

# Adiar

## Binary Decision Diagrams in External Memory

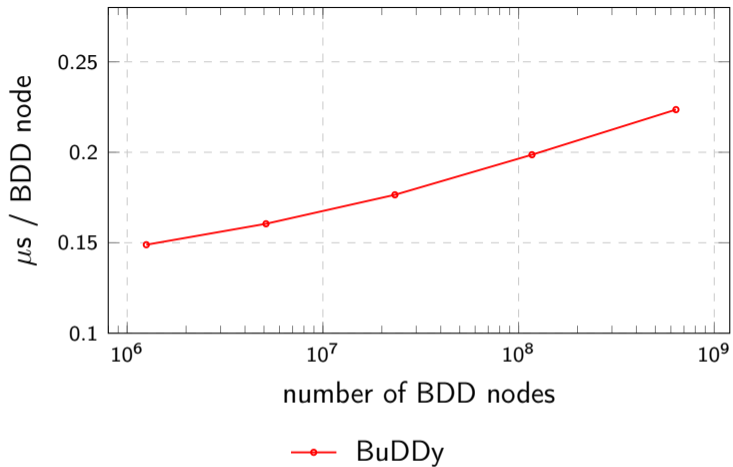
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**Steffan Christ Sølvesten**, Jaco van de Pol,  
Anna Blume Jakobsen, and Mathias Weller Berg Thomasen

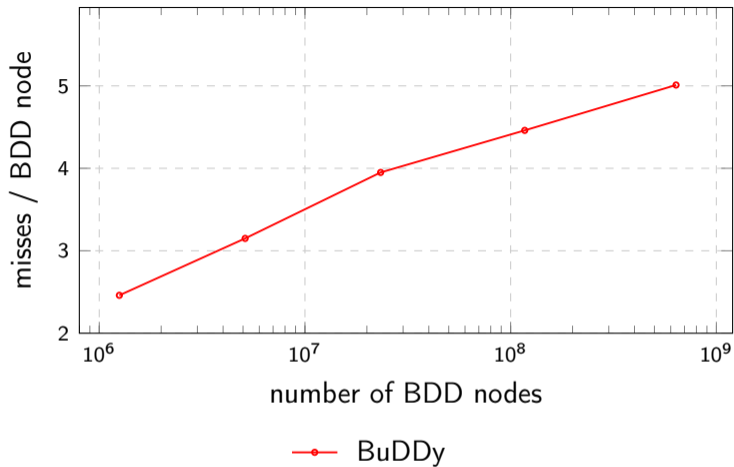
TACAS 2022





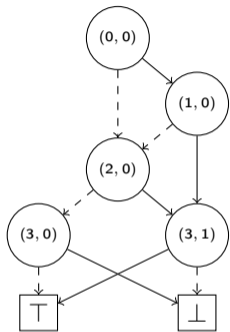


Minimal running time for the *Queens* problems.

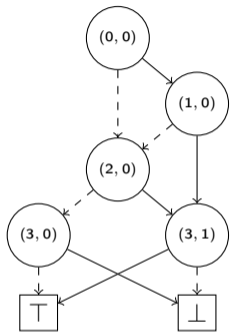


Cache-misses for the *Queens* problems.





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

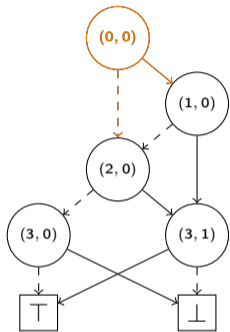


**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[

]



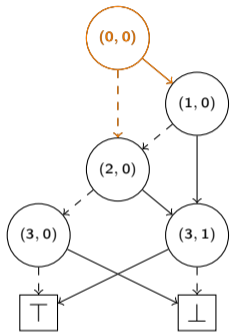
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Priority Queue:  $Q_{count}$ :

[

]



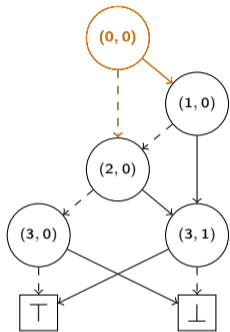


(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[  $((0,0) \xrightarrow{\top} (1,0), 1)$  ,  
 $((0,0) \xrightarrow{\perp} (2,0), 1)$  ,

