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SY BTech - CHEM

Question Bank

1 LAPLACE TRANSFORM:-

Find the Laplace transforms of each of the following functions:-

1. $2e^{3t} + 3e^{-2t}$ **Ans:** $\frac{2}{s-3} + \frac{3}{s+2}$

2. $(e^{-at} - e^{-bt})^2$ **Ans:** $\frac{1}{s+2a} + \frac{2}{s+a+b} + \frac{1}{s+2b}$

3. $(2e^{3t} + 5)^2$ **Ans:** $\frac{4}{s-6} + \frac{20}{s-3} + \frac{25}{s}$

4. c^{at+b} **Ans:** $\frac{c^b}{s-a\,logc}$

5. $t^2 - 3t + 5$ **Ans:** $\frac{5s^2-3s+2}{s^3}$

6. $t^4 + 5t^3 + t^{\frac{1}{2}}$ **Ans:** $\frac{24}{s^5} + \frac{30}{s^4} + \frac{\sqrt{\pi}}{2s^{3/2}}$

7. $a + \frac{b}{\sqrt{t}}$ **Ans:** $\frac{a}{s} + b\sqrt{\frac{\pi}{s}}$

8. $(t+2)^3 + (e^{2t}+3)^2$
Ans: $\frac{6}{s^4} + \frac{12}{s^3} + \frac{12}{s^2} + \frac{17}{s} + \frac{1}{s-4} + \frac{6}{s-2}$

9. $3\,cos\,2t - sin\,2t$ **Ans:** $\frac{3s}{s^2+4} - \frac{2}{s^2+4}$

10. $cosh\,5t + cos\,5t$ **Ans:** $\frac{s}{s^2-25} + \frac{s}{s^2+25}$

11. $cos\,3t\,cos\,2t$ **Ans:** $\frac{s\,(s^2+13)}{(s^2+1)(s^2+25)}$

12. $sin\,2t\,cos\,5t$ **Ans:** $\frac{2\,(s^2-21)}{(s^2+9)(s^2+49)}$

13. $cos\,(\omega t + \alpha)$ **Ans:** $\frac{s\,cos\,\alpha - \omega\,sin\,\alpha}{s^2+\omega^2}$

14. $sin\,(\omega t + \alpha)$ **Ans:** $\frac{\omega\,cos\,\alpha + s\,sin\,\alpha}{s^2+\omega^2}$

15. $cos^2\,bt$ **Ans:** $\frac{s^2+2\,b^2}{s(s^2+4\,b^2)}$

16. $sin^3\,2t$ **Ans:** $\frac{48}{(s^2+4)(s^2+36)}$

17. $sinh^3\,2t$ **Ans:** $\frac{48}{(s^2-4)(s^2-36)}$

18. $(sin\,t - cos\,t)^2$ **Ans:** $\frac{s^2-2s+4}{s\,(s^2+4)}$

19. $e^{2t} + 4\,t^3 - 2\,sin\,3t + 3\,cos\,3t$
Ans: $\frac{1}{s-2} + \frac{24}{s^4} - \frac{6}{s^2+9} + \frac{3\,s}{s^2+9}$

20. $4\,cos\,2t - 5\,t^2 + 2\,e^{3t}$ **Ans:** $\frac{4\,s}{s^2+4} - \frac{10}{s^3} + \frac{2}{s-3}$

21. $3\,t^4 - 2\,t^3 + 4\,e^{-3t} - 2\,sin\,5t + 3\,cos\,2t$
Ans: $\frac{72}{s^5} - \frac{12}{s^4} + \frac{4}{s+3} - \frac{10}{s^2+25} + \frac{3\,s}{s^2+4}$

22.
$$f(t) = \begin{cases} a, & 0 < t < b \\ 0, & t > b \end{cases}$$
Ans: $\frac{a}{s}\,(1 - e^{-sb})$

23.
$$f(t) = \begin{cases} cos\,t, & 0 < t < 2\pi \\ 0, & t > 2\pi \end{cases}$$
Ans: $\frac{s(1-e^{-2\pi s})}{s^2+1}$

24.
$$f(t) = \begin{cases} cos\,t, & 0 < t < \pi \\ sin\,t, & t > \pi \end{cases}$$
Ans: $\frac{s+(s-1)e^{-\pi s}}{s^2+1}$

