

# Tram and Trolleybus Net Traction Energy Consumption Comparison

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**Abstract--**Electric driven public transportation systems have shown to be a solution to traffic congestion and its related pollution level in cities. Additionally, when compared with the traditional internal combustion engine (ICE), the electric driven mode of transport is much more efficient especially when regenerative braking is used. This paper took a step farther and compared two most common electric public transportation systems in cities. Energy consumption per passenger of a trolleybus was compared to that of a tram, both using regenerative braking, travelling from point A to point B, assuming the same route. For the sake of energy consumption comparison an 11.95 km route with six (6) stations from Ubungo maziwa to Central railway station in Dar es Salaam, Tanzania was chosen. With their particular specifications such as effective mass and coefficient of friction, the two vehicles were simulated using MATLAB, in such a way their acceleration, braking deceleration, maximum speed, and travelling time are all the same, their speed profiles which were more or less the same are presented in this paper alongside their corresponding energy consumption.

**Keywords:** Tram, Trolleybus, Energy consumption per passenger

## I. INTRODUCTION

Exponentially growing traffic congestion and its related pollution level in cities is becoming serious problem that needs to be addressed. Electric driven public transportation systems have shown to be a solution to traffic congestion and its related pollution level in cities. Additionally, in comparison with the traditional internal combustion engine (ICE), the electric driven mode of transport is much more efficient especially when regenerative braking system is used, as part of energy is recovered during braking when motors are used as generators and the magnetic torque, provides the necessary braking force.

Trams and trolleybuses are very popular mode of transport in most of the cities. The choice as to whether city planners should go for trams or trolleybuses as means of public transport depends on many factors such as capital and life time running cost of trams and trolleybuses. As far as running cost is concerned, energy consumed by a tram or a trolleybus per passenger kilometer is a major contributing factor.

Almost in all cases a tram is heavier than a trolleybus in terms of weight per passenger. This might tempt and

mislead people to conclude that the tram just because is heavier, it then consumes more energy per passenger kilometer than a trolleybus. Despite the fact that the tram is heavier, it is not necessarily true that it consumes more energy per passenger kilometer than the trolleybus, due to the fact that, on the other hand the tram having steel wheels running on steel rail has a rolling resistance that is far much lower than that of the trolleybus having rubber tyres running on tarmac or concrete road.

This paper then compares the traction energy per passenger consumed by a tram to that consumed by a trolleybus, both using regenerative braking and travelling the same distance from point A to point B, assuming the same route.

The rest of the content in this paper has been structured as follows: The motive of the paper is explained in section II, vehicle kinematics and vehicle dynamics are presented in section III and section IV respectively, the two altogether give the general overview on vehicle speed profiles and associated forces. Details associated with the tram and trolleybus, alongside the selected route are described in section V, the tram and trolleybus simulation results are presented and discussed in section VI, and finally a conclusion is drawn in section VII.

## II. MOTIVE OF THE PAPER

Because of the rising energy prices and environmental concerns, the choice of an energy-efficient mode of transport is extremely important. Trams and trolleybuses are by far the most preferable modes of public transport in cities; where a tram is always heavier than a trolleybus in terms of weight per passenger but, on the other hand the tram has lower coefficient of friction than the trolleybus. This paper then compares the energy consumption per passenger of the two modes of transport to see which of them uses less energy per passenger, taking the same route.

## III. VEHICLE KINEMATICS

When a vehicle travels from point A (Starting point) to point B (Stopping point ) it generally passes through four stages namely i) Accelerating mode, ii) Constant speed

mode, iii) Coasting mode, and iv) braking mode; as shown in Fig. 1.

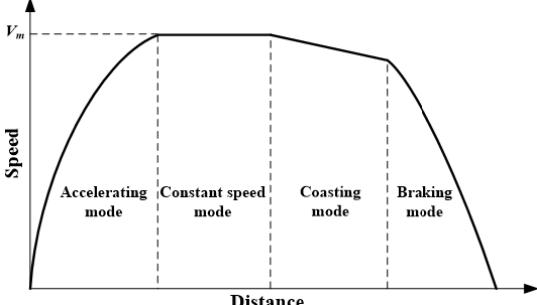


Fig. 1 General vehicle speed profile

**Acceleration mode:** Under the acceleration mode a vehicle accelerates from rest to a maximum operating speed ( $V_m$ ). The tractive force ( $F_T$ ) needed is given by  $F_T = Ma + F_R$ ;  $F_T \leq F_{T\_max}$ , where  $M$  is the effective vehicle mass,  $a$  the acceleration, and  $F_R$  the net force opposing the motion. The tractive power drawn is such that to overcome the resistive forces and accelerate the vehicle.

