Two-Stage Design for Demonstration

Interim Analysis for Futility Using Bayesian Predictive Probability

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Summary of Interim Analysis for Futility

Futility evaluation is implemented in one interim analysis with 25 patients, and 25 patients in the last stage, for a total of 50 patients. With an unfavorable rate set at 30% (null hypothesis) and posterior probability of 0.95 as the threshold, a total of at least 21 of the 50 patients must have response to be able to claim treatment efficacy. Given a 20% cutoff of the predictive probability (i.e. chance to stop the trial in the interim analysis) the stopping rule (Table 1) will be: the trial will be stopped if there are 8 or less patients with response in the 1st interim analysis.

Performance of the design (Figure 2 and Table 3) shows that if the true rate of response is 30%, the chance to reach at least a total of 21 patients with response at end of the study is 4% (Type I error), however the probability to stop the trial early is 68%. If true rate of response is indeed 50%, then the chance to reach at least 21 patients with response at end of the study is 88% (power), and the corresponding probability to stop the treatment early is 5%.

Details

A Bayesian approach for futility analysis is used to calculate posterior probability and predictive probability for the rate of response with a non-informative beta prior, beta(1,1), using the analytical form. We consider a 30% rate or lower of response as ineffective for the treatment. Thus, we expect the treatment arm is promising if the posterior probability of the rate (response) greater than 30% is higher than 0.95 (i.e., prob(rate of response>30% |data)>0.95)).

With a total 50 patients in treatment arm, the number of patients with response needs to be 21 or more in order to meet the criteria. Therefore, we use the number of 21 patients to guide the predictive probability. Specifically, given the number of patients with response, s, in the first 25 patients, we calculate predictive probability of $21 - s$ or more patients with response in the future remaining 25 patients, i.e., $\sum_{i=21-s}^{25} \binom{25}{i} \frac{beta(1+s+i,1+(25-s)+(25-i))}{beta(1+s,1+(25-s))}$. Calculation of predictive probability is based on beta binominal distribution for the number of patients with response in the future remaining 25 patients given a beta distribution for the rate of response, $beta(1+s, 1+25-s)$. For example, if there are 8 patients with response in the first 25 patients, the predictive probability of 13 or more patients with response in the future remaining 25 patients would be $\sum_{i=13}^{25} \binom{25}{i} \frac{beta(1+8+i,1+(25-8)+(25-i))}{beta(1+8,1+(25-8))} = 0.105$.

The predictive probability is also calculated for each of the remaining interim analyses to evaluate the chance of 21-s or more patients with response in the future remaining patients given s patients with response in the current stage of interim analysis. Figure 1 and Table 2 lists predictive probability for all scenarios of number of patients with response in each interim analysis and the associated largest number of patients with response needed in the future remaining patients to have at least a total of 21 patients with response.

We consider that a 20% cutoff of the predictive probability will give unlikely chance to have 21 patients or more with response at the end of study. Thus with this cutoff, the stopping rule (Table 1) will be: the trial will be stopped if there are 8 or less patients with response in the 1st interim analysis. Performance of this

stopping rule (Figure 2 and Table 3) shows that if the true rate of response is 30%, the chance to reach at least a total of 21 patients with response at end of the study is 4% (Type I error), however the probability of early termination (PET) is 68%. When the true rate of response is 50%, then the chance to reach at least 21 patients with response at end of the study is 88% (power), and the corresponding probability to stop the treatment early is 5%.

Sensitivity analysis (Table 4-7 and Figure 4-7) evaluates four parameters for their impact on performance (PET, type I error, and power): cutoff for the predictive probability, threshold for posterior probability of response rate, sample size, and beta prior distribtuion of the response rate. Evaluation is conducted for each parameter when the values of other parameters are fixed. When the cutoff of the predictive probability for the stopping rule is 0.01-0.3, the range is 0.34-0.81 for PET, 0.04-0.05 for type I error, and 0.84-0.9 for power (Table 4 and Figure 4). When the threshold for posterior probability is 0.8-0.99, the range is 0.51-0.81 for PET, 0.01-0.19 for type I error, and 0.73-0.97 for power (Table 5 and Figure 5). When the sample size of each stage is in the magnitude from decrease by -5 to increase by 5, the range is 0.61-0.77 for PET, 0.04-0.07 for type I error, and 0.82-0.92 for power (Table 6 and Figure 6). When the beta prior varies from non-informative prior to the one with a response rate at the null or alternative hypothesis and a series of standard deviation (SD), the range is 0.19-0.98 for PET, 0-0.31 for type I error, and 0.45-0.99 for power (Table 7 and Figure 7).

