Stratified wakes past an isolated topography: coherent structures and vortex dynamics

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Introduction

- Hills and seamounts are stirring rods in the ocean or atmosphere. They trigger unsteady vortical motions, internal waves, turbulence, and mixing.

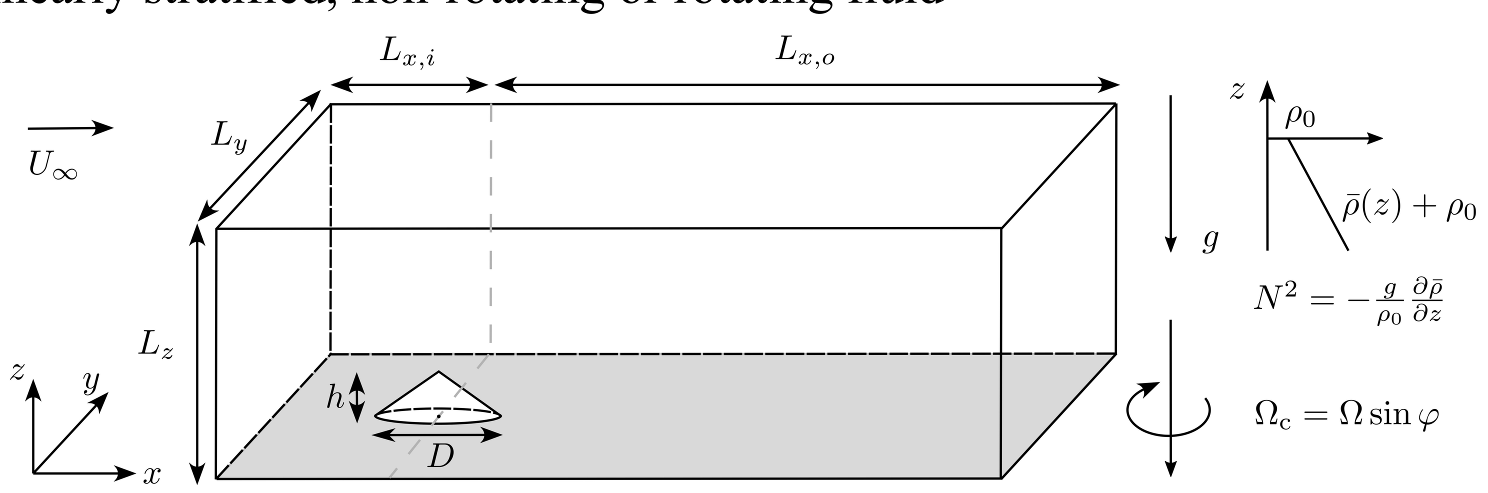
- The understanding of their roles in the dynamics of oceanic or atmospheric bottom flows, which are typically stratified, is crucial to the parameterization and prediction of their environmental impact.

- The presence of rotation leads to even richer dynamics, such as the asymmetry between the cyclones and the anticyclones. The instability of the anticyclonic vortices is a common source of turbulence generation.

Physical and numerical modeling

Flow configuration:

- uniform current past an isolated conical hill on a flat bottom
- hill slope is 30 degree
- linearly stratified, non-rotating or rotating fluid



Governing equations and large-eddy simulations (LES):

- incompressible Navier-Stokes equations under Boussinesq approximation
- sub-grid-scale motions modeled by an eddy-viscosity approach (WALE)
- non-hydrostatic simulations solving all three momentum equations

$$\frac{\partial u_i}{\partial x_i}=0,$$

$$\frac{\partial u_i}{\partial t}+\frac{\partial u_i u_j}{\partial x_j}-f_c\epsilon_{ij3}(u_j-U_\infty\delta_{j1})=-\frac{1}{\rho_0}\frac{\partial p^*}{\partial x_i}+\frac{\partial \tau_{ij}}{\partial x_j}-\frac{\rho^* g}{\rho_0}\delta_{i3},$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u_i}{\partial x_i}=\frac{\partial J_{\rho,i}}{\partial x_i},$$

$$\tau_{ij}=(\nu+\nu_{\mathrm{sgs}})\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),\ J_{\rho,i}=(\kappa+\kappa_{\mathrm{sgs}})\frac{\partial \rho}{\partial x_i}$$