25.
$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t^2 - 2t + 2, & t \ge 1 \end{cases}$$
Ans: $e^{-s}\left(\frac{1}{s} + \frac{2}{s^3}\right)$

26.
$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t, & 1 < t < 2 \\ 0, & t > 2 \end{cases}$$

Ans: $e^{-s}\left(\frac{1}{s^2}+\frac{1}{s}\right) - e^{-2s}\left(\frac{1}{s^2}+\frac{2}{s}\right)$

27. $t\,(3\,sin\,2t - 2\,cos\,2t)$ **Ans:** $\frac{8+12s-2s^2}{(s^2+4)^2}$

28. $t\,cos\,(4t+3)$ **Ans:** $\frac{s^2\,cos\,3-8s\,sin\,3-16\,cos\,3}{(s^2+16)^2}$

29. $t^2\,sin\,2t$ **Ans:** $\frac{4\,(3s^2-4)}{(s^2+4)^3}$

30. $t^2\,sinh\,t$ **Ans:** $\frac{6s^2+2}{(s^2-1)^3}$

31. $(t^2-3t+2)\,sin\,3t$ **Ans:** $\frac{6s^4-18s^3+126s^2-162s+432}{(s^2+9)^3}$

32. $t^3\,cos\,t$ **Ans:** $\frac{6s^4-36s^2+6}{(s^2+1)^4}$

33. $\frac{e^{2t}-1}{t}$ **Ans:** $log\frac{s}{s-2}$

34. $\frac{1-cos\,3t}{t}$ **Ans:** $log\frac{\sqrt{s^2+9}}{s}$

35. $\frac{cos\,2t-cos\,3t}{t}$ **Ans:** $\frac{1}{2}\,log\,\frac{s^2+9}{s^2+4}$

36. $t\,e^{3t}\,cos\,2t$ **Ans:** $\frac{s^2-6s+5}{(s^2-6s+13)^2}$

37. $\int_0^t \frac{e^t-cos\,2t}{t}\,dt$ **Ans:** $\frac{1}{s}\,log\,\frac{\sqrt{s^2+4}}{(s-1)}$

38. $\frac{e^{-3t}\,sin\,2t}{t}$ **Ans:** $cot^{-1}\,\frac{s+3}{2}$

39. $e^{-3t}\int_0^t \frac{sin\,2t}{t}\,dt$ **Ans:** $\frac{1}{s+3}\,cot^{-1}\,\frac{s+3}{2}$

40. $\int_0^\infty t\,e^{-2t}\,cos\,t\,dt$ **Ans:** $\frac{3}{25}$

41. $\int_0^\infty \frac{e^{-t}-e^{-3t}}{t}\,dt$ **Ans:** $\log 3$

42. $\int_0^\infty \frac{cos\,3t - cos\,2t}{t}\,dt$ **Ans:** $log\,\frac{2}{3}$

43. $\int_0^\infty \frac{1-cos\,t}{t^2}\,dt$ **Ans:** $\frac{\pi}{2}$

44. $\int_0^\infty e^{-2t}\,\frac{sinh\,t\,sin\,t}{t}\,dt$ **Ans:** $\frac{1}{2}\,tan^{-1}\,\frac{1}{2}$

45. $\int_0^\infty \frac{sin^3 t}{t}\,dt$ **Ans:** $\frac{\pi}{4}$

46. $\int_0^\infty e^{-t}\,\frac{1-cos\,t}{t}\,dt$ **Ans:** $\frac{1}{2}\,log\,2$

47. t^4 U(t - 2) **Ans:** $e^{-2s}\left(\frac{24}{s^5} + \frac{48}{s^4} + \frac{48}{s^3} + \frac{32}{s^2} + \frac{16}{s}\right)$

48. $(1 + 2t - 3t^2 + 4t^3)$ U(t - 2)
Ans: $e^{-2s}\left(\frac{25}{s} + \frac{38}{s^2} + \frac{42}{s^3} + \frac{24}{s^4}\right)$

49. $sin\,t$ U(t - 4) **Ans:** $e^{-4s}\left(\frac{cos\,4 + s\,sin\,4}{s^2+1}\right)$

