

# Computationally Efficient Relational Reinforcement Learning

Mitchell Keith Bloch

University of Michigan  
2260 Hayward Street  
Ann Arbor, MI. 48109-2121  
bazald@umich.edu

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# The Problem from 10,000 Feet

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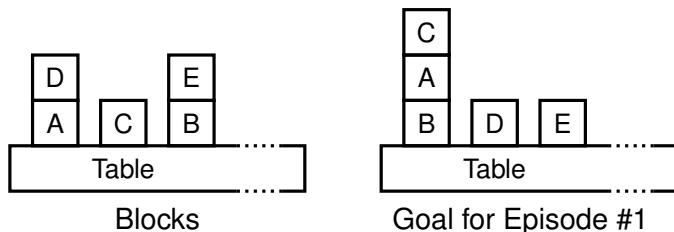
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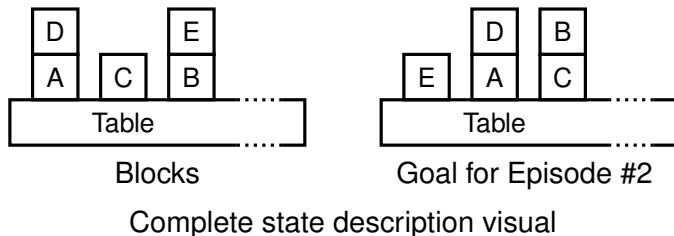
# Blocks World – Objective: **Exact**



Complete state description visual

- 1 Full representation of the goal presented by the environment
- 2 Variable goals and potentially numbers of blocks each episode
- 3 Relatively complex training goal vs
  - **Stack** – Creating a tower out of all the blocks
  - **Unstack** – Placing all blocks on the table
  - **On (a, b)** – Placing one specific block on top of another
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## Greedy-GQ( $\lambda$ )

[Maei and Sutton, 2010]

# Q-functions

TD methods over ...

What is  $Q(s, a)$ ?

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## Tabular RL

$Q(s, a)$  can map each state-action pair to a unique value called a *Q-value*

## Linear Function Approximation

$Q(s, a)$  can map each state-action pair to a sum of *weights* that are shared between different state-action pairs

$$Q(s, a) = \sum_{i=1}^n \phi_i(s, a) w_i$$

# Features?

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## Tile Codings

One answer to this problem is to use tile codings to partition the state-action space

# Tile Coding

## One 8x8 tiling

-1.0	-1.1	-1.2	-1.0	2.1	2.0	3.1	5.1
-0.7	-1.2	-1.0	-1.1	1.8	2.0	2.9	4.1
-2.9	-2.8	-1.0	-1.1	2.0	1.9	2.9	3.2
-4.9	-3.1	-1.2	-0.9	2.1	2.1	2.1	2.0
-3.2	-2.8	0.2	-0.1	0.9	1.0	1.1	1.2
-2.1	-1.8	0.0	0.1	1.0	0.9	1.1	1.0
-1.1	-0.9	0.0	-0.2	0.9	1.0	1.1	0.8
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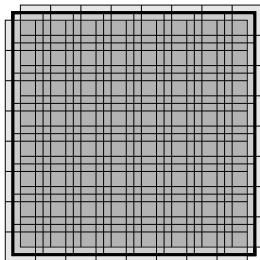


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## Three 8x8 tilings

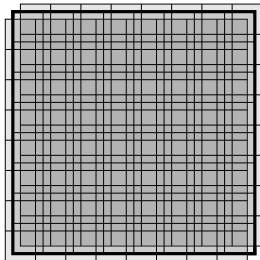


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## Adaptive Tile Coding (ATC)

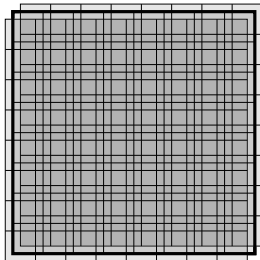
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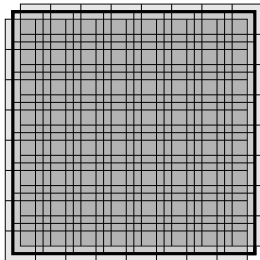
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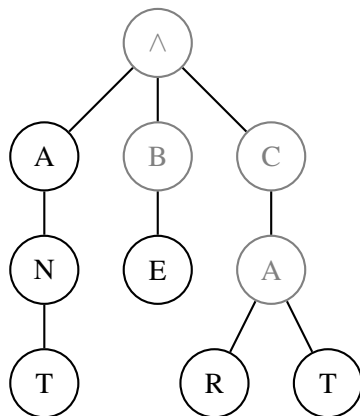
## Learning Efficiency

