Instrumental Variables



Roadmap

Where do (Good) Instruments Come From? True Lotteries Natural Experiments Panel Data

2SLS Mechanics Just-Identified IV Overidentification

Weak and Many Instruments Weak IVs Many IVs

To apply IV, we need to make a good case for instrument validity (note we can always check relevance!)

To apply IV, we need to make a good case for instrument validity (note we can always check relevance!)

Consider our simple causal model, $Y_i = \alpha + \beta D_i + \varepsilon_i$. Validity, $Cov(Z_i, \varepsilon_i) = 0$, intuitively requires two distinct assumptions:

To apply IV, we need to make a good case for instrument validity (note we can always check relevance!)

Consider our simple causal model, $Y_i = \alpha + \beta D_i + \varepsilon_i$. Validity,

 $Cov(Z_i, \varepsilon_i) = 0$, intuitively requires two distinct assumptions:

- As-good-as-random assignment : individuals with higher/lower potential earnings face the same distribution of Z_i
- Exclusion : the "assignment" of Z_i only affects Y_i through D_i

To apply IV, we need to make a good case for instrument validity (note we can always check relevance!)

Consider our simple causal model, $Y_i = \alpha + \beta D_i + \varepsilon_i$. Validity, $Cov(Z_i, \varepsilon_i) = 0$, intuitively requires two distinct assumptions:

• As-good-as-random assignment : individuals with higher/lower potential earnings face the same distribution of Z_i

• Exclusion : the "assignment" of Z_i only affects Y_i through D_i

Confusingly, old-school econometrics texts sometimes refer to $Cov(Z_i, \varepsilon_i) = 0$ as the "exclusion restriction"

To apply IV, we need to make a good case for instrument validity (note we can always check relevance!)

Consider our simple causal model, $Y_i = \alpha + \beta D_i + \varepsilon_i$. Validity, $Cov(Z_i, \varepsilon_i) = 0$, intuitively requires two distinct assumptions:

- As-good-as-random assignment : individuals with higher/lower potential earnings face the same distribution of Z_i
- Exclusion : the "assignment" of Z_i only affects Y_i through D_i

Confusingly, old-school econometrics texts sometimes refer to $Cov(Z_i, \varepsilon_i) = 0$ as the "exclusion restriction"

• More modern IV texts take care to distinguish between these two conceptually distinct requirements...

A Valid Instrument



A Violation of As-Good-As-Random Assignment



A Violation of Exclusion



1. True Lotteries

One sure-fire way to ensure that Z_i is as-good-as-randomly assigned is...

1. True Lotteries

One sure-fire way to ensure that Z_i is as-good-as-randomly assigned is... to randomly assign it!

1. True Lotteries

One sure-fire way to ensure that Z_i is as-good-as-randomly assigned is... to randomly assign it!

- Some of the best IVs come from lotteries, either run by the researcher (e.g. an RCT) or so-called "natural experiments"
- We still need to worry about violations of the exclusion restriction
- Relevance holds when Z_i has some effect on D_i

1. True Lotteries

One sure-fire way to ensure that Z_i is as-good-as-randomly assigned is... to randomly assign it!

- Some of the best IVs come from lotteries, either run by the researcher (e.g. an RCT) or so-called "natural experiments"
- We still need to worry about violations of the exclusion restriction
- Relevance holds when Z_i has some effect on D_i

"Gold standard" IV: a randomized offer to participate in a program, with D_i recording program participation

- Exclusion restriction likely to hold for any Y_i , by construction
- Relevance almost guaranteed (provided people want the program!)