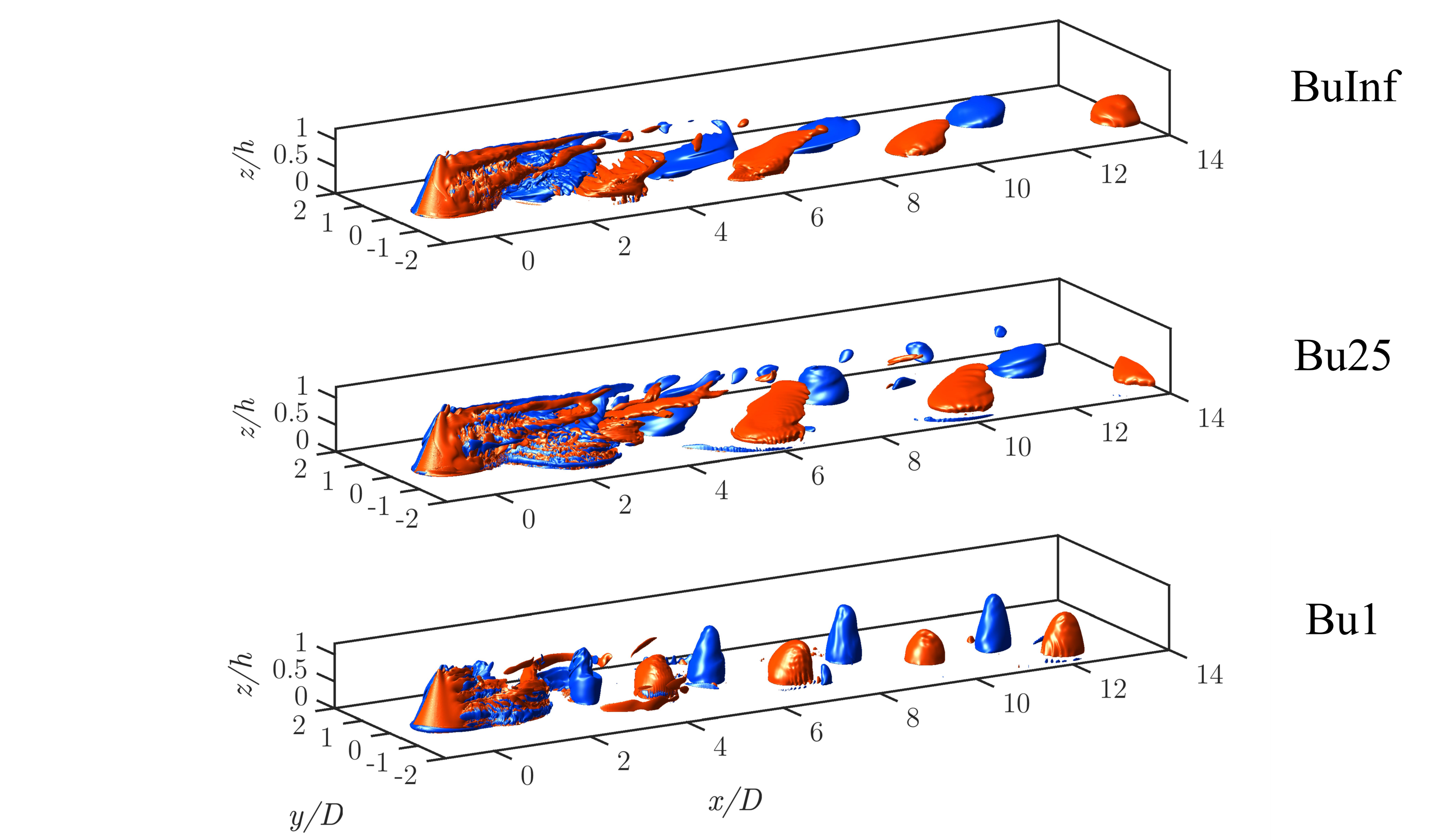
Non-dimensional parameters and numerical database:

- moderately high Reynolds number (Re = 10 000)
- moderately strong stratification (Fr = 0.15)
- mesoscale, submesoscale, and non-rotating (Ro = 0.15, 0.75, inf)
- roughly 0.6 billion grid points, time-resolved database