The offers generalization for states that share a tile

# Implementation: $k$ -Dimensional Tries ( $k$ -d Tries)

## Typical Use

Efficient English dictionary storage and lookup



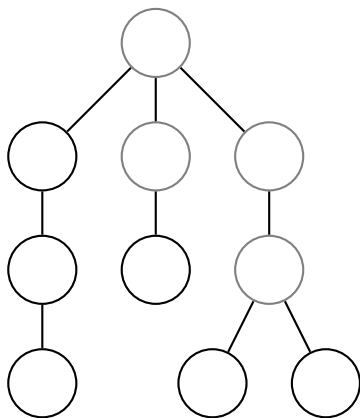
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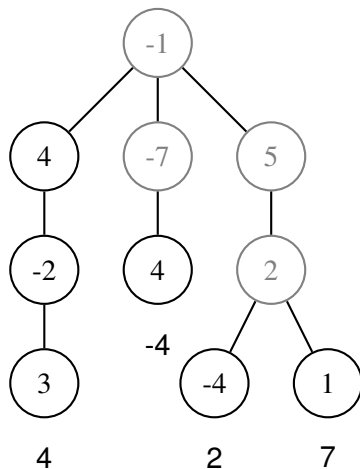
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# Adaptive Tile Coding

## ATC/HTC Trie Mapping

- Less refined tilings correspond to conjunctions of few features
- More refined tilings correspond to conjunctions of many features
- The most refined tilings correspond to *fringe* nodes  
i.e. candidate conjunctions for inclusion in the value function



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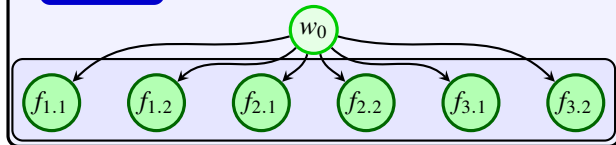
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### aHTC View

$w_0$   
 $f_{1.1}, f_{1.2}, f_{2.1},$   
 $f_{2.2}, f_{3.1}, f_{3.2}$

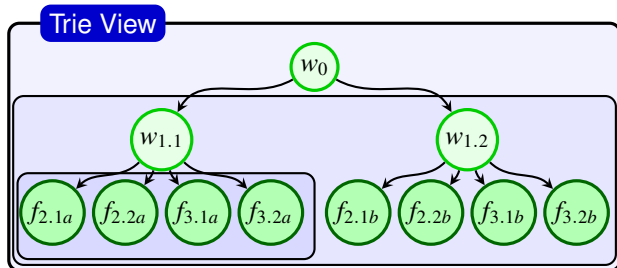
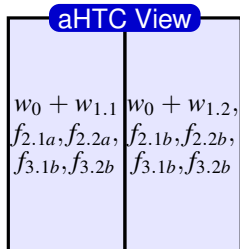
### Trie View



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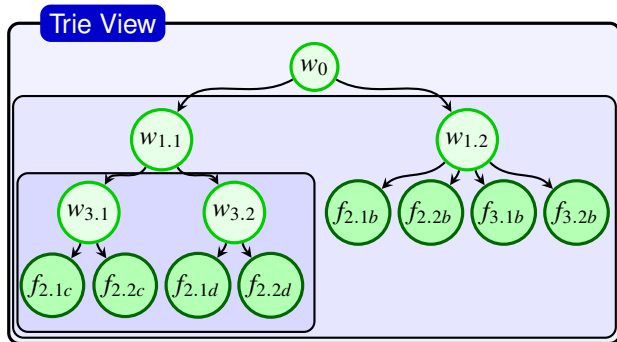


