Safe-by-design control with robust MPC

Quantifying, predicting & optimizing over uncertainty

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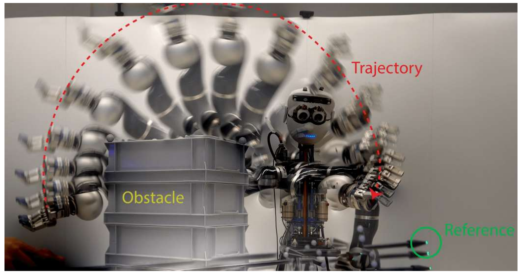
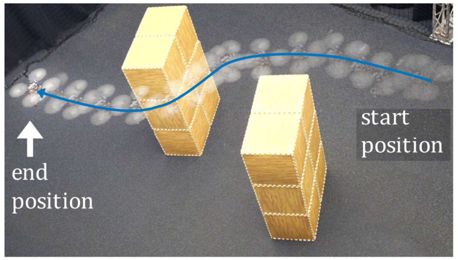
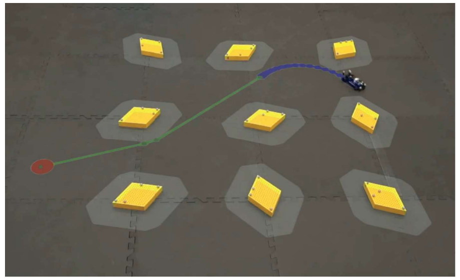
European Conference on Control (ECC), Rekjavic, July 2026

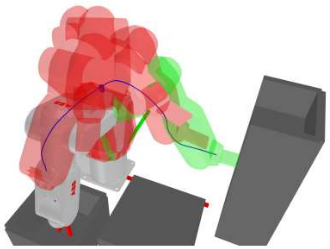
Motivation

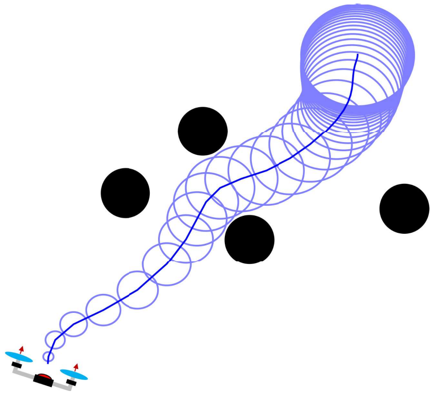
Real systems must satisfy safety-critical **constraints**.
Safe operation requires accounting for **uncertainty**.

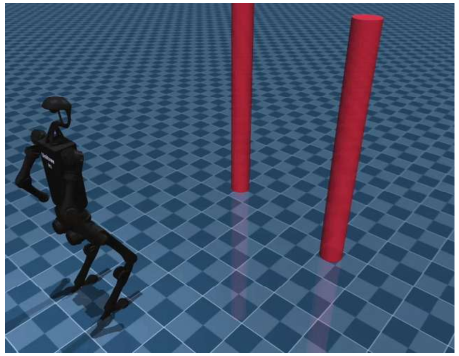












[Nubert et al., 2020, Benders et al., 2025, Köhler et al., 2025, Wullt et al., 2026, Sasfi et al., 2023, Fang and Chou, 2026]

Safe-by-design control using robust MPC

Dynamical Model & Uncertainty bounds → Robust MPC design → Safe-by-design controller

- **Robust MPC design**: Uses model & uncertainty bounds
- **Safe controller**: Guarantees constraint satisfaction

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↑

System Identification & Uncertainty Quantification

↑

Experimental Data

- **Robust MPC design**: Uses model & uncertainty bounds
- **Safe controller**: Guarantees constraint satisfaction
- **Data-driven:** Model & uncertainty learned from data

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Benefits: safe-by-design control

systematic, uncertainty-aware,

failures are diagnosable and correctable

Outline of tutorial

1 Safe-by-design control with robust MPC

2-3 Certified uncertainty propagation I-II

4 Uncertainty quantification from data

5 Safety under Interaction-Driven Distribution Shifts

6 Model predictive control for robot motion planning near humans

Focus of this talk

- Main components of robust MPC
- Role of "*quantify, predicting, and optimizing over uncertainty*"
- Theoretical properties of robust MPC