50. $e^{-t}\,sin\,t$ U(t - π) **Ans:** - $e^{-\pi(s+1)}\,\frac{1}{(s+1)^2+1}$

51. $cos\,(t-1)$ U(t - 1) **Ans:** $e^{-s}\,\frac{s}{s^2+1}$

52. $(t-\pi)$ U(t - π) **Ans:** $e^{-\pi}\,\frac{1}{s^2}$

53. $sin\,2t\,\delta(t-2)$ **Ans:** $e^{-2s}\,sin\,4$

54. $t^4\,\delta(t-5)$ **Ans:** $e^{-5s}\,5^4$

55. $sin\,2t\,\delta(t-\frac{\pi}{4}) - t^2\,\delta(t-4)$ **Ans:** $e^{-\frac{\pi s}{4}} - 16\,e^{-4s}$

56. t U(t - 4) - $t^3\,\delta(t-2)$
Ans: $e^{-4s}\left(\frac{1}{s^2} + \frac{4}{5}\right) - 8\,e^{-2s}$

57. t^2 U(t - 2) - cosh t $\delta(t-4)$
Ans: $2\,e^{-2s}\left(\frac{1+2s+2s^2}{s^3}\right) - e^{-4s}\,cosh\,4$

58. $t\,e^{-2t}\,\delta(t-2)$ **Ans:** $2\,e^{-2(s+2)}$

59. $e^{-at}\,sin\,bt$ **Ans:** $\frac{b}{(s+a)^2+b^2}$

60. $e^{-at}\,cos\,bt$ **Ans:** $\frac{s+a}{(s+a)^2+b^2}$

61. $e^{-at}\,sinh\,bt$ **Ans:** $\frac{b}{(s+a)^2-b^2}$

62. $e^{-at}\,cosh\,bt$ **Ans:** $\frac{s+a}{(s+a)^2-a^2}$

63. $e^{-at}\,t^n$ **Ans:** $\frac{\Gamma(n+1)}{(s+a)^{n+1}}$

64. $e^{4t}\,cosh\,5t$ **Ans:** $\frac{s-4}{s^2-8s-9}$

65. $(t+2)^2\,e^{4t}$ **Ans:** $\frac{4s^2-28s+50}{(s-4)^3}$

66. $e^{-2t}\,(3\,cos\,6t-5\,sin\,6t)$ **Ans:** $\frac{3s-24}{s^2+4s+40}$

67. $e^{-3t}\,sin^2t$ **Ans:** $\frac{2}{(s+3)(s^2+6s+13)}$

68. $cosh\,at\,sin\,at$ **Ans:** $\frac{a\,(s^2+2a^2)}{s^4+4a^4}$

69.
$$F(t)=\begin{cases} cos\,(t-\frac{2\pi}{3}), & t>\frac{2\pi}{3} \\ 0, & t<\frac{2\pi}{3} \end{cases}$$
Ans: $e^{\frac{-2\pi s}{3}}\left(\frac{s}{s^2+1}\right)$

70.
$$F(t)=\begin{cases} (t-1)^3, & t>1 \\ 0, & t<1 \end{cases}$$
Ans: $e^{-s}\left(\frac{6}{s^4}\right)$

71.
$$F(t)=\begin{cases} e^{-4(t-3)}\,sin\,3(t-3), & t>3 \\ 0, & t<3 \end{cases}$$
Ans: $e^{-3s}\left(\frac{3}{s^2+8s+25}\right)$

72.
$$F(t)=\begin{cases} sin\,2(t-\pi), & t>\pi \\ 0, & t<\pi \end{cases}$$
Ans: $e^{-\pi s}\left(\frac{2}{s^2+4}\right)$

73. Obtain the Laplace transforms of

    (a) $\frac{d^5x}{dt^5}$

    (b) $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 5y$

    given that y(0)=2 and y'(0)= -4.

74. Obtain the Laplace transform of y(t), if $\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 4y = t$, given that y(0)=y'(0)=y"(0)=1.

    Ans: $\frac{1}{s-1} + \frac{1}{s^2(s-1)(s^2+4)}$

75. $\frac{t \sin at}{2a}$ **Ans:** $\frac{s}{(s^2+a^2)^2}$

76. $\frac{1}{2a^3}(\sin at - at\cos at)$ **Ans:** $\frac{1}{(s^2+a^2)^2}$

77. $\frac{1}{2a}(\sin at + at\cos at)$ **Ans:** $\frac{s^2}{(s^2+a^2)^2}$

78. $t^2 \sin 4t$ **Ans:** $\frac{24s^2-128}{(s^2+16)^3}$

79. $t^2 \cos at$ **Ans:** $\frac{2s(s^2-3a^2)}{(s^2+a^2)^3}$

80. $t^3 e^{2t}$ **Ans:** $\frac{6}{(s-2)^4}$

81. Find the Laplace transform of $\frac{\sin at}{t}$ and hence show that $\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$
    Ans: $\cot^{-1}\frac{s}{a}$

