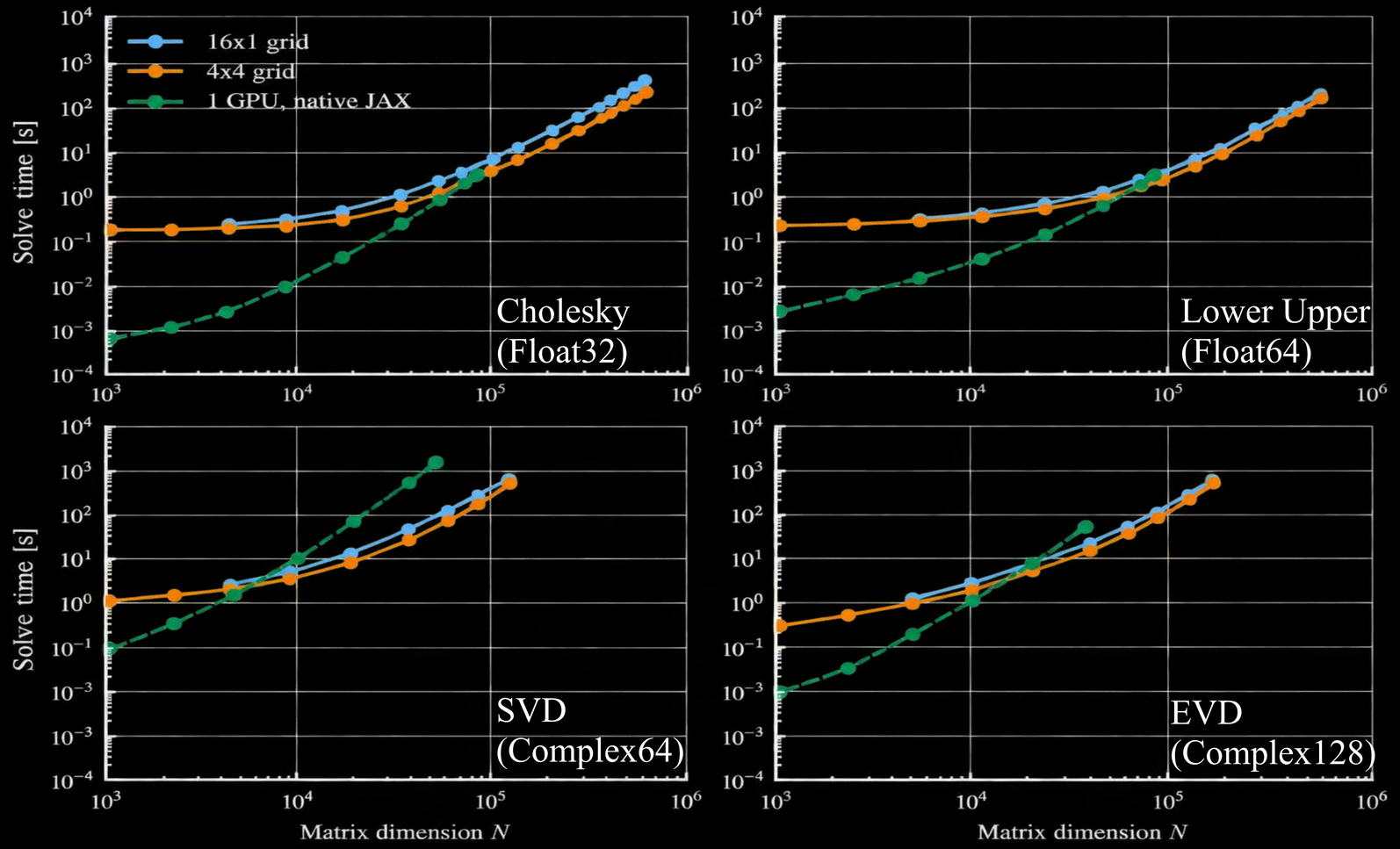


Credit: Tutt et al. (In Prep)

Unlocking Inference at Scale
JAXMg: a multi-GPU linear Solver



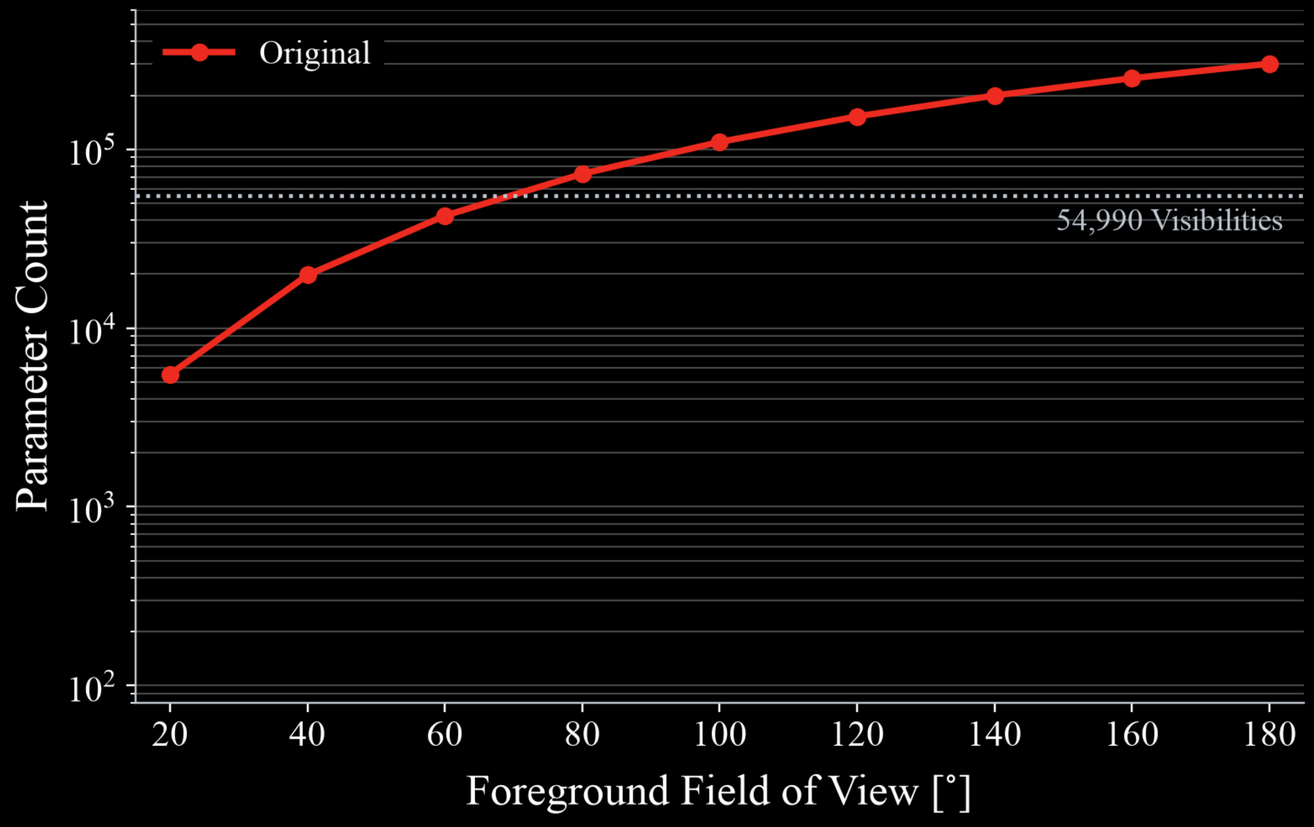
- Available on pip and compatible with all CUDA hardware and JAX versions >=0.10.0
- Currently supports:
  - Least Squares, Cholesky, LU Solve
  - Singular value, Cholesky, Polar, QR Decomposition
- Applications:
  - Interferometric Bayesian forward model
  - Computational EMs (PDE Solving)
  - Gaussian Processes
  - Natural Gradient Optimisation