Charter School Lotteries

Abdulkadiroglu et al. (2016) are interested in whether going to a "charter" middle school increases standardized test scores

- Charter students tend to score better, but we worry about selection
- History of doubting educational inputs, since Coleman (1966)

Charter School Lotteries

We leverage an institutional feature of charters: admission lotteries

• When more kids want to enroll than there are seats, admission offers $Z_i \in \{0, 1\}$ are effectively drawn from a hat

Charter School Lotteries

We leverage an institutional feature of charters: admission lotteries

- When more kids want to enroll than there are seats, admission offers $Z_i \in \{0, 1\}$ are effectively drawn from a hat
- Offers plausibly only affect later test scores Y_i by changing charter enrollment $D_i \in \{0, 1\}$, so are plausibly valid instruments

Charter School Lotteries

We leverage an institutional feature of charters: admission lotteries

- When more kids want to enroll than there are seats, admission offers $Z_i \in \{0, 1\}$ are effectively drawn from a hat
- Offers plausibly only affect later test scores Y_i by changing charter enrollment $D_i \in \{0, 1\}$, so are plausibly valid instruments
- We need to control for lottery fixed effects ("risk sets") to make Z_i as-good-as-randomly assigned – more on this soon

Charter School Lotteries

We leverage an institutional feature of charters: admission lotteries

- When more kids want to enroll than there are seats, admission offers $Z_i \in \{0, 1\}$ are effectively drawn from a hat
- Offers plausibly only affect later test scores Y_i by changing charter enrollment $D_i \in \{0, 1\}$, so are plausibly valid instruments
- We need to control for lottery fixed effects ("risk sets") to make Z_i as-good-as-randomly assigned – more on this soon

We study a particular charter (UP Academy), which is "takeover"

• Two offer IVs: "immediate" (on lottery night) and from a waitlist

Lottery IV Estimates of UP Test Score Effects

TABLE 8—LOTTERY IV ESTIMATES OF UP EFFECTS

				2SLS			
				First			
		Comparison group mean (1)	OLS (2)	Immediate offer (3)	Waitlist offer (4)	Enrollment effect (5)	
Panel A. All grades (Sixth through eighth)	Math (N = 2,202)	0.059	0.301 (0.022)	0.760 (0.063)	0.562 (0.067)	0.270 (0.056)	
	ELA (N = 2,205)	0.103	0.148 (0.020)	0.759 (0.063)	0.562 (0.067)	0.118 (0.051)	

2. Natural Experiments

Without appealing to literal randomization, we may credibly argue Z_i is as-good-as-randomly assigned conditional on some \mathbf{W}_i

- Such "natural experiments" rely on a selection-on-observables argument (for Z_i , instead D_i)
- Still worry about exclusion: Z_i cannot affect Y_i except through D_i

Quarter-of-Birth

Angrist and Krueger (1991) famously estimate labor market returns to schooling with a creative IV: student quarter-of-birth

- Compulsory schooling requirements prevent students from dropping before the day they turn 16 (used to be more binding)
- Fixed school start dates mean students who drop out at 16 get more or less schooling depending on their birth date

Quarter-of-Birth

Angrist and Krueger (1991) famously estimate labor market returns to schooling with a creative IV: student quarter-of-birth

- Compulsory schooling requirements prevent students from dropping before the day they turn 16 (used to be more binding)
- Fixed school start dates mean students who drop out at 16 get more or less schooling depending on their birth date
- Quarter-of-birth seems quasi-randomly assigned is it excludable?
 See Buckles and Hungerman (2013)...

The Quarter-of-Birth Natural Experiment: Visualized

A. Average Education by Quarter of Birth (first stage)



Quarter-of-Birth IV Estimates of Returns to Schooling

	OI	JS			2SLS			
	(1)	(2)	(3)	(4)	(5)	(6)		
Years of education	0.075 (0.0004)	0.072 (0.0004)	0.103 (0.024)	$0.112 \\ (0.021)$	$0.106 \\ (0.026)$	$0.108 \\ (0.019)$		
Covariates:								
9 year of birth dummies 50 state of birth dummies		\checkmark			\checkmark	\checkmark		
Instruments:			dummy for QOB=1	dummy for QOB=1 or QOB=2	dummy for QOB=1	full set of QOB dummies		

Table 4.1.1: 2SLS estimates of the economic returns to schooling

3. Panel Data

We might also combine IV + difference-in-differences identification

- E.g. instrument with $Z_i \times Post_t$, controlling for Z_i and $Post_t$ FEs
- This requires two parallel trends assumptions, for the RF and FS
- Still need to worry about the exclusion restriction, as always