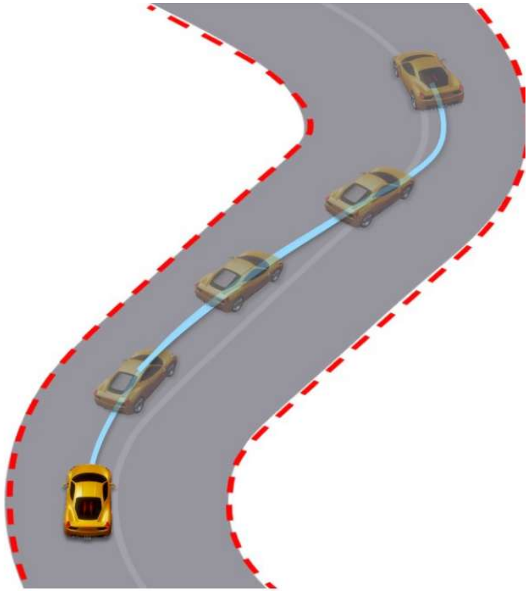
Model predictive control (MPC)

Finite-horizon MPC problem

$$\min_{u.} \sum_{k=0}^{N-1} \ell(x_k, u_k) \qquad \text{Cost}$$

$$\text{s.t.} \quad x_{k+1} = f(x_k, u_k), \quad x_0 = x(t), \qquad \text{Dynamics}$$

$$(x_k, u_k) \in \mathbb{X} \times \mathbb{U}, \ k \in [0, N-1], \qquad \text{Constraints}$$



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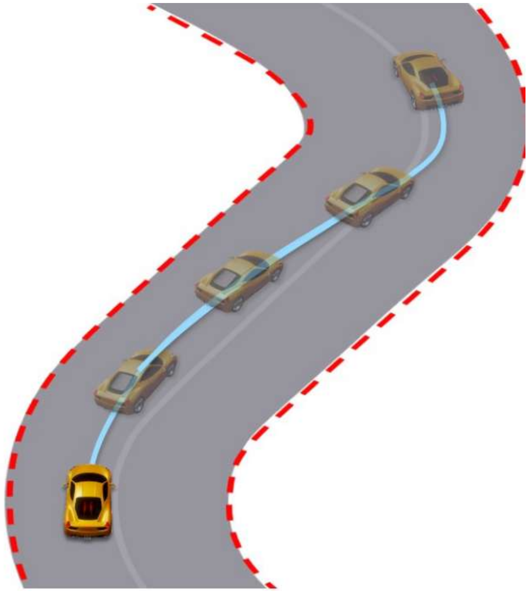

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  Solve $\rightarrow$ apply first input $\rightarrow$ shift horizon $\rightarrow$ repeat

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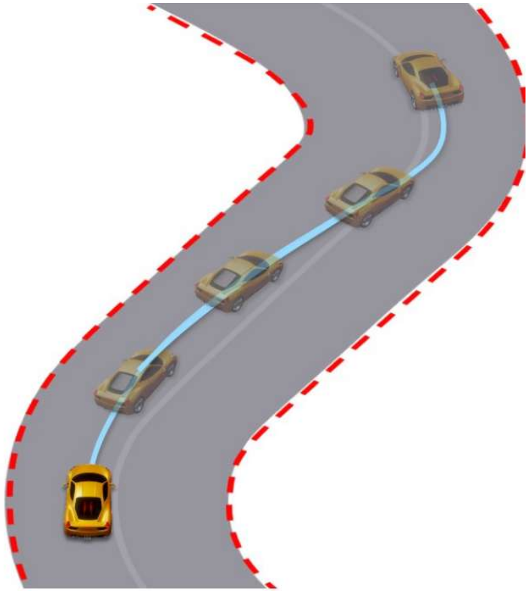

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Model predictive control (MPC)

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$$\min_{u.} \sum_{k=0}^{N-1} \ell(x_k, u_k) + \ell_{\mathrm{f}}(x_N) \qquad \text{Cost}$$

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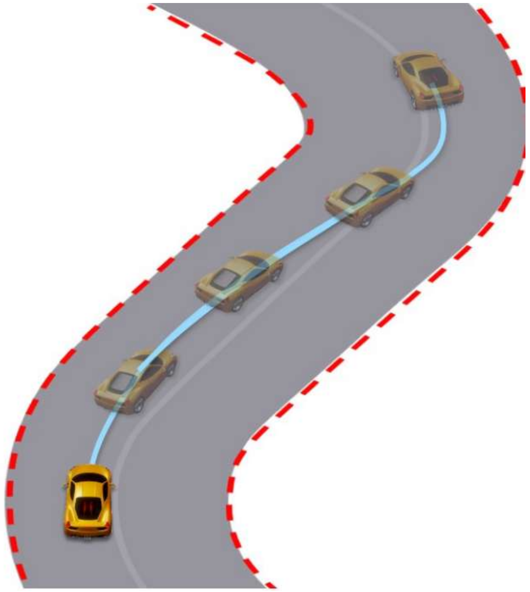