- Up to: 1.5 x 1.5 Million Solve (64 H200's, ~10mins)

Tutt and Wiersema, *arXiv:2601.14466* (2026)

Bayesian Forward Modelling (BayesEoR)
Latent Space Foreground Modelling



Inference Costs:

Before:

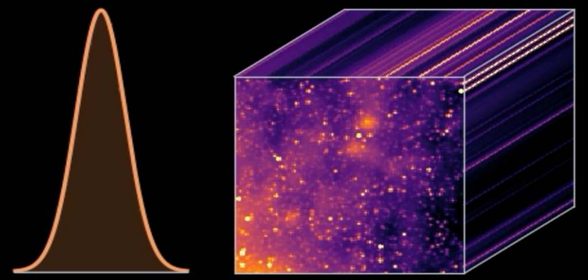
- ~300,000 Params
- ~6 GPUs (H100s)
- ~261 GPU Hours (H100s)

Credit: Tutt et al. (In Prep)

Bayesian Forward Modelling (BayesEoR)
Latent Space Foreground Modelling

Physically Motivated
Priors

Foregrounds



Credit: Tutt et al. (In Prep)

Bayesian Forward Modelling (BayesEoR)
Latent Space Foreground Modelling

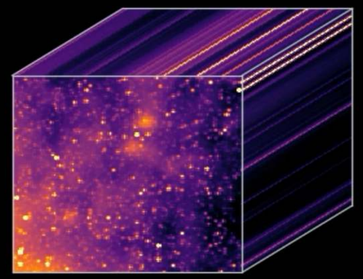
Physically Motivated Priors

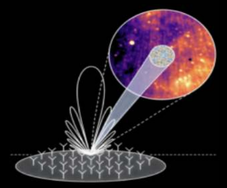
Instrumentally Rooted Operators

$$m_{\mathrm{FG}} = \mathbf{T}_{\mathrm{FG}} q = \mathbf{F}^{-1}\mathbf{P}\mathbf{Q}_z q$$

Foregrounds







Credit: Tutt et al. (In Prep)

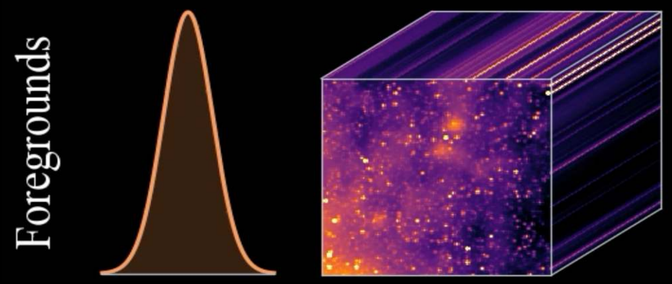
Bayesian Forward Modelling (BayesEoR)
Latent Space Foreground Modelling

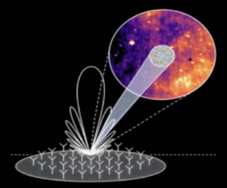
Physically Motivated Priors

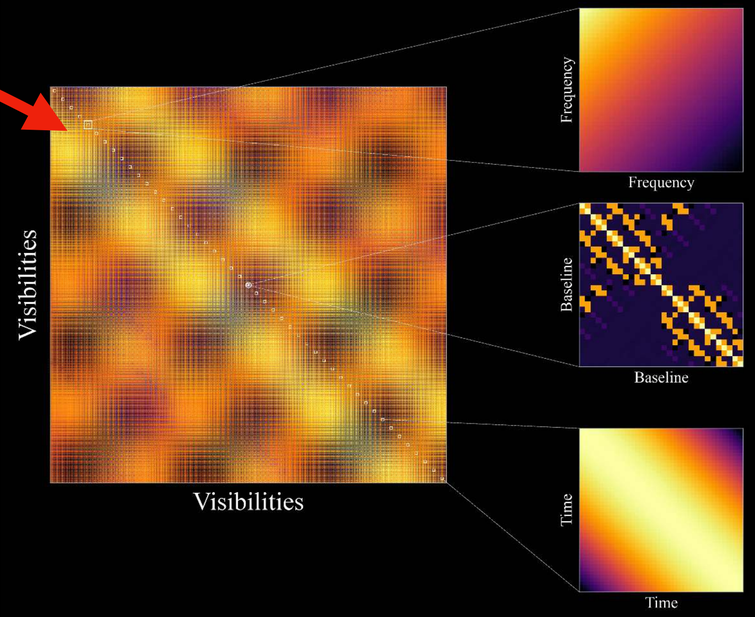
Instrumentally Rooted Operators

Prior-Predictive Foreground Covariance

$$m_{\mathrm{FG}} = \mathbf{T}_{\mathrm{FG}} q = \mathbf{F}^{-1}\mathbf{P}\mathbf{Q}_z q$$







Credit: Tutt et al. (In Prep)