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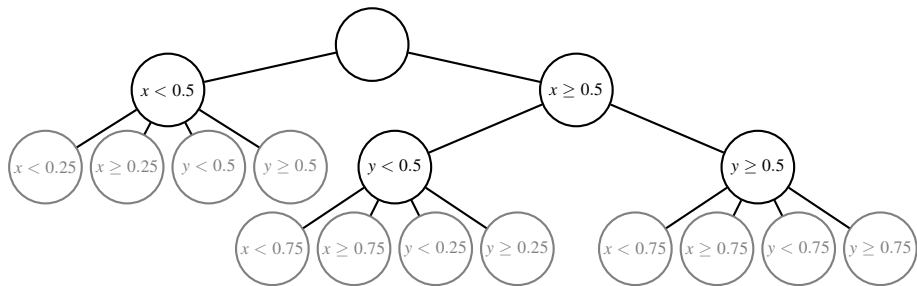
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aHTC View	
$w_0 + w_{1.1}$ $+w_{3.1},$ $f_{2.1c}, f_{2.2c}$	$w_0 + w_{1.2},$ $f_{2.1b}, f_{2.2b},$ $f_{3.1b}, f_{3.2b}$
$w_0 + w_{1.1}$ $w_{3.2},$ $f_{2.1d}, f_{2.2d}$	



# Adaptive $k$ -d Trie Representation



## Concept

First Order Logical Decision Tree (FOLDT)

# Relational Representations

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## Concept

First Order Logical Decision Tree (FOLDT)

## Problem

*k*-d Tries do not effectively support FOLDT implementation due to variable binding problem!

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# Relational Representations

## Concept

First Order Logical Decision Tree (FOLDT)

## Problem

$k$ -d Tries do not effectively support FOLDT implementation due to variable binding problem!

## Solution

I observed that a FOLDT could be embedded in a Rete for efficient RRL implementation

## Feature 2

$\text{on}(b, a)$

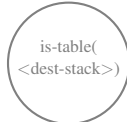
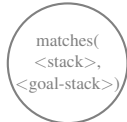
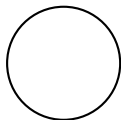
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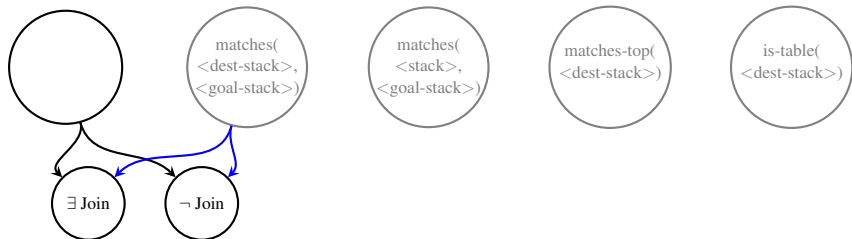
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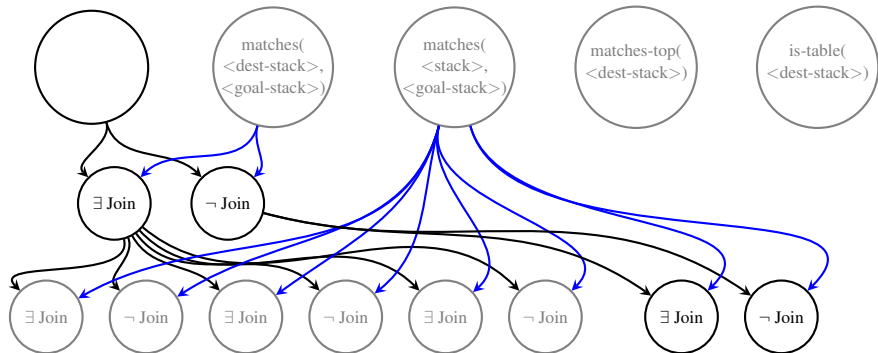
# Adaptive Rete Representation