82. $\frac{e^{-at}-e^{-bt}}{t}$ **Ans:** $\log\frac{s+b}{s+a}$

83. $\frac{\cos at-\cos bt}{t}$ **Ans:** $\frac{1}{2}\log\frac{s^2+a^2}{s^2+b^2}$

84. $e^{-3t}\,t^{-\frac{1}{2}}$ **Ans:** $\sqrt{\frac{\pi}{s+3}}$

85. $e^{-2t}(4\cosh 3t - 2\sinh t)$
    Ans: $\frac{4(s+2)}{s^2+4s-5} - \frac{2}{s^2+4s+3}$

86. $e^{-at}\,\frac{t^{n-1}}{(n-1)!}$ **Ans:** $\frac{1}{(s+a)^n}$

87.
$$F(t) = \begin{cases} (t-1)^2, & t > 1 \\ 0, & 0 < t < 1 \end{cases}$$
Ans: $e^{-s}\left(\frac{2}{s^3}\right)$

88. If L [f(t)]=$\frac{1}{s}\,e^{-1/s}$, find L $[e^{-t}\,f(3t)]$.
Ans: $\frac{e^{-3/s+1}}{s+1}$

89. $\int_0^t sin\,2u\,du$ **Ans:** $\frac{1}{s}\left(\frac{2}{s^2+4}\right)$

90. $t\,cos\,at$ **Ans:** $\frac{s^2-a^2}{(s^2+a^2)^2}$

91. $\frac{t\,sinh\,at}{2a}$ **Ans:** $\frac{s}{(s^2-a^2)^2}$

92. $t\,sin^3 t$ **Ans:** $\frac{3s}{2}\left[\frac{1}{(s^2+1)^2} - \frac{1}{(s^2+9)^2}\right]$

93. $\frac{1-cos\,t}{t}$ **Ans:** $\frac{1}{2}\,log\,\frac{s^2+1}{s^2}$

94. $\frac{1-cos\,t}{t^2}$ **Ans:** $\frac{1}{2}\,s\,log\,\frac{s^2}{s^2+1} + cot^{-1}\,s$

95. $\frac{sin^2\,t}{t^2}$ **Ans:** $\frac{1}{4}\,s\,log\,\frac{s^2}{s^2+4} + cot^{-1}\,\frac{s}{2}$

96. $t\,e^{3t}\,sin\,2t$ **Ans:** $\frac{4\,(s-3)}{(s^2-6s+10)^2}$

97. $t\,e^{-2t}\,(2\,cosh\,3t - 4\,sinh\,2t)$
Ans: $\frac{2s^2+8s+26}{(s^2+4s-5)^2} - \frac{16\,(s+2)}{(s^2+4s)^2}$

98. $\frac{d}{dt}\left(\frac{sin\,t}{t}\right)$ **Ans:** $s\,cot^{-1}\,s - 1$

99. $\int_0^t\ \frac{sin\,t}{t}\,dt$ **Ans:** $\frac{1}{s}\,cot^{-1}\,s$

100. $\int_0^t\ \frac{1-e^{-x}}{x}\,dx$ **Ans:** $\frac{1}{s}\,log\,\frac{s+1}{s}$

101. $\int_0^t\ u\,cosh\,u\,du$ **Ans:** $\frac{1}{s}\left(\frac{s^2+1}{(s^2-1)^2}\right)$

102. $t \int_0^t e^{-4t} \sin 3t \, dt$ **Ans:** $\frac{3\,(3s^2+16s+25)}{s^2\,(s^2+8s+25)^2}$

103. $e^{-4t} \int_0^t t \sin 3t \, dt$ **Ans:** $\frac{6}{(s^2+8s+25)^2}$

104. $\int_0^t t\, e^{-4t} \sin 3t \, dt$ **Ans:** $\frac{1}{s} \frac{6\,(s+4)}{(s^2+8s+25)^2}$

105. $\cosh t \int_0^t e^t \cosh t \, dt$
Ans: $\frac{1}{2}\left(\frac{s-2}{(s-1)^2(s-3)} + \frac{s}{(s+1)^2(s-1)}\right)$

