# **Instrumental Variables UNDERSTANDING IV**



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• More modern IV texts take care to distinguish between these two conceptually distinct requirements...

# A Valid Instrument



#### A Violation of As-Good-As-Random Assignment



# A Violation of Exclusion



*1. True Lotteries*

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"Gold standard" IV: a randomized offer to participate in a program, with  $D_i$  recording program participation

- $\bullet\,$  Exclusion restriction likely to hold for any  $Y_i$ , by construction
- Relevance almost guaranteed (provided people want the program!)

*Charter School Lotteries*

Abdulkadiroglu et al. (2016) are interested in whether going to a "charter" middle school increases standardized test scores

- Charter students tend to score better, but we worry about selection
- History of doubting educational inputs, since Coleman (1966)

*Charter School Lotteries*

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We study a particular charter (UP Academy), which is "takeover"

• Two offer IVs: "immediate" (on lottery night) and from a waitlist

#### Lottery IV Estimates of UP Test Score Effects

#### TABLE 8-LOTTERY IV ESTIMATES OF UP EFFECTS



<span id="page-20-0"></span>*2. Natural Experiments*

Without appealing to literal randomization, we may credibly argue  $Z_i$  is as-good-as-randomly assigned conditional on some  $W_i$ 

- Such "natural experiments" rely on a selection-on-observables argument (for  $Z_i$ , instead  $D_i)$
- Still worry about exclusion:  $Z_i$  cannot affect  $Y_i$  except through  $D_i$

*Quarter-of-Birth*

Angrist and Krueger (1991) famously estimate labor market returns to schooling with a creative IV: student quarter-of-birth

- Compulsory schooling requirements prevent students from dropping before the day they turn 16 (used to be more binding)
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- Fixed school start dates mean students who drop out at 16 get more or less schooling depending on their birth date
- Quarter-of-birth seems quasi-randomly assigned is it excludable? See Buckles and Hungerman (2013)...

# The Quarter-of-Birth Natural Experiment: Visualized



# Quarter-of-Birth IV Estimates of Returns to Schooling

|  | <b>OLS</b>        |                   |                                     |  | $\circ$                           | 2SLS                             |
|--|-------------------|-------------------|-------------------------------------|--|-----------------------------------|----------------------------------|
|  | (1)               | (2)               | (3)                                 | (4)  | (5)                               | (6)                              |
| Years of education                                   | 0.075<br>(0.0004) | 0.072<br>(0.0004) | 0.103<br>(0.024)                    | 0.112<br>(0.021)                             | 0.106<br>(0.026)                  | 0.108<br>(0.019)                 |
| Covariates:  |                   |                   |                                     |  |                                   |                                  |
| 9 year of birth dummies<br>50 state of birth dummies |                   |                   |                                     |  | $\checkmark$                      |                                  |
| <i>Instruments:</i>                                  |                   |                   | dummy<br>$f_{\rm O\!}$<br>$QOB = 1$ | dummy<br>for<br>$QOB = 1$<br>or<br>$QOB = 2$ | dummy<br>$f_{\rm O}$<br>$QOB = 1$ | full<br>set<br>of QOB<br>dummies |

Table 4.1.1: 2SLS estimates of the economic returns to schooling

<span id="page-25-0"></span>*3. Panel Data*

We might also combine IV + difference-in-differences identification

- E.g. instrument with  $Z_i \times Post_t$ , controlling for  $Z_i$  and  $Post_t$  FEs
- This requires two parallel trends assumptions, for the RF and FS
- Still need to worry about the exclusion restriction, as always

*Charter School Takeovers*

In Abdulkadiroglu et al. (2016), we complement the lottery analysis of takeover charters with an instrumented diff-in-diff analysis

- Students enrolled in the "legacy" public school were eligible for being "grandfathered" into UP, without having to apply to the charter
- We compare their trends in test scores & enrollment to a matched comparison group of observably-similar students at other schools

#### Grandfathering IV: Visualized



#### Grandfathering IV Estimates of UP Test Score Effects

#### TABLE 7-GRANDFATHERING IV ESTIMATES OF UP EFFECTS



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$$
Y_i = \beta D_i + \mathbf{W}_i' \boldsymbol{\gamma} + \varepsilon_i \text{ (second stage)}
$$
  

$$
D_i = \pi Z_i + \mathbf{W}_i' \boldsymbol{\mu} + \eta_i \text{ (first stage)}
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where  $\mathbf{W}_i$  includes a constant.

