

Random Access on Narrow Decision Diagrams in External Memory

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SPIN 2024



Adiar

I/O-efficient Decision Diagrams

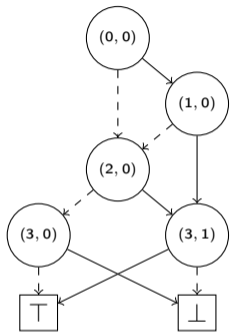
github.com/ssoelvsten/adiar

 Features

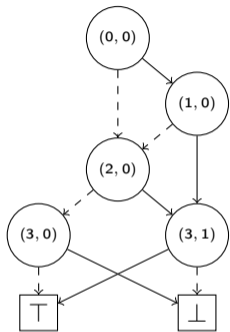
 Optimisations

 Features

 Optimisations



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

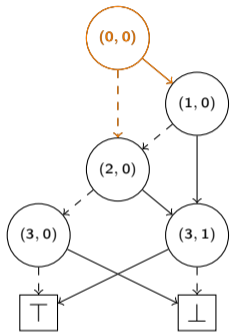


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[

]

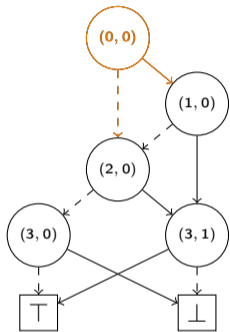


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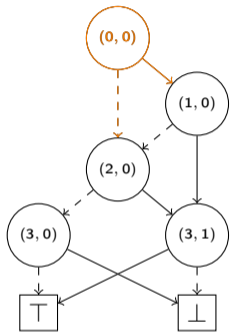


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[$((0,0) \xrightarrow{\top} (1,0), 1)$,
 $((0,0) \xrightarrow{\perp} (2,0), 1)$,

]



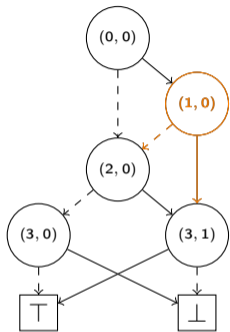
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{\top} (1, 0), 1)$,
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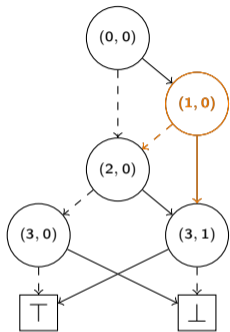
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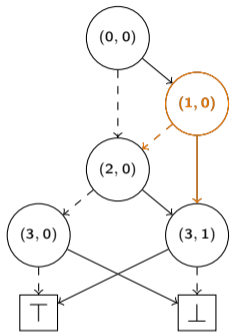


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(1, 0)	1	0

Priority Queue: Q_{count} :

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 $((0,0) \xrightarrow{\perp} (2,0), 1)$,
]

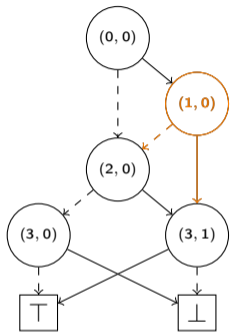


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(1, 0)	1	0

Priority Queue: Q_{count} :

[
	$((0, 0) \xrightarrow{\perp} (2, 0), 1)$,	
	$((1, 0) \xrightarrow{\perp} (2, 0), 1)$,	
	$((1, 0) \xrightarrow{\top} (3, 1), 1)$,	
]

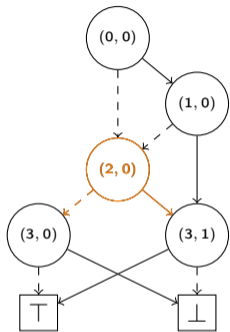


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
	$((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
	$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]

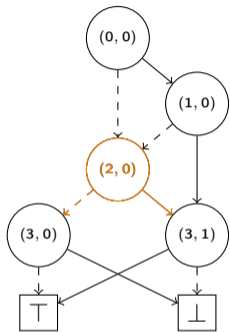


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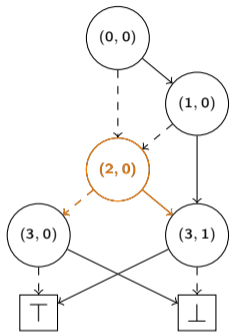


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

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(2, 0)	1	0

Priority Queue: Q_{count} :

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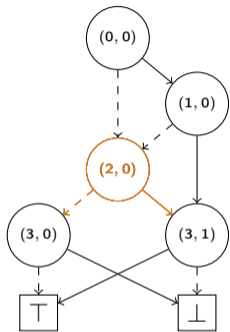
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	2	0