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  Solve → apply first input → shift horizon → repeat
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- **Terminal ingredients:** $\mathbb{X}_{\mathrm{f}}$, ℓ_{f} account for performance beyond finite horizon N

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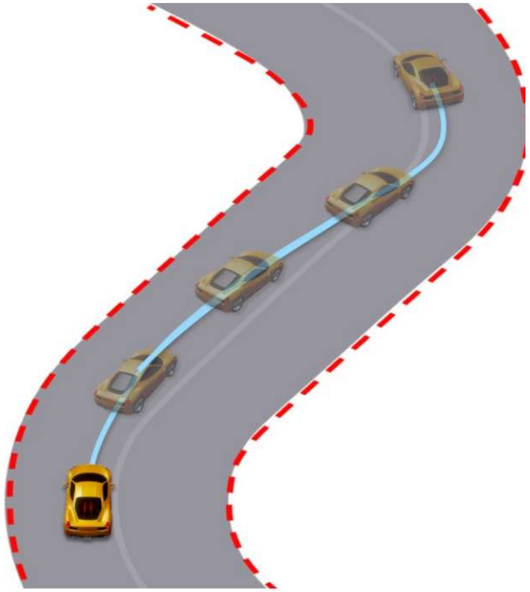

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- **Well-established theory and algorithms** [Rawlings et al., 2017];
  assuming accurate model . . .

When models are wrong

When models are wrong

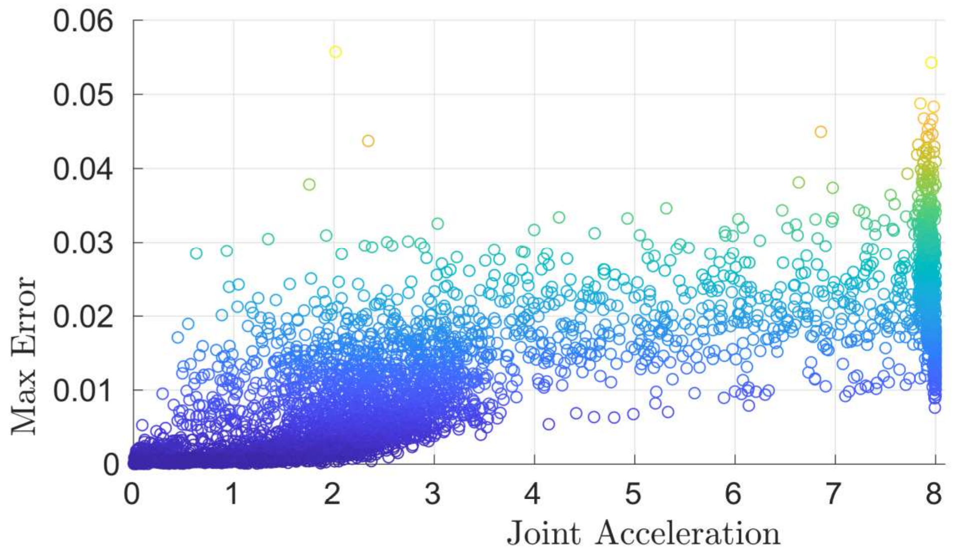
- Model mismatch

$\Rightarrow$ Inaccurate predictions

$\Rightarrow$ Constraint violations

Safe-by-design control needs to explicitly **account for uncertainty**.

Modeling uncertainty



Experimental model error in a robotic manipulator: correlated + random error [Nubert et al., 2020].

[a] Probabilistic safety guarantees are discussed later.