**Constant speed mode:** Under the constant speed mode, a vehicle maintains the speed reached by the end of the accelerating mode, that is to say a vehicle's acceleration is zero. The tractive force ( $F_T$ ) needed is given by  $F_T = F_R$ , where  $F_R$  is the net force opposing the motion. The vehicle draws tractive power only to overcome the resistive forces.

**Coasting mode:** In coasting mode a vehicle draws no tractive power, it moves by its own momentum, that is tractive force is zero ( $F_T = 0$ ); due to forces opposing the motion, the vehicle starts decelerating; the vehicle's acceleration is thus given by  $a = \frac{-F_R}{M}$ .

**Braking mode:** In the braking mode, brakes are applied to stop the vehicle; braking system can be regenerative or non-regenerative. This paper considers a regenerative braking system, with which part of energy is recovered during braking by using the traction motors as generators and the vehicle's kinetic energy is converted into electrical energy.

A vehicle speed profile will always consist of the accelerating and braking modes but not necessarily the constant speed and coasting modes. Depending on the distance between point A (Starting point) and point B (Stopping point), and time limit for a vehicle to travel from point A to Point B, the constant speed mode or coasting mode or both may not be included in the vehicle speed profile. Fig. 2 shows a vehicle speed profile with only accelerating, coasting and braking modes.

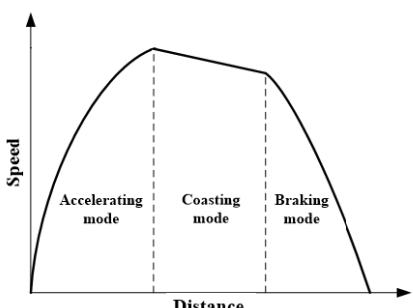


Fig. 2 Vehicle speed profile without a constant speed mode

#### IV. VEHICLE DYNAMICS

Regardless of whether the vehicle is a simple private-car, bus or train, their movement calculations are basically the same, the differences between them will only be their respective parameters such as mass, maximum velocity, maximum tractive effort, coefficient of friction, aerodynamic drag coefficient and frontal area. A vehicle movement calculations are based on the Newton's laws of motion. A free body diagram showing the forces acting upon an uphill moving vehicle is shown in Fig. 3

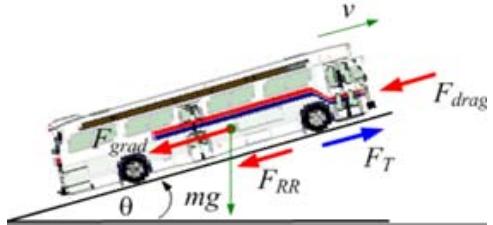


Fig. 3: Free body diagram of a vehicle uphill movement [1]

The relationship between the forces is as expressed in Eq. (1) and Eq. (2)

$$F_T = ma + F_R \quad (1)$$

$$F_R = F_{RR} + F_{grad} + F_{drag} \quad (2)$$

where  $F_T$  is the vehicle tractive force (N),  $F_R$  is the net resistance force against the vehicle motion (N),  $F_{RR}$  is the vehicle rolling resistance force (N),  $F_{drag}$  is the aerodynamic drag force (N),  $F_{grad}$  is the vehicle gravitational (gradient) force (N), it is positive when the vehicle is moving uphill and negative when the vehicle is moving downhill,  $m$  is the vehicle effective mass (kg), and  $a$  is the vehicle acceleration ( $\text{ms}^{-2}$ ).

**Tractive force,  $F_T$ :** A vehicle accelerates through the application of tractive forces. The tractive force of a vehicle is produced at a tyre-road interface [1]; it is normally restricted in such a way  $F_T \leq F_{T\_Max}$ , where  $F_{T\_Max}$  is the maximum tractive force; otherwise the vehicle wheels or tyres will be slipping instead of spinning. The maximum tractive force depends on the coefficient of adhesion between a wheel or tyre and a track or road respectively, the greater the coefficient of adhesion the greater the maximum tractive force.

