# Understanding the Capacity and Delay Scaling Laws of Delay Tolerant Networks: A Unified Approach

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***Abstract*— Mobile wireless networks with intermittent connectivity, often called Delay/Disruption Tolerant Networks (DTNs), have recently received a lot of attention because of their applicability in various applications. It has been shown that DTN routing and transport protocols can benefit from node mobility by letting the nodes carry and forward data to overcome partial connectivity. The scalability of DTN protocols is very important for protocol design and evaluation, yet so far little work has been done to study the scaling properties using a unified framework that formalizes the primary characteristics of a DTN, i.e., inter-contact behavior of nodes. In this paper, we represent DTNs as a class of wireless mobile networks with intermittent connectivity, where the inter-contact behavior of an arbitrary pair of nodes follows a homogeneous Poisson process. Using this DTN model, we make the following contributions. First, we generalize the throughput and delay scaling results of Grossglauser and Tse. Second, we introduce an optimal single-copy/multi-hop relay routing scheme and report its capacity and delay scaling properties. Third, we analyze the impact of various network parameters and routing strategies (such as buffer constraints, data replication, intermittent connectivity, and node speed) on the capacity/delay scaling properties of DTNs. Finally, we validate our analytical results with a simulation study.**

**Categories and Subject Descriptors:** C.2.1 [Network Architecture and Design]: Wireless Communication

**General Terms:** Design, Analysis

**Keywords:** Delay Tolerant Networks, Wireless Networks, Scaling Properties

## I. INTRODUCTION

Mobile wireless networks that can withstand intermittent connectivity, often called Delay/Disruption Tolerant Networks (DTNs), are becoming increasingly popular because of their applicability to various scenarios ranging from inter-vehicle communications [4] to content distribution in challenged networks [21]. It has been shown that DTN routing and transport protocols can benefit from node mobility and overcome the capacity bound of $\Theta(1/\sqrt{n \log n})$[1] originally established by Gupta and Kumar [17] for a fixed wireless network. Noting that the average hop length of a path is the key limiting factor, Grossglauser and Tse proposed a 2-hop relay routing algorithm that exploits node mobility to effectively reduce the hop length, and utilizes relay nodes to deliver data to the destination when they meet [16].

This result has been followed by a flurry of research activities that tried to characterize the delay/capacity relationship with respect to node mobility. Various mobility models have been considered, from a simple independent and identically distributed (I.I.D.) mobility model [29], [27], [38], [41], to more complex random mobility models, such as random waypoint [34], random direction [33], uniform mobility [2], [8], Brownian mobility [26], and random walk [11]. Sharma et al. systematically studied the impact of different mobility models on delay/capacity trade-offs [33]. Garetto et al. studied a home-point mobility model where each node moves around its home-point, and studied its impact on capacity scaling properties [12], [13]. In addition, the impact of finite buffer constraints on each node to the capacity of network has also been studied [18].

Despite the wealth of analytic results about the capacity scaling properties and related trade-offs of wireless networks under various constraints, our understanding of the basic scaling properties of DTNs is still limited and fragmented. This is in part due to the fact that so far little work has been done to study the scaling properties of DTNs in a unified framework, which formalizes the primary characteristics of a delay tolerant network.

In this paper, we represent DTNs as a class of wireless mobile networks with intermittent connectivity, where the inter-contact behavior of an arbitrary pair of nodes follows a homogeneous Poisson process (i.e., exponential pairwise inter-contact time). Groenevelt et al. [15] showed that *random* mobility models such as random direction and random waypoint can be modeled using a homogeneous Poisson process where the pairwise inter-contact rate of arbitrary nodes results to be proportional to *radio range* and *node speed*. When the radio range scales below the critical connectivity threshold $\Theta(\sqrt{\log n/n})$, the network becomes disconnected with high probability [17]. Under such a circumstance, we notice that the radio range and node speed mainly characterize how frequently a node meets some other node. Thus, our DTN model provides us a fundamental insight that we can represent an arbitrary delay tolerant network with certain *mobility* and *radio* characteristics using a single parameter capturing the

[1]Here, $n$ is the number of nodes. Recall that (i) $f(n) = O(g(n))$ means that $\exists c$ and $\exists N$ such that $f(n) \leq cg(n)$ for $n > N$ (i.e., asymptotic upper bound); (ii) $f(n) = \Omega(g(n))$ means that $\exists c$ and $\exists N$ such that $f(n) \geq cg(n)$ for $n > N$ (i.e., asymptotic lower bound); (iii) $f(n) = \Theta(g(n))$ means that $f(n) \in O(g(n)) \cap \Omega(g(n))$ (i.e., asymptotic tight bound); (iv) $f(n) = o(g(n))$ means that $\lim_{n\to\infty} f(n)/g(n) = 0$ (i.e., asymptotic insignificance); and (v) $f(n) = \omega(g(n))$ means $\lim_{n\to\infty} f(n)/g(n) = \infty$ (i.e., asymptotic dominance).

pairwise inter-contact rate $\lambda$.[2]

Using this DTN model, we make the following contributions in this paper. First, we report a generalized capacity and delay scaling laws of the 2-hop routing algorithm based on the inter-contact rate parameter to provide a new insight to the fundamental properties of DTNs. Second, we study an optimal multi-hop relay routing scheme and report its asymptotic capacity and delay bounds. Third, we analyze the impact of various network parameters and routing strategies on the capacity/delay scaling properties of DTNs, namely buffer constraints, data replication, intermittent connectivity, and node speed, and discuss capacity/delay/buffer trade-offs. Finally, we validate our analytical results with a simulation study.

### *Summary of Results and Organization*

The following is the preview of the key results reported in this paper. For a DTN with $n$ nodes with pairwise node inter-contact rate of $\lambda$, we report the following:

- The per-node throughput of a 2-hop relay routing protocol scales with $\Theta(n\lambda)$ and the delay of the same routing scheme is $\Theta(1/\lambda)$. Grossglauser and Tse's result is a special case when the radio range is $\Theta(1/\sqrt{n})$, i.e., $\lambda = \Theta(1/n)$. We report that for any $n$, there exists a delay tolerant network with $\lambda = \Theta(1/n)$ that achieves the per node throughput of $\Theta(1)$.
- The throughput and delay of an optimal single-copy/multi-hop relay DTN routing protocol are $O(\frac{n\lambda}{\log n})$ and $\Theta(\frac{\log n}{n\lambda})$ respectively. Our results show that the proposed scheme achieves better throughput-delay trade-offs than any known schemes based on 2-hop relaying with replication [29], [38], [27], [41].
- Given a finite buffer size of $K$ per communication pair, we find that the throughput is $\Theta(n\rho\lambda\frac{K}{K+n\rho})$ where $\rho$ is a constant utilization factor. Our results provides a tighter bound than the previous results by Herdtner et al. [18].
- Given intermittent connectivity with the radio range between $\omega(1/\sqrt{n})$ and $o(\sqrt{\log n/n})$, we show that a *hybrid DTN multi-hop routing protocol* where "electronic" multi-hopping is used within a connected sub-graph, and "mechanical" carry-and-forward is used to deal with network partitioning has the per node throughput bound of $o(1/\log^2 n)$; thus, intermittent connectivity improves the throughput at the cost of delay increment.

The rest of the paper is organized as follows. In Section II, we present the network model. In Section III, we show the analysis of the DTN routing protocols, namely 2-hop and optimal multi-hop relay DTN routing protocols. In Section IV, we formally investigate various DTN design parameters and their impacts on the scaling properties. In Section V, we validate our results via simulations. Finally, we present the conclusion in Section VI.

[2]We show the validity of this model using a simulation study in this paper.

## II. NETWORK MODEL

In this section, we review the system model used for analysis. We present the communication model and traffic patterns and define the throughput and delay. We then provide a simple mobility model through which we represent a DTN in general. Finally, we show the buffer model.

**Communication Model and Traffic Patterns**: We use the protocol model to abstract interference between transmissions [17]. Suppose that node $i$ transmits to node $j$. Node $j$ receives the transmission successfully if every other node that transmits simultaneously is at a distance of at least $(1+\Delta)r(n)$ from $j$ where $\Delta$ is some positive number and $r(n)$ is the radio range. Each node $i \in \{1, 2, \cdots, n\}$ has a designated destination node $d_i \neq i$, so there are a total of $n$ source-destination pairs.

**Definition of Throughput and Delay**: For a given scheduling algorithm $\pi$, a throughput $\gamma > 0$ is said to be feasible/achievable if every node can send at a rate of $\gamma$ bits per seconds to its chosen destination. Let $T^{\pi}(n)$ denote the maximum feasible per-node throughput under scheduling algorithm $\pi$. The delay of a packet in a network is the time for a packet to reach the destination after it leaves the source. Let $D^{\pi}(n)$ denote the average packet delay for a network with $n$ nodes under scheduling algorithm $\pi$. Note that a scheduling algorithm is $stable$ if the rate $T^{\pi}(n)$ is satisfied by all users such that one's queue does not grow infinity, i.e., $D^{\pi}(n)$ is bounded.