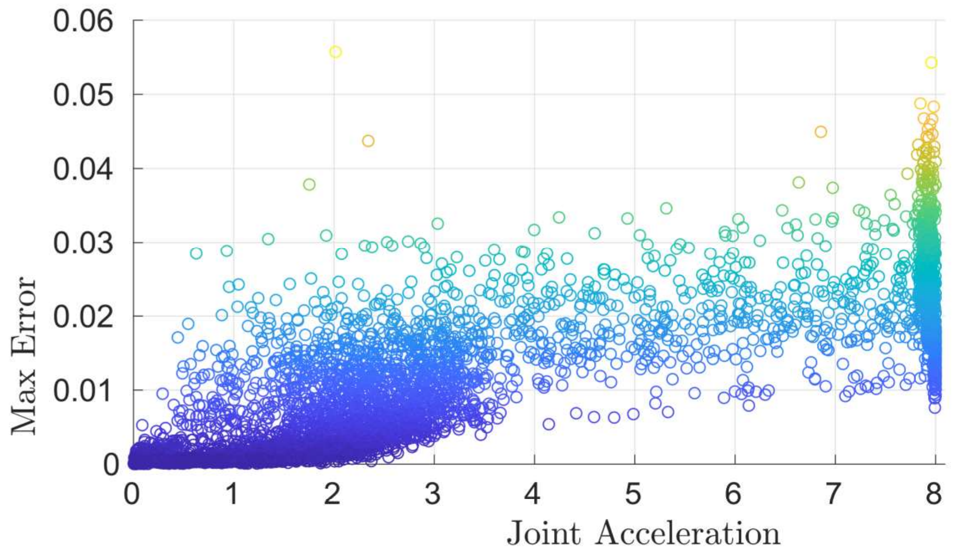
Modeling uncertainty

Mathematical model

$$x_{k+1} = f(x_k, u_k, w_k, \theta)$$
$$y_k = h(x_k, \theta, v_k)$$

Epistemic
model not precisely known $\rightarrow \theta$

Aleatoric
random disturbances $\rightarrow w_k$, v_k



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Given known deterministic[a] bounds:

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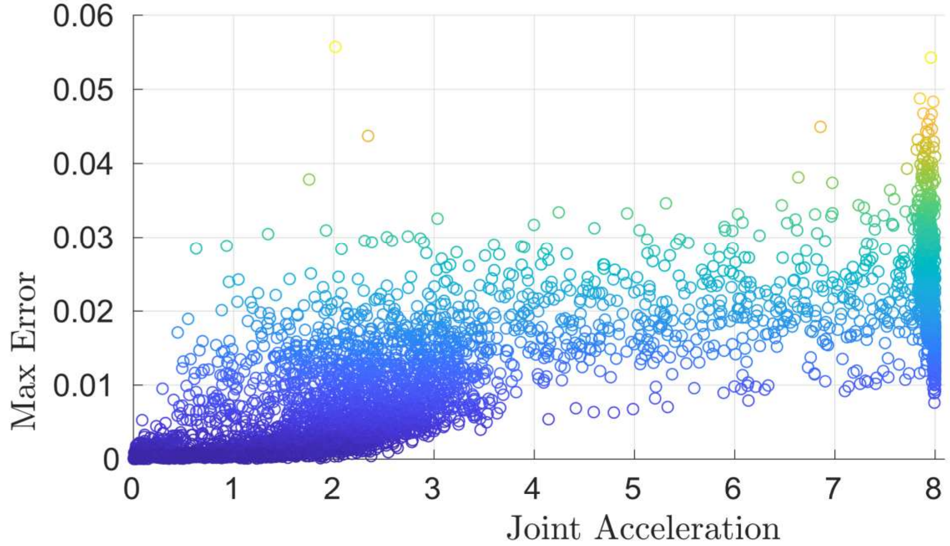

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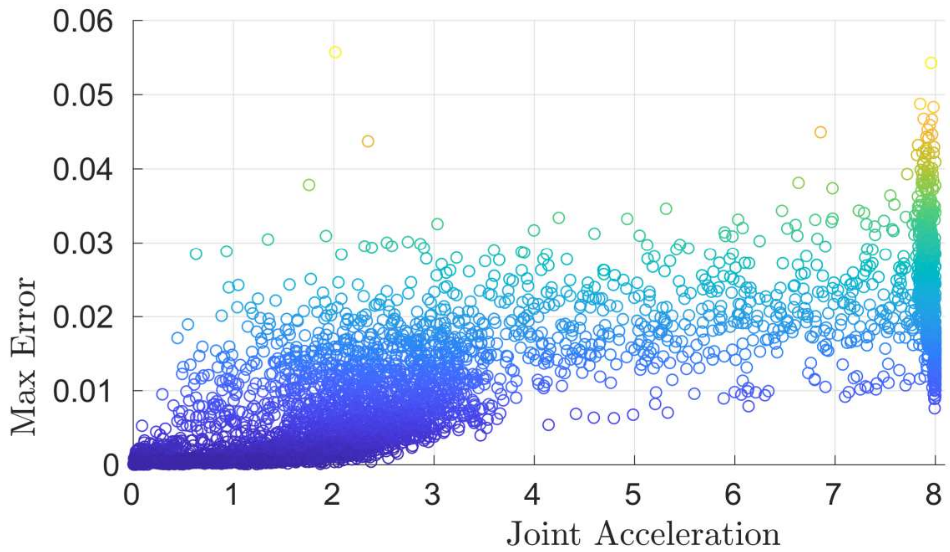


Experimental model error in a robotic manipulator: correlated + random error [Nubert et al., 2020].