106. $\frac{e^{-4t} \sin 3t}{t}$ **Ans:** $\cot^{-1} \frac{s+4}{3}$

107. $\int_0^t \frac{e^{-4t} \sin 3t}{t} \, dt$ **Ans:** $\frac{1}{s} \cot^{-1} \frac{s+4}{3}$

108. $e^{-4t} \int_0^t \frac{\sin 3t}{t} \, dt$ **Ans:** $\frac{1}{s+4} \cot^{-1} \frac{s+4}{3}$

2 INVERSE LAPLACE TRANSFORM:-

Find the inverse Laplace transforms of each of the following functions:-

1. $\frac{5}{s+3}$ **Ans:** $5\, e^{-3t}$
2. $\frac{1}{2\,s-3}$ **Ans:** $\frac{1}{2}\, e^{(3/2)t}$
3. $\frac{4}{3\,s-1}$ **Ans:** $\frac{4}{3}\, e^{(1/3)t}$
4. $\frac{2\,s+1}{s\,(s+1)}$ **Ans:** $1 + e^{-t}$
5. $\frac{2}{s^2+16}$ **Ans:** $\frac{\sin 4t}{2}$
6. $\frac{4s}{s^2-16}$ **Ans:** $4 \cosh 4t$
7. $\frac{2s-5}{4s^2+25}$ **Ans:** $\frac{1}{2}\left(\cos \frac{5t}{2} - \sin \frac{5t}{2}\right)$
8. $\frac{3s-12}{s^2+8}$ **Ans:** $3 \cos (2\sqrt{2})t - 3\sqrt{2} \sin (2\sqrt{2})t$
9. $\frac{s-4}{s^2-4}$ **Ans:** $\cosh 2t - 2 \sinh 2t$

10. $\frac{s\cos\alpha+\omega\sin\alpha}{s^2+\omega^2}$ **Ans:** $\cos(\omega t-\alpha)$

11. $\frac{a_1}{s}+\frac{a_2}{s^2}+\frac{a_3}{s^3}$ **Ans:** $a_1+a_2\,t+\frac{a_3\,t^2}{2}$

12. $\frac{1}{s^4}$ **Ans:** $\frac{t^3}{6}$

13. $\frac{3\,(s^2-1)^2}{2\,s^5}$ **Ans:** $\frac{3}{2}\left(1-t^2+\frac{t^4}{24}\right)$

14. $\frac{3}{s+2}-\frac{2s}{s^2+25}+\frac{3}{s^2+9}$
Ans: $3\,e^{-2t}-2\cos 5t+\sin 3t$

15. $\frac{5s+4}{s^3}-\frac{2s-18}{s^2+9}+\frac{24-30\sqrt{s}}{s^4}$
Ans: $5t+2t^2-2\cos t+6\sin 3t+4t^3-\frac{16\,t^{5/2}}{\sqrt{\pi}}$

16. $\frac{6}{2s-3}-\frac{3+4s}{9s^2-16}+\frac{8-6s}{16s^2+9}$
Ans: $3\,e^{3t/2}-\frac{1}{4}\sinh\frac{4t}{3}-\frac{4}{9}\cosh\frac{4t}{3}+\frac{2}{3}\sin\frac{3t}{4}-\frac{3}{8}\cos\frac{3t}{4}$

17. $\frac{1}{(s+4)^6}$ **Ans:** $e^{-4t}\,\frac{t^5}{120}$

18. $\frac{s}{(s-3)^5}$ **Ans:** $e^{3t}\,t^3\left(\frac{1}{6}+\frac{t}{8}\right)$

19. $\frac{3s+1}{(s+1)^4}$ **Ans:** $e^{-t}\,t^2\left(\frac{3}{2}-\frac{t}{3}\right)$

20. $\frac{s}{s^2+6s+25}$ **Ans:** $e^{-3t}\left(\cos 4t-\frac{3}{4}\sin 4t\right)$