]



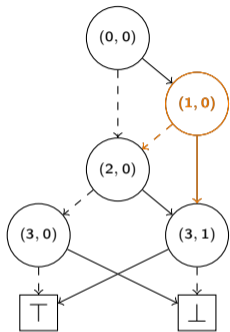
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(1, 0)	0	0

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{\top} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]



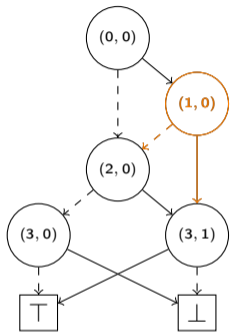
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{\top} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]

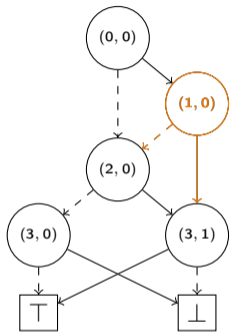


**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue:  $Q_{count}$ :

[  
 $((0,0) \xrightarrow{\perp} (2,0), 1)$  ,  
 ]

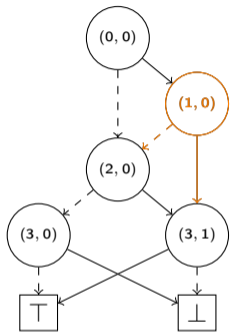


(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(1, 0)	1	0

Priority Queue:  $Q_{count}$ :

[			
	$((0, 0) \xrightarrow{\perp} (2, 0), 1)$	,	
	$((1, 0) \xrightarrow{\perp} (2, 0), 1)$	,	
	$((1, 0) \xrightarrow{\top} (3, 1), 1)$	,	
			]

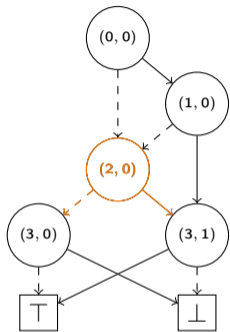


(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue:  $Q_{count}$ :

[			
	$((0, 0) \xrightarrow{\perp} (2, 0), 1)$	,	
	$((1, 0) \xrightarrow{\perp} (2, 0), 1)$	,	
	$((1, 0) \xrightarrow{\top} (3, 1), 1)$	,	
			]

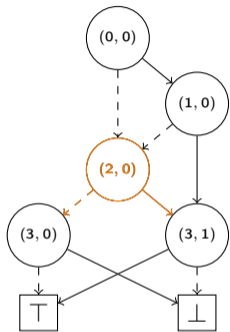


(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue:  $Q_{count}$ :

[	$((0, 0) \xrightarrow{\perp} (2, 0), 1)$	,
	$((1, 0) \xrightarrow{\perp} (2, 0), 1)$	,
	$((1, 0) \xrightarrow{\top} (3, 1), 1)$	,
		]



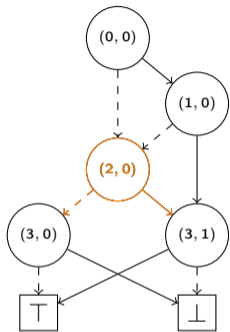
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	1	0

Priority Queue:  $Q_{count}$ :

[			
	$((1, 0) \xrightarrow{\perp} (2, 0), 1)$	,	
	$((1, 0) \xrightarrow{\top} (3, 1), 1)$	,	
			]



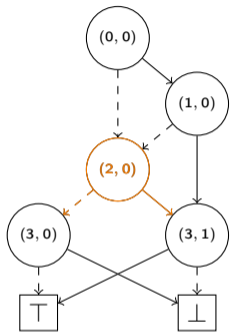


(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	2	0

Priority Queue:  $Q_{count}$ :

[  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]



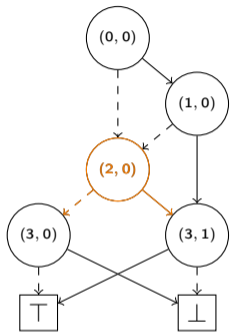
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	2	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$  ,  
 $((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



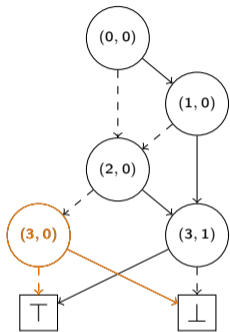
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]