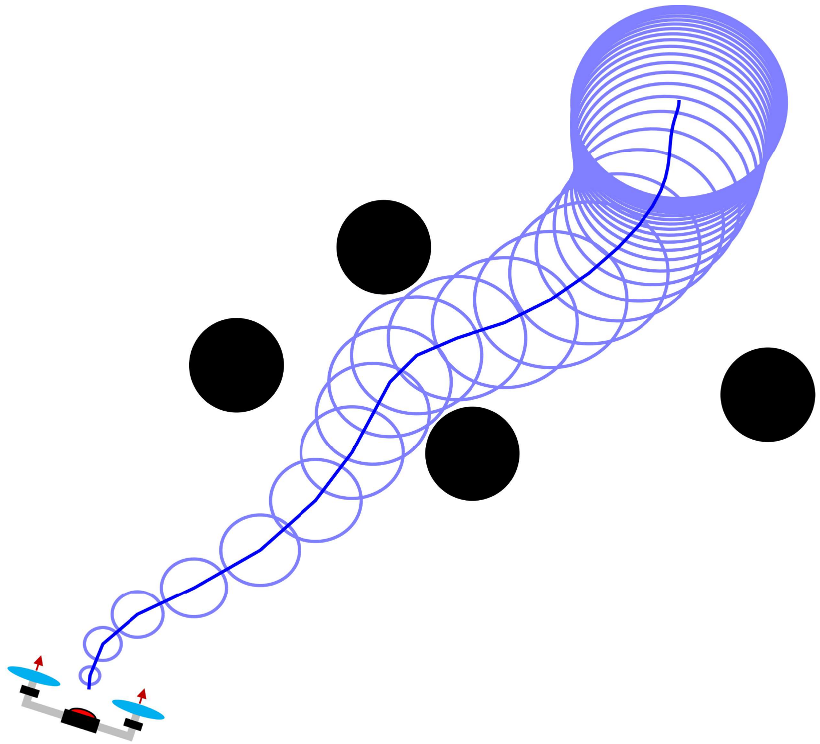
For illustration, we primarily focus on lumped errors $x_{k+1} = f(x_k, u_k, w_k)$, $w_k \in \mathbb{W}$.

Robust predictions in MPC

Predictions under uncertainty

- **Multiple trajectories**
  plausible under uncertain dynamics
- **Robust prediction**
  predict the set $\mathcal{R}_k(x_0, \mathbf{u})$ containing all trajectories

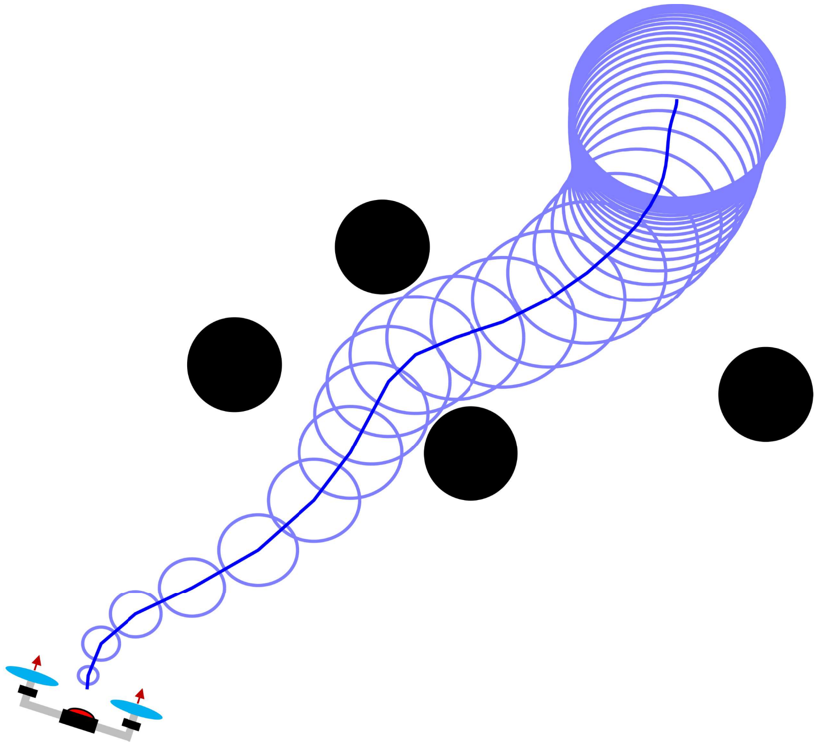
$$x_k \in \mathcal{R}_k(x_0, \mathbf{u}) \quad \forall w \in \mathbb{W}$$