Table 1: Stopping Boundary for Futility

Stage of interim analysis	1	Final
Sample size up to the current stage	25	50
Sample size at each stage	25	25
Stopping boundary	8	20

Table 2: Bayesian Predictive Probability for Stopping Rule

number of patients with response in the 1st interim analysis	minimum number of patients with response needed in the future remaining patients	predictive probability
0	21	0.000
1	20	0.000
2	19	0.000
3	18	0.000
4	17	0.000
5	16	0.001
6	15	0.008
7	14	0.034
8	13	0.105
9	12	0.246
10	11	0.453
11	10	0.674
12	9	0.846
13	8	0.945
14	7	0.985
15	6	0.997
16	5	1.000
17	4	1.000
18	3	1.000
19	2	1.000
20	1	1.000

Table 3: Performance (Probability of Early Termination, Type I error, and Power)

true rate	overall probability of early stopping the trial	probability to have at least 21 patients with response
0.05	1.000	0.000
0.10	1.000	0.000
0.15	0.992	0.000
0.20	0.953	0.000
0.25	0.851	0.006
0.30	0.677	0.044
0.35	0.467	0.172
0.40	0.274	0.412
0.45	0.134	0.683
0.50	0.054	0.876
0.55	0.017	0.966
0.60	0.004	0.993

Table 4: Sensitivity Analysis: Predictive Probability

Cutoff of predictive probability	PET	typeI	power
0.01	0.34	0.05	0.90
0.05	0.51	0.05	0.89
0.10	0.51	0.05	0.89
0.15	0.68	0.04	0.88
0.20	0.68	0.04	0.88
0.25	0.81	0.04	0.84
0.30	0.81	0.04	0.84

Table 5: Sensitivity Analysis: Posterior Probability

threshold of posterior probability	PET	typeI	power
0.80	0.51	0.19	0.97
0.81	0.51	0.19	0.97
0.82	0.51	0.19	0.97
0.83	0.51	0.19	0.97
0.84	0.51	0.13	0.95
0.85	0.51	0.13	0.95
0.86	0.51	0.13	0.95
0.87	0.51	0.13	0.95
0.88	0.51	0.13	0.95
0.89	0.51	0.13	0.95
0.90	0.68	0.07	0.91
0.91	0.68	0.07	0.91
0.92	0.68	0.07	0.91
0.93	0.68	0.07	0.91
0.94	0.68	0.07	0.91
0.95	0.68	0.04	0.88
0.96	0.68	0.04	0.88
0.97	0.81	0.02	0.79
0.98	0.81	0.02	0.79
0.99	0.81	0.01	0.73

Table 6: Sensitivity Analysis: Sample Size

n1	n2	PET	typeI	power
20	20	0.61	0.06	0.85
21	21	0.72	0.05	0.82
22	22	0.67	0.04	0.83
23	23	0.62	0.06	0.89
24	24	0.73	0.05	0.87
25	25	0.68	0.04	0.88
26	26	0.63	0.07	0.92
27	27	0.73	0.05	0.90
28	28	0.68	0.05	0.91
29	29	0.77	0.04	0.89
30	30	0.73	0.05	0.92

Table 7: Sensitivity Analysis: Beta Prior Distribution

	beta.a	beta.b	PET	typeI	power
0/0	0.00	0.00	0.68	0.04	0.88
0/1	0.00	1.00	0.81	0.04	0.84
1/0	1.00	0.00	0.68	0.07	0.91
1/1	1.00	1.00	0.68	0.04	0.88
0.3 (SD=0.05)	24.90	58.10	0.98	0.00	0.45
0.3 (SD=0.1)	6.00	14.00	0.90	0.02	0.73
0.3 (SD=0.2)	1.27	2.97	0.81	0.04	0.84
0.3 (SD=0.3)	0.40	0.93	0.68	0.04	0.88
0.5 (SD=0.1)	12.00	12.00	0.19	0.31	0.99
0.5 (SD=0.2)	2.62	2.62	0.68	0.07	0.91
0.5 (SD=0.3)	0.89	0.89	0.68	0.04	0.88