Charter School Takeovers

In Abdulkadiroglu et al. (2016), we complement the lottery analysis of takeover charters with an instrumented diff-in-diff analysis

- Students enrolled in the "legacy" public school were eligible for being "grandfathered" into UP, without having to apply to the charter
- We compare their trends in test scores & enrollment to a matched comparison group of observably-similar students at other schools

Grandfathering IV: Visualized



Grandfathering IV Estimates of UP Test Score Effects

TABLE 7—GRANDFATHERING IV ESTIMATES OF UP EFFECTS

				2SLS		
		Comparison group mean (1)	OLS (2)	First stage (3)	Enrollment effect (4)	
Panel A. All grades						
(Seventh through eighth)	Math $(N = 1,543)$	-0.233	0.400 (0.032)	1.051 (0.040)	0.321 (0.039)	
	ELA (N = 1,539)	-0.214	0.296 (0.035)	1.040 (0.041)	0.394 (0.044)	

Roadmap

Where do (Good) Instruments Come From? True Lotteries Natural Experiments Panel Data

2SLS Mechanics Just-Identified IV Overidentification

Weak and Many Instruments Weak IVs Many IVs Tthe Stata ivregress/ivreg2 commands (or fixest::feols in R) allows for controls and multiple treatments / instruments

• When # treatment = # instruments, we say the IV is "just-identified":

Tthe Stata ivregress/ivreg2 commands (or fixest::feols in R) allows for controls and multiple treatments / instruments

• When # treatment = # instruments, we say the IV is "just-identified":

$$Y_i = \beta D_i + \mathbf{W}'_i \boldsymbol{\gamma} + \varepsilon_i \text{ (second stage)}$$
$$D_i = \pi Z_i + \mathbf{W}'_i \boldsymbol{\mu} + \eta_i \text{ (first stage)}$$

where \mathbf{W}_i includes a constant.

The reduced form is:

$$Y_i = \rho Z_i + \mathbf{W}_i' \boldsymbol{\kappa} + \nu_i$$

The reduced form is:

$$Y_i = \rho Z_i + \mathbf{W}'_i \boldsymbol{\kappa} + \nu_i$$

Same identification logic as before:

• Validity: $Cov(Z_i, \varepsilon_i) = 0$, allowing $Cov(Z_i, \mathbf{W}_i) \neq 0$

The reduced form is:

$$Y_i = \rho Z_i + \mathbf{W}_i' \boldsymbol{\kappa} + \nu_i$$

Same identification logic as before:

- Validity: $Cov(Z_i, \varepsilon_i) = 0$, allowing $Cov(Z_i, \mathbf{W}_i) \neq 0$
- Relevance: $\pi \neq 0$, so Z_i and D_i are correlated controlling for \mathbf{W}_i

The reduced form is:

$$Y_i = \rho Z_i + \mathbf{W}'_i \boldsymbol{\kappa} + \nu_i$$

Same identification logic as before:

- Validity: $Cov(Z_i, \varepsilon_i) = 0$, allowing $Cov(Z_i, \mathbf{W}_i) \neq 0$
- Relevance: $\pi \neq 0$, so Z_i and D_i are correlated controlling for \mathbf{W}_i

IV is still "reduced form over first stage": ($\beta^{IV} = \rho^{OLS} / \pi^{OLS}$)

The reduced form is:

$$Y_i = \rho Z_i + \mathbf{W}_i' \boldsymbol{\kappa} + \nu_i$$

Same identification logic as before:

- Validity: $Cov(Z_i, \varepsilon_i) = 0$, allowing $Cov(Z_i, \mathbf{W}_i) \neq 0$
- Relevance: $\pi \neq 0$, so Z_i and D_i are correlated controlling for \mathbf{W}_i

IV is still "reduced form over first stage": ($\beta^{IV} = \rho^{OLS} / \pi^{OLS}$)

• Can use Frisch-Waugh-Lovell to "partial out" \mathbf{W}_i from Y_i , D_i , Z_i , and so get back to an IV regression without controls