21. $\frac{s}{a^2s^2+b^2}$ **Ans:** $\frac{1}{a^2}\cos\left(\frac{bt}{a}\right)$

22. $\cot^{-1}s$ **Ans:** $\frac{\sin t}{t}$

23. $\log\left(\frac{s+b}{s+a}\right)$ **Ans:** $\frac{e^{-at}-e^{-bt}}{t}$

24. $\frac{s}{(s^2+a^2)^2}$ **Ans:** $\frac{t\sin at}{2a}$

25. $\frac{2s}{(s^2-4)^2}$ **Ans:** $\frac{t\sinh 2t}{2}$

26. $\frac{2s+1}{(s^2+s+1)^2}$ **Ans:** $\frac{2t}{\sqrt{3}}\,e^{-t/2}\sin\frac{\sqrt{3}}{2}t$

27. $\frac{s}{s^2+4}$ **Ans:** $\cos 2t$

28. $\frac{s}{s^2-4}$ **Ans:** $cosh\,2t$

29. $\frac{1}{s\,(s+2)}$ **Ans:** $\frac{1-e^{-2t}}{2}$

30. $\frac{1}{s\,(s^2+4)}$ **Ans:** $\frac{1}{2}\,sin^2 t$

31. $\frac{1}{s^3\,(s^2+1)}$ **Ans:** $\frac{t^2}{2} + cos\,t - 1$

32. $\frac{1}{s(s^2+a^2)}$ **Ans:** $\frac{1-cos\,at}{a^2}$

33. $\frac{s^2}{(s^2+a^2)(s^2+b^2)}, a \neq b$ **Ans:** $\frac{a\,sin\,at - b\,sin\,bt}{a^2-b^2}$

34. $\frac{1}{(s+1)(s^2+1)}$ **Ans:** $\frac{1}{2}\,(sin\,t - cos\,t + e^{-t})$

35. $\frac{3s+7}{s^2-2s-3}$ **Ans:** $4\,e^{3t} - e^{-t}$

36. $\frac{11s^2-2s+5}{(s-2)(2s-1)(s+1)}$ **Ans:** $5\,e^{2t} - \frac{3}{2}\,e^{t/2} + 2\,e^{-t}$

37. $\frac{3s+5}{4s^2+12s+9}$ **Ans:** $\frac{e^{-3t/2}}{4}\,(3 + \frac{t}{2})$

38. $\frac{2s+5}{s^2+4s+13}$ **Ans:** $e^{-2t}\,(2\,cos\,3t + \frac{1}{3}\,sin\,3t)$

39. $\frac{s+7}{s^2+2s+2}$ **Ans:** $e^{-t}\,(cos\,t + 6\,sin\,t)$

40. $cot^{-1}(\frac{s-2}{3})$ **Ans:** $\frac{e^{2t}\,sin\,3t}{t}$

41. $log\,(1 + \frac{a^2}{s^2})$ **Ans:** $\frac{2\,(1-cos\,at)}{t}$

3 Z - TRANSFORM:-

Find the Z - transforms of each of the following functions:-

1. $2^k,\ k \geq 0$ **Ans:** $\frac{z}{z-2}$
2. $(\frac{1}{3})^k,\ k \geq 0$ **Ans:** $\frac{z}{z-\frac{1}{3}}$
3. $(\frac{1}{4})^{|k|},\ \forall k$ **Ans:** $\frac{1}{4}\,\frac{z}{1-\frac{z}{4}} + \frac{z}{z-\frac{1}{4}}$

4. $\frac{1}{k},\ k \geq 1$ **Ans:** $-log\,(1-\frac{1}{z})$

5. $\frac{a^k}{k},\ k \geq 1$ **Ans:** $-log\,(1-\frac{a}{z})$

6. $\frac{2^k}{k!},\ k \geq 0$ **Ans:** $e^{2/z}$

7. $e^{-ak},\ k \geq 0$ **Ans:** $\frac{z}{z-e^{-a}}$

8. $\frac{sin\,ak}{k},\ k > 0$ **Ans:** $cot^{-1}\,(\frac{z-cos\,a}{sin\,a})$

9. $\frac{2^k}{k},\ k \geq 1$ **Ans:** $-log\,(1-\frac{2}{z})$

10. $sin\,(\frac{k\pi}{4}+\alpha),\ k \geq 0$ **Ans:** $\frac{z}{\sqrt{2}}\left(\frac{cos\,\alpha+sin\,\alpha\,(\sqrt{2}\,z-1)}{z^2-\sqrt{2}\,z+1}\right)$

11. $cos\,(\frac{k\pi}{4}+\alpha),\ k \geq 0$ **Ans:** $\frac{z}{\sqrt{2}}\left(\frac{cos\,\alpha\,(\sqrt{2}\,z-1)-sin\,\alpha}{z^2-\sqrt{2}\,z+1}\right)$