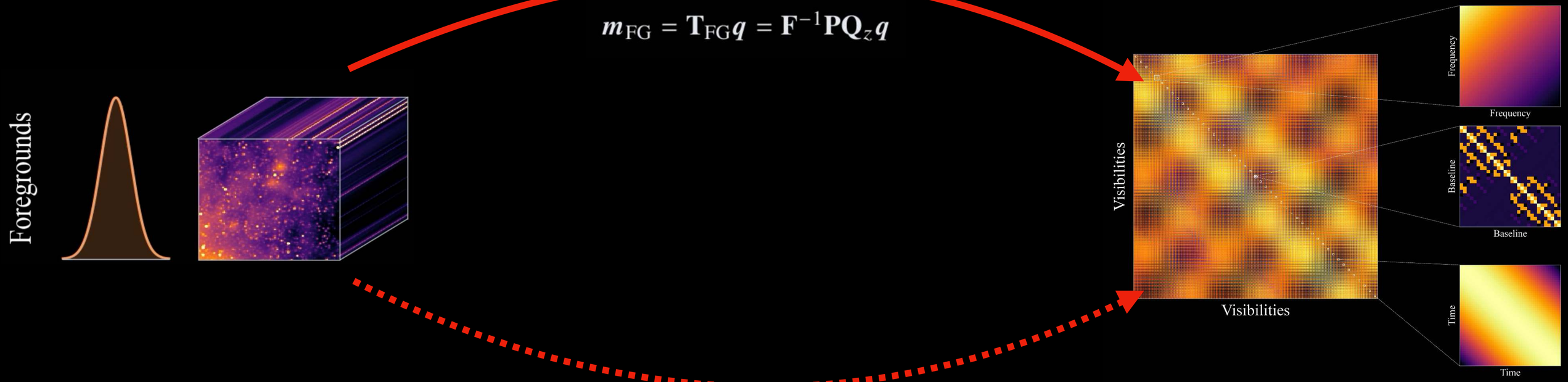
Bayesian Forward Modelling (BayesEoR)
Latent Space Foreground Modelling

Physically Motivated Priors

Instrumentally Rooted Operators

Prior-Predictive Foreground Covariance

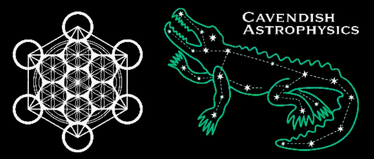
$$m_{\mathrm{FG}} = \mathbf{T}_{\mathrm{FG}} q = \mathbf{F}^{-1}\mathbf{P}\mathbf{Q}_z q$$



A latent representation spanning the noise-relevant space of foreground uncertainty

Credit: Tutt et al. (In Prep)

Bayesian Forward Modelling (BayesEoR)
A Prior and Noise Informed KL Basis



Weight the Physical Operator

$$\mathbf{A} = \mathbf{N}^{-1/2}\,\mathbf{T}_{\mathrm{FG}}\,\boldsymbol{\Sigma}_q^{1/2}$$

Construct Orthonormal Basis

$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^{\dagger}$$

Expensive (Amortised)

Truncate Based on Noise

$$\varepsilon_{\mathrm{RMS}}(r) = \sqrt{\frac{1}{N_{\mathrm{d}}}\sum_{i>r} s_i^2} \leq 0.1$$

Construct Reduced Operator

$$\mathbf{T}_{\mathrm{FG}}^{\mathrm{KL}} = \mathbf{N}^{1/2}\mathbf{U}_r\mathbf{S}_r, \qquad z \sim N(\mathbf{0}, \mathbf{I})$$

Credit: Tutt et al. (In Prep)

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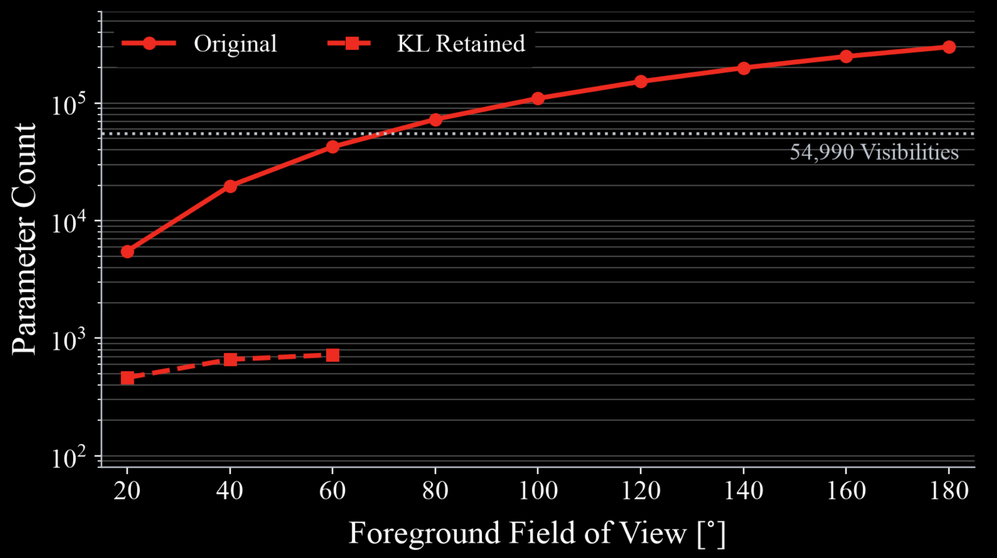
Expensive (Amortised)

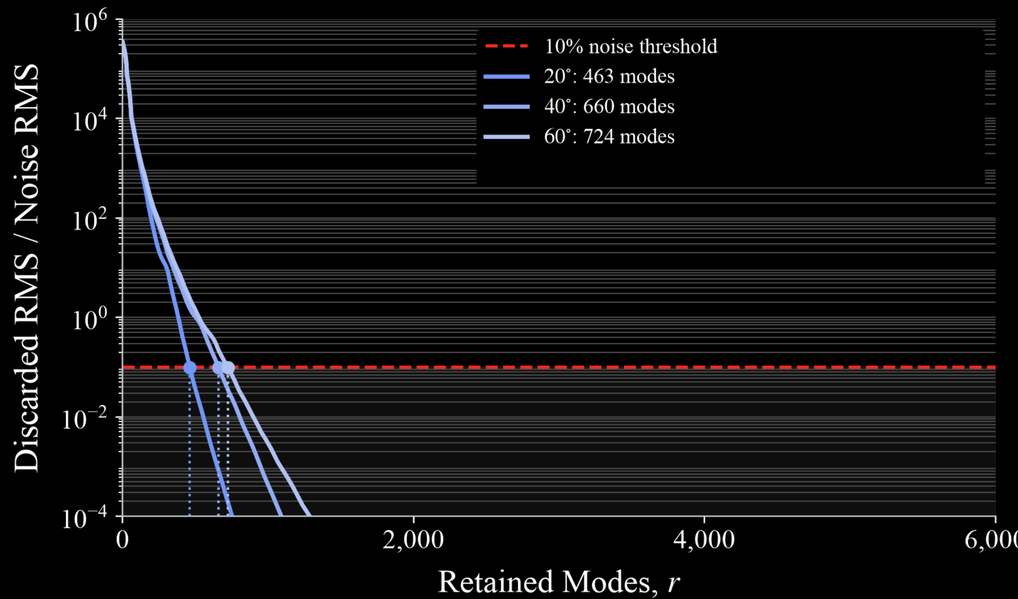
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Credit: Tutt et al. (In Prep)

