Coherent structures and their minimal flow units in strongly stratified Kármán wakes

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Large-scale Kármán vortices have been commonly observed behind islands in satellite images. It is intuitive and tempting to relate them to the canonical Kármán vortex shedding (KVS) in a cylinder wake at laboratory scale. On one hand, this phenomenon akin to two-dimensional wakes is a result of strong stratification (with respect to the scales of the flow), which suppresses vertical perturbations. On the other hand, the role of density perturbations can not be neglected. Density modes interact closely with vortical modes, leading to coherent structures that inherit features from the canonical KVS but are distinct to stratified flows.

In this work, the wake of a vertical cylinder placed in a vertically stratified fluid is studied with direct numerical simulations (DNS). The Froude number, $Fr = U/ND$, describes the relative strength of buoyancy to inertia. Here U is the freestream, D is the cylinder diameter, and N is the buoyancy frequency. We consider the regime of strong stratification characterized by $Fr \leq O(0.1)$, which is relevant to most geophysical wakes where the dimensions of the obstacles are typically large, and a range of Reynolds numbers from three-dimensional instability to fully turbulent flow.

Two basic types of coherent structures are found, called 'surfboard' and 'pancake' vortices by their shapes. They both stem from a vertical instability of the primary Kármán wake with a stratification-imposed vertical length scale (U/N), similar to the zig-zag instability previously found in other stratified shear flows. The mode A and mode B instabilities in the transition to three-dimensionality of unstratified cylinder wakes are found to be stabilized.

The difference in coherent structures and unstable modes compared to unstratified wakes implies a different route of transition to turbulence and the necessity of new modeling strategies. The minimal flow unit approach is applied to isolate individual surfboard and pancake modes, and to identify the critical parameters for the transition to turbulence through vertical shear (Kelvin-Helmholtz) instability. Spectral proper orthogonal decomposition (SPOD) is also used to statistically extract the coherent vorticity-density structures in full and minimal domains.

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