# Instrumental Variables <br> Intraduction 



## Roadmap

Introductions
Who Am I?
What is This Course?

Regression Review
Models vs. Estimands vs. Estimators
Regression Identification and Endogeneity

Introduction to IV
Instrument Validity and Relevance
The 2SLS Estimator

## Who Am I?

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- Lottery- and non-lottery IVs in studies of educational quality (Angrist et al. 2016, 2017, 2021, 2022; Abdulkadiroğlu et al. 2016)
- Quasi-experimental evaluations of healthcare quality (Hull 2020; Abaluck et al. 2021, 2022)
- IV-based analyses of discrimination and bias
(Arnold et al. 2020, 2021, 2022; Hull 2021; Bohren et al. 2022)
- Shift-share instruments and related designs
(Borusyak et al. 2022; Borusyak and Hull 2021, 2022; Goldsmith-Pinkham et al. 2022)


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(Borusyak et al. 2022; Borusyak and Hull 2021, 2022; Goldsmith-Pinkham et al. 2022)
A constant student of IV (and econometrics more generally)


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- Far from comprehensive - stay tuned for more "mixtape tracks" that take deeper dives on particular topics (judge IV, shift-share, etc)
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Four one-hour lectures: from IV basics to recent topics

- Please ask questions in the Discord chat!
- I will try to stick to the schedule but may improvise slightly


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Two 40-minute coding labs, applying what we've learned

- 20 min: you seeing how far you can get on your own, or with your classmate's help (use Discord rooms!)
- 20 min: me live-coding solutions in Stata (we will also post $R$ code)


## Schedule

| Tuesday $9 / 27$ | $6: 00-7: 00 \mathrm{pm}$ | Lecture 1: Regression Review; Regression Endogeneity; Introduction to IV |
| :--- | :--- | :--- |
|  | $7: 00-7: 10 \mathrm{pm}$ | Break |
|  | $7: 10-8: 10 \mathrm{pm}$ | Lecture 2: Understanding Instrument Validity; 2SLS Mechanics; Applications |
|  | $8: 10-8: 20 \mathrm{pm}$ | Break |
|  | $8: 20-9: 00 \mathrm{pm}$ | Coding Lab 1: Angrist and Krueger (1991) |
| Wednesday $9 / 28$ | $6: 00-7: 00 \mathrm{pm}$ | Lecture 3: Heterogeneous Treatment Effects; Characterizing Compliers; MTEs |
|  | $7: 00-7: 10 \mathrm{pm}$ | Break |
|  | $7: 10-8: 10 \mathrm{pm}$ | Lecture 4: Judge Leniency Designs; Shift-Share IV; New IV Frontiers |
|  | $8: 10-8: 20 \mathrm{pm}$ | Break |
|  | $8: 20-9: 00 \mathrm{pm}$ | Coding Lab 2: Stevenson (2018) |
|  | $9: 00-9: 15 \mathrm{pm}$ | Closing Remarks |

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$\rightarrow$ Make assumptions to link parameters \& estimands ("identification")


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$\rightarrow$ Make assumptions to link parameters \& estimands ("identification")
- Estimators are functions of the observed data itself (the "sample")
$\rightarrow$ E.g. a difference in sample means or ratio of OLS coefficients
$\rightarrow$ Since data are random, so are estimators. Each has a distribution
$\rightarrow$ Use knowledge of estimator distributions to learn about estimands ("inference") and-hopefully-identified parameters


## The Lay of the Land



This course will mostly focus on identification, but we'll cover some IV estimation / inference issues as well

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Human capital theory (e.g. Becker, 1957) tells us that taking two-day IV intensives are likely to boost later-life productivity

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- Parameter: returns to taking this class $\beta$, measured in some outcome $Y_{i}$ (e.g. lifetime top-5 pubs / earnings / twitter followers)
- Simple causal/structural model: $Y_{i}=\alpha+\beta D_{i}+\varepsilon_{i}$, where $D_{i} \in\{0,1\}$ indicates taking this class


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We see a sample of $Y_{i}, D_{i}$, and some other covariates $W_{1 i}, \ldots, W_{K i}$

- We fire up Stata and type reg y d w*, r
- How do we interpret the output?