Figure 1: Bayesian Predictive Probability for Stopping Rule

1st Interim Analysis of for Futility
minimum number of patients with response needed in the future remaining 25 patients

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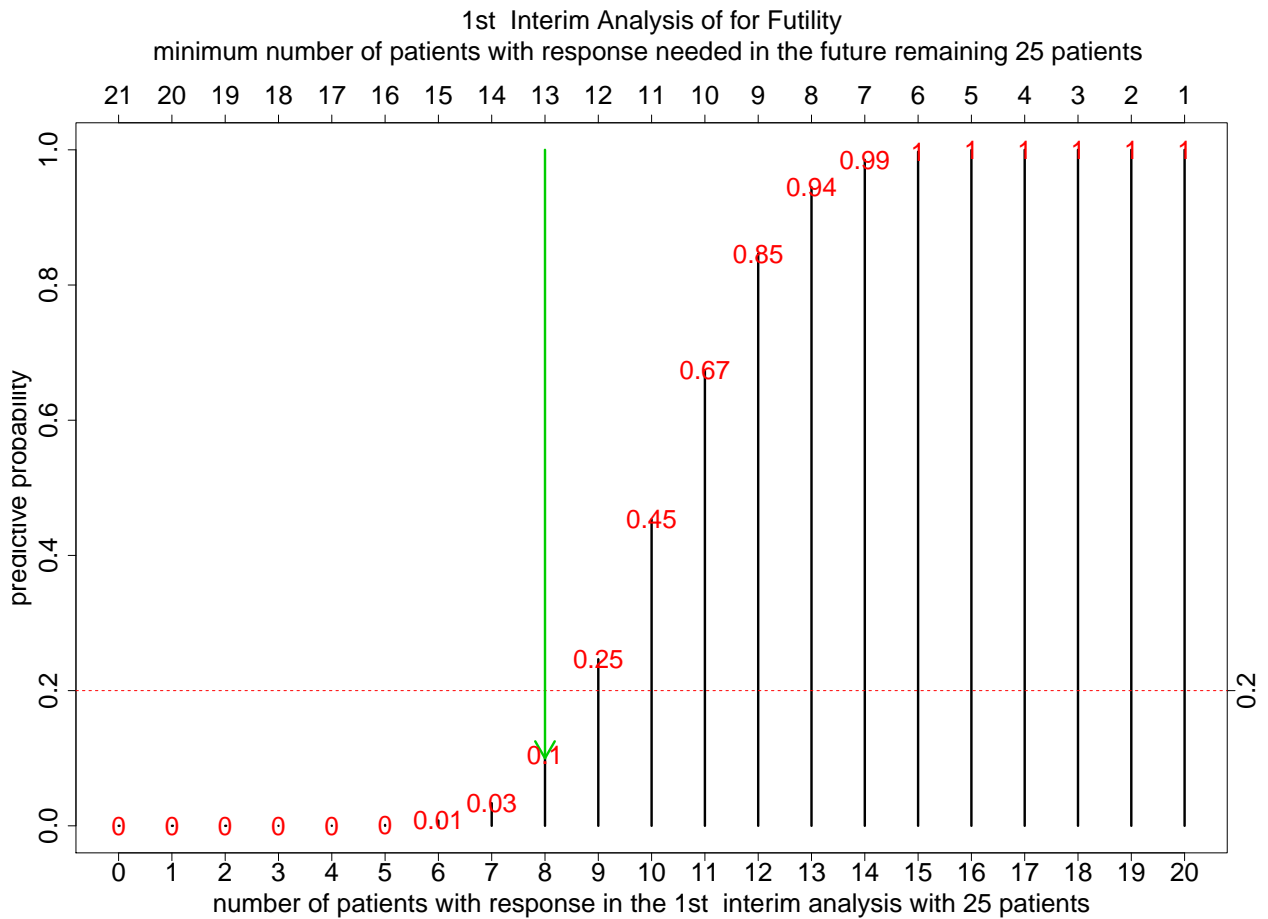
Figure 2: Performance (Probability of Early Termination (PET), Type I error, and Power)