Key idea: ensure constraints for $\mathcal{R}_k$ $\Rightarrow$ real trajectory is safe

Optimizing over robust predictions

- Input sequence $\mathbf{u}$ determines the uncertainty tube $\mathcal{R}_k$
- Choose $\mathbf{u}$ such that $\mathcal{R}_k$ satisfies constraints



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- Input sequence $\mathbf{u}$ determines the uncertainty tube $\mathcal{R}_k$
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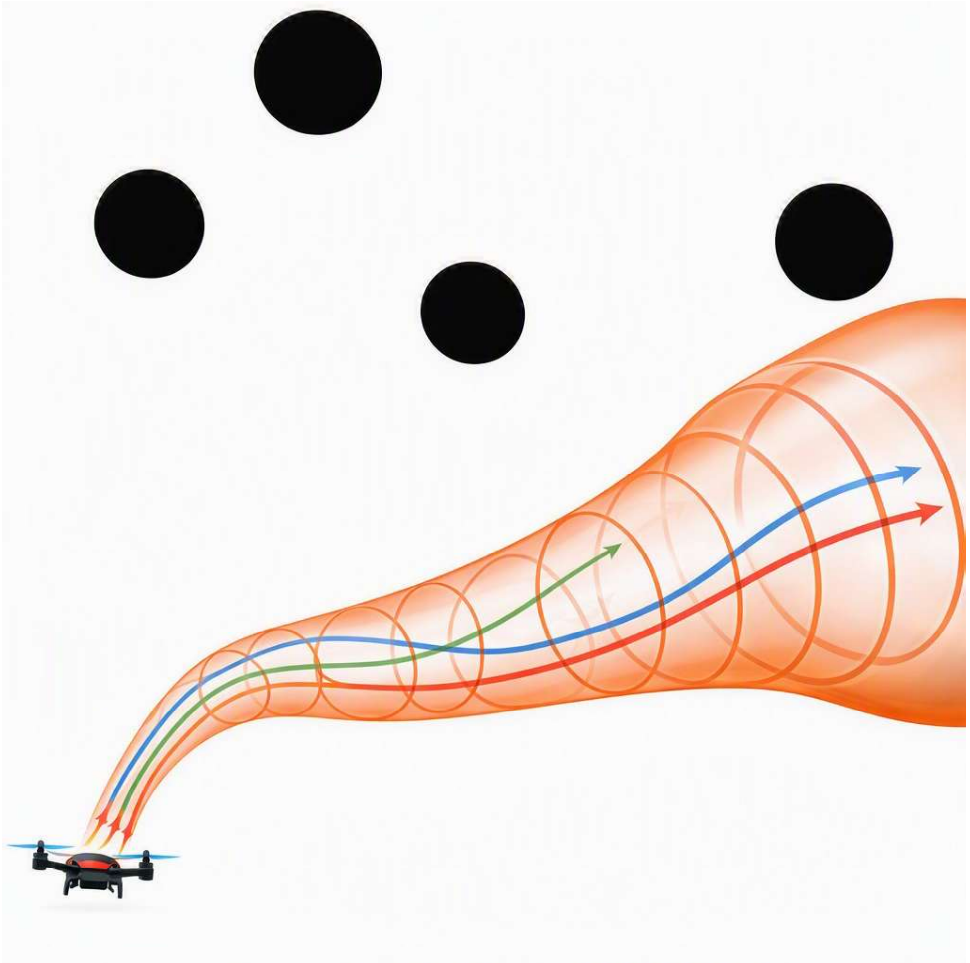
- Uncertain trajectories diverge $\Rightarrow$ large sets $\mathcal{R}_k$ $\Rightarrow$ conservative behavior