**Modeling Mobility**: DTN protocols leverage node mobility as a means of data delivery (i.e., *carry-and-forward*) and thus, the performance mainly depends on the encounter pattern. In this paper, we describe the mobility model using the pairwise inter-contact time, which is the time interval between two successive encounters of a pair of nodes. For analysis, we consider a class of random mobility models where each node independently makes its own decision, e.g., each node independently chooses a random *destination* (Random Waypoint) or a random *direction* (Random Direction). Groenevelt et al. showed that the inter-contact stochastic process of these mobility models can be captured using an independent homogeneous Poisson process with meeting rate $\lambda$ [15], [14]. In other words, inter-contact time distributions of any pairs are exponentially distributed with rate $\lambda$. This concept can be generalized using heterogeneous meeting rates with $\lambda_{ij}$ for $i, j = 1, \cdots, n$. We present the Theorem 4.2.1 from [14] to provide a basis for estimating the $\lambda$ value for different mobility models.

*Theorem 1:* Given that two nodes move randomly in an L×L square (L×L$m^2$) with the average speed $v$, if the transmission range $r \ll L$ and the position of a node at time $t+\Delta$ is independent of its position at time $t$ for small $\Delta$, then the inter-contact time between two nodes is exponentially distributed with parameter $\lambda = \alpha rv/L^2$ where $\alpha$ is a constant and $\alpha = 2\omega\int_0^1\int_0^1 \pi^2(x,y)dxdy$. Here, $\pi(x,y)$ is the steady state distribution of node position and $\omega$ is a constant compensation factor for the average relative speed. For Random Direction and Random Waypoint, we have $\alpha = 2\omega$ and $\alpha = 2\omega \times 1.3863$ respectively.

We note that the inter-contact behavior of an I.I.D. mobility model, which is much simpler than random mobility models, can also be approximated using the above model. Given a network with $1/r \times 1/r$ grids ($r < 1$) in a unit area (1x1), every node is completely re-shuffled in each time slot in I.I.D. mobility. Since a random node encounters a destination node with probability $r^2$, the number of encounters before meeting the destination follows a geometric distribution with success probability $r^2$, and by re-scaling time, it can be approximated to an exponential distribution with rate $\lambda = r^2$. The pairwise inter-contact behavior of I.I.D mobility is a special case of our model in that the speed of a node is in the same order as the radio range.

In various empirical studies, the inter-contact time distribution has been reported to follow an exponential distribution in real-life mobility patterns. Conan et al. showed that several mobility traces contain significant fraction of contact pairs following exponential distributions [7]. For instance, in the Dartmouth College WiFi trace, out of 13,482 pairs 62.3% pairs have been found to follow an exponential distribution. Karagiannis et al. found an invariant property that there is a time granule in the order of half a day, up to which the distribution of inter-contact time is well approximated by a power law and beyond it decays exponentially [22]. They also found that the aggregate inter-contact distribution does not deviate significantly from the individual pairwise inter-contact time distribution. In general, when a mobility model is defined in a finite domain, it has been mathematically proven that the inter-contact time distribution has an exponential tail [5].

**DTN Model**: We model an arbitrary DTN in a unit area of (1×1) using the pairwise inter-contact rate $\lambda = \Theta(rv)$ where $r$ is radio range and $v$ is speed. We can map an arbitrary delay tolerant network to a unit area by relatively scaling the radio range and average speed. In our study, we consider two cases: (a) when $\lambda$ is given and fixed and (b) when $\lambda$ scales according to $r$ and $v$.

Theorem 1 shows that the contact rate is independent of the number of nodes. However, when $\lambda$ is given, increasing the number of nodes over a certain limit will reduce the effective capacity due to wireless interferences. Also it changes the operating mode from DTN to mostly/full connected networks. Thus, in this case, we need to bound the number of nodes. To identify this threshold, consider the following. Assume that nodes are uniformly distributed on a unit square. The radio range determines the number of simultaneous transmissions, or the network-wide aggregate throughput. Since it is approximately the same as the total number of non-overlapping circles with radius $r$ that fills 1×1 area, the network-wide aggregate throughput $\mathcal{T}$ is bounded by $\Theta(1/r^2)$. Per-node throughput bound is simply given by dividing the aggregate throughput by the total number of communication pairs. Therefore, the aggregate throughput can be expressed in terms of $\lambda$: i.e., $\mathcal{T} \leq \Theta(1/r^2) = \Theta(1/\lambda)$. For a DTN with arbitrary $\lambda$, per node throughput is maximized when the number of nodes is in the same order as the aggregate throughput. Thus, for a DTN with a given $\lambda$, we analyze the scaling property for $n \leq 1/\lambda$.

On the other hand, if $\lambda$ scales with the node speed and the radio range (that are the functions of the number of nodes), we have $\lambda = \Theta(rv)$. Unless otherwise mentioned, we assume that the speed of a node is in the same order as the radio range such that the contact duration of two nodes is constant (and so is the packet size) as in [8], [11], [33].[3] For instance, with radio range $r = 1/\sqrt{n}$, we set the speed $v = 1/\sqrt{n}$ and thus, $\lambda = 1/n$. Grossglauser and Tse showed that when we scale the radio range as $r = \Theta(1/\sqrt{n})$ (a class of DTNs with $\lambda = \Theta(1/n)$), we can achieve the throughput to $\Theta(1)$.

In this paper, we slightly abuse the asymptotic notation for simplicity as follows. For instance, when we say that the per-node throughput of the 2-hop relay scheme is $\Theta(n\lambda)$, this statement is always true only when $\lambda$ scales with $n$. However, when $\lambda$ is fixed, it is true only when $n \leq 1/\lambda$. This conditional rule applies to all the asymptotic notations in this paper. Unless otherwise mentioned, we assume that the network area is partitioned into $C$ non-overlapping cells with size $s_n \times s_n$ where we have $s_n = 1/\sqrt{n}$ to have the node density per cell $O(1)$.

**Shared Buffer Model**: In delay tolerant networking, as we will see, any node can be a potential relay node (i.e., a data carrier). We assume that each node maintains a queue per each source and destination pair. Let each node have a finite buffer space of $K$ packets and then, the total buffer space in the network is $nK$. Assuming that the network-wide buffer space is fairly shared among all communication pairs, we model the per-pair buffer behavior using a single global queue with size $nK/M$ where M denotes the number of sources. Since we have $M = n$ in our traffic scenario, the size of a global queue is $K$. A source can send packets to relay nodes as long as the global queue is not full; and the occupied buffer space is released whenever relay nodes deliver packets to the destination.

## III. DTN Routing Analysis

In this section, we first present the capacity and delay scaling laws of the 2-hop relay routing model using our DTN model. This result is a generalized version of previous results by Grossglauser and Tse [16]. We then present the capacity and delay scaling laws for an optimal single-copy/multi-hop relay DTN routing and propose an algorithm to achieve the optimal bounds.

### A. 2-hop Relay DTN Routing

We briefly review the 2-hop relay algorithm proposed by Grossglauser and Tse [16] for completeness. In each time slot a cell becomes active if it contains at least a pair of nodes that are within their radio ranges. In each active cell, if there is a source-destination pair, such a pair is randomly picked and the source transmits a packet to the destination (*direct*

[3]In Section IV-D, we investigate the case when the node speed is not in the same order as the radio range.

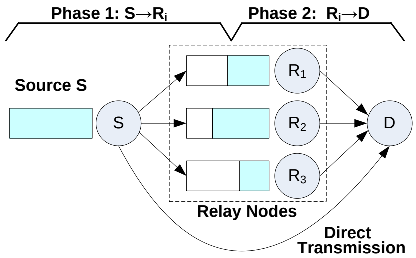


Fig. 1. An illustration of the 2-hop relay algorithm.

*transmission*). Otherwise, we randomly select a pair of nodes, set one as a sender and the other one as a receiver, and do the following: either a source node sends a new relay packet to a relay node (*Phase 1: Relay*), or a relay node delivers a packet to the destination (*Phase 2: Delivery*). The overall procedure is illustrated in Figure 1. Recall that a relay node has a separate queue for each source and destination pair and it may have multiple packets in the queue.

*Theorem 2:* The capacity of the 2-hop relay scheme is $\Theta(n\lambda)$.

*Proof:* Consider a pair of nodes: source $i$ and destination $d_i$. During a small time interval $\Delta t$, a random node $j$ encounters the destination with probability $\lambda\Delta t + o(\Delta t)$. In our network setting, there are a constant number of nodes in each cell. Since the chance of transmission is equally shared by $k$ interfering nodes under the protocol model [24], [11], [8], [18], node $j$ can successfully deliver a packet with the probability $\lambda\Delta t/k$ ($k = O(1)$). Here, we are interested in the event that the destination $d_i$ is scheduled to receive node $i$'s packet at time $t$. Let an indicator random variable $M_i(\Delta t, n)$ denote this event. Since $d_i$ can meet any of the relay nodes, we have:

$$Pr\{M_i(\Delta t, n) = 1\} \tag{1}$$

$$= \sum_{j=1, j\neq d_i}^{n} Pr\{\text{node } j \text{ delivers a packet during } \Delta t\} \tag{2}$$

$$\approx \frac{(n-1)\lambda\Delta t}{k} \tag{3}$$

Thus, the throughput is given as

$$T(n) = \frac{\mathbb{E}[M_i(t,n)]}{\Delta t} = \frac{(n-1)\lambda\Delta t}{k}\frac{1}{\Delta t} \tag{4}$$

$$= \Theta(n\lambda) \tag{5}$$

■

If $\lambda$ is fixed, the throughput linearly increases as $n$ increases. When $n$ becomes approximately $1/\lambda$, the throughput is maximized to $\Theta(1)$. This special case matches with the Grossglauser and Tse's result with radio range $\Theta(1/\sqrt{n})$, i.e., $\lambda = \Theta(1/n)$. Thus, there exists a delay tolerant network with $\lambda = \Theta(1/n)$ that achieves the throughput of $\Theta(1)$.