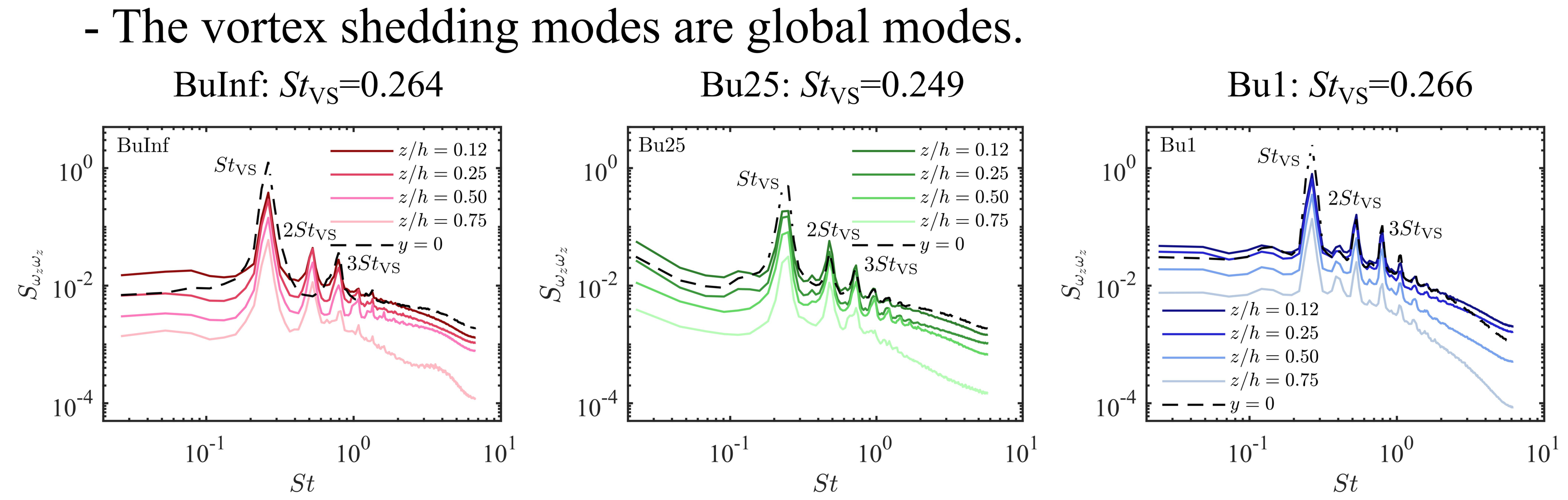
$$Re_D=\frac{U_\infty D}{\nu},\ Fr=\frac{U_\infty}{Nh},\ Ro=\frac{U_\infty}{f_c D},\ Bu=\left(\frac{Ro}{Fr}\right)^2=\left(\frac{Nh}{f_c D}\right)^2$$

Case	Bu	Ro	Fr	(N_x, N_y, N_z)	N_t	TU_∞/D
BuInf	∞	∞				295
Bu25	25	0.75	0.15	(1536,1280,320)	4000	345
Bu1	1	0.15				322