12. $e^{-ak}\,cos\,bk,\ k \geq 0$ **Ans:** $\frac{z\,(z-e^{-a}\,cos\,b)}{z^2-(2\,e^{-a}\,cos\,b)\,z+e^{-2a}}$

13. $e^{-ak}\,sin\,bk,\ k \geq 0$ **Ans:** $\frac{z-e^{-a}\,sin\,b}{z^2-(2\,e^{-a}\,cos\,b)\,z+e^{-2a}}$

14. $2^k\,cos\,(3k+2),\ k \geq 0$ **Ans:** $\frac{z(z\,cos\,2-2\,cos\,1)}{z^2-4\,z\,cos\,3+4}$

15. $4^k\,sin\,(2k+3),\ k \geq 0$ **Ans:** $\frac{z\,(z\,sin\,3-4\,sin\,1)}{z^2-8z\,cos\,2+16}$

16. $3^k\,sinh\,\alpha k,\ k \geq 0$ **Ans:** $\frac{z\,sinh\,\alpha}{z^2-6z\,cosh\,\alpha+9}$

17. $2^k\,cosh\,\alpha k,\ k \geq$ **Ans:** $\frac{z\,(z-2\,cosh\,\alpha)}{z^2-4z\,cosh\,\alpha+4}$

18. k, $k \geq 0$ **Ans:** $\frac{z}{(z-1)^2}$

19. $k\,5^k,\ k \geq 0$ **Ans:** $\frac{5\,z}{(z-5)^2}$

20. $(k+1)\,a^k,\ k \geq 0$ **Ans:** $\frac{z^2}{(z-a)^2}$

21. $k^2\,e^{-ak},\ k \geq 0$ **Ans:** $\frac{z\,e^{-a}\,(e^{-a}+z)}{(z-e^{-a})^3}$

22. $k^2\,a^{k-1},\ k \geq 0$ **Ans:** $\frac{z\,(z+a)}{(z-a)^3}$

23. $k^2\,a^{k-1}\,U(k-1)$ **Ans:** $\frac{z\,(z+a)}{(z-a)^3}$

4 INVERSE Z - TRANSFORM:-

Find the inverse Z - transforms of each of the following functions:-

1. $\frac{z}{z-a}$, $|z| > |a|$ **Ans:** a^k, $k \geq 0$
2. $\frac{z}{z-a}$, $|z| < |a|$ **Ans:** $-a^k$, $k < 0$
3. $\frac{1}{z-a}$, $|z| > |a|$ **Ans:** a^{k-1}, $k \geq 1$
4. $\frac{1}{z-a}$, $|z| < |a|$ **Ans:** $-a^{k-1}$, $k \leq 0$
5. $\frac{1}{(z-a)^2}$, $|z| < a$ **Ans:** $\frac{-k+1}{a^{-k+2}}$, $k \leq 0$
6. $\frac{1}{(z-5)^3}$, $|z| > 5$ **Ans:** $\frac{(k-2)(k-1)}{2}\, 5^{k-3}$, $k \geq 3$
7. $\frac{1}{(z-\frac{1}{2})(z-\frac{1}{3})}$, $\frac{1}{3} < |z| < \frac{1}{2}$
   Ans: $-12\,(2^{-k}) - \frac{6}{3^{k-1}}$
8. $\frac{z}{(z-1)(z-2)}$, $|z| \geq 2$ **Ans:** $2^k - 1$, $k > 0$
9. $\frac{z}{(z-\frac{1}{4})(z-\frac{1}{5})}$, $|z| > \frac{1}{4}$
   Ans: $20\,(\frac{1}{4})^k - 20\,(\frac{1}{5})^k$, $k \geq 0$
10. $\frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{3})}$, $|z| > \frac{1}{2}$
    Ans: $3\,(\frac{1}{2})^k - 2\,(\frac{1}{3})^k$, $k \geq 0$
11. $\frac{z^3}{(z-1)(z-\frac{1}{2})^2}$, $|z| > 1$
    Ans: $4 - (k+3)\,(\frac{1}{2})^k$, $k \geq 0$
12. $\frac{z(z+1)}{z^2-2z+1}$, $|z| > 1$ **Ans:** $1 + 2k$, $k \geq 0$