Figure 4: Sensitivity Analysis: Predictive Probability

PET

Cutoff of predictive probability

type I error

Cutoff of predictive probability

power
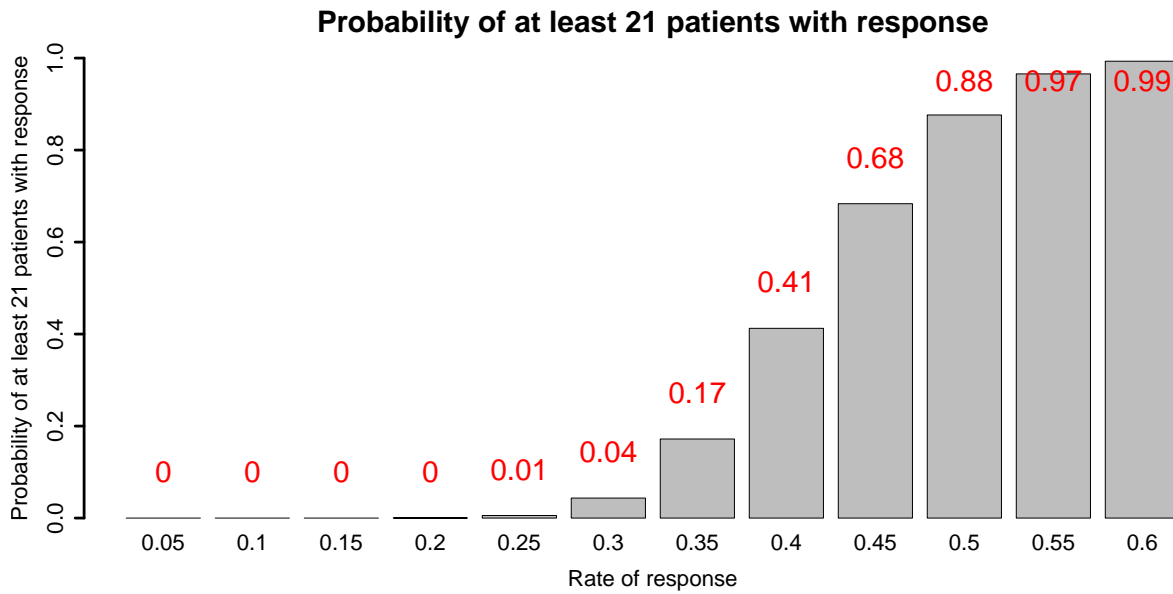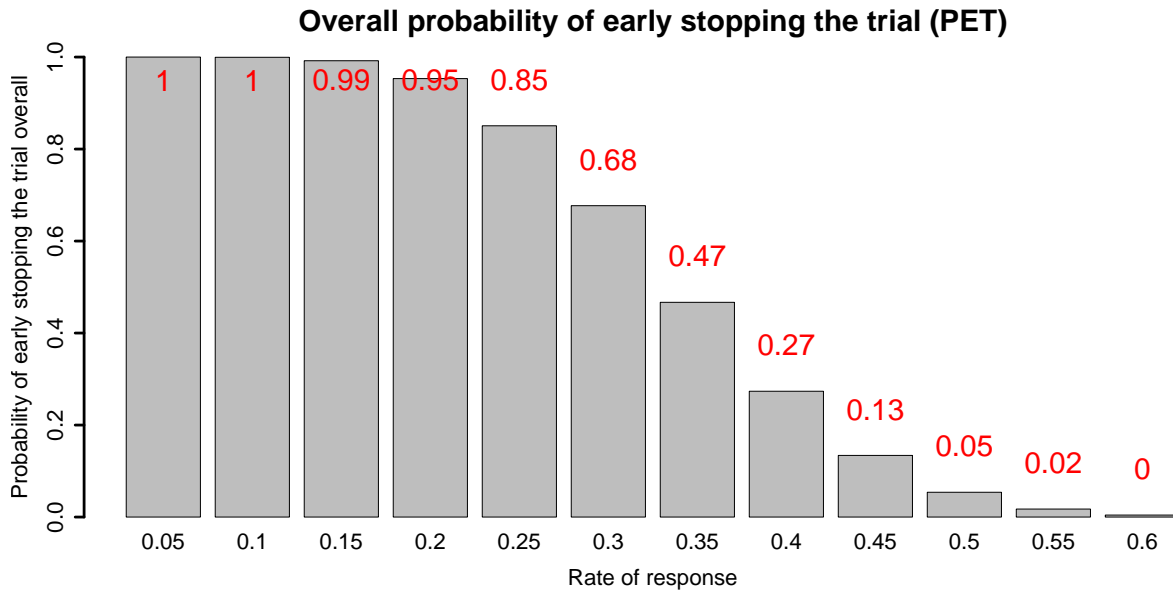
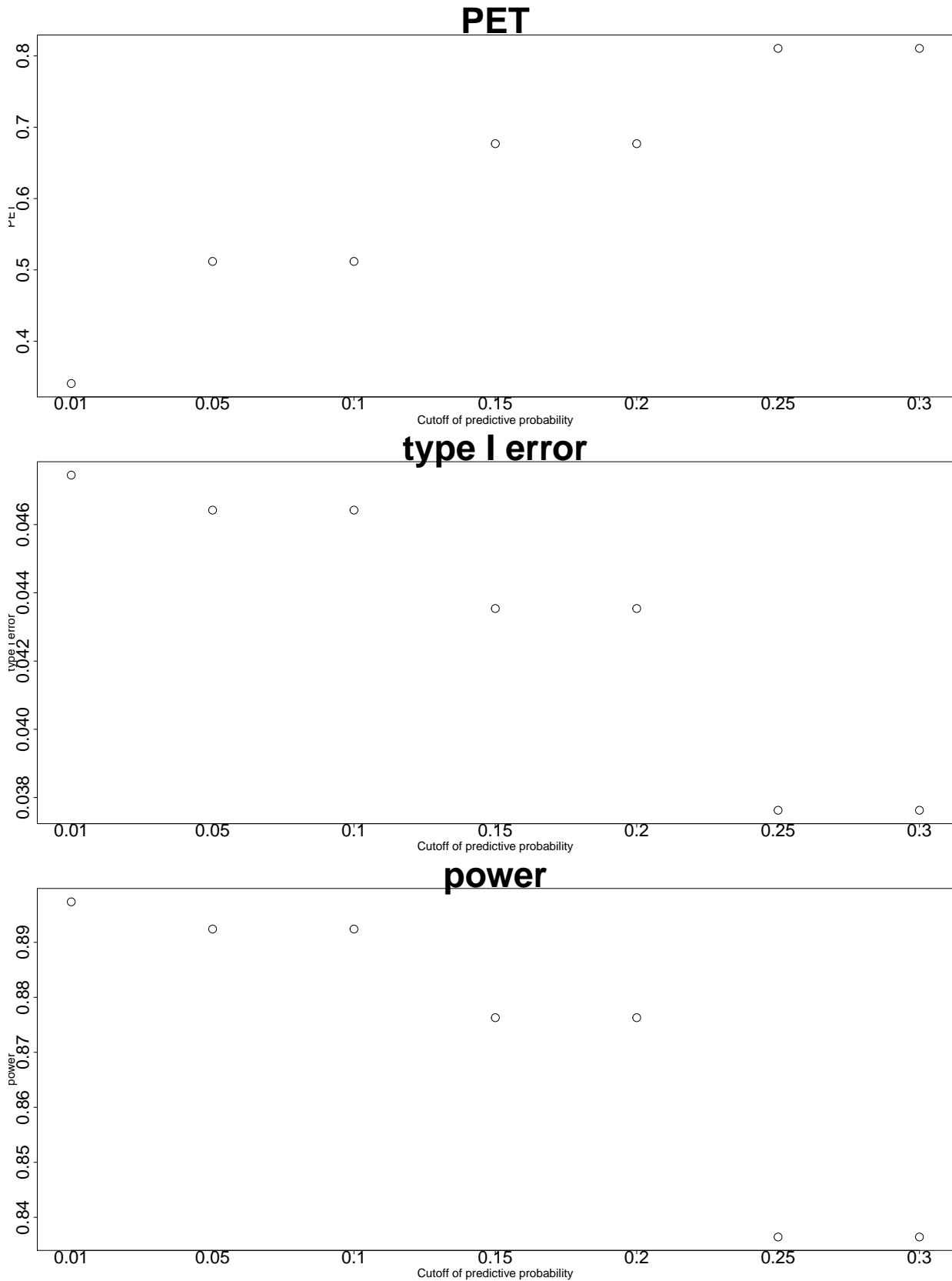Cutoff of predictive probability