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Construct Orthonormal Basis

$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^{\dagger}$$

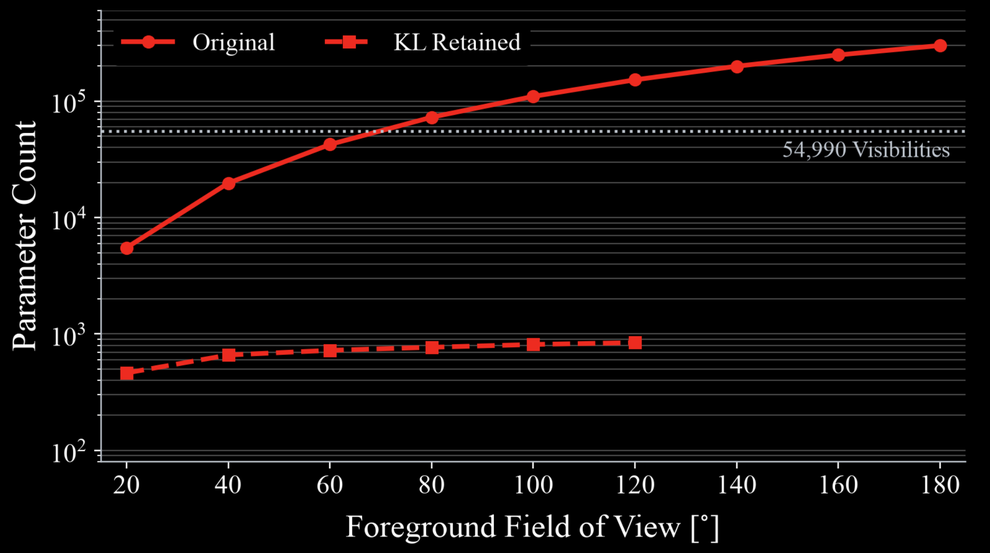
Expensive (Amortised)

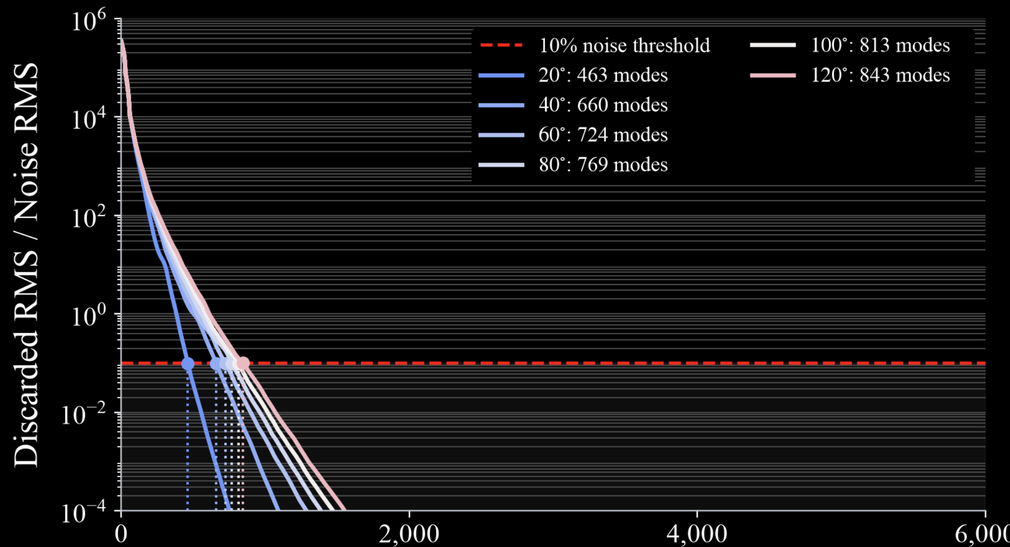
Truncate Based on Noise

$$\varepsilon_{\mathrm{RMS}}(r) = \sqrt{\frac{1}{N_{\mathrm{d}}}\sum_{i>r} s_i^2} \leq 0.1$$

Construct Reduced Operator

$$\mathbf{T}_{\mathrm{FG}}^{\mathrm{KL}} = \mathbf{N}^{1/2}\mathbf{U}_r\mathbf{S}_r, \qquad \boldsymbol{z} \sim N(\mathbf{0}, \mathbf{I})$$





Credit: Tutt et al. (In Prep)

Bayesian Forward Modelling (BayesEoR)

A Prior and Noise Informed KL Basis

Weight the Physical Operator

$$\mathbf{A} = \mathbf{N}^{-1/2}\,\mathbf{T}_{\mathrm{FG}}\,\boldsymbol{\Sigma}_q^{1/2}$$

Construct Orthonormal Basis

$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^{\dagger}$$

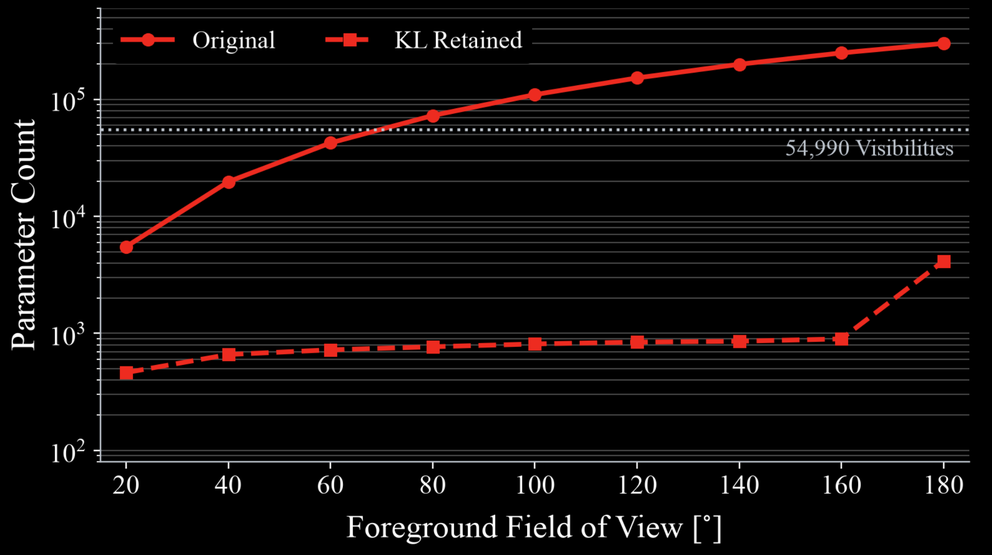
Expensive (Amortised)