Now we look at the delay of the 2-hop relay scheme. For now, we assume that a communication pair is given an infinite size buffer. A source node encounters a destination node with rate $\lambda$ and any potential relay nodes with rate $(n-2)\lambda$ (i.e., minimum of $n-2$ random variables). The total output rate of the source is $(n-1)\lambda$. In the same way, the destination will encounter a random node carrying the source's packets with aggregate rate of $(n-1)\lambda$. Note that if we let the source to transmit in every chance it meets a relay the system is unstable because the arrival rate is the same as the service rate. In order to stabilize the system, we assume that the source sends a packet with probability $\rho < 1$ and the utilization is $\rho$.

*Theorem 3:* The delay of the 2-hop relay scheme is $\Theta(1/\lambda)$.

*Proof:* Consider a pair of nodes: source $i$ and destination $d_i$. The relay delay is the expected time for a packet at the source to be delivered to the destination $d_i$. If the source encounters the destination first, it can be delivered directly. This happens with probability $1/n$ and the average delay is $\frac{1}{n\lambda}$. Otherwise, the packet will be delivered via a relay node with probability $1-1/n$. Recall that a communication pair has its own queue at each relay node and a packet is served based on the First In First Out (FIFO) policy. In a nut shell, a queue will be incremented with rate $\rho\lambda$ (i.e., when it encounters the source) or decremented with rate $\lambda$ (i.e., when it encounters the destination). Therefore, the queueing behavior can be modeled using the standard M/M/1 queue. The average sojourn time is simply given as $\mathbb{E}[W] = \frac{1}{(1-\rho)\lambda}$ [23]. The expected delay can be expressed as follows:

$$D(n) = \frac{1}{n}\frac{1}{n\lambda} + \frac{n-1}{n}\frac{1}{(1-\rho)\lambda} \tag{6}$$

$$\approx \frac{1}{(1-\rho)\lambda} \tag{7}$$

Here, $D(n)$ can be approximated to $\mathbb{E}[W]$ as $n$ tends to infinity. Thus, we have $D(n) = \Theta(1/\lambda)$. ■

### B. Optimal Multi-hop Relay DTN Routing

Most DTN routing protocols use *multi-hop relaying* in order to reduce the latency [20], [28], [36], [25]. A packet from source $s$ is delivered to destination $d$ via a sequence of relay nodes as $s \to R_1 \to R_2 \to \cdots \to d$; i.e., the packet is to be moved from a source to destination over a time-varying connectivity graph [20], [30], [3].

In this section, we first derive the capacity and delay scaling laws of an optimal multi-hop relay DTN routing algorithm assuming the existence of an oracle that knows the complete contact information in the future. In order to derive the capacity/delay bounds of this single-copy/multi-hop scheme, we first analyze the multi-copy/multi-hop scheme, which is similar to epidemic data dissemination. We can describe the operation of a multi-copy/hop scheme as a node coloring process. Initially, the source's color is red, and all the other nodes are blue. Whenever a red node encounters a blue node, the latter is colored red. The overall process stops when one of the red nodes encounters the destination. The following theorem from [15] presents the expected delay and the expected number of copies (which corresponds to the number of transmissions).

*Theorem 4:* The expected number of copies in the multi-copy/hop scheme is $\frac{n+1}{2}$. The expected delay is approximately $\frac{\log n}{n\lambda}$.

Now we note that the multi-copy/hop scheme in fact embeds the optimal single-copy/multi-hop DTN routing scheme. We can visualize the overall replication process of the multi-copy/hop scheme using a binary replication tree as shown in

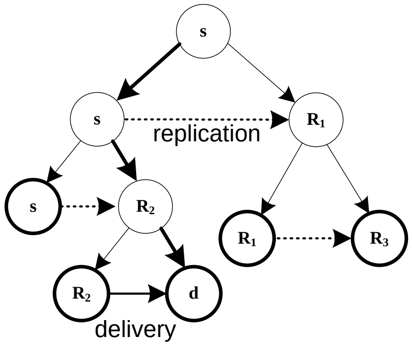


Fig. 2. Binary replication tree: it shows the realization of a replication process.

Figure 2. For example, when the source $s$ encounters node $R_1$, it is also colored red. After that moment both $s$ and $R_1$ start coloring other nodes. In a replication tree, whenever replication happens (per encounter), the original node is placed on the left and the newly colored node is placed on the right. For instance, $s$ is placed on the left and $R_1$ is placed on the right at the second level. In the figure, the leaf nodes with bold circles denote the colored nodes and we have total five nodes colored red. Because the optimal single-copy DTN routing knows the future contact, it will take a single path in the replication tree. In Figure 2 this optimal path is given as $s \rightarrow R_2 \rightarrow d$. From this observation, we have the following lemma.

*Lemma 1:* The expected number of transmissions in the optimal DTN routing is $\frac{\log n}{2}$.

*Proof:* Since we assume independent and homogeneous Poisson process, it is expected that the resulting binary replication tree is well balanced. Let $X_i$ denote an indicator random variable denoting whether a transmission has happened or not at level $i$ of the replication tree. If a path traverses level $i$ to the right, there is a transmission; otherwise, no transmissions has happened. The optimal path will be a random path in the replication tree and thus, this happens with probability $Pr\{X_i = 1\} = 1/2$. The depth of the tree is $\log n$ on average. The total number of transmissions is given as $X = X_1 + \cdots + X_{\log n}$. Thus, $\mathbb{E}[X] = \sum_{i=1}^{\log n} Pr\{X_i = 1\} = \frac{\log n}{2}$ ■

We now find the capacity and delay of the optimal multi-hop relay DTN routing.

*Theorem 5:* The capacity of the optimal multi-hop relay DTN routing is $O(\frac{n\lambda}{\log n})$.

*Proof:* Our goal is to find an upper bound on the throughput capacity of the optimal DTN routing. The derivation is similar to that in [17]. Consider a bit $b$ originating at source $i$. Lemma 1 shows that the number of hops required by $b$ to reach its destination is $H(b) = \log n/2$. Node $i$ can encounter another node and transmit data at a rate of $n\lambda$. We define this using an indicator random variable $S_i$. The total number of simultaneous transmissions per unit time is given as $S = \sum_{j=1}^{n} S_j$. The expected value of random variable $S$ is given as $\mathbb{E}[S] = n\mathbb{E}[S_i] \approx n^2\lambda$. For the sake of simplicity, we ignore the "constant" interference factor under the protocol model. Each source generates bits with rate $T(n)$. For a given period $\tau$, the total number of bits generated in the network is $nT(n)\tau$. The total number of hops required to support these bits during time interval $\tau$ is $nT(n)\tau H(b)$. This is bounded by the total number of feasible transmissions in the network during time interval $\tau$ that is $\tau S$. Hence, we have $nT(n)\tau H(b) \leq \tau S$. By substituting $H(b)$ and $S$, we have $T(n) \leq \frac{2n\lambda}{\log n}$ and thus, $T(n) = O(\frac{n\lambda}{\log n})$. ■

To achieve the throughput bound, we design an algorithm that is similar to the random network scheduling algorithm in [17]. The key distinction is that in our case the packets now routed over a *time-varying connectivity* graph instead of a *spatial* graph. Since there are a constant number of interfering nodes for a given *contact* opportunity [24], time-division multiple-access (TDMA) is used among these nodes; i.e., a node transmits at regularly scheduled *contact* time slots. The optimal multi-hop routing scheme with the contact oracle allows us to find $n$ source-destination spatial/temporal paths. For a given contact and its allocated contact time slot, a node again divides a contact time slot into packet time slots and performs another TDMA.[4] This routing over time-divided spatial/temporal paths enables us to show the achievability of the optimal throughput bound using the same techniques used by El Gamal et al. [11].[5]

*Theorem 6:* The delay of the optimal multi-hop relay DTN routing is $\Theta(\frac{\log n}{n\lambda})$.