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
]



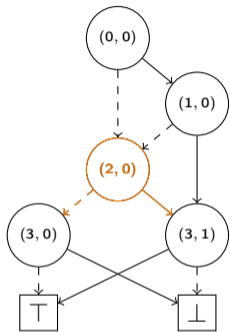
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	2	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$,
 $((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



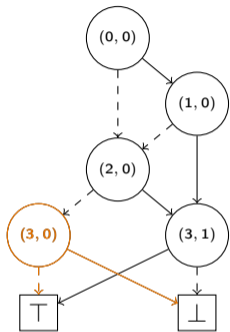
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]



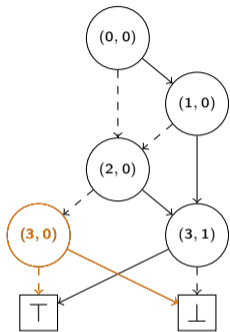
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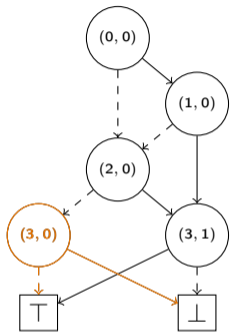
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

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(3, 0)	2	0

Priority Queue: Q_{count} :

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$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



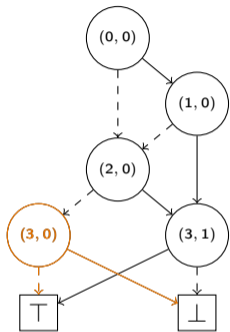
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



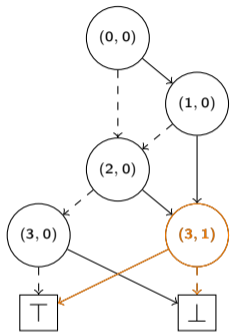
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



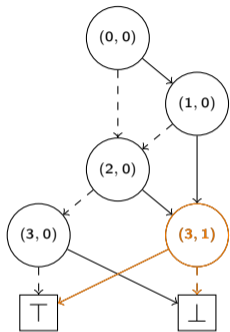
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



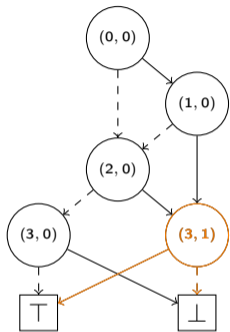
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	1	2

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{T} (3, 1), 2) \]$



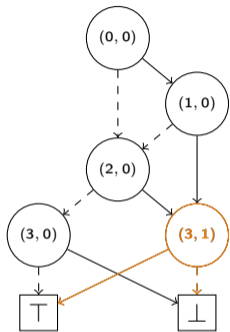
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	3	2

Priority Queue: Q_{count} :

[

]



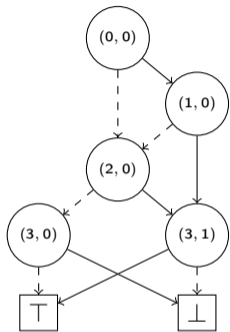
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	3	5

Priority Queue: Q_{count} :

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

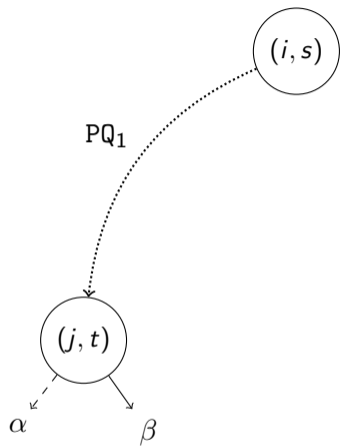
Result
5

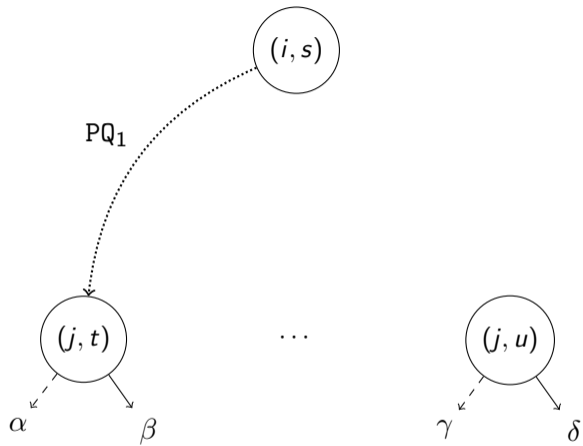
Priority Queue: Q_{count} :