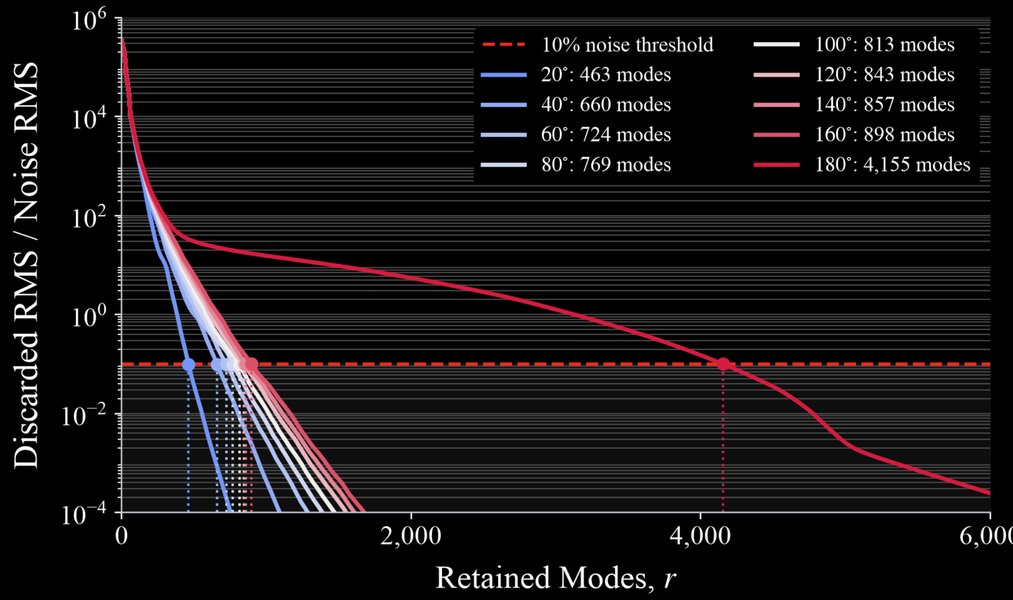
Truncate Based on Noise

$$\varepsilon_{\mathrm{RMS}}(r) = \sqrt{\frac{1}{N_{\mathrm{d}}}\sum_{i>r} s_i^2} \leq 0.1$$

Construct Reduced Operator

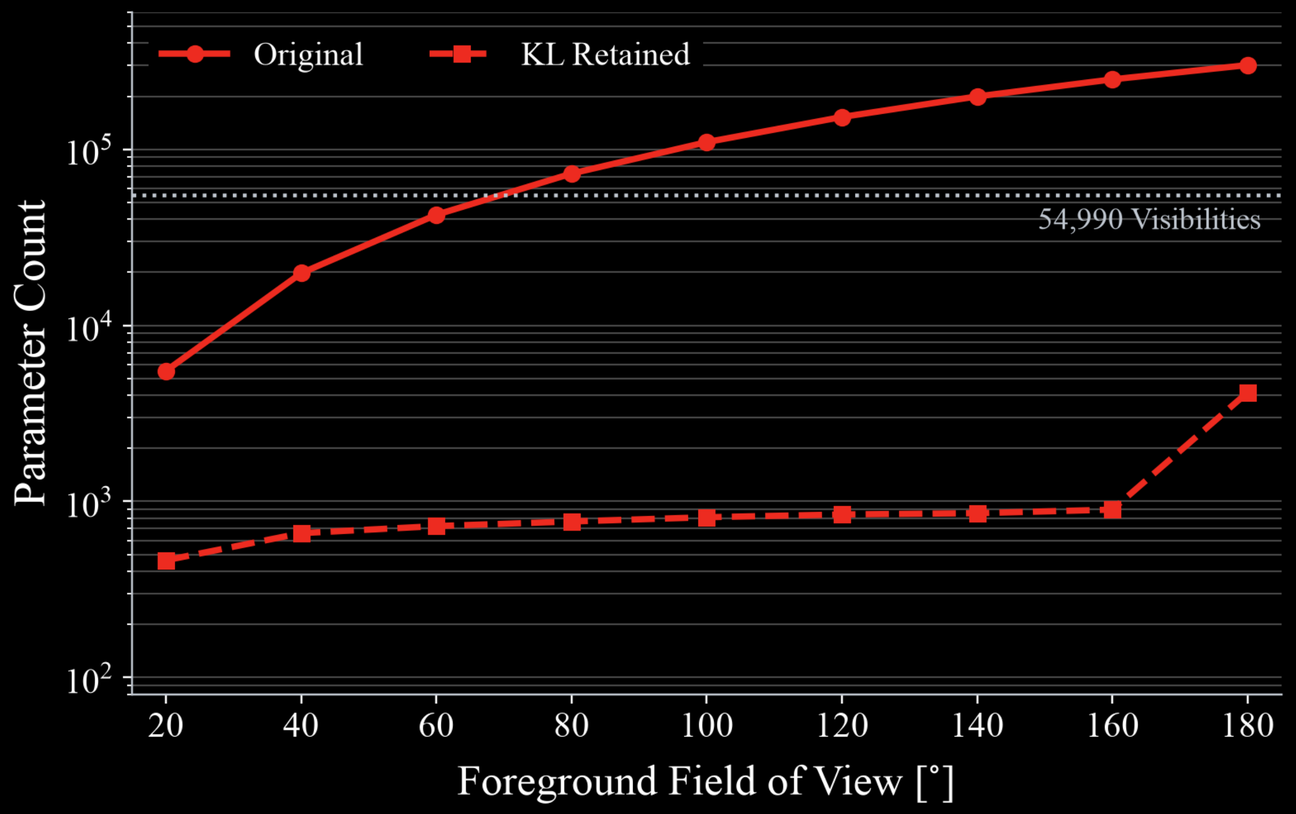
$$\mathbf{T}_{\mathrm{FG}}^{\mathrm{KL}} = \mathbf{N}^{1/2}\mathbf{U}_r\mathbf{S}_r, \qquad z \sim N(\mathbf{0}, \mathbf{I})$$