**Aerodynamic drag force,  $F_{drag}$ :** The motion of a vehicle is taking place in the air and the force exerted by air on the vehicle will influence the motion. The aerodynamic resistance force results from three basic effects: i) the pressure difference in front and behind the vehicle due to the separation of the air flow and the vortex creation behind the vehicle, ii) skin friction representing the surface roughness of the vehicle body and iii) internal flow of air entering the internal parts of the vehicle [1]. It is common to express the aerodynamic resistance force in the basic form as in Eq. (3).

$$F_{drag} = \frac{1}{2} \rho_{air} C_d A_f V_{air}^2 \quad (3)$$

where  $\rho_{air}$  is the air density ( $\text{kgm}^{-3}$ ),  $C_d$  is an aerodynamic drag coefficient,  $A_f$  is the projected vehicle frontal area ( $\text{m}^2$ ) and,  $V_{air}$  is the speed of air relative to the vehicle body ( $\text{ms}^{-1}$ ) [1].

Gravitational (gradient) force,  $F_{grad}$ : The force is positive when the vehicle is moving uphill and negative when the vehicle is moving downhill, mathematically expressed as in Eq. (4)

$$F_{grad} = \pm mgsin\theta \quad (4)$$

where  $m$  is the vehicle effective mass (kg),  $g$  is the acceleration due to gravity ( $9.81\text{ms}^{-2}$ ) and,  $\theta$  is the slope angle.

Rolling resistance,  $F_{RR}$ : is the resistance to motion of rotating parts. It can be categorized into two main resistances: i) frictional torques (bearing torques, gear teeth friction, brake pads) and ii) tyre deformation. At the most elementary level the rolling resistance of a moving vehicle is mathematically expressed as in Eq. (5) [3]

$$F_{RR} = \mu W \quad (5)$$

where  $W$  is the wheel load and  $\mu$  is the rolling resistance coefficient.

In some literatures, Eq.(3) and Eq.(5) are combined in one equation expressing the resistance to motion due to aerodynamics and coefficient of friction as a function of velocity, which is called Davis' equation as in [4]

Mathematically the approach to calculate the energy consumed by a vehicle per trip is as summarized in step (i) through step (vii) below

- i. Variables are initialized, for example,  $t = 0$ ;  $v(t) = 0$ ;  $s(t) = 0$ .
- ii. Vehicle acceleration ( $a$  ( $\text{m/s}^2$ )) is determined, which depends of the mode of operation.
- iii. At every time step ( $dt$ ), a value of velocity ( $v$ ) and change in distance ( $ds$ ) are calculated as  $v(t) = v(t - dt) + adt$  (this shows that change in velocity is directly proportional to change in time when acceleration is constant) and  $ds = v(t - dt) + 0.5adt^2$ , where  $v(t - dt)$  and  $v(t)$  are previous and current velocities respectively.
- iv. Resistance forces (rolling resistance force, aerodynamic drag force and, gradient force) are calculated, and then the tractive force ( $T_F$ ) is obtained.
- v. The output mechanical power ( $P_M$ ), and input electrical power ( $P_E$ ) are calculated as  $P_M = T_Fv$  and  $P_E = \frac{P_M}{\eta}$  respectively and then electrical energy consumption ( $E$ ) is computed as  $E = P_E dt$ ; where  $\eta$  is the input electrical power ( $P_E$ ) to the output mechanical power ( $P_M$ ) conversion efficiency.
- vi. The data are stored
- vii. The values of time ( $t$ ), velocity ( $v$ ), and distance ( $s$ ) are updated as  $t = t + dt$ ,  $v(t - dt) = v(t)$  and  $s(t) = s(t - dt) + ds$  respectively.

viii. The cycle (step ii. to step vii.) repeats till a vehicle reaches a stopping point. The procedures are presented in a flow chart as shown in Fig. 4