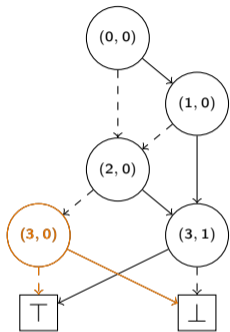
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]



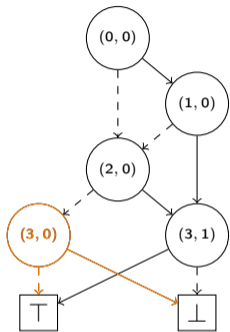
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	0

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



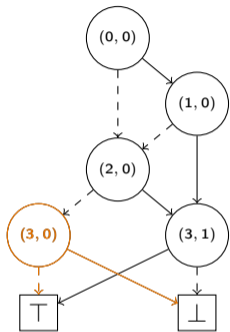
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



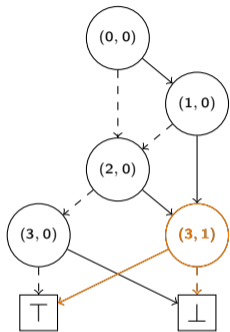
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

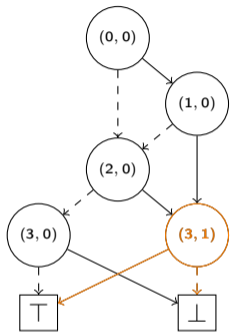
Seek	Sum	Result
(3, 1)	0	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]





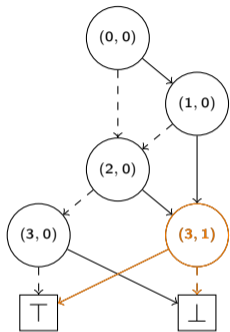
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	1	2

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{T} (3, 1), 2) \ ]$



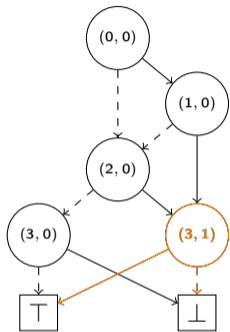
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	3	2

Priority Queue:  $Q_{count}$ :

[

]



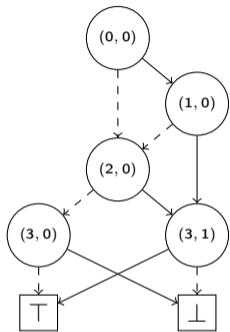
**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	5

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result  
5

Priority Queue:  $Q_{count}$ :

[

]

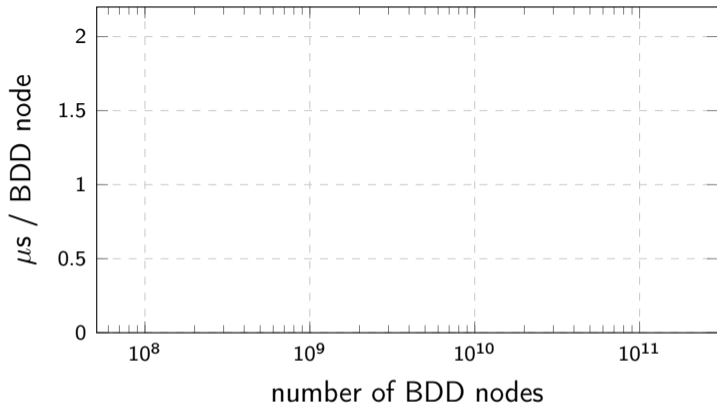


# Adiar

*I/O-efficient Decision Diagrams*

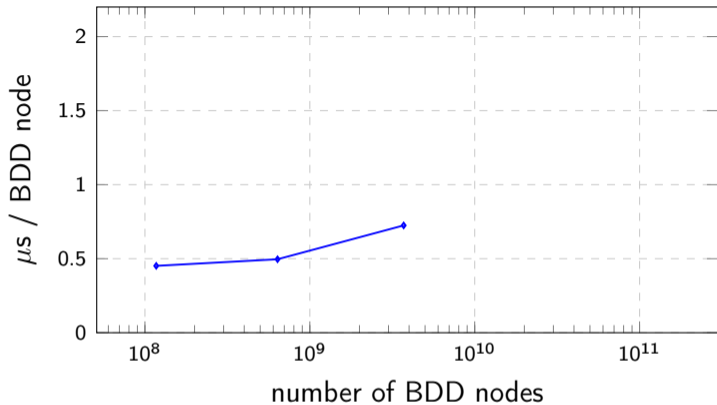
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[github.com/ssoelvsten/adiar](https://github.com/ssoelvsten/adiar)



