# Instrumental Variables



#### Roadmap

Introductions Who Am I? What is This Course?

**Regression Review** 

Models vs. Estimands vs. Estimators

Regression Identification and Endogeneity

Introduction to IV

Instrument Validity and Relevance

The 2SLS Estimator



Groos Family Assistant Professor of Economics, Brown University

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- Lottery- and non-lottery IVs in studies of educational quality (Angrist et al. 2016, 2017, 2021, 2022; Abdulkadiroğlu et al. 2016)
- Quasi-experimental evaluations of healthcare quality (Hull 2020; Abaluck et al. 2021, 2022)
- IV-based analyses of discrimination and bias

(Arnold et al. 2020, 2021, 2022; Hull 2021; Bohren et al. 2022)

Shift-share instruments and related designs

(Borusyak et al. 2022; Borusyak and Hull 2021, 2022; Goldsmith-Pinkham et al. 2022)

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A constant student of IV (and econometrics more generally)

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Two 40-minute coding labs, applying what we've learned

- 20 min: you seeing how far you can get on your own, or with your classmate's help (use Discord rooms!)
- 20 min: me live-coding solutions in Stata (we will also post R code)

## Schedule

Tuesday 9/27	6:00-7:00pm 7:00-7:10pm 7:10-8:10pm 8:10-8:20pm	Lecture 1: Regression Review; Regression Endogeneity; Introduction to IV Break Lecture 2: Understanding Instrument Validity; 2SLS Mechanics; Applications Break
	8:20-9:00pm	Coding Lab 1: Angrist and Krueger (1991)
Wednesday 9/28	6:00-7:00pm 7:00-7:10pm	Lecture 3: Heterogeneous Treatment Effects; Characterizing Compliers; MTEs $Break$
	7:10-8:10pm	Lecture 4: Judge Leniency Designs; Shift-Share IV; New IV Frontiers
	8:10-8:20pm	Break
	8:20-9:00pm	Coding Lab 2: Stevenson (2018)
	9:00-9:15pm	Closing Remarks

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  - ightarrow Make assumptions to link parameters & estimands ("identification")
- Estimators are functions of the observed data itself (the "sample")
  - ightarrow E.g. a difference in sample means or ratio of OLS coefficients
  - $\rightarrow~$  Since data are random, so are estimators. Each has a distribution
  - → Use knowledge of estimator distributions to learn about estimands ("inference") and—hopefully—identified parameters

## The Lay of the Land



This course will mostly focus on identification, but we'll cover some IV estimation / inference issues as well

## Let's Get Specific

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- Parameter: returns to taking this class  $\beta$ , measured in some outcome  $Y_i$  (e.g. lifetime top-5 pubs / earnings / twitter followers)
- Simple causal/structural model:  $Y_i = \alpha + \beta D_i + \varepsilon_i$ , where  $D_i \in \{0, 1\}$  indicates taking this class

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We see a sample of  $Y_i$ ,  $D_i$ , and some other covariates  $W_{1i}, \ldots, W_{Ki}$ 

- We fire up Stata and type reg y d w\*, r
- How do we interpret the output?

The OLS estimator  $\hat{\beta}^{OLS}$  consistently estimates the regression estimand  $\beta^{OLS}$  under relatively weak conditions (e.g. *i.i.d.* data)

- Stata tells us  $\hat{\beta}^{OLS}$  and what we can infer about  $\beta^{OLS}$  from it
- It doesn't directly tell us about the relationship between  $\beta^{OLS}$  and  $\beta$

The population regression of  $Y_i$  on  $\mathbf{X}_i = [1, D_i, W_{1i}, \dots, W_{Ki}]'$  is given by  $Y_i = \mathbf{X}'_i \beta^{OLS} + U_i$  where  $E[\mathbf{X}_i U_i] = 0$ 

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- $\beta^{OLS}$  contains regression *coefficients*;  $U_i$  is the regression *residual*

Key point: we can always define  $\beta^{OLS}$  for any  $Y_i$  and  $\mathbf{X}_i$  (assuming no perfect collinearity); this is what Stata estimates

• Specifically it computes  $\hat{\beta}^{OLS} = (\frac{1}{N}\sum_{i} \mathbf{X}_{i} \mathbf{X}_{i}')^{-1} (\frac{1}{N}\sum_{i} \mathbf{X}_{i} Y_{i})$  and uses large-sample asymptotics (LLN/CLT) to get a standard error

But what if this estimand is not what we want?

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 What if β<sup>OLS</sup> fails to coincide with our economic parameter of interest (e.g. returns to mixtape workshops)?