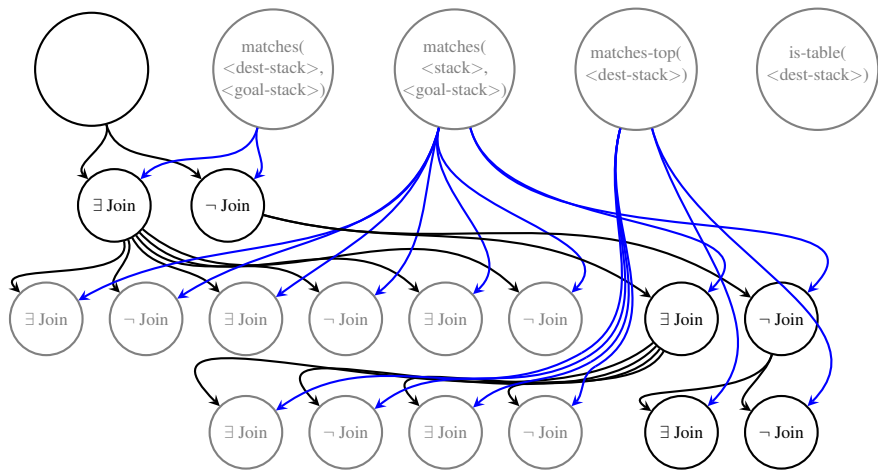
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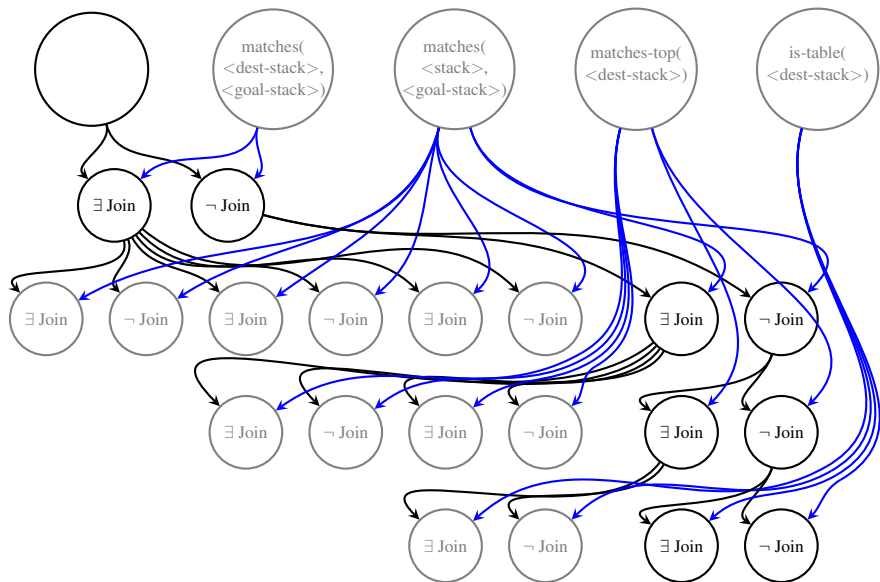
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## Criteria

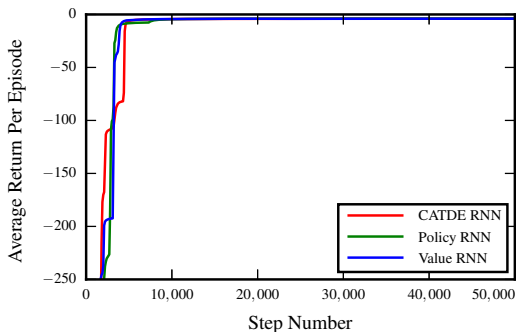
- 1 Cumulative Absolute Temporal Difference Error (CATDE) – Maximal error accumulation
  - Focus on regions of high activity and error.
  - Track TD error experienced at each leaf in the value function.
  - The nodes with highest error are eligible for refinement when their features match.
- 2 Policy – Maximal change in  $\pi(s, a)$ 
  - Focus on modifying policy (Whiteson 2007)
  - Choose features which maximize the change in the greedy set of actions.
- 3 Value – Maximal change in  $Q(s, a)$ 
  - Focus on improving value estimates (Whiteson 2007)
  - Choose features which maximize value spread on refinement.



# Exact with Refinement Only – No Distractors

- “Refinement only” gives a baseline
- All criteria okay with no distractors

## Learning



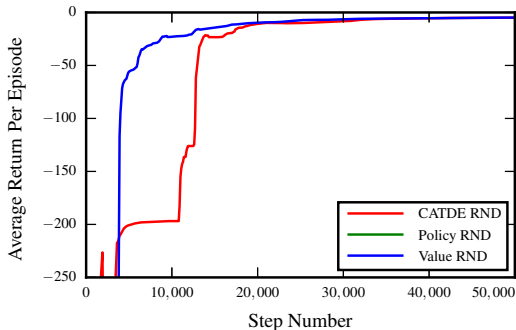
## Average Return Per Episode & Wall Clock Time Per Step