*Proof:* The multi-copy/hop scheme embeds the optimal multi-hop relay DTN routing. Since the optimal DTN routing takes the optimal path out of the replication tree, the expected latency should be the same as the multi-copy/hop scheme. From Theorem 4, we have $D(n) = \Theta(\frac{\log n}{n\lambda})$. Now we consider the queueing delay. Each node generates traffic with a rate less than $\rho\frac{n\lambda}{\log n}$ where $\rho$ is a utilization factor ($\rho < 1$). The aggregate rate is given as $\rho n^2\lambda/\log n$. Since the average path length is $\Theta(\log n)$, a random node is in a multi-hop path with probability $\log n/n$. The aggregate incoming rate to a node is given as $\rho\frac{n^2\lambda}{\log n}\frac{\log n}{n} = \rho n\lambda$. Given the knowledge of an oracle, we assume that each relay node can fully utilize every encounter for the purpose of data forwarding. Thus, the service rate is $n\lambda$ as well. As in Theorem 3, the average sojourn time at an intermediate node is $\mathbb{E}[W] = \frac{1}{(1-\rho)n\lambda}$. Since the average path length is $\Theta(\log n)$, the average delay is given as $\frac{\log n}{(1-\rho)n\lambda}$. Thus the order of delay does not change even after considering the queueing delay. ■

The above results show that single-copy/multi-hop relay DTN routing reduces the capacity to $O(\frac{n\lambda}{\log n})$, yet it improves the latency to $\Theta(\frac{\log n}{n\lambda})$ compared to the 2-hop relaying scheme. This new type of single-copy/multi-hop relaying scheme has a different characteristics compared to the traditional multi-copy/multi-hop routing scheme [29]. In DTNs

[4] To be precise, a contact time slot is divided into $\log n$ packet time slots.

[5] In fact, the Oracle algorithm can be realistic in some cases; e.g., a packet is routed bus to bus assuming that the bus schedule is known [1]. If we do not have the contact oracle, DTN protocols should make a forwarding decision using a utility function. For instance, the age of last encounter containing relative location information of nodes can be used [28], [36], or a packet can be forwarded to a relay, which has a similar mobility pattern as the destination [25].

there are two main resource constraints, per-node bandwidth and finite buffer. The single-copy schemes only use only bandwidth whereas the multi-copy schemes use both bandwidth and buffer. We study the impact of various replication schemes on the capacity/delay scaling in Section IV-B and the relationship among capacity, delay, and buffer in Section IV-E.

## IV. DTN Design Parameter Analysis

We analyze DTN design parameters and their impacts on the scaling properties of DTN routing. First, we study the impact of finite buffer on the capacity of 2-hop relay DTN routing. Second, we analyze the capacity/delay scaling properties of multi-copy DTN routing with replication. Third, we study the impact of intermittent connectivity on the capacity/delay. Fourth, we show the impact of node speed on the capacity/delay. Finally, we discuss the relationship between buffer, delay, throughput and the delay-throughput trade-offs in DTN routing.

### A. Finite Buffer Constraint

A source node encounters a potential relay node with rate $m\lambda$ where $m = n - 2$. We assume that a single packet is sent per contact with probability $\rho$ and thus, the packet arrival rate is $m\lambda\rho$. If $k$ relay nodes are delivering packets to the destination, the destination encounters a relay node with rate $k\lambda$ where $k = 1, 2, \cdots, m$: i.e., the delivery rate is mainly dependent on the number of relay nodes that carry the packets from the source. Note that the number of distinct relay nodes does not grow linearly with the number of outstanding packets in the network, because the source node can meet the same node multiple times.

Given that there are $\ell$ outstanding packets (or $\ell$ encounters with actual packet transfers), we want to find the expected number of relay nodes. In fact, the problem is the same as the coupon collection problem; i.e., given $m$ coupons, find out how many trials one has to make to collect all $m$ coupons. Each trial is a random encounter in our scenario and is independent of one another. Let $X_i^\ell$ for $i = 1, 2, \cdots, m$ denote an indicator random variable that shows the existence of an encounter after $\ell$ trials: $X_i^\ell = 1$ if the source encountered node $i$; otherwise, $X_i^\ell = 0$. Since a node encounters a random node $i$ with probability $1/m$ in each trial, the probability that the node fails to meet the node $i$ after $\ell$ trials is $P(X_i^\ell = 0) = (1 - 1/m)^\ell$. The encounter probability is $P(X_i^\ell = 1) = 1 - P(X_i^\ell = 0)$ and let $g_\ell$ denote this probability:

$$g_\ell = P(X_i^\ell = 1) = 1 - (1 - \frac{1}{m})^\ell \quad (8)$$

We can represent the number of distinct nodes after $\ell$ trials as $X^\ell = X_1^\ell + X_2^\ell + \cdots + X_m^\ell$ where $X_i^\ell$ for all $i$ are IID. By the linearity of expectation, we have $\mathbb{E}[X^\ell] = m\mathbb{E}[X_1^\ell] = mg_\ell$. After $\ell$ trials $\mathbb{E}[X^\ell]$ nodes relay $\ell$ packets to the destination. Thus, the delivery rate is $\mathbb{E}[X^\ell]\lambda = mg_\ell\lambda$.

The system can be modeled using a standard Markovian queueing model (M/M/1). We first investigate the case where the queue capacity is infinite. The state transition diagram is shown in Figure 3. By writing down the global balance

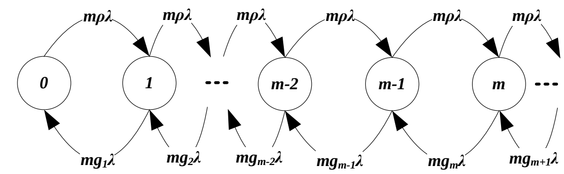


Fig. 3. State transition diagram. Each state denotes the number of outstanding packets in the networks. The service capacity at state $k$ is $m\lambda g_k$. We can model the finite buffer system by simply limiting the number of outstanding packets.

equations of the steady-state probability $p_k$ for all $k \geq 0$, we obtain

$$m\rho\lambda p_{k-1} = mg_k\lambda p_k \quad (9)$$

By solving the global balance equations we obtain the following steady state probability $p_k$.

$$p_k = p_0 \frac{\rho^k}{\prod_{i=1}^k g_i} \quad \textit{where} \quad p_0 = \left[1 + \sum_{j=1}^{\infty} \frac{\rho^j}{\prod_{i=1}^j g_i}\right]^{-1} \quad (10)$$

We consider the case where the buffer size is limited to $K$; i.e., there are at most $K$ outstanding packets in the system. Recall that we assume a shared buffer model such that there are at most $K$ packets per communication pair. The transition diagram in Figure 3 has now only $K + 1$ states, and the resulting equations are the same as in Equation (10) except that the summation to the infinity is bounded to $K$. In the system, an incoming packet is dropped when the buffer is full, i.e., the number of outstanding packets is $K$. This happens with probability $p_K$ and dropping probability monotonically decreases as the buffer size $K$ increases.

Given this, we consider the effective arrival rate. In a stable system, the effective arrival rate is the same as the effective service rate. In other words, this allows us to calculate the relay capacity; i.e., how many relay nodes are available in the network. The effective arrival rate is simply given as $\mathbb{E}[S_K] = m\rho\lambda(1 - p_K)$.

*Theorem 7:* The average service rate $\mathbb{E}[S_K]$ of a 2-hop relay system with finite buffer size of $K$ is determined as

$$\mathbb{E}[S_K] = m\rho\lambda \frac{K}{K + \rho m}$$

*Proof:* See the Appendix. ■

We then find the relationship between the service rate and the actual throughput. The following theorem shows that the "service rate" is proportional to the "actual throughput."

*Theorem 8:* The capacity of 2-hop relay with buffer space $K$ is given as $\Theta(\mathbb{E}[S_K]\lambda)$.

*Proof:* The derivation is quite similar to Theorem 2. The only difference is that the number of relay nodes is now restricted to $\mathbb{E}[S_K]$. Given $\mathbb{E}[S_K]$ relay nodes, the throughput is given as:

$$T(n) = \frac{\mathbb{E}[M_i(t,n)]}{\Delta t} = \frac{\mathbb{E}[S_K]\lambda\Delta t}{k}\frac{1}{\Delta t} \quad (11)$$

$$= \Theta(\mathbb{E}[S_K]\lambda) \quad (12)$$

■

Compared to [18] where a loose upper bound under $r(n) = \Theta(1/\sqrt{n})$ is presented, we provide a tight bound under general DTN scenarios by showing the relationship between service capacity and buffer size. For instance, if the buffer size is in the order of $\Theta(n)$ we see that the capacity is also $\Theta(n\lambda)$. Given that the buffer size is in the order of $\Theta(\sqrt{n})$, the capacity is also $\Theta(\sqrt{n}\lambda)$. In the connectivity regime with $r(n) = \Theta(1/\sqrt{n})$ (i.e., $\lambda = 1/n$), our results show that the buffer size should scale as $\Theta(n)$ to achieve $\Theta(1)$ per node throughput.