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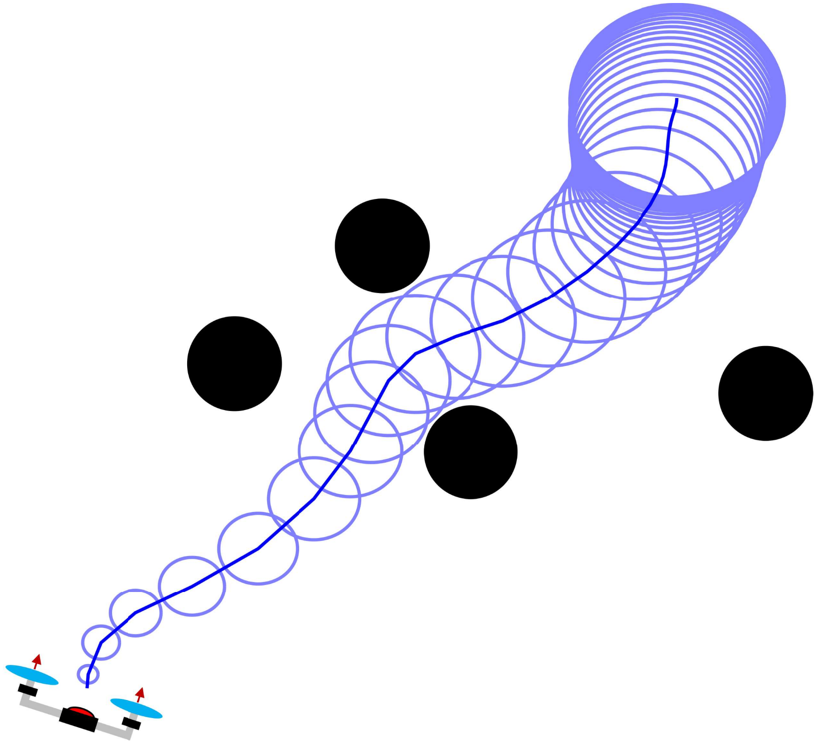

Closed-loop predictions

Use feedback to react to deviations:

$$u_k = \pi_k(x_k)$$

Takeaway: Feedback in predictions reduces uncertainty and conservatism.

General robust MPC formulation

Finite-horizon robust MPC problem

$$
\begin{aligned}
\min_{\pi} \sum_{k=0}^{N-1} \max_{x\in\mathcal{X}_k} \ell(x,\pi_k(x)) + \max_{x\in\mathcal{X}_N} \ell_{\mathrm{f}}(x) \qquad & \text{Worst-case cost} \\
\text{s.t. } \mathcal{X}_k = \mathcal{R}_k(x(t),\pi) \qquad & \text{Closed-loop prediction} \\
\mathcal{X}_k \subseteq \mathbb{X}, \quad k=0,\dots,N-1, \qquad & \text{Constraint satisfaction} \\
\mathcal{X}_N \subseteq \mathbb{X}_{\mathrm{f}} \qquad & \text{Terminal constraint}
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Receding horizon

Solve $\Rightarrow$ Apply $u = \pi_0^\star(x(t))$
$\Rightarrow$ Measure $\Rightarrow$ Repeat

Terminal ingredient

$\mathbb{X}_{\mathrm{f}}$ robust control invariant:

$\Rightarrow$ Deterministic safety guarantees for all times.

General robust MPC - Theoretical properties

Theorem [Guay et al., 2015, Köhler et al., 2021]

Design conditions

- The true model error is captured by $w_k \in \mathbb{W}$
- The terminal set $\mathbb{X}_\mathrm{f}$ and terminal cost ℓ_f are properly design
- The robust MPC problem is feasible at initialization $x(0)$

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Guarantees:

- Recursive feasibility
- Safety: $x_k \in \mathbb{X}$
- Stability/performance

$\Rightarrow$ Robust MPC provides a systematic way to construct safe-by-design controllers!

General robust MPC - sketch of proof

The proof reveals the key ingredients required for robust MPC design.