## Population Regression

The OLS estimator $\widehat{\beta} O L S$ consistently estimates the regression estimand $\beta^{O L S}$ under relatively weak conditions (e.g. i.i.d. data)

- Stata tells us $\widehat{\beta}^{O L S}$ and what we can infer about $\beta^{O L S}$ from it
- It doesn't directly tell us about the relationship between $\beta^{O L S}$ and $\beta$


## Population Regression

The population regression of $Y_{i}$ on $\mathbf{X}_{i}=\left[1, D_{i}, W_{1 i}, \ldots, W_{K i}\right]^{\prime}$ is given by $Y_{i}=\mathbf{X}_{i}^{\prime} \beta^{O L S}+U_{i}$ where $E\left[\mathbf{X}_{i} U_{i}\right]=0$

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- Equivalently, $\beta^{O L S}=E\left[\mathbf{X}_{i} \mathbf{X}_{i}^{\prime}\right]^{-1} E\left[\mathbf{X}_{i} Y_{i}\right]$ and $U_{i}=Y_{i}-\mathbf{X}_{i}^{\prime} \beta^{O L S}$
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Key point: we can always define $\beta^{O L S}$ for any $Y_{i}$ and $\mathbf{X}_{i}$ (assuming no perfect collinearity); this is what Stata estimates

- Specifically it computes $\widehat{\beta} O L S=\left(\frac{1}{N} \sum_{i} \mathbf{X}_{i} \mathbf{X}_{i}^{\prime}\right)^{-1}\left(\frac{1}{N} \sum_{i} \mathbf{X}_{i} Y_{i}\right)$ and uses large-sample asymptotics (LLN/CLT) to get a standard error


## You Can't Always Get What you Want...

But what if this estimand is not what we want?

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- What if $\beta^{O L S}$ fails to coincide with our economic parameter of interest (e.g. returns to mixtape workshops)?


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The model parameter in $Y_{i}=\alpha+\beta D_{i}+\varepsilon_{i}$ need not coincide with the regression coefficient in $Y_{i}=\alpha^{O L S}+\beta^{O L S} D_{i}+U_{i}$

- I.e. we may not have $\operatorname{Cov}\left(D_{i}, \varepsilon_{i}\right)=0$ (always have $\left.\operatorname{Cov}\left(D_{i}, U_{i}\right)=0\right)$


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Selection bias (a.k.a. omitted variables bias): students with higher latent earnings potential $\varepsilon_{i}$ are more likely to take this class $D_{i}$

- $\operatorname{Cov}\left(D_{i}, \varepsilon_{i}\right)>0$ means $\beta^{O L S}>\beta$ : overstate the returns-to-mixtape


## Can I just Control My Way Out of This?

Adding more controls (e.g. demographics) may or may not help

- Projecting $\varepsilon_{i}$ on $X_{i}$, we get $Y_{i}=\alpha+\beta D_{i}+\gamma X_{i}+\tilde{\varepsilon}_{i}, \operatorname{Cov}\left(X_{i}, \tilde{\varepsilon}_{i}\right)=0$
- Whether or not $\operatorname{Cov}\left(D_{i}, \tilde{\varepsilon}_{i}\right)=0$ depends on whether $X_{i}$ sufficiently accounts for the confounding relationship $\operatorname{Cov}\left(D_{i}, \varepsilon_{i}\right) \neq 0$

Regression "Exogeneity"


## Regression "Endogeneity"



## ...But Sometimes, You Get What you Need

Imagine this course was "oversubscribed," and admission was determined by lottery

- Among those interested in taking the course, a random sample denoted by $Z_{i}=1$ was given access
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Indeed, this leads us to IV estimands (and estimators)

The IV Solution


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& =\beta \operatorname{Cov}\left(Z_{i}, D_{i}\right) \Longrightarrow \beta=\frac{\operatorname{Cov}\left(Z_{i}, Y_{i}\right)}{\operatorname{Cov}\left(Z_{i}, D_{i}\right)}
\end{aligned}
$$

The IV Estimand

The (simple) IV estimand is:

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\beta^{I V}=\frac{\operatorname{Cov}\left(Z_{i}, Y_{i}\right)}{\operatorname{Cov}\left(Z_{i}, D_{i}\right)}
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- Compare to the OLS estimand: $\beta^{O L S}=\frac{\operatorname{Cov}\left(D_{i}, Y_{i}\right)}{\operatorname{Var}\left(D_{i}\right)}$


## "Reduced Form" and "First Stage"

Note that we can write:

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where $\rho^{O L S}$ and $\pi^{O L S}$ are two OLS estimands:

$$
\begin{aligned}
Y_{i} & =\kappa^{O L S}+\rho^{O L S} Z_{i}+V_{i} \quad \text { "reduced form" } \\
D_{i} & =\mu^{O L S}+\pi^{O L S} Z_{i}+W_{i} \quad \text { "first stage" }
\end{aligned}
$$