### B. Replication

We show that the delay of 2-hop relay DTN routing is $\Theta(1/\lambda)$. Considering a DTN with $\lambda = 1/n$, the delay linearly scales with the number of nodes, i.e., $\Theta(n)$. Neely et al. showed the $\Theta(n)$ delay can be improved using replication [29]. They showed that the 2-hop relay with $\sqrt{n}$ and unrestricted replication can reduce the delay to $O(\sqrt{n})$ and $O(\log n)$ respectively. In this section, we generalize capacity/delay tradeoffs of different replication methods used in DTN routing. In particular, we consider the "probabilistic" replication where a packet is replicated to a relay node with probability $p$, and the "$k$-copy" replication where a packet is replicated exactly $k$ times. Instead of a single source making $k$ copies, we assume that each replica holder can replicate the packet as long as the total number of replicas is less than $k$ [37], [35]. For instance, Spyropoulos et al. proposed a binary spraying method [37]; i.e., a counter value that is initially set to $k$ to generate $k$ replicas is halved for each encounter and is distributed to nodes; a node finishes replication if its counter value reaches to zero. Capacity/delay tradeoffs can be summarized as follows.

*Theorem 9:* The capacity of probabilistic and limited replication are given as $O(\frac{\lambda(1+p)}{p})$ and $O(\frac{n\lambda}{k})$ respectively. The delay of probabilistic and $k$-copy replication are given as $O(\frac{\log n}{pn\lambda})$ and $\mathrm{O}\left(\frac{k\log k+n-k}{nk\lambda}\right)$ respectively.

*Proof:* See the Appendix. ∎

As shown earlier, the number of transmissions to deliver a packet (i.e., the number of replicas) is the critical factor of determining the capacity. For instance, probabilistic replication generates on average $pn/(1+p)$ replicas and thus, the per node throughput is given as $O(\frac{\lambda(1+p)}{p})$. Theorem 9 clearly shows that as the degree of replication increases, the delay decreases. Note that the results that Neely et al. reported are the special case with $\lambda = \Theta(1/n)$ [29]. Given $k = \sqrt{n}$ and $k = n$, the delay is given as $O(\sqrt{n})$ and $O(\log n)$ respectively, and the capacity is given $O(1/\sqrt{n})$ and $O(1/n)$ respectively.

In reality, it is non-trivial to determine the degree of replication. Spyropoulos et al. proposed a method of choosing the number of replicas for a given *delay constraint* [37]. Similarly, Small et al. showed the relationship between the number of replicas and the delivery probability. This probability is then used to set the expiration time of a replica [35]. Unlike these methods, the number of replicas is not strictly enforced in a marginal utility-based strategy [1], because a packet with the highest marginal utility is always replicated first. Walker et al. proposed a simple counting algorithm that limits the fraction of nodes carrying replicas using a simple counting protocol [39].

Not only does the number of transmissions reduce the capacity, but the finite buffer size also decreases the capacity as discussed above. If the buffer space is limited, excessive replication may cause severe contention, thus diminishing the effectiveness of replication. Therefore, cleaning up replicas is also important since unnecessary replicas take up valuable buffer space. Small et al. showed that replicas can be discarded after a certain time threshold (or TTL), or via an anti-packet generated/disseminated by the destination [35]. Neely et al. proposed to use a sequence number such that a destination node sends its current sequence number to clean all the packets whose sequence numbers are less than the destination's current sequence number [29].

### C. Multi-hop Routing with Intermittent Connectivity

A wireless network is connected with high probability when the radio range scales with $\Theta(\sqrt{\log n/n})$ [17]. We first study the throughput bounds of such a wireless network, using a similar argument as in Theorem 5. Consider a bit $b$ originating at source $i$. Given that we have a unit network, the average hop length is given as $H(b) = \Theta(1/r)$. The total number of simultaneous transmissions in the network is $S = \Theta(1/r^2)$. Each source generates bits with rate $T(n)$. The total number of hops required to support these bits during time interval $\tau$ is bounded by the total number of feasible transmissions in the network during time interval $\tau$; i.e., $nT(n)\tau H(b) < \tau S$. Hence, we have $T(n) < \frac{S}{nH(b)}$. By substituting $S$ and $H(b)$, we have the capacity bound of $O(\frac{1}{nr}) = O(1/\sqrt{n\log n})$.

If the radio range scales below this critical connectivity threshold, the network is disconnected with high probability. In Section III, we mainly study the scaling properties under the radio range of $O(1/\sqrt{n})$. We now want to find the capacity bounds of DTN multi-hop routing with intermittent connectivity under the radio range, from $w(1/\sqrt{n})$ to $o(\sqrt{\log n/n})$. We assume that the routing goal is to minimize the average delay. Given that a network is composed of a set of connected subgraphs depending on the radio range, we use a *hybrid DTN multi-hop routing protocol* where "electronic" multi-hopping is used within a connected sub-graph, and "mechanical" carry-and-forward is used to deal with network partitioning. Note that we assume that the radio range is asymptotically greater than the speed: $r(n) = \omega(v(n))$ in order to guarantee a progress in multi-hopping [11].

*Theorem 10:* Given intermittent connectivity under the radio range, from $w(1/\sqrt{n})$ to $o(\sqrt{\log n/n})$, throughput upper bound of hybrid DTN multi-hop routing is $o(\frac{1}{\log^2 n})$.

*Proof:* The key factor of determining the throughput is the average hop length. Since we use mobility to route packets, the average hop length is upper bounded by that of "electronic" multi-hopping. For instance, given that the distance between two connected sub-graphs is $w(r(n))$, electronic multi-hopping requires $w(1)$ transmissions, whereas mechanical carry-and-forward reduces it to $\Theta(1)$ transmissions. Also, the average hop length cannot be lower than the optimal hop length of $\log n$ as shown in Section III-B. Thus, the hop length ranges from $\Theta(\log n)$ to $\Theta(1/r) = o(\sqrt{n/\log n})$. From the above argument, per node throughput is bounded by $T(n) < \frac{1}{r^2 n H(b)}$, ranging from $o(\sqrt{\log n/n})$ to $o(\frac{1}{\log^2 n})$. ■

The above results show that the intermittent connectivity provides a better throughput from $O(1/\sqrt{n\log n})$ to $o(\frac{1}{\log^2 n})$, mainly because the average hop length can be shorted by exploiting mobility. However, the throughput improvement comes at the cost of an increased delay, because the delay caused by the mobility dominates the delay caused by the electronic multi-hopping.

### D. Node Speed

So far we assume that the node speed is in the same order as the radio range. We now want to show the impact of the node speed on the throughput and delay. For a fixed radio range, the speed determines the contact duration denoted as $D_c(n) = r(n)/v(n)$ as follows: $v(n) = \Theta(r(n)) \Leftrightarrow D_c(n) = \Theta(1)$, $v(n) = o(r(n)) \Leftrightarrow D_c(n) = w(1)$, and $v(n) = w(r(n)) \Leftrightarrow D_c(n) = o(1)$.[6]

In our 2-hop relay analysis, we show that a source node encounters a relay node with rate $n\lambda$. The actual data rate per contact is proportional to the contact duration, i.e., $n\lambda D_c(n)$. As a special case, when the contact duration is constant, i.e., $D_c(n) = \Theta(1)$, the actual rate is the same as the meeting rate, $n\lambda$. By substituting $\lambda$ and $D_c(n)$, the actual rate is given as $nr(n)^2$ in general. This shows that the node speed is independent of the throughput. For example, consider the case where node speed is asymptotically greater than radio range, $v(n) = w(r(n))$ (i.e., $D_c(n) = o(1)$). Here, the increased meeting rate compensates for a short contact duration of $o(1)$; i.e., the achievable throughput is invariant with the node speed. Note that the contact duration is closely related to the packet size. In 2-hop relaying, we simply assume that the packet size scales as the contact duration, particularly when node speed is asymptotically greater than radio range, $v(n) = w(r(n))$. The packet size must be scaled down to $D_c(n) = o(1)$ accordingly, so that for a given contact duration, a node can have enough time to pick up a packet.

Since the average delay is a function of the meeting rate $\lambda = r(n)v(n)$, we notice that the average delay is directly related to the node speed. For instance, the average delay of 2-hop relaying is $\Theta(1/\lambda)$. In general, as node speed increases, the delay decreases. In the following section, we show that throughput-delay product determines the buffer requirement for a given communication pair. When the delay changes, the buffer requirement also changes; i.e., the slower the mobility, the greater the buffer requirement.

### E. Discussion

We discuss the relationship among buffer, delay, throughput and the delay-throughput trade-offs in DTN routing.

*1) Throughput-Delay Product, Buffer Requirements, and Congestion Control in DTNs:* In this section, we use the previous results to investigate the relationship between delay, capacity, and buffer requirements. First, for a given node pair, we find the average packet queue in the network using Little's law: the product of per node throughput and average packet life time. Since the packet life time is equal to the average delay, the average number of packets for a given pair (i.e., the number of buffers required to support a given pair flow) is determined by the throughput-delay product in a DTN. From this observation, the average buffer requirements for 2-hop relay and optimal multi-hop relay DTN routing can be computed as $O(n\lambda) \times O(1/\lambda) = O(n)$ and $O(n\lambda/\log n) \times O(\log n/n\lambda) = O(1)$ respectively (See Section III). Using a similar technique, we can find the average buffer size of replication schemes. At first glance, excessive replication will tend to waste buffer space. However, the results presented in Section IV-B show that replication reduces both throughput and delay because we trade throughput for delay improvement. As a result, the throughput-delay product decreases as well. More precisely, the average buffer requirements for replication scheme with $k = \sqrt{n}$ and $k = n$ are $O(1)$ and $O(\log n/n)$, respectively.