Large-scale coherent vortex structures

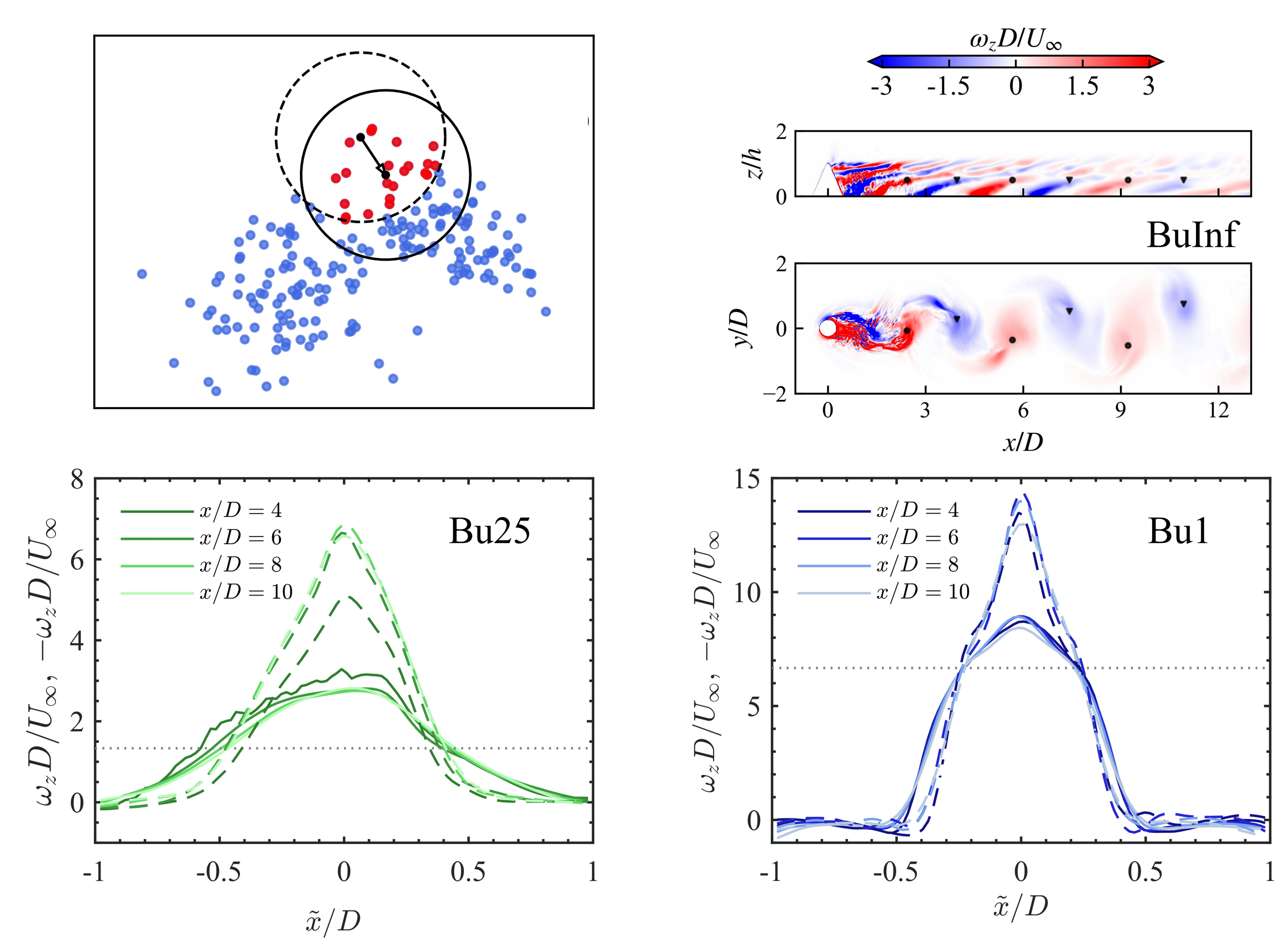


- In all three cases, von Kármán vortex shedding (VS) structures are observed.
- Structures change from slanted 'tongues' to tall columns as rotation increases.
- Spectral proper orthogonal decomposition (SPOD) reveals that:
  - In each case, the shedding frequency is a global constant in the wake and is little affected by the rotation strength.
  - The vortex shedding modes are global modes.