Figure 5: Sensitivity Analysis: Posterior Probability

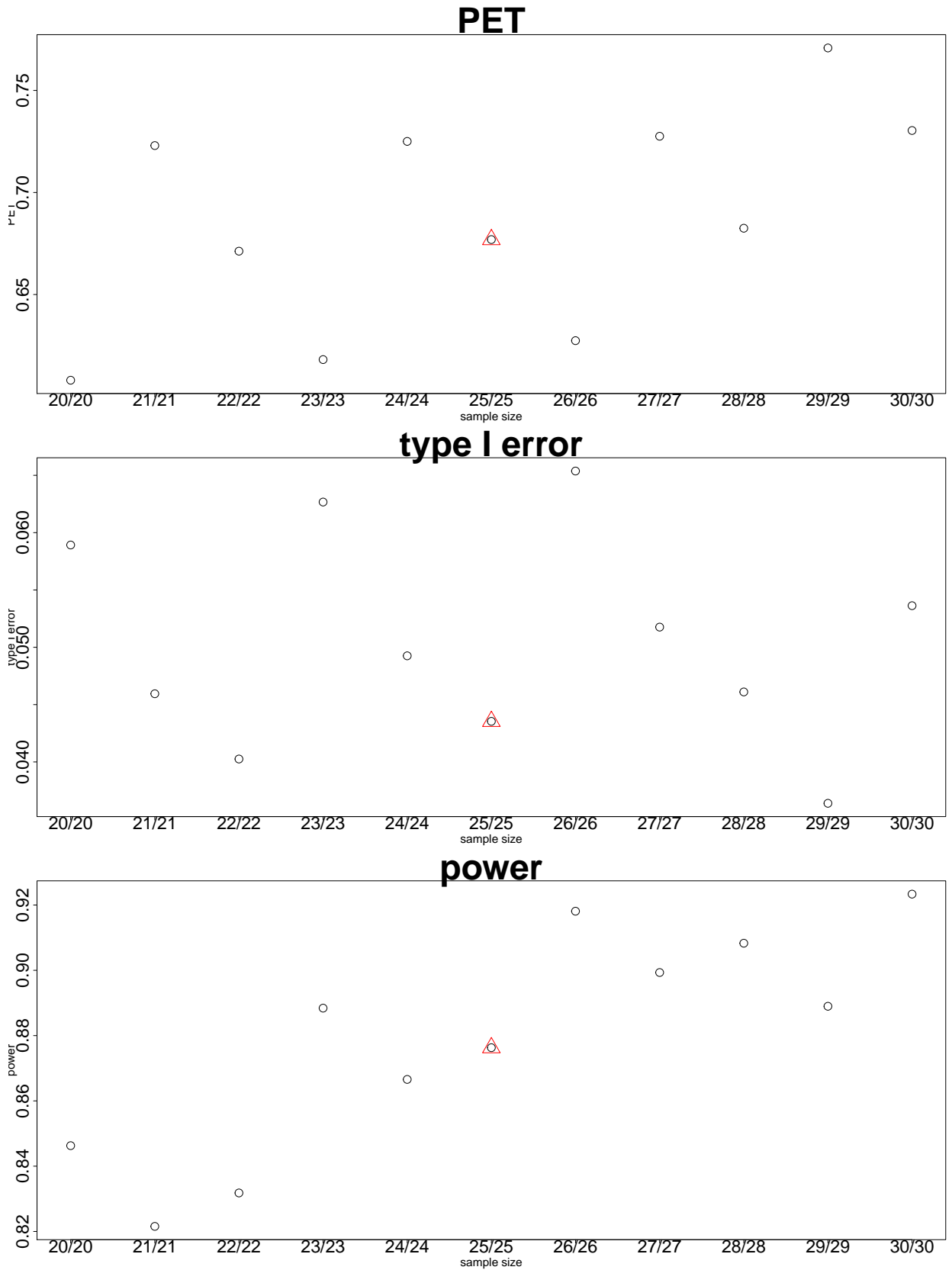## PET

type I error

power
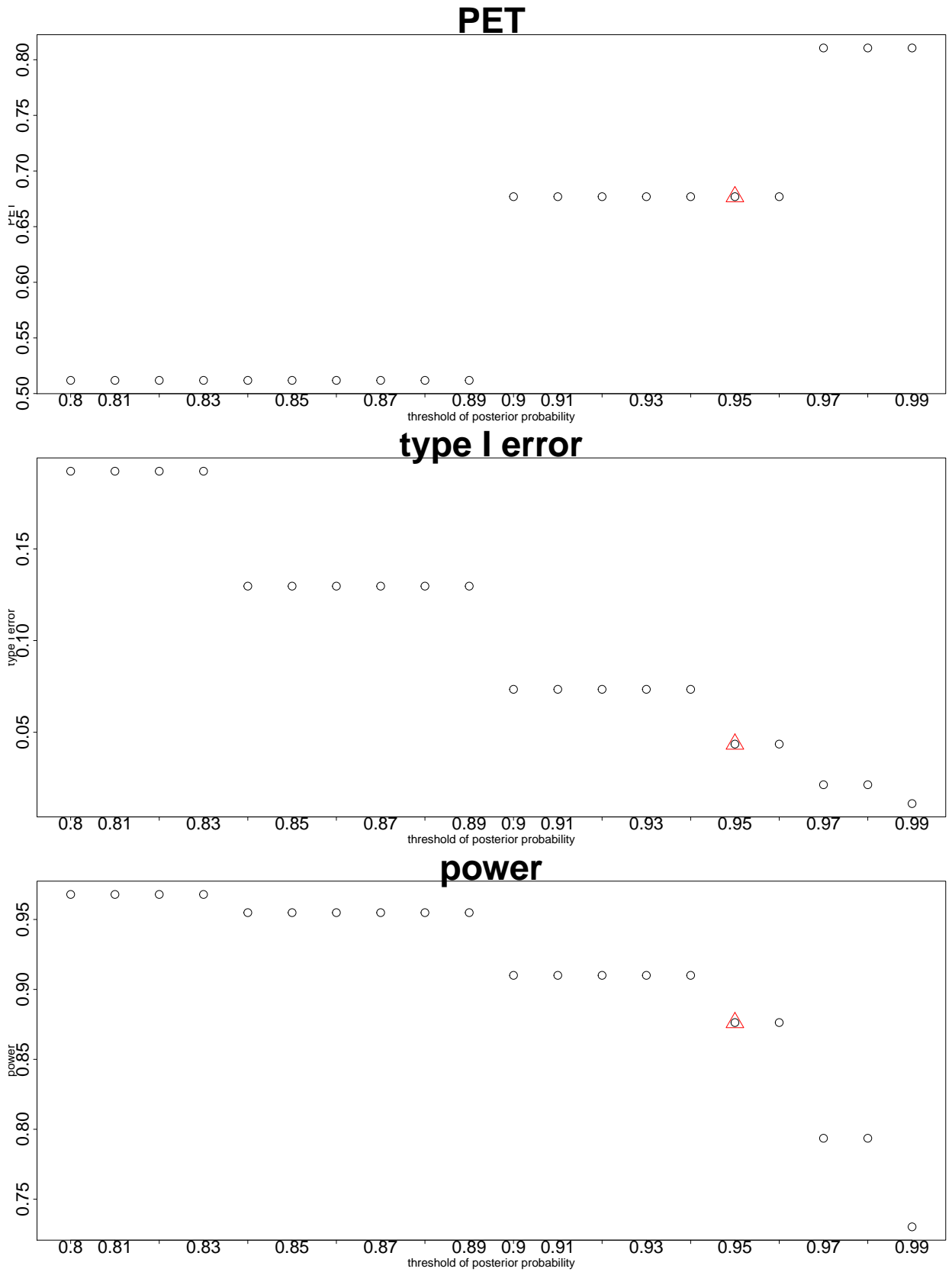