Theorem 8 in Section IV-A shows that the achievable throughput decreases when the available buffer size is smaller than the average buffer requirements. Assuming that buffer space is a critical resource in DTNs, Jain et al. claimed that *minimizing the delay of a message in routing is a good approximation for maximizing message throughput because it reduces contention for resources* [20]. Our study reveals that we can minimize delay by replication and multi-hop relaying (i.e., by trading capacity for delay improvement). As a result, the network-wide buffer occupancy will decrease and subsequently, the packet drop rate will decrease as well.

This anecdotal evidence elicits the need for "congestion control" of buffer space in DTNs. Fall [9] proposed proactive methods that involve admission control and expiration timers, and reactive methods that reserve buffer space based on the class of a service. Seligman et al. [32] proposed a custodian migration method where packets (bundles) are migrated to nodes with available space (or custody transfer) to prevent packet drops. However, these approaches do not consider the *bandwidth-delay product* that measures the capacity of a *network pipe*. The bandwidth-delay product has been used for setting up the TCP congestion window such that the number of outstanding (or unacknowledged) packets does not exceed the TCP flow's share of the product. In the same way, in DTNs bandwidth-delay product (or throughput-delay product) gives the required buffer space to accommodate "in-flight" packets. Thus, the congestion control algorithm should be able to adjust source's rate based on the buffer space. Naturally, the assumption of uniform traffic and uniform random mobility are good to the analysis, but are not very realistic. If traffic and motion are not uniform, the optimal operating conditions vary from flow to flow. One needs end-to-end feedbacks to implement mechanisms such as equation-based flow control [10].[7]

*2) Delay-Throughput Trade-offs:* Given the fact that the delay improvement comes at the cost of throughput reduction,

[6]Note that readers can find the distribution of contact duration (or link duration) in [31], [40].

[7]In DTNs the end-to-end argument is mitigated such that hop-by-hop reliability via custody transfer is used to provide end-to-end reliability [9], [32]

understanding fundamental delay-throughput trade-offs has been an active area of research [29], [38], [27], [41]. One can represent the trade-offs using either the ratio of delay to throughput (or $D/T$ ratio), or the throughput bound for a given delay constraint $D$.

Neely et al. showed that the ratio of delay to throughput is bounded by $\Theta(n)$ (i.e., $D/T \geq \Theta(n)$) using 2-hop relay-based routing protocols, namely without redundancy, $\Theta(\sqrt{n})$ replicas, with unrestricted replication [29]. However, we showed that optimal multi-hop DTN routing can achieve a better lower bound: $D/T \geq \Theta(\frac{\log^2 n}{n^2 \lambda^2})$. Recall that the key limiting factor of determining the capacity is the number of transmissions to deliver a packet to a destination node. The number of replicas generated by the unrestricted replication is $\Theta(n)$, requiring at least $\Theta(n)$ transmissions. For $r(n) = \Theta(1/\sqrt{n})$, we show that there can be $\Theta(n)$ simultaneous transmissions in the network. Thus, in any period of $\tau$, there are total $\tau\Theta(n)$ transmission opportunities in the network. Unrestricted replication can deliver at most $\tau$ packets whereas the optimal DTN multi-hop routing can deliver up to $\tau n/\log n$. By effectively reducing the number of transmissions, we can improve the capacity by a factor of $n/\log n$.

Ying et al. recently showed that message splitting/coding can achieve the per node throughput of $O(\sqrt{D/n})$; i.e., $D/T \geq \sqrt{Dn}$ [41]. The following is the description of their model. A packet expiration timer is set based on a delay constraint of $\Theta(D)$: when a packet fails to meet the deadline, it will be automatically removed from the relay node's buffer. They showed that this communication system can be modeled using an abstract channel with erasure probability $e^{-\lambda D}$, the probability that a relay node fails to encounter a destination node in time $D$. To achieve optimal capacity/delay tradeoffs, they used erasure coding and showed that the per node throughput scales with $O(\sqrt{D/n})$. Given the radio range $r(n) = \Theta(1/\sqrt[4]{nD})$, a node first generates $D/M$ coded packets using $\frac{6D}{25M}$ original data packets where $M$ is the number of neighboring nodes, i.e., $\Theta(nr(n)^2)$. A node then broadcasts $D/M$ coded packets to relay nodes during $D$ time slots of *broadcasting period*. This is followed by $5D$ time slots of *message delivery period*.

The throughput bound of this procedure can be found using our framework as follows. Each broadcasting generates $O(M)$ relay nodes. $M$ neighboring nodes equally share $D$ steps of the broadcasting period. After this, there will be at most $O(D)$ relay nodes and they will deliver packets to a destination node during the message delivery period. Since the number of relay nodes determines the capacity, it is simply given as $O(D\lambda)$. By substituting $\lambda = r(n)^2$, the per node throughput is given as $O(\sqrt{D/n})$. We now compare the delay/throughput ratio bound. By plugging the delay of the optimal multi-hop routing from Theorem 6, the ratio is given as $D/T \geq \Theta(\sqrt{\log n/\lambda})$. For a DTN with $\lambda = 1/n$, the ratio becomes $D/T \geq \Theta(\sqrt{n \log n})$. The reason why it has a loose lower bound is that the communication system did not use multi-hop routing. Recall that the erasure probability is mainly determined by the delay of 2-hop relaying. Although multi-user packet reception via wireless broadcasting may help, its improvement is limited. Given a wireless mobile network with intermittent connectivity where the radio range scales as $o(\sqrt{\log n/n})$, the average number of neighboring nodes is given as $M = o(\log n)$. From $M = \sqrt{n/D} = o(\log n)$, the delay bound in this case is given as $D = w(n/\log^2 n)$ which is much greater than $\Theta(\frac{\log n}{n\lambda})$.

## V. Simulations

In this section, we validate our framework using QualNet v3.9.5, a packet level network simulator. We measure the inter-contact time and average contact duration through which we analyze the throughput of 2-hop relay routing. We then study the impact of finite buffer and validate our model.

### A. Simulation Setup

We use the random waypoint mobility model with 0 pause time and two sets of speed ranges ([min speed, max speed]): [1,20]m/s and [1,30]m/s. Nodes are moving in an area of size 3000m × 3000m. We use 802.11b with two-ray ground path-loss propagation model, 250m transmission range, and 2Mbps transmission rate. We vary the number of nodes from 10 to 50 by 10 node increments. We implemented the 2-hop relay routing protocol, in which a node maintains a separate queue for each destination. When a node encounters another node, if the encountered node is a destination, the node will keep sending packets until the link breaks; otherwise, the node makes a packet by packet forwarding decision; i.e., with the same probability, the node either relays its own packet to a relay node or it delivers a relay packet to the encountered destination. When a link becomes available, scheduling too many packets at the same time causes MAC layer buffer overflow. Thus we implemented a flow control mechanism such that a new packet is scheduled only after an earlier packet is successfully delivered or dropped. To support this operation, the MAC layer interacts with the network layer, and notifies packet delivery and packet drop events. To measure the maximum throughput, we randomly choose a single pair of nodes and generate packets using the Constant Bit Rate (CBR) traffic in QualNet. We warm up simulations for 10,000s to remove the initial startup phase. Unless otherwise mentioned, reported results are the averages of 50 runs with different random seeds and are presented with the 95% confidence interval. The duration of a simulation is 40,000 seconds.

### B. Simulation Results

We measure the pairwise inter-contact time and show the Complementary Cumulative Distribution Functions (CCDF) in Figure 4. The figure confirms that the inter-contact time follows an exponential distribution and is independent of the number of nodes. The mean inter-contact time is given as 1282s and 990s for the maximum speed of 20m/s and 30m/s respectively. Figure 5 shows the average contact duration between nodes. A node with the maximum speed of 20m/s and 30m/s has the average contact duration of about 32s and 25s respectively.