## IV estimand as the "Second Stage"

Sometimes we refer to the IV estimand as the "second stage":

$$
Y_{i}=\alpha^{I V}+\beta^{I V} D_{i}+U_{i}
$$

where now $\operatorname{Cov}\left(Z_{i}, U_{i}\right)=0$. Thus "IV=RF/FS" $\left(\beta^{I V}=\rho^{O L S} / \pi^{O L S}\right)$

## The 2SLS Estimator

As with OLS, we estimate IV by sample analog:

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\widehat{\beta}^{I V}=\frac{\widehat{\operatorname{Cov}}\left(Z_{i}, Y_{i}\right)}{\widehat{\operatorname{Cov}}\left(Z_{i}, D_{i}\right)}=\frac{\widehat{\rho}^{O L S}}{\widehat{\pi}^{O L S}}
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We will soon consider extensions of all of this, with controls / multiple instruments / etc

## Angrist (1990): The "Draft Lottery Paper"

Angrist famously used Vietnam-era draft eligibility as an instrument to estimate the earnings effects of military service

- Let $Z_{i} \in\{0,1\}$ be an indicator for draft eligibility, $D_{i} \in\{0,1\}$ be an indicator for military service, and $Y_{i}$ measure later-life earnings


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- First stage $E\left[D_{i} \mid Z_{i}=1\right]-E\left[D_{i} \mid Z_{i}=0\right]$ : effect of eligibility on the probability of military service (b/c $D_{i}$ is binary)
- Reduced form $E\left[Y_{i} \mid Z_{i}=1\right]-E\left[Y_{i} \mid Z_{i}=0\right]$ : effect of eligibility on adult earnings (measured in 1971, 1981...)


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Here $\beta^{I V}=\frac{\operatorname{Cov}\left(Z_{i}, Y_{i}\right) / \operatorname{Var}\left(Z_{i}\right)}{\operatorname{Cov}\left(Z_{i}, D_{i}\right) / \operatorname{Var}\left(Z_{i}\right)}=\frac{E\left[Y_{i} \mid Z_{i}=1\right]-E\left[Y_{i} \mid Z_{i}=0\right]}{E\left[D_{i} \mid Z_{i}=1\right]-E\left[D_{i} \mid Z_{i}=0\right]}$ has a special name, because $Z_{i}$ is binary: the Wald estimand

- First stage $E\left[D_{i} \mid Z_{i}=1\right]-E\left[D_{i} \mid Z_{i}=0\right]$ : effect of eligibility on the probability of military service (b/c $D_{i}$ is binary)
- Reduced form $E\left[Y_{i} \mid Z_{i}=1\right]-E\left[Y_{i} \mid Z_{i}=0\right]$ : effect of eligibility on adult earnings (measured in 1971, 1981...)

IV interprets the latter causal effect in terms of the former

IV Estimates of the Effects of Military Service on the Earnings of White Men born in 1950

| Earnings <br> year | Earnings |  | Veteran Status |  | Wald Estimate of Veteran Effect |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Eligibility Effect | Mean | Eligibility Effect |  |
|  | (1) | (2) | (3) | (4) |  |
| 1981 | 16,461 | $\begin{aligned} & -435.8 \\ & (210.5) \end{aligned}$ | . 267 | $\begin{aligned} & .159 \\ & (.040) \end{aligned}$ | $\begin{aligned} & -2,741 \\ & (1,324) \end{aligned}$ |
| 1971 | 3,338 | $\begin{array}{r} -325.9 \\ (46.6) \end{array}$ |  |  | $\begin{array}{r} -2050 \\ (293) \end{array}$ |
| 1969 | 2,299 | $\begin{gathered} -2.0 \\ (34.5) \end{gathered}$ |  |  |  |

Note: Adapted from Table 5 in Angrist and Krueger (1999) and author tabulations. Standard errors are shown in parentheses. Earnings data are from Social Security administrative records. Figures are in nominal dollars. Veteran status data are from the Survey of Program Participation. There are about 13,500 individuals in the sample.


Figure 3. Earnings and the Probability of Veteran Status by
Lottery Number
Notes: The figure plots mean W-2 compensation in 1981-4 against probabilities of veteran status by cohort and groups of five consecutive lottery numbers for white men born 1950-3. Plotted points consist of the average residuals (over four years of earnings) from regressions on period and cohort effects. The slope of the least-squares regression line drawn through the points is $-2,384$, with a standard error of 778 , and is an estimate of $\alpha$ in the equation

$$
\bar{y}_{c t j}=\beta_{c}+\delta_{t}+\hat{p}_{c j} \alpha+\bar{u}_{c t j} .
$$