The reduced form is:

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Y_i = \rho Z_i + \mathbf{W}_i' \boldsymbol{\kappa} + \nu_i
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 $\bullet~$  Can use Frisch-Waugh-Lovell to "partial out"  $\textbf{W}_i$  from  $Y_i$ ,  $D_i$ ,  $Z_i$ , and so get back to an IV regression without controls

<span id="page-37-0"></span>Sometimes we have more than one instrument  $Z_{i\ell}$ , for  $\ell = 1, \ldots, L$ . This leads to an "overidentified" IV regression:

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where  $\mathbf{Z}_i = [Z_{i1}, \ldots, Z_{iL}]'$ . Reduced form:  $Y_i = \mathbf{Z}_i' \boldsymbol{\rho} + \mathbf{W}_i' \boldsymbol{\kappa} + \nu_i$ 

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Overidentification can yield tests of IV validity

• Intuitively, 2SLS checks whether all the  $Z_{i\ell}$  yields the same IV estimate, which is sensible in a constant-effects model...

# Putting the "2S" in "2SLS"

You'll notice I haven't actually defined 2SLS beyond the simple case

- Before we had  $\beta^{IV} = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)}$  $\frac{Cov(Z_i,Y_i)}{Cov(Z_i,D_i)}$  leading to  $\widehat{\beta}^{IV} = \frac{Cov(Z_i,Y_i)}{\widehat{Cov}(Z_i,D_i)}$  $Cov(Z_i, D_i)$
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A more useful way to define 2SLS is by a two-step procedure:

- First regress  $D_i$  on all instruments  $Z_{i\ell}$  and controls  $W_{i\ell}$ .
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- Then regress  $Y_i$  on the "fitted values"  $\widehat{D}_i$  and controls  $W_{ik}$ The proof of this follows from some (simple) linear algebra
	- Intuitively, regressing  $Y_i$  on  $\widehat{\pi}^{OLS}Z_i$  gives a scaled RF:

$$
\widehat{\beta}^{IV} = \frac{\widehat{\rho}^{OLS}}{\widehat{\pi}^{OLS}}
$$

Although easy, you should never do such "manual 2SLS" yourself!

- Your point estimates will be right, but your SEs won't be!
- Also might forget to include some controls in the second stage, etc

Just let Stata/R do everything for you...

#### 2SLS Done Right

IV (2SLS) estimation

Estimates efficient for homoskedasticity only Statistics robust to heteroskedasticity



and a



Excluded instruments: rep78

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#### <span id="page-47-0"></span>Weak Instruments

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Much made of this over the years, but Angrist and Kolesár (2022) argue recently that we shouldn't worry too much

- The SE increase tends to be large enough to "cover up" the bias
- Just-id. 2SLS is "approximately median-unbiased"

#### Weak Instruments: Visualized

Monte Carlo:  $Y_i = \varepsilon_i$ ,  $D_i = \Pi Z_i + \eta_i$ :  $\Pi = Var(\varepsilon_i) = Var(\eta_i) = 1$ 



#### Weak Instruments: Visualized

Monte Carlo:  $Y_i = \varepsilon_i$ ,  $D_i = \Pi Z_i + \eta_i$ :  $\Pi = 0.1$  (Weaker)



#### Weak Instruments: Visualized

Monte Carlo:  $Y_i = \varepsilon_i$ ,  $D_i = \Pi Z_i + \eta_i$ :  $\Pi = 0.01$  (Very Weak)



# <span id="page-53-0"></span>Many IVs

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As we'll see, this bias is especially relevant in judge IV designs

- Potentially many judge assignment indicators as the instrument
- Leave-out corrections (e.g. Angrist et al. 1999) have been adapted to this setting in recent years (e.g. Kolesár 2013)

#### Many Instruments: Visualized

Monte Carlo:  $Y_i = \varepsilon_i$ ,  $D_i = \Pi Z_{i1} + \eta_i$ : IV with one  $Z_{i1}$ 



#### Many Instruments: Visualized

Monte Carlo:  $Y_i=\varepsilon_i$ ,  $D_i=\Pi Z_{i1}+\eta_i$ : IV with ten  $Z_{ij}$ 



#### Many Instruments: Visualized

Monte Carlo:  $Y_i=\varepsilon_i$ ,  $D_i=\Pi Z_{i1}+\eta_i$ : IV with 100  $Z_{ij}$ 



#### What to Do?

Check your F's after every IV regression

- State of the art: Montiel Olea and Pflueger '15; weakivtest in Stata
- Staiger-Stock rule-of-thumb ( $F > 10$ ) still seems widely held
- See Lee et al. (2020) and Keane and Neal (2022) for some discussions of additional subtleties

If your F is small, some things to consider:

- Is there a different instrument that's stronger?
- Is there a better functional form for the instrument you have?
- Do interactions with covariates help? (note: beware many-weak!)
- Does changing the covariate set help? (note: beware invalidity!)