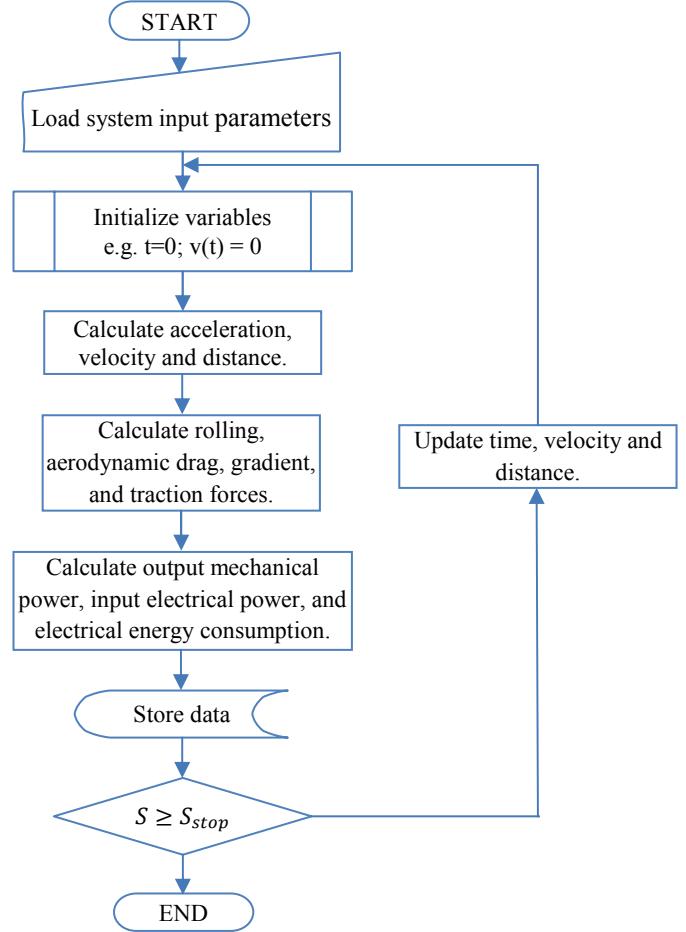


Fig. 4 Vehicle energy consumption, calculation flow chart

## V. DETAILS ASSOCIATED WITH THE TRAM AND THE TROLLEYBUS

The tram and trolleybus details are as shown in Table 1 [6] [7]

Table 1: Tram and trolleybus details

Attribute	Value	
	Tram	Trolleybus
Maximum operational speed	88.5km/h	75km/h
Service acceleration	$1.34\text{m/s}^2$	$1.3 \text{ m/s}^2$
Service deceleration	$1.34\text{m/s}^2$	$1.4 \text{ m/s}^2$
Vehicle empty weight	43409 kg	19100 kg
Passenger capacity @ $3\text{p/m}^2$	Approx. 220	Approx. 138

Taking a person average weight as 62 kg [4], aerodynamic drag coefficient as 0.6 [5], and assuming an equal frontal area of  $8.5 \text{ m}^2$ , the simulation details associated with the tram and trolleybus were as shown in Table 2 [3] [6] [7]

Table 2: Tram and trolleybus simulation details

Attribute	Value	
	Tram	Trolleybus
Maximum operational speed	60 km/h	60 km/h
Acceleration	1.2 m/s <sup>2</sup>	1.2 m/s <sup>2</sup>
Braking deceleration	1.2 m/s <sup>2</sup>	1.2 m/s <sup>2</sup>
Passenger capacity @ 3p/m <sup>2</sup>	Approx. 220	Approx. 138
Vehicle effective weight @ 62kg/p	57049 kg	27656 kg
Rolling resistance coefficient	0.006	0.015
Aerodynamic drag coefficient	0.6	0.6
Frontal area	8.5 m <sup>2</sup>	8.5 m <sup>2</sup>
Air density	1.225kg/m <sup>3</sup>	1.225kg/m <sup>3</sup>

For the sake of comparison, the same tram route from Ubungo maziwa to Central railway station in Dar es Salaam Tanzania which is fairly flat was chosen for both tram and trolleybus (assuming the trolleybus has the same route). As shown in Fig. 5 [8], the route is 11.95 km long, having six(6) stations.