Sometimes we have more than one instrument $Z_{i\ell}$, for $\ell = 1, ..., L$. This leads to an "overidentified" IV regression:

$$Y_i = \beta D_i + \mathbf{W}'_i \boldsymbol{\gamma} + \varepsilon_i \text{ (second stage)}$$
$$D_i = \mathbf{Z}'_i \boldsymbol{\pi} + \mathbf{W}'_i \boldsymbol{\mu} + \eta_i \text{ (first stage)}$$

where $\mathbf{Z}_i = [Z_{i1}, \dots, Z_{iL}]'$. Reduced form: $Y_i = \mathbf{Z}'_i \boldsymbol{\rho} + \mathbf{W}'_i \boldsymbol{\kappa} + \nu_i$

Sometimes we have more than one instrument $Z_{i\ell}$, for $\ell = 1, ..., L$. This leads to an "overidentified" IV regression:

$$Y_i = \beta D_i + \mathbf{W}'_i \boldsymbol{\gamma} + \varepsilon_i \text{ (second stage)}$$
$$D_i = \mathbf{Z}'_i \boldsymbol{\pi} + \mathbf{W}'_i \boldsymbol{\mu} + \eta_i \text{ (first stage)}$$

where $\mathbf{Z}_i = [Z_{i1}, \dots, Z_{iL}]'$. Reduced form: $Y_i = \mathbf{Z}'_i \boldsymbol{\rho} + \mathbf{W}'_i \boldsymbol{\kappa} + \nu_i$ Validity: $Cov(Z_{i\ell}, \varepsilon_i) = 0$ for all ℓ

• "Overidentified" b/c we could use any $Z_{i\ell}$ to identify $\beta = \rho_\ell / \pi_\ell$

Sometimes we have more than one instrument $Z_{i\ell}$, for $\ell = 1, ..., L$. This leads to an "overidentified" IV regression:

$$Y_i = \beta D_i + \mathbf{W}'_i \boldsymbol{\gamma} + \varepsilon_i \text{ (second stage)}$$
$$D_i = \mathbf{Z}'_i \boldsymbol{\pi} + \mathbf{W}'_i \boldsymbol{\mu} + \eta_i \text{ (first stage)}$$

where $\mathbf{Z}_i = [Z_{i1}, \dots, Z_{iL}]'$. Reduced form: $Y_i = \mathbf{Z}'_i \boldsymbol{\rho} + \mathbf{W}'_i \boldsymbol{\kappa} + \nu_i$ Validity: $Cov(Z_{i\ell}, \varepsilon_i) = 0$ for all ℓ

- "Overidentified" b/c we could use any $Z_{i\ell}$ to identify $\beta = \rho_\ell/\pi_\ell$
- Relevance: $\pi_\ell
 eq 0$ for at least some ℓ

Sometimes we have more than one instrument $Z_{i\ell}$, for $\ell = 1, ..., L$. This leads to an "overidentified" IV regression:

$$Y_i = \beta D_i + \mathbf{W}'_i \boldsymbol{\gamma} + \varepsilon_i \text{ (second stage)}$$
$$D_i = \mathbf{Z}'_i \boldsymbol{\pi} + \mathbf{W}'_i \boldsymbol{\mu} + \eta_i \text{ (first stage)}$$

where $\mathbf{Z}_i = [Z_{i1}, \dots, Z_{iL}]'$. Reduced form: $Y_i = \mathbf{Z}'_i \boldsymbol{\rho} + \mathbf{W}'_i \boldsymbol{\kappa} + \nu_i$ Validity: $Cov(Z_{i\ell}, \varepsilon_i) = 0$ for all ℓ

- "Overidentified" b/c we could use any $Z_{i\ell}$ to identify $\beta = \rho_\ell/\pi_\ell$
- Relevance: $\pi_\ell
 eq 0$ for at least some ℓ

Overidentification can yield tests of IV validity

• Intuitively, 2SLS checks whether all the $Z_{i\ell}$ yields the same IV estimate, which is sensible in a constant-effects model...