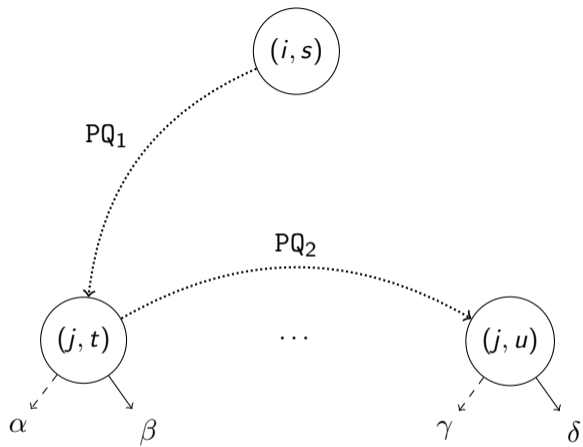
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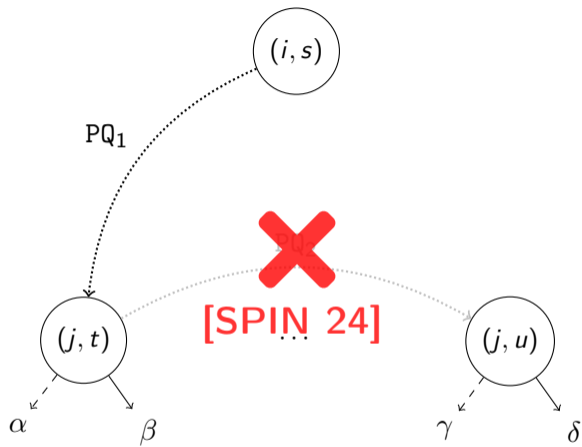
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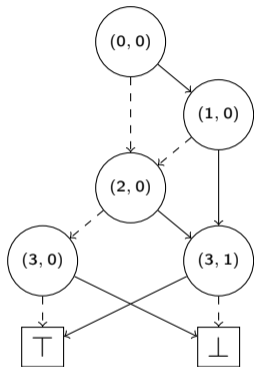
(i, s)