Credit: Tutt et al. (In Prep)

Bayesian Forward Modelling (BayesEoR)
Latent Space Foreground Modelling



Inference Costs:

Before:

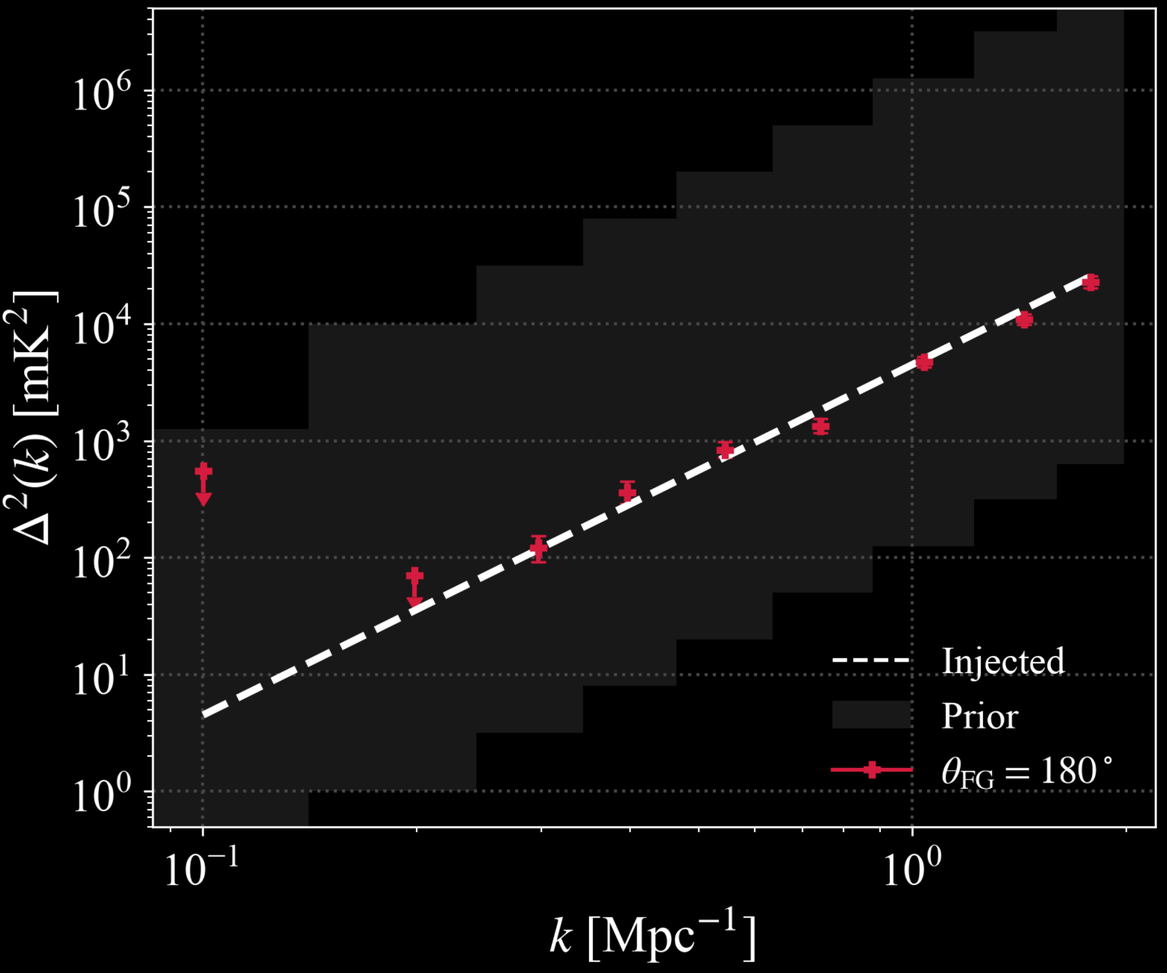
- ~300,000 Params
- ~6 GPUs (H100s)
- ~261 GPU Hours

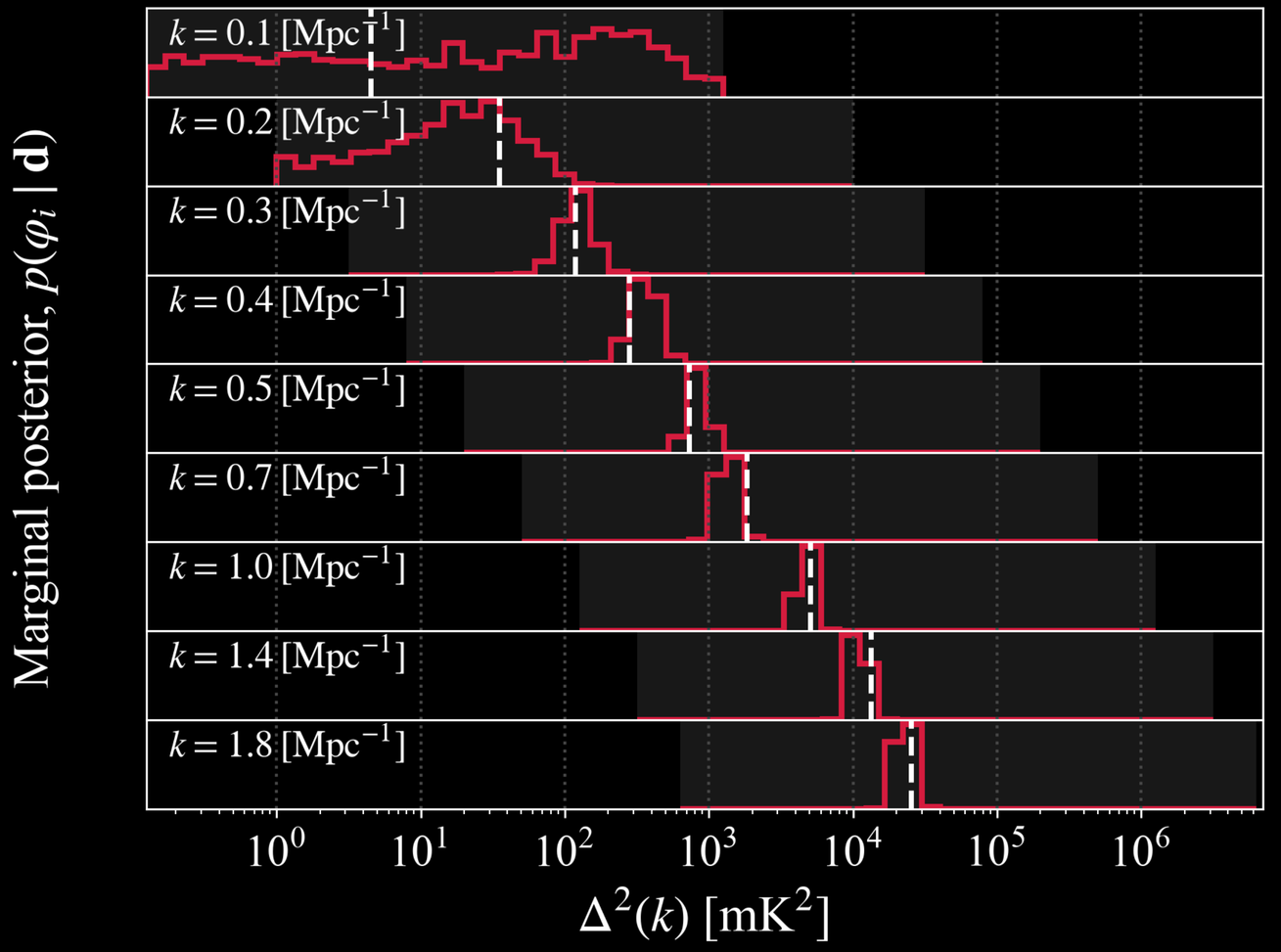
After:

- ~4,000 Params
- ~1 GPU (1.5%) (H100s)
- ~0.34 GPU Hours (H100s)

Credit: Tutt et al. (In Prep)

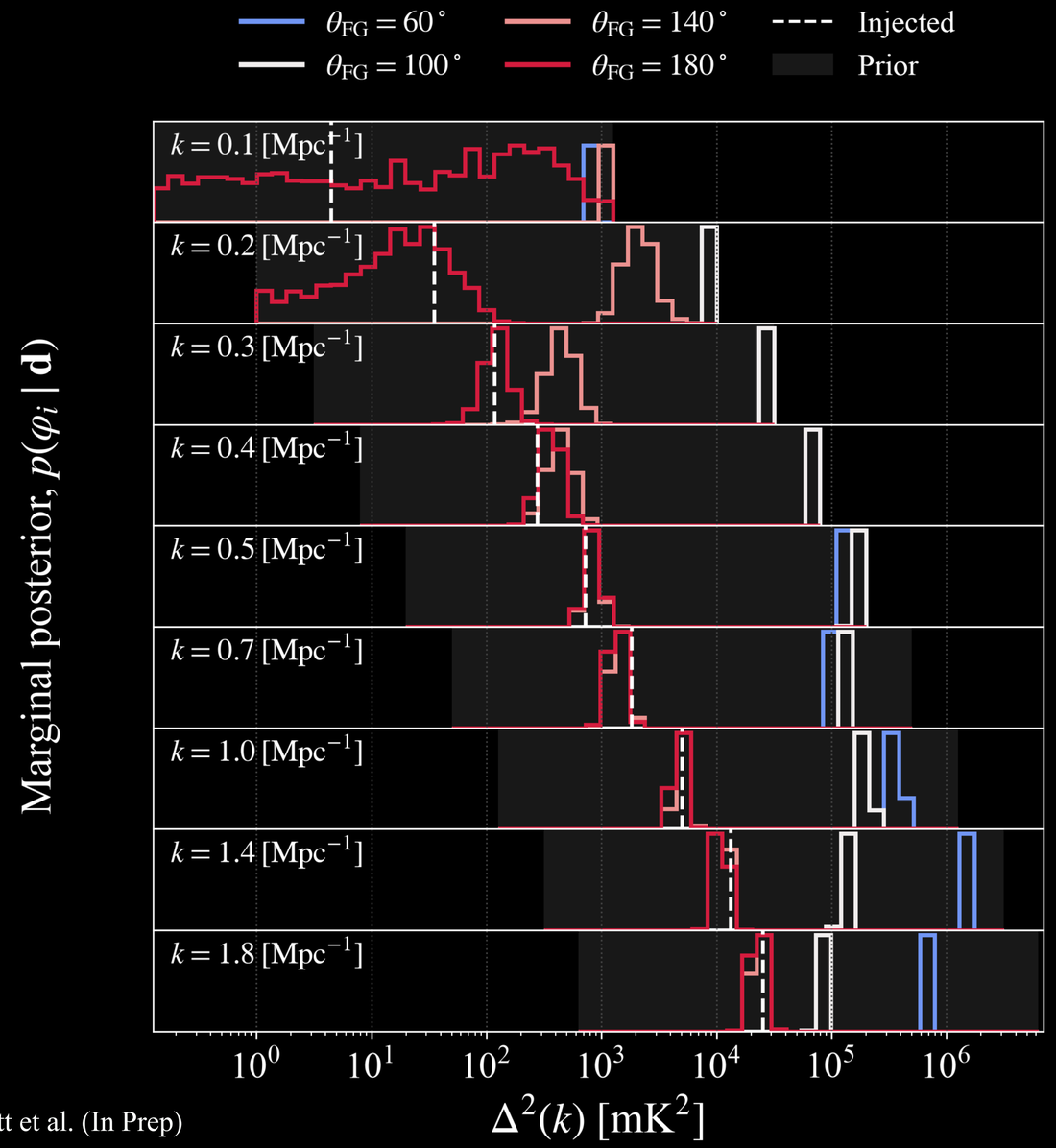
End to End Signal Recovery
Posterior Recovery