Criterion at 50,000	ARtPE	WCTPS	# Weights
CATDE	-4.06	1.38ms	23.0
Policy Criterion	-3.84	0.91ms	24.7
Value Criterion	-3.97	1.33ms	25.6

# Exact with Refinement Only – With Distractors

- Value criterion does best with distractors
- Number of weights skyrockets
- Policy criterion performance too low to appear

## Learning

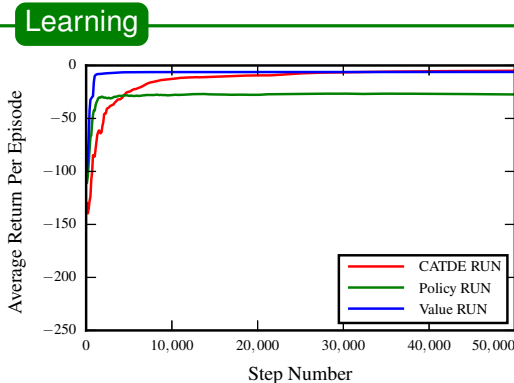


## Average Return Per Episode & Wall Clock Time Per Step

Criterion at 50,000	ARtPE	WCTPS	# Weights
CATDE	-4.93	18.8ms	1,487.8
Policy Criterion	-639	21.7ms	1,318.4
Value Criterion	-4.83	19.0ms	1,459.5

# Exact with Rerefinement – No Distractors

- CATDE does best with unrestricted rerefinement
- Policy does not converge



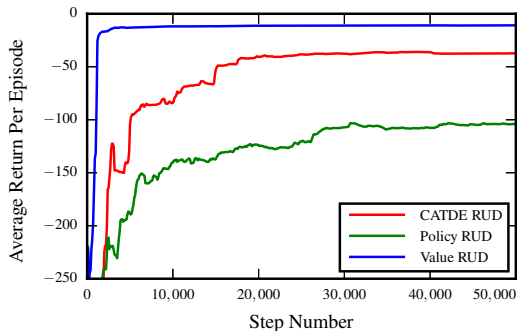
## Average Return Per Episode & Wall Clock Time Per Step

Criterion at 50,000	ARtPE	WCTPS	# Weights
CATDE	-5.03	1.65ms	16.3
Policy Criterion	-27.2	1.06ms	4.1
Value Criterion	-6.13	1.10ms	5.94

# Exact with Rerefinement – With Distractors

- Until you include distractors – then value does best
- Number of weights persistent in the system is low due to thrashing
- Fast execution as a result of few weights

## Learning



## Average Return Per Episode & Wall Clock Time Per Step

Criterion at 50,000	ARtPE	WCTPS	# Weights
CATDE	-37.3	3.27ms	7.58
Policy Criterion	-104	1.79ms	3.15
Value Criterion	-10.9	2.20ms	4.75

# Exact with Rerefinement

## Functionality So Far

- Demonstrated efficacy in absence of distractors
- Demonstrated computationally efficient refinement and rerefinement

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- Incomplete convergence without distractors
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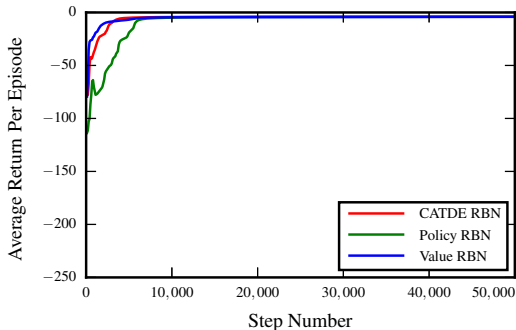
## What's Next

- Demonstrate flexibility of the architecture
- Blacklists
- Boost
- Concrete

# Exact with Rerefinement and Blacklists – No Dist.

- Most obvious strategy to avoid thrashing
- Works fairly well in the absence of distractors