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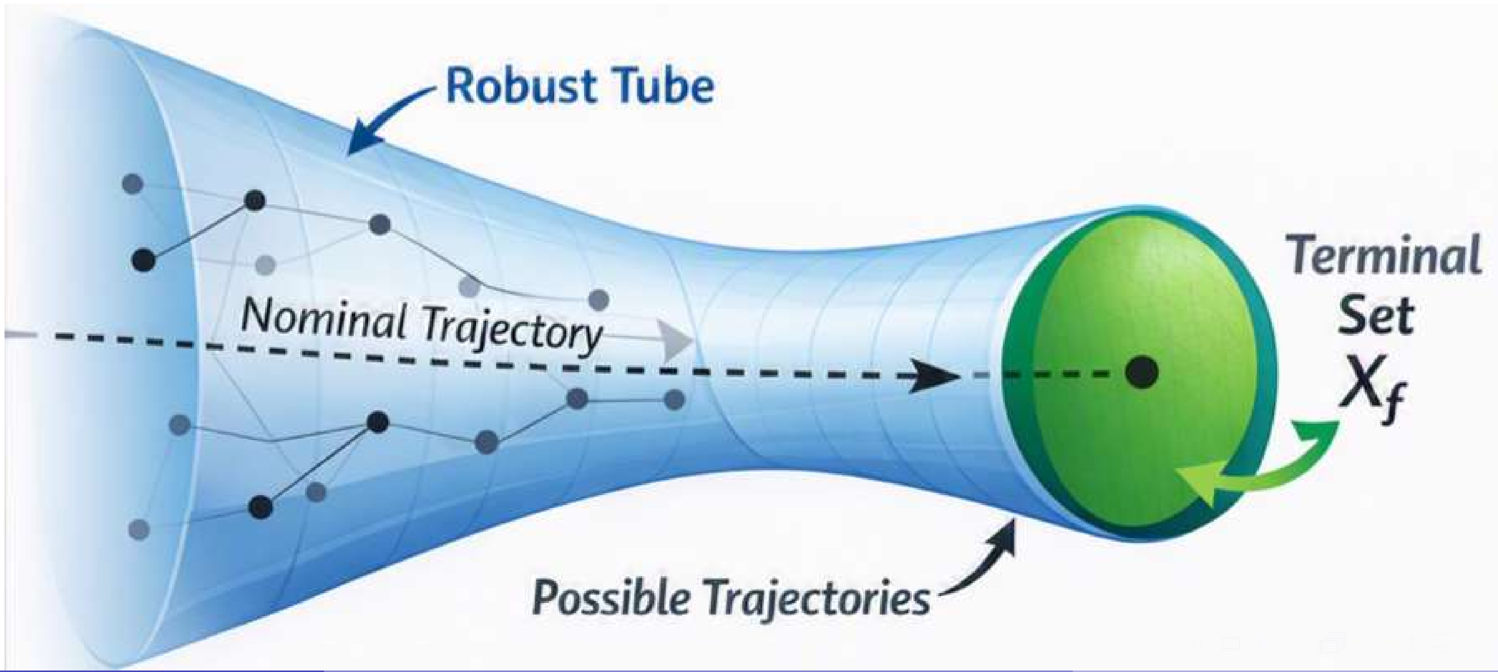
- Suppose robust MPC problem is feasible at some time k
- Policy $\pi^\star$ ensures future states satisfy constraints and end in a terminal set $\mathbb{X}_\mathrm{f}$ $\forall w \in \mathbb{W}$
- The terminal policy π_f can keep the state in the terminal set



General robust MPC - sketch of proof

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- Suppose robust MPC problem is feasible at some time k
- Policy $\pi^\star$ ensures future states satisfy constraints and end in a terminal set $\mathbb{X}_\mathrm{f}$ $\forall w \in \mathbb{W}$
- The terminal policy π_f can keep the state in the terminal set
- **Key idea: shift the policy**

$$\hat{\pi} = (\pi_1^\star, \ldots, \pi_{N-1}^\star, \pi_\mathrm{f})$$

- Feasible for all possible next states $x(k+1) = f(x(k), \pi_0^\star(x(k)), w(k))$, $w(k) \in \mathbb{W}$

$\Rightarrow$ feasibility is preserved at the next step

- Repeating this argument $\Rightarrow$ feasibility for all times

Challenges in robust MPC

General robust MPC provides all guarantees,
but implementation faces the following challenges.

Key challenges in robust MPC

- **Uncertainty propagation:** exact computation of robust reachable sets $\mathcal{R}_k$ is intractable
- **Policy optimization:** $\pi \in \{$all feedback policies$\}$ is infinite-dimensional
- **Model uncertainty:** bound on model mismatch $\mathbb{W}$ typically unknown

$\Rightarrow$ The next talks in this tutorial will focus on different methods to address these challenges

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- Role of main components in robust MPC

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Tutorial material available online:

`https://kohlerjohannes.github.io/ECC_Tutorial_SafeByDesignMPC`

*Appendix provides more source material

Appendix
Main literature

- MPC theory [Rawlings et al., 2017]
  MPC theory for "dynamic operation" [Köhler et al., 2024]
- Theoretical guarantees for general min-max robust MPC formulation [Guay et al., 2015]
- General robust MPC with sequential propagation [Köhler et al., 2021, Thm. 1]
- Recent overview on robust MPC [Houska and Villanueva, 2019]

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