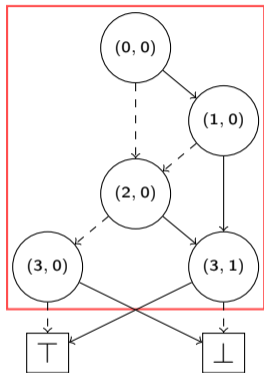


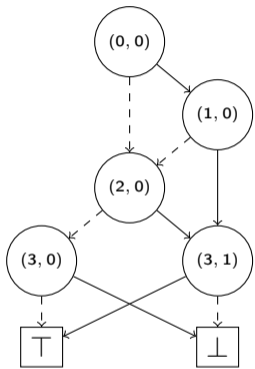


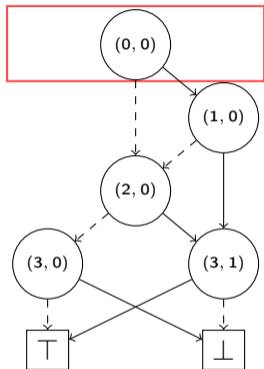


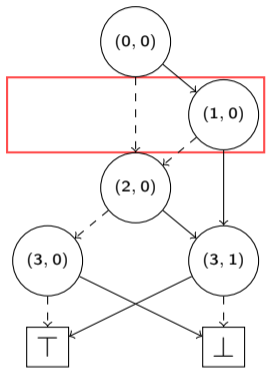


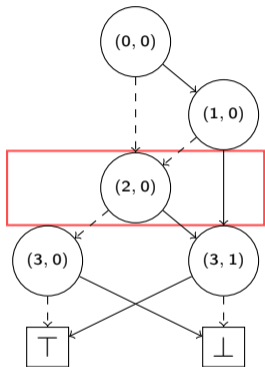


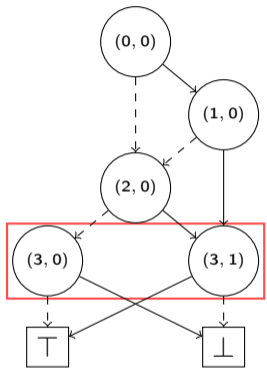


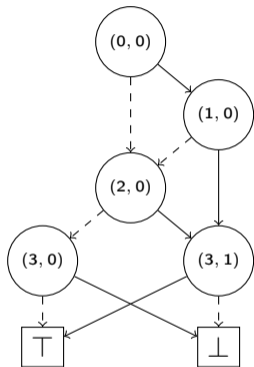


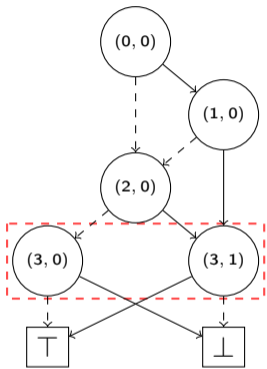




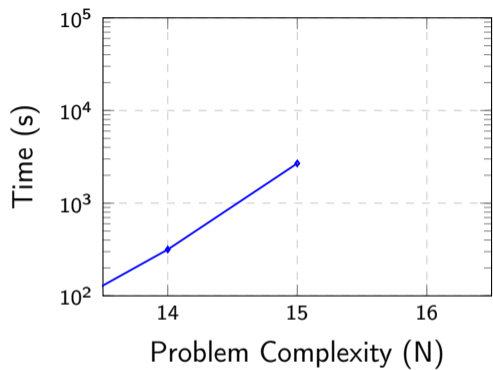









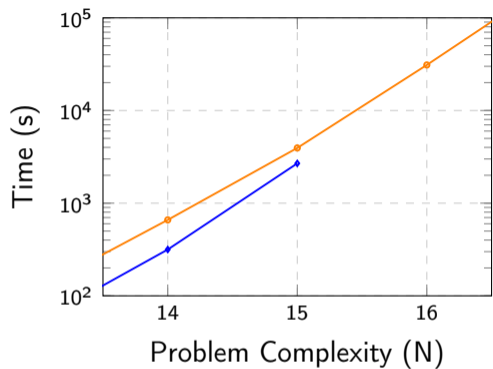
$$\frac{\text{Width}}{2}$$




 $N = 15$

◇ CUDD v3.0 : 44.8 min

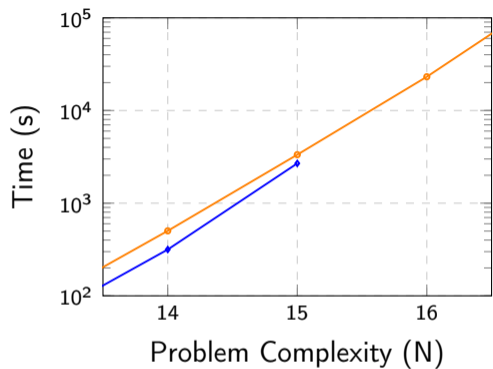
Queens | 300 GiB of RAM



🕒
 $N = 15$

◇	CUDD	v3.0	:	44.8 min
○	Adiar	v1.0	:	66.7 min

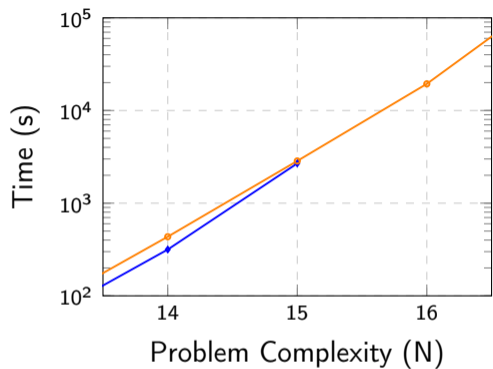
Queens | 300 GiB of RAM



⌚
 $N = 15$

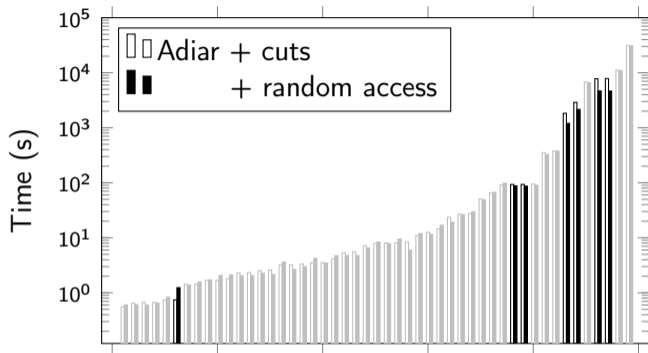
◇	CUDD v3.0	:	44.8 min
○	Adiar v1.0	:	66.7 min
	+ cuts	:	56.8 min

Queens | 300 GiB of RAM



			i
			$N = 15$
◇	CUDD v3.0	:	44.8 min
○	Adiar v1.0	:	66.7 min
	+ cuts	:	56.8 min
	+ random access	:	47.2 min

Queens | 300 GiB of RAM



^
Speed Ups
6 (+26.9%)

v
Slowdowns
1 (-66.8%)

EPFL Circuit Verification | 300 GiB of RAM

Steffan Christ Sølvsten

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🌐 ssoelvsten.github.io

Adiar

📄 github.com/ssoelvsten/adiar

📖 ssoelvsten.github.io/adiar