## Learning



## Average Return Per Episode & Wall Clock Time Per Step

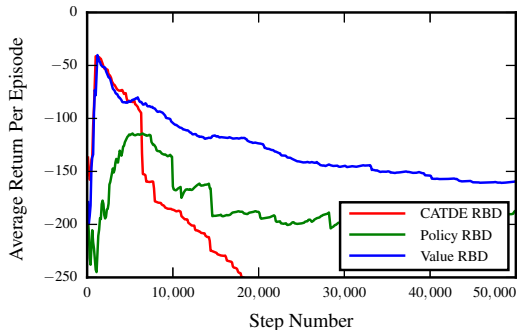
Criterion at 50,000	ARtPE	WCTPS	# Weights
CATDE	-3.94	1.51ms	26.5
Policy Criterion	-4.04	1.20ms	25.9
Value Criterion	-4.13	1.41ms	26.8



# Exact with Rerefinement and Blacklists – w/ Dist.

- Works poorly with distractors
- Best features likely to be tried and blacklisted first
- Converge on worst value function structure

## Learning



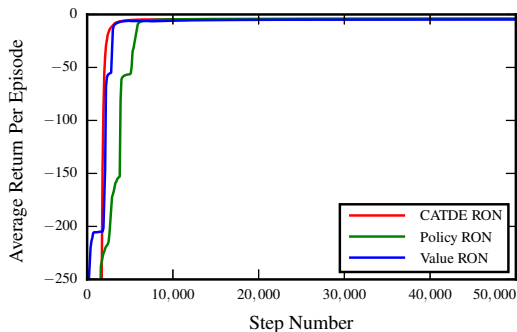
## Average Return Per Episode & Wall Clock Time Per Step

Criterion at 50,000	ARtPE	WCTPS	# Weights
CATDE	-407	4.72ms	30.9
Policy Criterion	-187	8.55ms	87.1
Value Criterion	-159	5.55ms	44.7

# Exact with Rerefinement and Boost – No Dist.

- Gradually increase the likelihood of reselection instead – opposite of blacklists
- Generally a little slower to converge than when using blacklists

## Learning



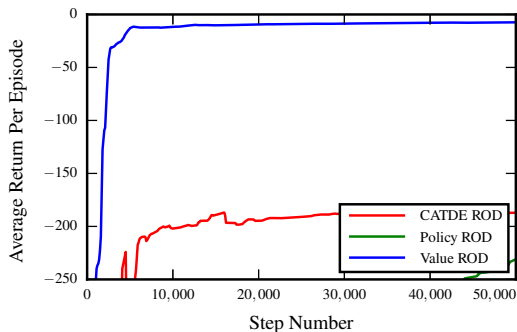
## Average Return Per Episode & Wall Clock Time Per Step

Criterion at 50,000	ARtPE	WCTPS	# Weights
CATDE	-4.14	1.77ms	22.7
Policy Criterion	-3.95	2.22ms	24.8
Value Criterion	-4.78	1.56ms	15.1

# Exact with Rerefinement and Boost – w/ Dist.

- Much slower with distractors
- However, now the value function comes close to converging

## Learning

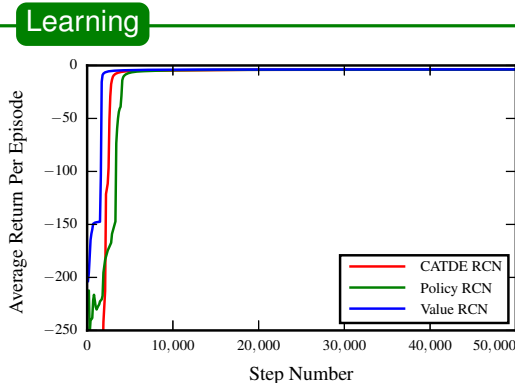


## Average Return Per Episode & Wall Clock Time Per Step

Criterion at 50,000	ARtPE	WCTPS	# Weights
CATDE	-187	12.4 ms	73.2
Policy Criterion	-231	33.5 ms	218
Value Criterion	-7.42	6.91ms	26.2

# Exact with Reref. & Boost & Concrete – No Dist.

- Value criterion gives best performance
- Lowest CPU cost



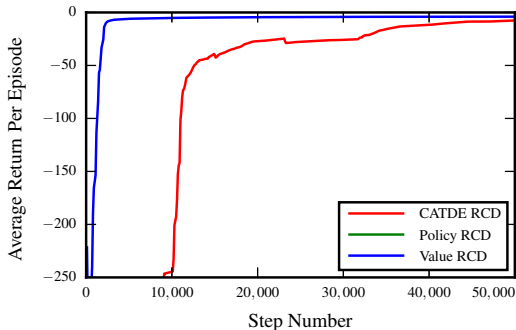
## Average Return Per Episode & Wall Clock Time Per Step