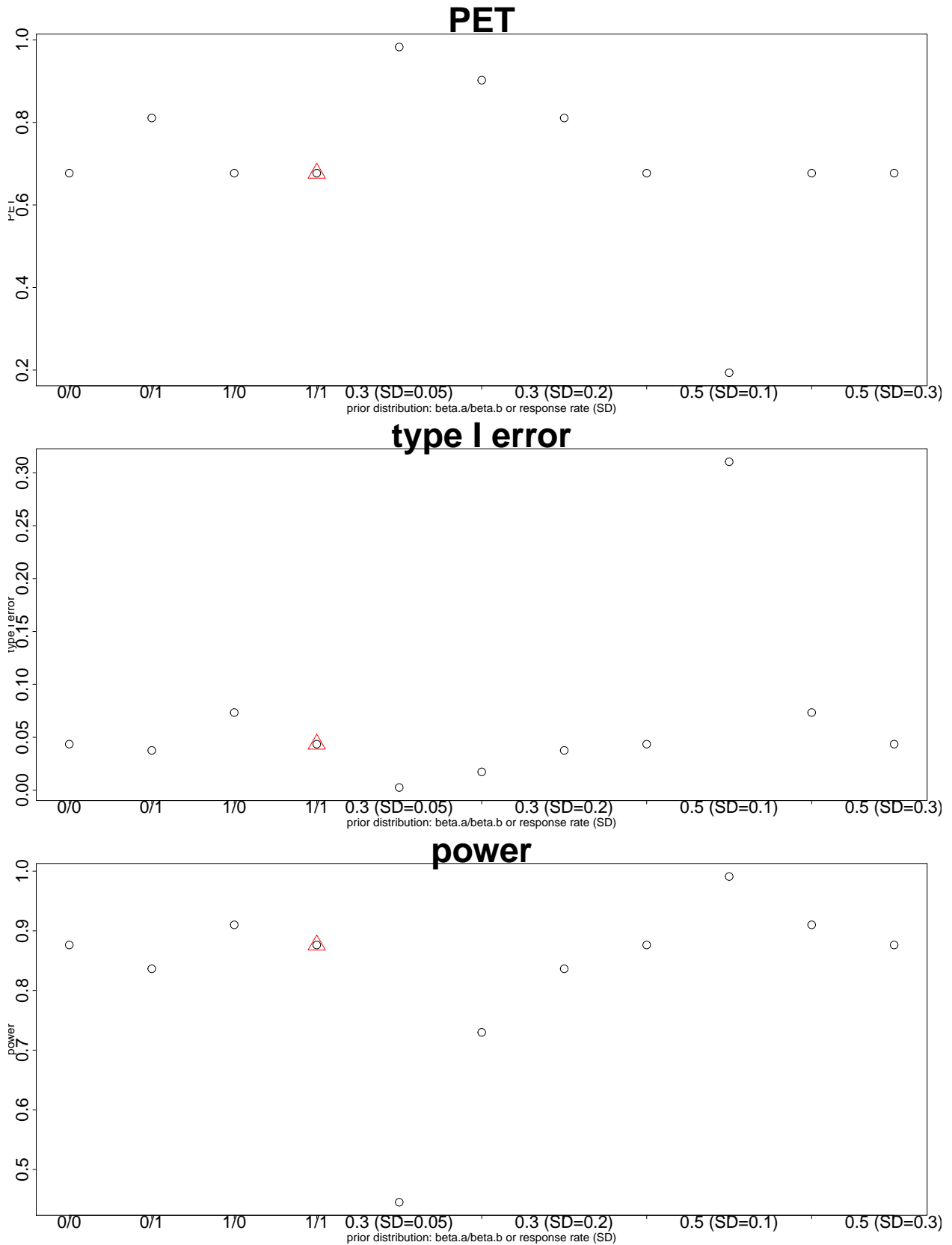
Figure 6: Sensitivity Analysis: Sample Size

Figure 7: Sensitivity Analysis: Beta Prior Distribution

References

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Lee JJ and Liu DD. A predictive probability design for phase II cancer clinical trials. Clinical trials. 2008; 5: 93-106.