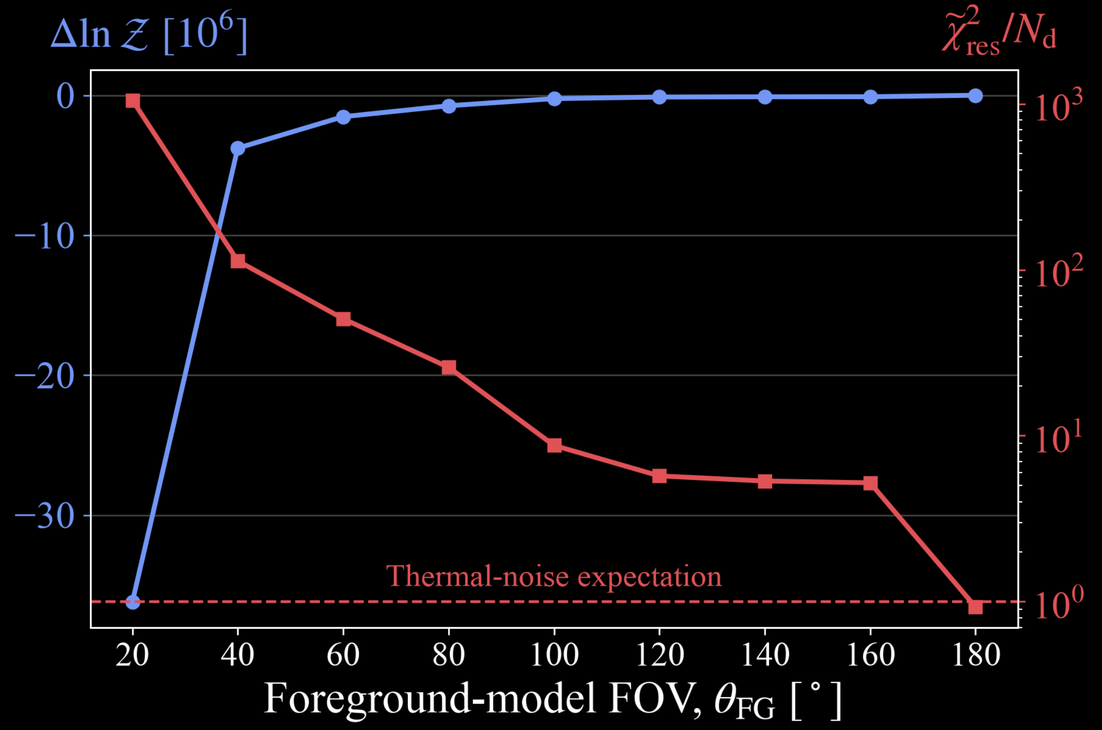


Credit: Tutt et al. (In Prep)

End to End Signal Recovery
Bayesian Validation

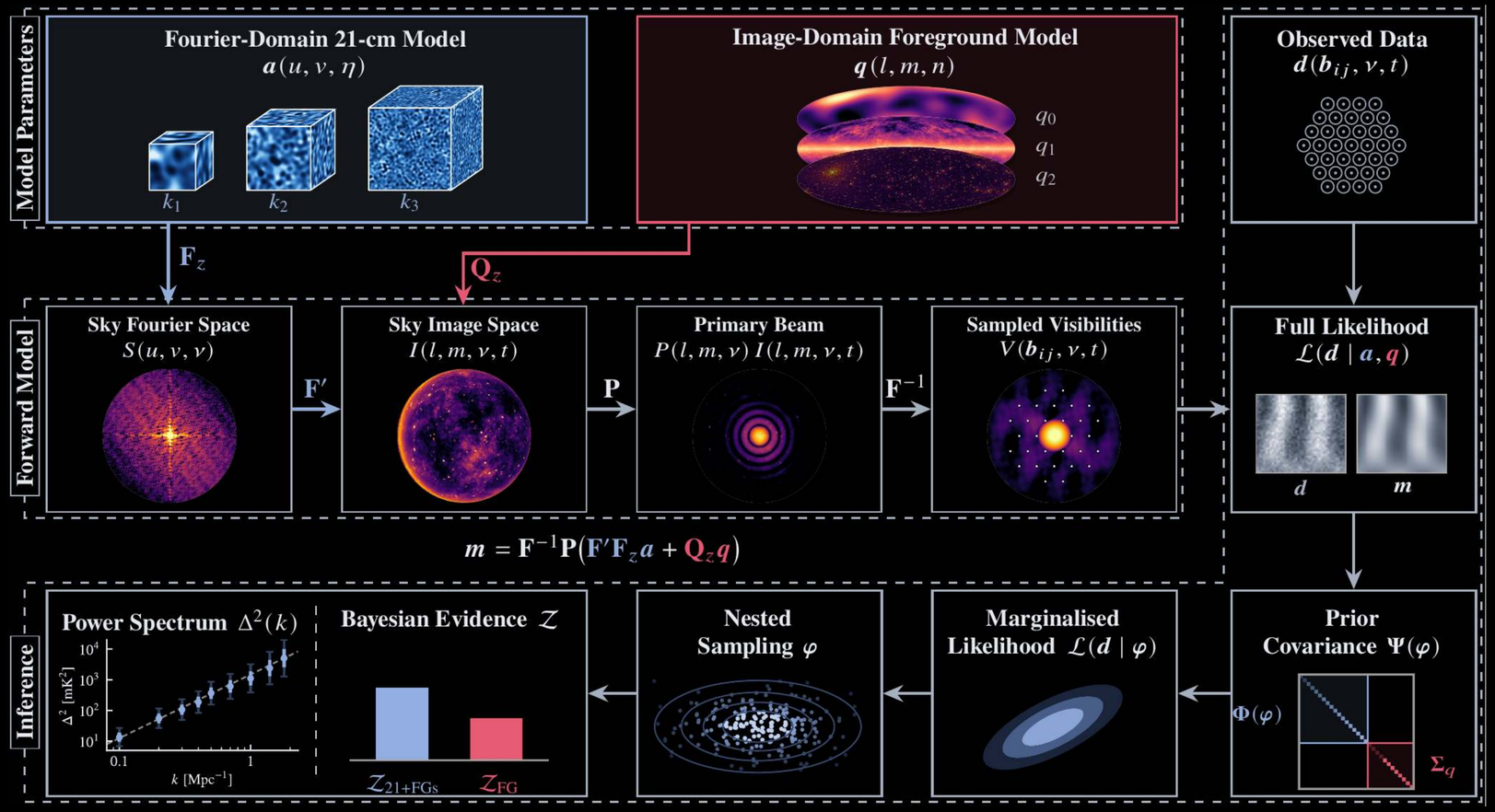


θ_{FG} [$^\circ$]	20	40	60	80	100	120	140	160	180
$\ln B_{val} < 0$	✗	✗	✗	✗	✗	✗	✗	✗	✓
$\ln B_{det} > 0$	✓	✓	✓	✓	✓	✓	✓	✓	✓
$q \geq 0.01$	✗	✗	✗	✗	✗	✗	✗	✗	✓



Credit: Tutt et al. (In Prep)

Future Work



1. Modelling Requirements
2. Learnt Compression/ SBI
3. Inference on Real Data

Conclussions:

Physically motivated wide-field modelling:

- Qualify foreground contamination from across the full visible sky

A statistically optimal latent basis:

- 306,163 to 9,551 parameter compression statistically the truncated

Statistical quantification of horizon complexity:

- 898 to 4,155 Orthonormal Prior-predictive covariance modes

Validated cosmological recovery:

- Calibrated and Validated Power Spectrum detections

Computationally tractable joint inference:

- 0.32 GPU-hours for physically motived Bayesian Inference