Criterion at 50,000	ARtPE	WCTPS	# Weights
CATDE	-3.99	0.605ms	23.6
Policy Criterion	-3.91	0.645ms	24.0
Value Criterion	-3.71	0.612ms	26.4

# Exact with Reref. & Boost & Concrete – w/ Dist.

- Value gives best performance even with distractors
- Does so with fewer weights and lower CPU cost

## Learning



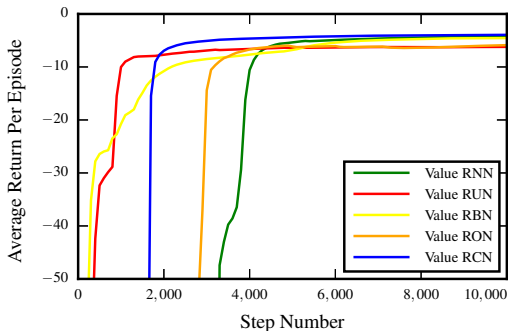
## Average Return Per Episode & Wall Clock Time Per Step

Criterion at 50,000	ARtPE	WCTPS	# Weights
CATDE	-7.60	19.6ms	596
Policy Criterion	-1,100	10.2ms	238
Value Criterion	-3.97	12.3ms	284

## Exact with the Value Criterion – No Dist.

- Unrestricted rerefinement does the best between 1,000 and 3,000 steps
- Rerefinement with boost and concrete overtakes it from 3,000 steps on

### Learning

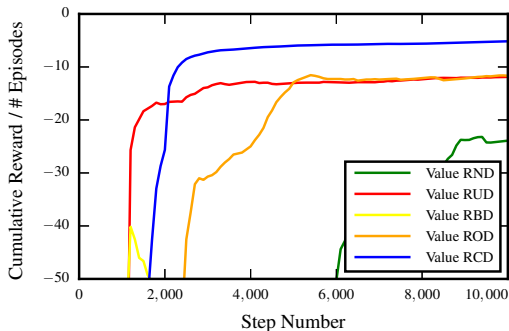


N – No rerefinement  
U – Unrestricted rerefinement  
B – Blacklists  
O – bOost  
C – boost with Concrete

## Exact with the Value Criterion – w/ Dist.

- Unrestricted rerefinement does the best between 1,000 and 2,000 steps
- Rerefinement with boost and concrete overtakes it from 2,000 steps on

### Learning

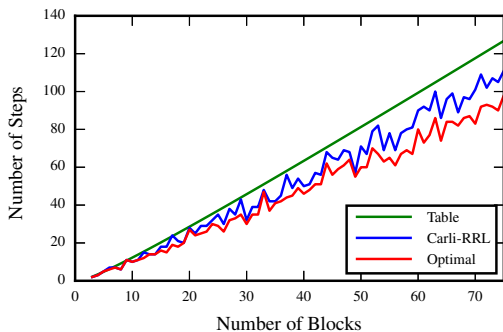


N – No rerefinement  
U – Unrestricted rerefinement  
B – Blacklists  
O – bOost  
C – boost with Concrete

# Policy Optimality Scaling

- The policy learned by my agents with a general solution to **exact** with variable target configurations, compared to
- Optimal calculated with  $A^*$  and
- An expected number of steps for a policy moving all blocks to the table and then into place

## As Blocks Increase





# Nuggets and Coal

## Nuggets

- 1 Successfully embedded an adaptive Hierarchical Tile Coding (aHTC) in a Rete
- 2 Demonstrated architectural flexibility using three different refinement criteria in addition to rerefinement, blacklist, boost, and concrete mechanisms
- 3 A general policy for **exact** that scales for arbitrary numbers of blocks

## Coal

- 1 Computational costs to execute policy are high for hundreds of blocks
- 2 Policy is only approximately optimal, but problem is NP-hard
- 3 I've been a student at U-M almost as long as Kenan Thompson has been a cast member of SNL



Hamid Reza Maei and Richard S Sutton.

Gq ( $\lambda$ ): A general gradient algorithm for temporal-difference prediction learning with eligibility traces.

*In Proceedings of the Third Conference on Artificial General Intelligence*, volume 1, pages 91–96, 2010.