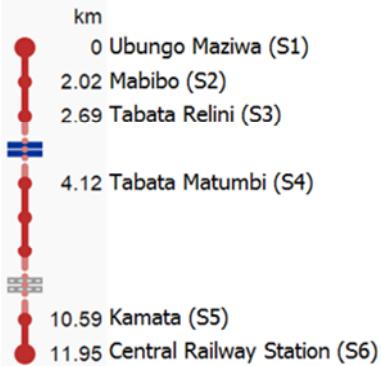


Fig. 5 Ubungo maziwa to Central railway station route

## VI. SIMULATION RESULTS AND DISCUSSION

When the two vehicles (i.e. Tram and Trolleybus) were modeled in such a way that their speed profiles are more or less the same, that is both vehicles accelerate at  $1.2 \text{ ms}^{-2}$  from rest to a maximum speed in acceleration mode, then they maintain the speed in constant speed mode, in the coasting mode the vehicles move by their own momenta, and lastly in braking mode brakes are applied and vehicles decelerate at  $1.2 \text{ ms}^{-2}$  to rest; such that the vehicles cover the same distance, 11.95 km, in 903 seconds as shown in Fig. 6. The tram used *approx.*  $16.7751 \text{ kWh}$  while the trolleybus used *approx.*  $19.7569 \text{ kWh}$  as shown in Fig. 7 and Table 3.

Given passenger carrying capacities in Table 2, the tram used *approx.*  $76.25 \text{ Wh}$  per passenger while the trolleybus used *approx.*  $143.17 \text{ Wh}$  per passenger. That is to say the tram consumes 53 percent of the energy consumed by the trolleybus per passenger. Assuming the electricity cost is  $0.25 \text{ \$/kWh}$ , then the tram uses *approx.*  $0.019 \text{ \$}$  per passenger while the trolleybus uses *approx.*  $0.036 \text{ \$}$  per passenger. That is, the tram costs 53 percent of the cost of the trolleybus per passenger.