Putting the "2S" in "2SLS"

You'll notice I haven't actually defined 2SLS beyond the simple case

- Before we had $\beta^{IV} = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)}$ leading to $\hat{\beta}^{IV} = \frac{\widehat{Cov}(Z_i, Y_i)}{\widehat{Cov}(Z_i, D_i)}$
- General form follows similarly (as a sample analog) but is notation-heavy, so we won't go into it here

Putting the "2S" in "2SLS"

You'll notice I haven't actually defined 2SLS beyond the simple case

- Before we had $\beta^{IV} = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)}$ leading to $\hat{\beta}^{IV} = \frac{\widehat{Cov}(Z_i, Y_i)}{\widehat{Cov}(Z_i, D_i)}$
- General form follows similarly (as a sample analog) but is notation-heavy, so we won't go into it here

A more useful way to define 2SLS is by a two-step procedure:

- First regress D_i on all instruments $Z_{i\ell}$ and controls W_{ik}
- Then regress Y_i on the "fitted values" \widehat{D}_i and controls W_{ik}

Putting the "2S" in "2SLS"

You'll notice I haven't actually defined 2SLS beyond the simple case

- Before we had $\beta^{IV} = \frac{Cov(Z_i, Y_i)}{Cov(Z_i, D_i)}$ leading to $\hat{\beta}^{IV} = \frac{\widehat{Cov}(Z_i, Y_i)}{\widehat{Cov}(Z_i, D_i)}$
- General form follows similarly (as a sample analog) but is notation-heavy, so we won't go into it here

A more useful way to define 2SLS is by a two-step procedure:

- First regress D_i on all instruments $Z_{i\ell}$ and controls W_{ik}
- Then regress Y_i on the "fitted values" \hat{D}_i and controls W_{ik} The proof of this follows from some (simple) linear algebra
- Intuitively, regressing Y_i on $\widehat{\pi}^{OLS} Z_i$ gives a scaled RF:

$$\widehat{\beta}^{IV} = \frac{\widehat{\rho}^{OLS}}{\widehat{\pi}^{OLS}}$$

Although easy, you should never do such "manual 2SLS" yourself!

- Your point estimates will be right, but your SEs won't be!
- Also might forget to include some controls in the second stage, etc

Just let Stata/R do everything for you...

2SLS Done Right

IV (2SLS) estimation

Estimates efficient for homoskedasticity only Statistics robust to heteroskedasticity

			Number of obs	=	69
			F(2, 66)	=	5.16
			Prob > F	=	0.0083
Total (centered) SS	=	576796958.9	Centered R2	=	-2.5922
Total (uncentered) SS	=	3183192639	Uncentered R2	=	0.3491
Residual SS	=	2071965250	Root MSE	=	5480

clear all sysuse auto	price	Coef.	Robust Std. Err.	z	P> z	[95% Conf.	Interval]		
ivreg2 price (mpg-pep78) weight p	mpg	1404.283	1499.569	0.94	0.349	-1534.819	4343.384		
IVIEBZ price (mpg=repro) weight, r	weight	10.38214	8.57869	1.21	0.226	-6.431778	27.19607		
	_cons	-55229.89	57542.19	-0.96	0.337	-168010.5	57550.73		
	Underidentification test (Kleibergen-Paap rk LM statistic): 1.200								
					Chi-	-sq(1) P-val =	0.2734		
	Weak identification test (Cragg-Donald Wald F statistic): 1.459								
	(Kleibergen-Paap rk Wald F statistic): 1.083								
	Stock-Yogo weak ID test critical values: 10% maximal IV size 16.38								
				15% m	aximal I\	/ size	8.96		
				20% m	aximal I\	/ size	6.66		
				25% m	aximal I\	/ size	5.53		
	Source: Stock-Yogo (2005). Reproduced by permission.								
	NB: Critical values are for Cragg-Donald F statistic and i.i.d. errors.								
	Hansen J statistic (overidentification test of all instruments): 0.000 (equation exactly identified)								
	Instrumented	mpg							

Included instruments: weight Excluded instruments: rep78

Roadmap

Where do (Good) Instruments Come From?
True Lotteries
Natural Experiments
Panel Data
2SLS Mechanics