We measure the per node throughput by increasing CBR traffic rate (i.e., packets/sec). The size of a packet is 1500B. Heusse et al. showed that the channel utilization of 802.11b

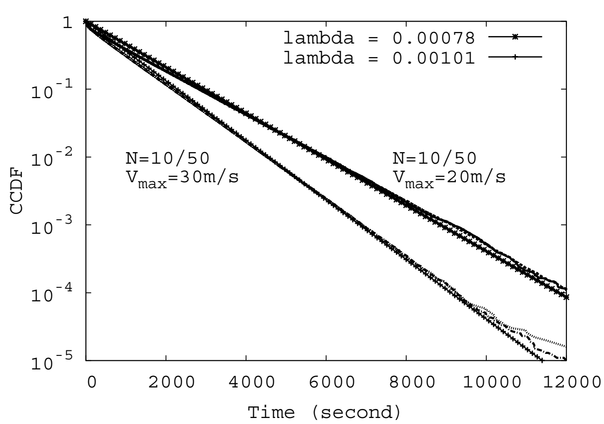


Fig. 4. Inter-contact time distribution

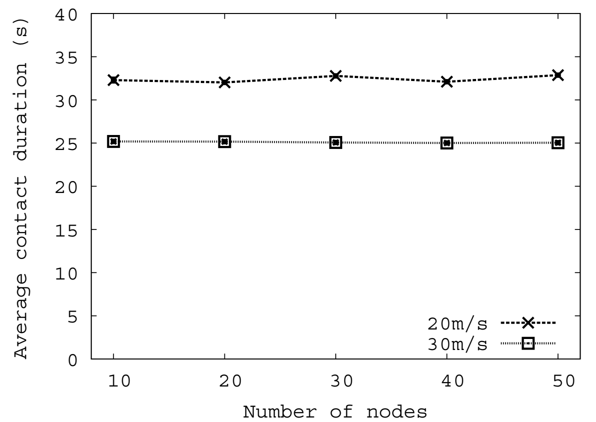


Fig. 5. Average contact duration

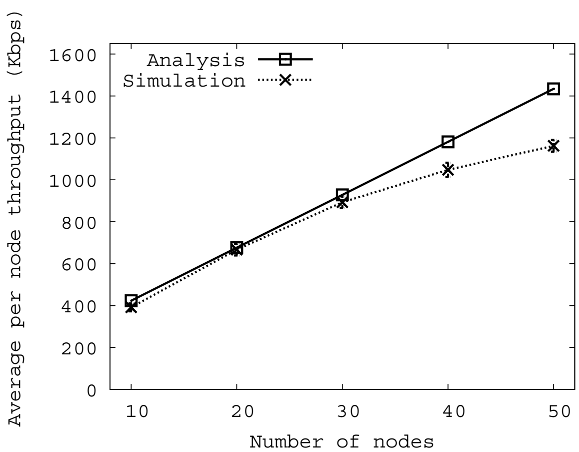


Fig. 6. Average per node throughput

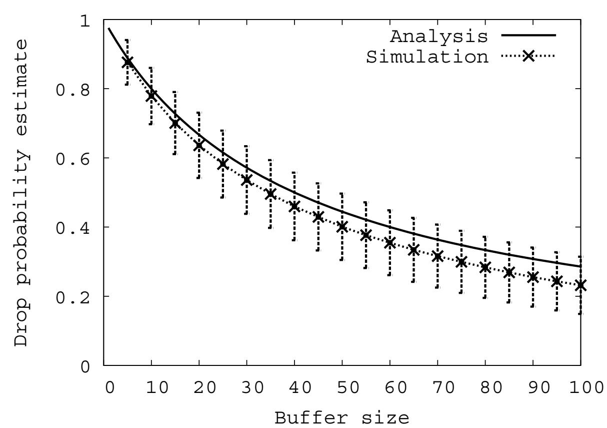


Fig. 7. Packet drop probability with finite buffer

with a packet of size 1500B is 70% (denoted as $u$) [19]. For a given bandwidth $B$, the effective bandwidth is given as $B_e = Bu$. We measure the channel utilization in our setting by placing two nodes at the same location. The measured utilization is about 80% (1.6Mbps). Given $n$ nodes and contact rate of $\lambda$, a node encounters another node with rate $n\lambda$, which is a renewal process with the mean inter-renewal interval of $1/n\lambda$. For a given contact duration $D_c$, a node can transfer on average $D_cB_e$. The average throughput is given as $n\lambda D_cB_e$. Assuming the interfering nodes to either source or destination equally share the bandwidth, the throughput becomes $n\lambda D_cB_eP_i$ where $P_i$ denotes the interference factor ($P_i \leq 1$). We use the approximated density distribution proposed by Chu et al. to find $P_i$ [6]. Figure 6 shows the measured throughput of the 20m/s case. When the number of nodes is small, the analytic throughput model matches well with the simulated estimate. However, the analytic throughput model deviates from the simulated estimate as the number of nodes increases, mainly because our interference model does not consider the MAC layer operations (e.g., random back-offs). Relay nodes competes with each other to deliver its relay packets as much as possible, and these nodes also interferes with the source. Recall that the interference range is about twice larger than the radio range. The simulation log shows that as the number of contending nodes increases, the number of packet drops (RTS/CTS and data packets) increases and this collision resolution process causes throughput loss. We only present the results for 20m/s. The 30m/s case exhibits almost identical behavior because the ratio of contact duration to inter-contact time is about the same.

We then simulate the finite buffer scenario. Since we measure the throughput by increasing the CBR rate, it is non-trivial to determine the buffer size. To accurately characterize the overall behavior, we modified the 2-hop relay protocol such that a node can only send a single packet for a given contact. We implement an ideal global buffer where for a given communication pair, the total number of packets in the network is limited by a pre-defined threshold. We measure the fraction of packets dropped. Figure 7 shows the drop probability with the following configuration: 40 nodes, 20 m/s maximum speed, and variable buffer size ranging from 1 to 100. We also plot the analytic results in Theorem 7. The figure shows that our model estimates the dropping probability accurately. It is interesting to note that the dropping probability is independent of the meeting rate. This is because the buffer space will be consumed faster when the rate is high, but the packet delivery is also faster, thus releasing buffer at a higher rate. Our simulation results with 30m/s maximum speed are similar to those with 20m/s.

## VI. CONCLUSION

We studied the capacity/delay scaling properties of DTN routing protocols in mobile ad hoc networks with intermittent connectivity. We considered a class of random mobility models where a pairwise inter-contact time can be modeled using an independent homogeneous Poisson process, e.g., Random Waypoint, Random Direction, etc. Using this unified framework, we generalized the scaling behavior of 2-hop relay routing and optimal single-copy/multi-hop routing. Our results showed that optimal single-copy/multi-hop routing provides a better delay-capacity trade-off than any known schemes based on 2-hop relay routing with replication. We studied various DTN design parameters such as finite buffer, replication, intermittent connectivity, and node speed, and reported their impacts on capacity/delay scaling behavior. In particular, we found that (1) analytic results of finite buffer and replication provide a better insight into the trade-offs between delay, capacity, and buffer; (2) intermittent connectivity improves the throughput at the cost of delay increment; and (3) node speed does not affect the achievable throughput, yet it is directly related to the delay.

## APPENDIX

This section provides the proofs of the theorems.

*Proof:* (Theorem 7) We know that $\mathbb{E}[S] = mp\lambda(1 - p_K)$. By Binomial expansion, we can rearrange Equation (8) as follows:

$$g_n = 1 - (1 - \frac{1}{m})^n = \sum_{i=1}^{n} \binom{n}{i} (-1)^{n-1} \frac{1}{m^n} \approx \frac{n}{m} \tag{13}$$

As $m$ goes to the infinity, $1/m$ is order of magnitude greater than $1/m^k$ for $k > 1$; thus, $g_n$ can be approximated to $n/m$. With the similar argument, we can rewrite $p_K$ as follows:

$$p_K = \frac{\rho^K \left[\prod_{i=1}^{K} g_i\right]^{-1}}{1 + \sum_{n=1}^{K} \rho^n \left[\prod_{i=1}^{n} g_i\right]^{-1}} \tag{14}$$

$$= \left[\frac{1}{\rho^K} \prod_{i=1}^{K} g_i + \frac{1}{\rho^{K-1}} \prod_{i=2}^{K} g_i + \cdots + \frac{1}{\rho} g_K + 1\right]^{-1} \tag{15}$$

$$\approx \left[\frac{1}{\rho} g_K + 1\right]^{-1} = \frac{\rho m}{K + \rho m} \tag{16}$$

$g_K$ is replaced in the last statement. Therefore, we can rewrite $\mathbb{E}[S]$ as follows

$$\mathbb{E}[S] = m\rho\lambda(1 - p_K) = m\rho\lambda \frac{K}{K + \rho m} \tag{17}$$

■

*Proof:* (Theorem 9) As shown in Theorem 5, the critical factor determining capacity is the average number of transmissions to deliver a packet (i.e., the average number of replicas). Zhang et al. showed that the average number of replicas in probabilistic replication is $\frac{np}{1+p}$ [42] and thus, the capacity is $O(\frac{(\lambda(1+p)}{p})$. Given $k$ replicas, the capacity is simply $O(\frac{n\lambda}{k})$.

Zhang et al. also showed that the average delay of probabilistic replication is upper bounded by $\Theta(\frac{\log n}{pn\lambda})$ [42]. The average delay bound of $k$ replica scheme can be calculated as follows. The proof is quite similar to Theorem 4, but we now have a Markov chain with $k + 1$ states instead of $n + 1$ states. Recall that each state denotes the number of replicas. The state change from $i$ to $i + 1$ happens with rate $i(n - i)\lambda$ whereas $i$ to the absorbing state $A$ happens with rate $i\lambda$. The event $0 \to i \to A$ happens with probability $\frac{1}{n-1}$ and the average delay is $\sum_{k=1}^{i} \frac{1}{\lambda k(n-k)}$. Note that one special event is $0 \to k \to A$ which happens with probability $1 - \sum_{i=0}^{k-1} \frac{1}{n-1} = 1 - \frac{k}{n-1}$.

$$D(n) = \frac{1}{n-1} \sum_{i=0}^{k-1} \sum_{k=1}^{i} \frac{1}{\lambda k(n-k)} + (1 - \frac{k}{n-1}) \frac{1}{\lambda k} \tag{18}$$

$$= \frac{1}{\lambda(n-1)} \sum_{i=1}^{k-1} \frac{k-i}{i(n-i)} + (1 - \frac{k}{n-1}) \frac{1}{\lambda k} \tag{19}$$

$$< \frac{1}{\lambda(n-1)} \log k + (1 - \frac{k}{n-1}) \frac{1}{\lambda k} \tag{20}$$

$$= O\left(\frac{k \log k + n - k}{n\lambda k}\right) \tag{21}$$

■

## REFERENCES


[1] A. Balasubramanian, B. N. Levine, and A. Venkataramani. DTN Routing as a Resource Allocation Problem. In *SIGCOMM'07*, Kyoto, Japan, Aug. 2007.