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• I.e. we may not have  $Cov(D_i, \varepsilon_i) = 0$  (always have  $Cov(D_i, U_i) = 0$ )

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Selection bias (a.k.a. omitted variables bias): students with higher latent earnings potential  $\varepsilon_i$  are more likely to take this class  $D_i$ 

•  $Cov(D_i, \varepsilon_i) > 0$  means  $\beta^{OLS} > \beta$ : overstate the returns-to-mixtape

#### Can I just Control My Way Out of This?

Adding more controls (e.g. demographics) may or may not help

- Projecting  $\varepsilon_i$  on  $X_i$ , we get  $Y_i = \alpha + \beta D_i + \gamma X_i + \tilde{\varepsilon}_i$ ,  $Cov(X_i, \tilde{\varepsilon}_i) = 0$
- Whether or not  $Cov(D_i, \tilde{\varepsilon}_i) = 0$  depends on whether  $X_i$  sufficiently accounts for the confounding relationship  $Cov(D_i, \varepsilon_i) \neq 0$

## Regression "Exogeneity"



## Regression "Endogeneity"



## ...But Sometimes, You Get What you Need

Imagine this course was "oversubscribed," and admission was determined by lottery

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- The rest, with  $Z_i = 0$  not initially given access (maybe got in later)

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Indeed, this leads us to IV estimands (and estimators)

## The IV Solution



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Causal/structural model  $Y_i = \alpha + \beta D_i + \varepsilon_i$  and a candidate IV  $Z_i$ 

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Two key assumptions:

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We then have identification:

 $Cov(Z_i, Y_i) = Cov(Z_i, \alpha + \beta D_i + \varepsilon_i)$ 

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$$= \beta Cov(Z_i, D_i) \implies \beta = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)}$$

#### The IV Estimand

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• Compare to the OLS estimand:  $\beta^{OLS} = \frac{Cov(D_i, Y_i)}{Var(D_i)}$ 

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where  $\rho^{OLS}$  and  $\pi^{OLS}$  are two OLS estimands:

$$Y_i = \kappa^{OLS} + \rho^{OLS} Z_i + V_i \quad \text{"reduced form"}$$
$$D_i = \mu^{OLS} + \pi^{OLS} Z_i + W_i \quad \text{"first stage"}$$

#### IV estimand as the "Second Stage"

Sometimes we refer to the IV estimand as the "second stage":

$$Y_i = \alpha^{IV} + \beta^{IV} D_i + U_i$$

where now  $Cov(Z_i, U_i) = 0$ . Thus "IV=RF/FS" ( $\beta^{IV} = \rho^{OLS} / \pi^{OLS}$ )

#### The 2SLS Estimator

As with OLS, we estimate IV by sample analog:

$$\widehat{\beta}^{IV} = \frac{\widehat{Cov}(Z_i, Y_i)}{\widehat{Cov}(Z_i, D_i)} = \frac{\widehat{\rho}^{OLS}}{\widehat{\pi}^{OLS}}$$

where 
$$\widehat{Cov}(X_i, W_i) = \frac{1}{N} \sum_i X_i W_i - \left(\frac{1}{N} \sum_i X_i\right) \left(\frac{1}{N} \sum_i W_i\right)$$
,  
 $\hat{\rho}^{OLS} = \widehat{Cov}(Z_i, Y_i) / \widehat{Var}(Z_i)$ , and  $\hat{\pi}^{OLS} = \widehat{Cov}(Z_i, D_i) / \widehat{Var}(Z_i)$ 

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We will soon consider extensions of all of this, with controls / multiple instruments / etc

Angrist famously used Vietnam-era draft eligibility as an instrument to estimate the earnings effects of military service

• Let  $Z_i \in \{0,1\}$  be an indicator for draft eligibility,  $D_i \in \{0,1\}$  be an indicator for military service, and  $Y_i$  measure later-life earnings

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- First stage  $E[D_i | Z_i = 1] E[D_i | Z_i = 0]$ : effect of eligibility on the *probability* of military service (b/c  $D_i$  is binary)
- Reduced form  $E[Y_i | Z_i = 1] E[Y_i | Z_i = 0]$ : effect of eligibility on adult earnings (measured in 1971, 1981...)

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IV interprets the latter causal effect in terms of the former

	Earnings		Veteran Status		Wald Estimate of
Earnings year	Mean	Eligibility Effect	Mean	Eligibility Effect	Veteran Effect
	(1)	(2)	(3)	(4)	(5)
1981	16,461	-435.8 (210.5)	.267	.159 (.040)	-2,741 (1,324)
1971	3,338	-325.9 (46.6)			-2050 (293)
1969	2,299	-2.0 (34.5)			

IV Estimates of the Effects of Military Service on the Earnings of White Men born in 1950

Note: Adapted from Table 5 in Angrist and Krueger (1999) and author tabulations. Standard errors are shown in parentheses. Earnings data are from Social Security administrative records. Figures are in nominal dollars. Veteran status data are from the Survey of Program Participation. There are about 13,500 individuals in the sample.



FIGURE 3. EARNINGS AND THE PROBABILITY OF VETERAN STATUS BY LOTTERY NUMBER

Notes: The figure plots mean W-2 compensation in 1981–4 against probabilities of veteran status by cohort and groups of five consecutive lottery numbers for white men born 1950–3. Plotted points consist of the average residuals (over four years of earnings) from regressions on period and cohort effects. The slope of the least-squares regression line drawn through the points is -2,384, with a standard error of 778, and is an estimate of  $\alpha$  in the equation

$$\bar{y}_{ctj} = \beta_c + \delta_t + \hat{p}_{cj}\alpha + \bar{u}_{ctj}$$