Just-Identified IV

Overidentification

Weak and Many Instruments Weak IVs

vveak IVs

Many IVs

Weak Instruments

When running just-identified IV, you should always worry about the "strength" of your instrument

• Specifically the first stage F-statistic , which tests $\pi^{OLS} = 0$

Weak Instruments

When running just-identified IV, you should always worry about the "strength" of your instrument

• Specifically the first stage F-statistic , which tests $\pi^{OLS} = 0$

If π^{OLS} is small relative to its standard error, the IV is "weak"

- Typically use the rule-of-thumb of F < 10 (Staiger and Stock 1997)
- In this case the second-stage SEs will be large and the 2SLS estimate will tend to be biased towards the corresponding OLS

Weak Instruments

When running just-identified IV, you should always worry about the "strength" of your instrument

• Specifically the first stage F-statistic , which tests $\pi^{OLS} = 0$

If π^{OLS} is small relative to its standard error, the IV is "weak"

- Typically use the rule-of-thumb of F < 10 (Staiger and Stock 1997)
- In this case the second-stage SEs will be large and the 2SLS estimate will tend to be biased towards the corresponding OLS

Much made of this over the years, but Angrist and Kolesár (2022) argue recently that we shouldn't worry too much

- The SE increase tends to be large enough to "cover up" the bias
- Just-id. 2SLS is "approximately median-unbiased"

Weak Instruments: Visualized

Monte Carlo: $Y_i = \varepsilon_i$, $D_i = \Pi Z_i + \eta_i$: $\Pi = Var(\varepsilon_i) = Var(\eta_i) = 1$



Weak Instruments: Visualized

Monte Carlo: $Y_i = \varepsilon_i$, $D_i = \Pi Z_i + \eta_i$: $\Pi = 0.1$ (Weaker)



Weak Instruments: Visualized

Monte Carlo: $Y_i = \varepsilon_i$, $D_i = \Pi Z_i + \eta_i$: $\Pi = 0.01$ (Very Weak)



Many IVs

A more pernicious problem is many-instrument bias, when overid

• Also tends to manifest in low first-stage F's, so also good to check

Many IVs

A more pernicious problem is many-instrument bias, when overid

• Also tends to manifest in low first-stage F's, so also good to check

Many-IV bias is also towards OLS. But unlike before, the SEs go down

- Intuitively, a more flexible FS tends to fit D_i better \rightarrow more power
- But we can have overfitting with lots of $Z_i \rightarrow$ essentially recreate D_i

Many IVs

A more pernicious problem is many-instrument bias, when overid

• Also tends to manifest in low first-stage F's, so also good to check

Many-IV bias is also towards OLS. But unlike before, the SEs go down

- Intuitively, a more flexible FS tends to fit D_i better \rightarrow more power
- But we can have overfitting with lots of $Z_i \rightarrow$ essentially recreate D_i

As we'll see, this bias is especially relevant in judge IV designs

- Potentially many judge assignment indicators as the instrument
- Leave-out corrections (e.g. Angrist et al. 1999) have been adapted to this setting in recent years (e.g. Kolesár 2013)

Many Instruments: Visualized

Monte Carlo: $Y_i = \varepsilon_i$, $D_i = \prod Z_{i1} + \eta_i$: IV with one Z_{i1}



Many Instruments: Visualized

Monte Carlo: $Y_i = \varepsilon_i$, $D_i = \Pi Z_{i1} + \eta_i$: IV with ten Z_{ij}



Many Instruments: Visualized

Monte Carlo: $Y_i = \varepsilon_i$, $D_i = \Pi Z_{i1} + \eta_i$: IV with 100 Z_{ij}



What to Do?

Check your F's after every IV regression

- State of the art: Montiel Olea and Pflueger '15; weakivtest in Stata
- Staiger-Stock rule-of-thumb (F > 10) still seems widely held
- See Lee et al. (2020) and Keane and Neal (2022) for some discussions of additional subtleties

If your F is small, some things to consider:

- Is there a different instrument that's stronger?
- Is there a better functional form for the instrument you have?
- Do interactions with covariates help? (note: beware many-weak!)
- Does changing the covariate set help? (note: beware invalidity!)