[2] N. Bansal and Z. Liu. Capacity, Delay and Mobility in Wireless Ad-Hoc Networks. In *INFOCOM'03*, San Francisco, CA, May 2003.

[3] V. Borrel, M. H. Ammar, and E. W. Zegura. Understanding the Wireless and Mobile Network Space: A Routing-centered Classification . In *CHANTS'07*, Montreal, Canada, Sept. 2007.

[4] J. Burgess, B. Gallagher, D. Jensen, and B. N. Levine. MaxProp: Routing for Vehicle-Based Disruption-Tolerant Networks. In *INFOCOM'06*, Barcelona, Spain, Apr. 2006.

[5] H. Cai and D. Y. Eun. Crossing Over the Bounded Domain: From Exponential To Power-law Inter-meeting Time in MANET. In *MobiCom'07*, Montréal, QC, Canada, Sept. 2007.

[6] T. Chu and I. Nikolaidis. Node Density and Connectivity Properties of the Random Waypoint Model. *Computer Communications*, 27:914–922, 2004.

[7] V. Conan, J. Leguay, and T. Friedman. Characterizing Pairwise Inter-contact Patterns in Delay Tolerant Networks. In *ACM Autonomics'07*, Rome, Italy, Oct. 2007.

[8] R. M. de Moraes, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves. Throughput-Delay Analysis of Mobile Ad-hoc Networks with a Multi-Copy Relaying Strategy. In *SECON'04*, Santa Clara, CA, Oct. 2004.

[9] K. Fall. A Delay Tolerant Networking Architecture for Challenged Internets. In *SIGCOMM'03*, Karlsruhe, Germany, Aug. 2003.

[10] S. Floyd, M. Handley, J. Padhye, and J. Widmer. Equation-Based Congestion Control for Unicast Applications. In *SIGCOMM'00*, Stockholm, Sweden, Aug. 2000.

[11] A. E. Gamal, J. Mammen, B. Prabhakar, and D. Shah. Throughput-Delay Trade-Off in Wireless Networks. In *INFOCOM'04*, Hong-Kong, Mar. 2004.

[12] M. Garetto, P. Giaccone, and E. Leonardi. Capacity Scaling in Delay Tolerant Networks with Heterogeneous Mobile Nodes. In *MobiHoc'07*, Quebec, Canada, Sept. 2007.

[13] M. Garetto, P. Giaccone, and E. Leonardi. Capacity Scaling of Sparse Mobile Ad Hoc Networks. In *INFOCOM'08 (Preprint)*, Phoenix, AZ, Apr. 2008.

[14] R. Groenevelt. Stochastic Models in Mobile Ad Hoc Networks. Technical report, University of Nice, Sophia Antipolis, INRIA, 2006.

[15] R. Groenevelt, P. Nain, and G. Koole. The Message Delay in Mobile Ad Hoc Networks. In *Performance'05*, Juan-les-Pins, France, Oct. 2005.

[16] M. Grossglauser and D. N. C. Tse. Mobility Increases the Capacity of Ad Hoc Wireless Networks. *IEEE Transactions on Networking*, 10(4), Aug. 2002.

[17] P. Gupta and P. R. Kumar. The Capacity of Wireless Networks. *IEEE Transactions on Information Theory*, 46(2), 2000.

[18] J. D. Herdtner and E. K. P. Chong. Throughput-Storage Tradeoff in Ad Hoc Networks. In *INFOCOM'05*, Miami, FL, Mar. 2005.

[19] M. Heusse, F. Rousseau, G. Berger-Sabbatel, and A. Duda. Performance Anomaly of 802.11b. In *INFOCOM'03*, San Francisco, CA, May 2003.

[20] S. Jain, K. Fall, and R. Patra. Routing in a Delay Tolerant Networking. In *SIGCOMM'04*, Portland, OR, Aug.-Sep. 2004.

[21] S. Jung, U. Lee, A. Chang, D. Cho, and M. Gerla. BlueTorrent: Cooperative Content Sharing for Bluetooth Users. In *PerCom'07*, White Plains, NY, Mar. 2007.

[22] T. Karagiannis, J.-Y. L. Boudec, and M. Vojnovic. Power Law and Exponential Decay of Inter Contact Times Between Mobile Devices. In *MobiCom'07*, Montréal, QC, Canada, Sept. 2007.

[23] L. Kleinrock. *Queueing Systems: Theory, Volume 1*. Wiley-Interscience, 1975.

[24] S. R. Kulkarni and P. Viswanath. A Deterministic Approach to Throughput Scaling in Wireless Networks. *IEEE Transactions on Information Theory*, 50(6):1041–1049, 2004.

[25] J. Leguay, T. Friedman, and V. Conan. Evaluating Mobility Pattern Space Routing for DTNs. In *INFOCOM'06*, Barcelona, Spain, Apr. 2006.

[26] X. Lin, G. Sharma, R. R. Mazumdar, and N. B. Shroff. Degenerate Delay-Capacity Trade-offs in Ad Hoc Networks with Brownian Mobility. *IEEE/ACM Transactions on Networking (TON)*, 14(SI):2777–2784, 2006.

[27] X. Lin and N. B. Shroff. The Fundamental Capacity-Delay Tradeoff in Large Mobile Wireless Networks. In *MedHocNet'04*, Bodrum, Turkey, Jun. 2004.

[28] A. Lindgren, A. Doria, and O. Schelén. PROPHET: Probabilistic Routing in Intermittently Connected Networks. In *SAPIR'04*, Fortaleza, Brazil, Aug. 2004.

[29] M. J. Neely and E. Modiano. Capacity and Delay Tradeoffs for Ad-Hoc Mobile Networks. *IEEE Transactions on Information Theory*, 46(2), June 2005.

[30] R. Ramanathan, P. Basu, and R. Krishnan. Towards a Formalism for Routing in Challenged Networks. In *CHANTS'07*, Montreal, Canada, Sept. 2007.

[31] P. Samar and S. B. Wicker. On the Behavior of Communication Links of a Node in a Multi-Hop Mobile Environment. In *MobiHoc'04*, Tokyo, Japan, May 2004.

[32] M. Seligman, K. Fall, and P. Mundur. Alternative Custodians for Congestion Control in Delay Tolerant. In *CHANTS'06*, Pisa, Italy, Sep. 2006.

[33] G. Sharma, R. Mazumdar, and N. Shroff. Delay and Capacity Trade-offs in Mobile Ad Hoc Networks: A Global Perspective. In *INFOCOM'06*, Barcelona, Spain, Apr. 2006.

[34] G. Sharma and R. R. Mazumdar. Delay and Capacity Trade-off in Wireless Ad Hoc Networks with Random Way-point Mobility. Technical report, Purdue University, 2005.

[35] T. Small and Z. J. Haas. Resource and Performance Tradeoffs in Delay-Tolerant Wireless Networks. In *CHANTS'05*, Philadelphia, PA, Aug. 2005.

[36] T. Spyropoulos, K. Psounis, and C. Raghavendra. Single-Copy Routing in Intermittently Connected Mobile Networks. In *SECON'04*, Santa Clara, CA, Oct. 2004.

[37] T. Spyropoulos, K. Psounis, and C. Raghavendra. Spray and Wait: An Efficient Routing Scheme for Intermittently Connected Mobile Networks. In *CHANTS'05*, Philadelphia, PA, Aug. 2005.

[38] S. Toumpis and A. J. Goldsmith. Large Wireless Networks Under Fading, Mobility, and Delay Constraints. In *INFOCOM'04*, Hong-Kong, Mar. 2004.

[39] B. Walker, J. Glenn, and T. Clancy. Analysis of Simple Counting Protocols for Delay-Tolerant Networks. In *CHANTS'07*, Montréal, QC, Canada, Sept. 2007.

[40] S. Xu, K. Blackmore, and H. Jones. Mobility Assessment for MANETs Requiring Persistent Links. In *WitMeMo'05*, Seattle, WA, Jun. 2005.

[41] L. Ying, S. Yang, and R. Srikant. Coding Achieves the Optimal Delay-Throughput Tradeoff in Mobile Ad Hoc Networks: Two-Dimensional I.I.D. Mobility Model with Fast Mobiles. In *WiOpt'07*, Limassol, Cyprus, Apr. 2007.

[42] X. Zhang, G. Neglia, J. Kurose, and D. Towsley. Performance Modeling of Epidemic Routing. In *IFIP'06*, Coimbra, Portugal, May 2006.