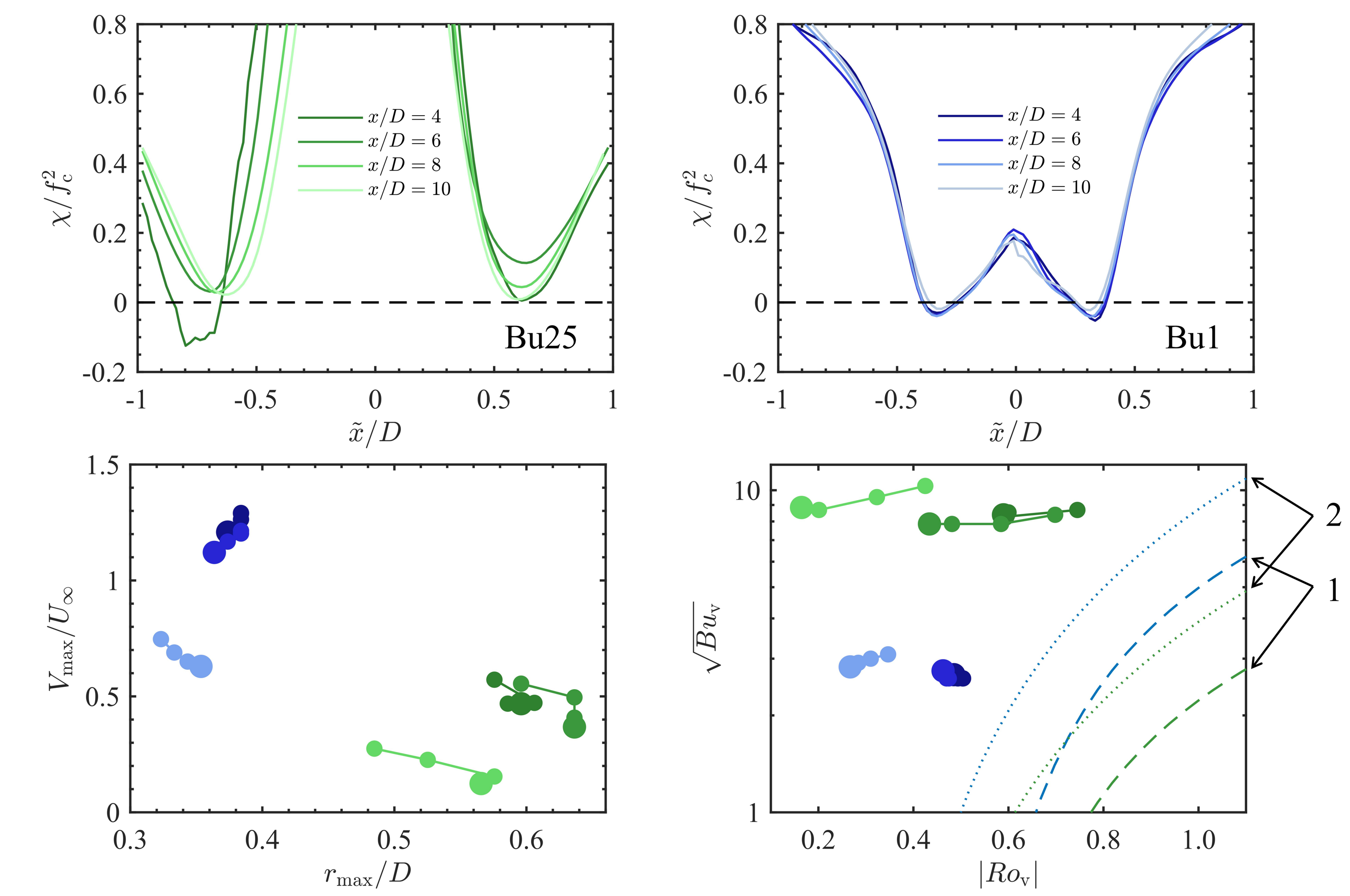
Vortex center tracking and conditional statistics

- Individual vortex centers in horizontal planes are tracked in time with the mean shift algorithm. Statistics are conditioned to the vortex centers.
- Asymmetry between cyclonic (CVs) and anticyclonic vortices (AVs) are observed in the rotating cases. Stable anticyclones with their vertical vorticity staying stronger than the rotating frequency are also found statistically.



Stability of the anticyclones

- Various criteria are applied to analyze the stability of the anticyclones:
  - the absolute vorticity criteria: $\omega_z/f_c < -1$ (instability)
  - the generalized Rayleigh discriminant: $(\omega_z+f_c)(2\frac{u_\theta}{r}+f_c)<0$ (instability)
  - new criteria[1,2] incorporating stratification and dissipation effects



$$Ro_v=\frac{V_{\max}}{f_c r_{\max}},\ Bu_v=\left(\frac{Nh}{f_c r_{\max}}\right)^2$$

[1]Lazar et al. (2013a); [2]Yim et al. (2019)

- Anticyclones stronger than the rotating frequencies (in Bu25 and Bu1) are found to stay marginally stable during the evolution. The most unstable regions are the peripheries of the vortices.
- The new criteria[1,2] have improved the stability of AVs, showing the stabilizing effects of stratification and turbulence.
- In the Ro_v-Bu_v parameter space, AVs tend to evolve towards more stable states (smaller Ro_v, away from the marginal stability curves).

Summary

- Non-hydrostatic large-eddy simulations are performed to study the wake of a conical hill. Dynamical regimes of moderately high Reynolds number, moderately strong stratification, and various rotation levels are explored.
- Coherent VS motions are identified with SPOD and are shown to be global modes. The VS frequency varies little as rotation changes.
- Vortex centers are extracted using the mean shift algorithm, and the statistics of vorticity and stability criteria are conditioned to the tracked centers. The asymmetry between the CVs and AVs is presented.
- The stability of AVs that have unstable absolute vorticity is observed, and analyzed using the generalized Rayleigh discriminant (that includes the centrifugal contribution) and two recently proposed criteria[1,2] (that include the effects of stratification and dissipation).

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