—◆— CUDD —■— Sylvan —●— Adiar

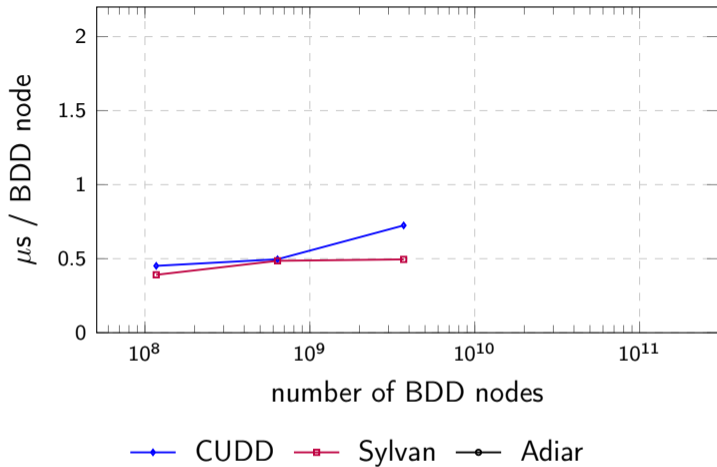
Minimal running time for the *Queens* problems.



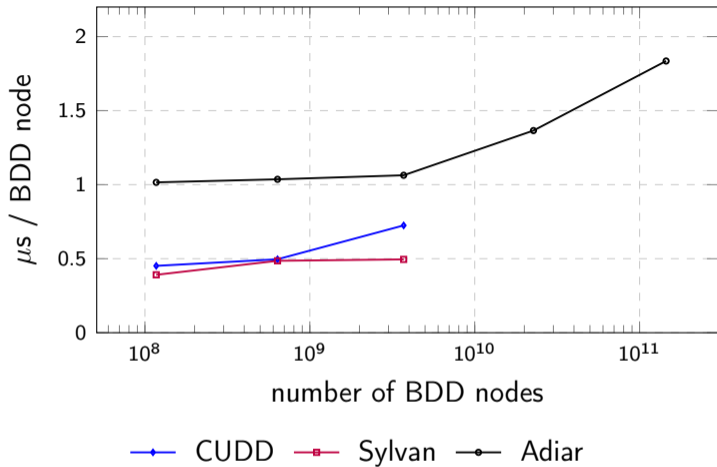
—◆— CUDD    —■— Sylvan    —○— Adiar

Minimal running time for the *Queens* problems.





Minimal running time for the *Queens* problems.



Minimal running time for the *Queens* problems.



Algorithm	Time (s)
$f \leftrightarrow g \equiv \top$	0.38

Checking the (EPFL Benchmark) *voter* circuit's single output gate ( $|N_f| = |N_g| = 5.76$  MiB).

Algorithm	Time (s)
$f \leftrightarrow g \equiv \top$	0.38
$O(N \log N)$	0.058

Checking the (EPFL Benchmark) *voter* circuit's single output gate ( $|N_f| = |N_g| = 5.76$  MiB).

Algorithm	Time (s)
$f \leftrightarrow g \equiv \top$	0.38
$O(N \log N)$	0.058
$O(N)$	0.006

Checking the (EPFL Benchmark) *voter* circuit's single output gate ( $|N_f| = |N_g| = 5.76$  MiB).



# Steffan Christ Sølvsten

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✉ [soelvsten@cs.au.dk](mailto:soelvsten@cs.au.dk)

🌐 [ssoelvsten.github.io](https://ssoelvsten.github.io)

## Adiar

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📄 [github.com/ssoelvsten/adiar](https://github.com/ssoelvsten/adiar)

📖 [ssoelvsten.github.io/adiar](https://ssoelvsten.github.io/adiar)