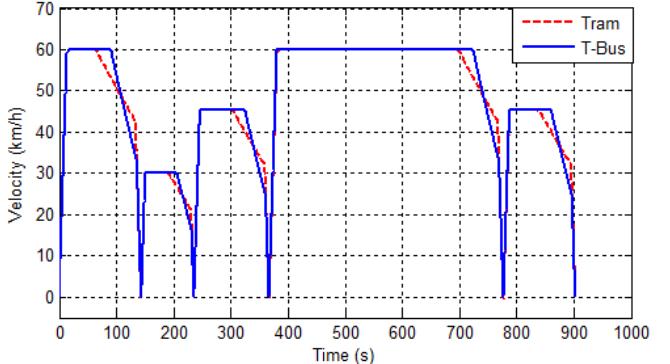


Fig. 6 Tram and Trolleybus speed profiles

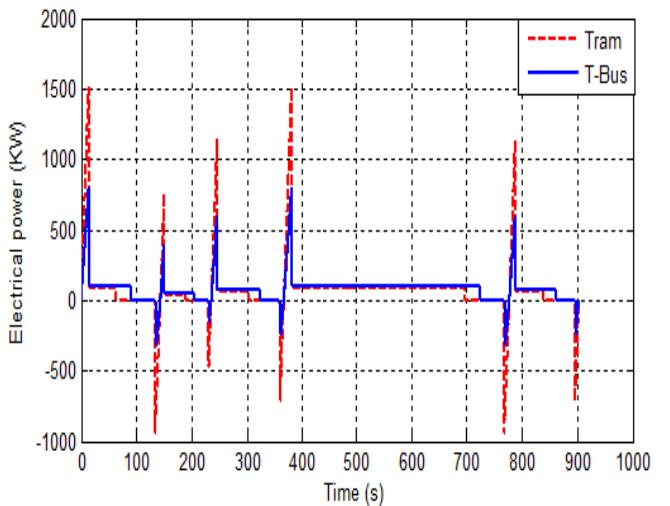


Fig. 7 Tram and Trolleybus power profile

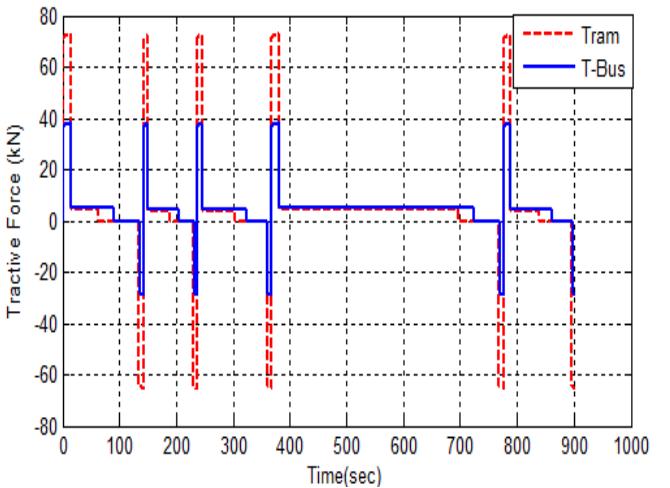


Fig. 8 Tram and Trolleybus Tractive Force Profile

Table 3: Energy consumed between stations

	Energy Consumed (kWh)		Travelling time (s)
	Tram	Trolleybus	
S1 – S2 (2.02 km)	2.8626	3.3782	143.7
S2 – S3 (0.67 km)	0.8357	0.9964	92
S3 – S4 (1.43 km)	1.8880	2.2419	131.1
S4 – S5 (6.47 km)	9.3942	11.0095	410.5
S5 – S6 (1.36 km)	1.7946	2.1309	125.6
<b>Total</b>	<b>16.7751</b>	<b>19.7569</b>	<b>903</b>

Though the tram has higher peak power than the trolleybus as shown in Fig. 7, yet it consumes lesser net energy than the trolleybus. This is due to two main reasons:-

First; despite the tram being heavier yet it has lower rolling resistance than the trolleybus. This gives the tram an advantage over the trolleybus especially when the vehicles are not accelerating. For example; when travelling from station 1 (S1) to station 2 (S2), when the vehicles are in a constant speed mode, the tractive force needed is only to counter balance the forces opposing the motion, the tractive force ( $F_T$ ) is thus given as  $F_T = F_R$  where  $F_R$  is the net force opposing the motion as given in Eq. (2). As assumed the track is flat, with details as shown in Table 2, the traction force needed by the tram is *approx.* 4226 N while that needed by the trolleybus is *approx.* 4937 N as shown in Fig. 8

On the other hand when the vehicles are in acceleration mode, the tractive force ( $F_T$ ) is as given in Eq. (1), and the tractive force needed by the tram is higher than that needed by the trolleybus at any given time with the acceleration of  $1.2 \text{ ms}^{-2}$  as shown in Fig. 8. The trolleybus will in turn consume more energy than the tram in the constant speed mode, and vice versa in the acceleration mode, the energy consumed has to do with not only the tractive force, but also velocity and time, as in Eq. (6)

$$dE = F_T(t) \times v(t) \times dt \quad (6)$$

where  $dE$  and  $dt$  are changes in energy and time respectively,  $F_T(t)$  and  $v(t)$  are tractive force and velocity respectively. In this particular case the constant speed mode last longer, with tram tractive effort lower than that of the trolleybus. This shows that vehicle speed profile has a significant effect on energy consumption.

Second; the fact that the tram being heavier than the trolleybus, with the same maximum velocity, the tram has more kinetic energy than the trolleybus, thus in the coasting mode (when the power is disconnected and a vehicle is moving by its own momentum), the tram has higher kinetic energy and lower coefficient of friction than the trolleybus; it can then travel a longer distance in the coasting mode, and even when the brakes are applied, the tram recovers more energy than the trolleybus. The trolleybus on the other hand has to travel longer in the constant speed mode, and during braking it recovers lesser energy.

## VII. CONCLUSION

In this paper mathematical approach in computing net traction energy consumed by a vehicle travelling from point A (starting point) to point B (stopping point) was highlighted, and traction energy per passenger consumed by the tram was compared to the traction energy per passenger consumed by the trolleybus, both using regenerative braking, and travelling the same distance, assuming the same route. It was found that with their respective details as shown in Table 2, despite the tram being heavier than the trolleybus in terms of weight per passenger, yet by taking the advantage of low coefficient of friction during constant speed mode, and high kinetic energy during coasting mode, the tram was found to consume only 53 percent of the energy consumed by the trolleybus per passenger. That is, the tram costs 53 percent of the cost of the trolleybus per passenger. Thus the tram was a better choice in this particular case

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