Simulation-In-Hardware Dynamic Model for Quadrotor X

This report describes the equations of motion and dynamic models used by the SIH for quadrotor X.

This model assumes a North-East-Down (NED) inertial reference frame $\mathcal{F}_I$ as well as a Front-Right-Down body frame $\mathcal{F}_B$ attached to the center of mass (CM) of the vehicle.

Equations of motion

The equations of motion of a rigid body are given by

$\dot{p}_I = v_I$	Inertial position
$\dot{v}_I = \frac{1}{m}(W_I + F_{a,I} + C_{IB}(q)\, T_B)$	Inertial velocity, conservation of linear momentum
$\dot{q} = \frac{1}{2} q \otimes \omega_B$	Quaternion (attitude)
$\dot{\omega}_B = I^{-1}(M_{t,B} + M_{a,B} - \omega_B \times I\omega_B)$	Body rates, conservation of angular momentum

q is the quaternion used for the representation of the attitude. The Direct Cosine Matrix (DCM) C_{IB} can be obtained from q . It allows performing transformations from the body to the inertial frame.

The inertia matrix is assumed constant, so its inversion is computed once. The inertia is a 3x3 matrix composed of six (repeated) entries

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

Weight force

The weight force in the inertial frame is a constant given by

$W_I = (0; 0; mg)$

where m is the mass in kg and $g = 9.81\ [m/s^2]$ is the gravity acceleration.

Aerodynamic forces

The aerodynamic forces are modeled as a first order drag in order to stop the vehicle in absence of horizontal thrust.

$F_{a,I} = -K_{DV}\, v_I$

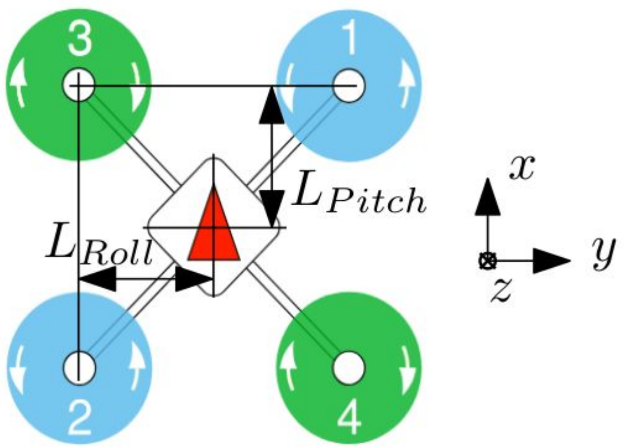
Thruster force

The quadrotor is equipped with 4 thrusters, that produce forces T1 to T4 pulling the vehicle upward (opposite to z-axis)

$T_B = [0; 0; -(T_1 + T_2 + T_3 + T_4)]$

where the thrust force T_i of each motor i is related to its normalized signal u_i as $T_i = T_{max}\, u_i$

Thruster moments



The quad thrusters produce moments in the body frame as

$M_{t,Bx} = L_{Roll}(-T_1 + T_2 + T_3 - T_4)$

$M_{t,By} = L_{Pitch}(T_1 - T_2 + T_3 - T_4)$

$M_{t,Bz} = Q_1 + Q_2 - Q_3 - Q_4$

Where L_{Roll} is the arm length generating the rolling moment, i.e. distance from the left motors to the CM, and L_{Pitch} is the arm length generating the pitching moment, i.e. the distance from the front motors to the CM.

The torque Q_i of each motor i is related to its normalized signal u_i as $Q_i = Q_{max}\, u_i$

Aerodynamic Moments

The aerodynamic moments are modeled as a first order drag moments to stop the vehicle rotation in the absence of thruster moments.

$M_{a,B} = -K_{D\omega}\, \omega_B$

Integration step

Forward Euler is used for the integration step. Rewriting the equations of motion in discrete time, with a sampling time Δt gives

$p_I^{k+1} = p_I^{\,k} + v_I^{\,k}\, \Delta t$

$v_I^{k+1} = v_I^{\,k} + \frac{1}{m}(W_I^{\,k} + F_{a,I}^{\,k} + C_{IB}(q^k)\, T_B^{\,k})\, \Delta t$

$q^{k+1} = \left(q^k + \frac{1}{2} q^k \otimes \omega_B^{\,k}\, \Delta t\right) / \left\| q^k + \frac{1}{2} q^k \otimes \omega_B^{\,k}\, \Delta t \right\|$

$\omega_B^{\,k+1} = \omega_B^{\,k} + I^{-1}(M_{t,B}^{\,k} + M_{a,B}^{\,k} - \omega_B^{\,k} \times I\omega_B^{\,k})\, \Delta t$

Note that the quaternion is normalized at every time step to ensure $\|q\| = 1$

Sensors Reconstruction

In this section $\eta_i \in N(0, \sigma_i)$ denotes a white Gaussian noise with a standard deviation σ_i.
The IMU[1] reconstruction and sensor noise standard deviations are taken from [1].

The IMU signals are reconstructed from the states as follows

$u_{acc} = C_{BI}(\dot{v}_I - g) + \eta_{acc}$

$u_{gyro} = \omega_B + \eta_{gyro}$

$u_{mag} = C_{BI}\mu_I + \eta_{mag}$

where g is the gravity acceleration vector, μ_I is the magnetic field assumed constant (i.e. taken at the initial location).

The AMSL[2] altitude is taken as

$h = h_0 - p_{I,z}$

where h_0 is the AMSL ground altitude at the initial location.

The barometric altitude, pressure, and temperature are reconstructed as follows

$h_{baro} = h + \eta_{h,baro}$

$P = P_0 (1 + ah/T_0)^{-g/(aR)}$

$\theta = \theta_0 + ah$

where $P_0 = 1013.25\ [mbar]$ is the MSL pressure, $a = -6.5 \cdot 10^{-3}\ [K/m]$ is the temperature gradient according to the standard atmosphere model, T_0 is the ground temperature in Kelvin, θ_0 its corresponding temperature in degrees Celsius, and $R = 287.1\ [J/(kg\ K)]$ is the ideal gas constant.

The GPS latitude, longitude, altitude, and velocity are reconstructed as follows

$lat = lat_0 + p_{I,x}/R_E + \eta_{lat}$

$lon = lon_0 + p_{I,y}/R_E/\cos lat_0 + \eta_{lon}$

$h_{gps} = h + \eta_{h,gps}$

$v_{gps} = v_I + \eta_{gps}$

where lat_0 and lon_0 are the initial geodetic locations in radians, and $R_E = 6371000\ [m]$ is the radius of Earth

Limitations:

1. The barometric model for the pressure and temperature are valid in the Troposphere only (up to $11000\ [m]$ AMSL), which is really not a limitation for a drone.
2. This set of models for the sensors reconstruction are defined under the assumption of a local flat Earth (i.e. the Earth is big enough that it can be approximated locally as a flat plane). Therefore those models are valid only for flight missions no more than a few km around the initial position.

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[1] Inertial Measurement Unit

[2] Above Mean Sea Level

3. The initial position cannot be at the geographic North or South pole. The GPS model for the latitude and longitude assumes small angles and therefore the division by $\cos lat_0$ would become a singularity at the geographic poles.

Timing

The inner loop (i.e. forces, moments, equations of motion, sensors reconstruction, and publishing) takes about 420us to be executed. The *sih* task is given maximum priority by the scheduler as all the other tasks depend on this one (if the simulator stops, the controller and navigation systems will be stuck).

References

[1] Bulka, Eitan, and Meyer Nahon. "Autonomous fixed-wing aerobatics: from theory to flight." *2018 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2018.