AIDS

Practice Questions
Unit - 2

* Examples on One dimensional Heat eqn :- (Partial Differential Equation)

*) Solve the one-dimensional heat flow equation.

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$ for the function

$u(x,t)$ subject to following condns.

1) i) $u(0,t)=0 \; \forall \; t$

ii) $u(l,t)=0 \; \forall \; t$

iii) $u(x,0)= x \; ; \; \forall \; x \in (0,l)$.

iv) $u(x,\infty)$ is finite.

2) i) $u(0,t)=0 \; \forall \; t$

ii) $u(20,0)=0 \; \forall \; t$

iii) $u(x,0) = 20x - x^2 \; ; \forall x \in (0,20)$

iv) $u(x,t)$ is finite.

3) i) $u(0,t)=0 \; \forall \; t$

ii) $u(\pi,t)=0 \; \forall \; t$.

iii) $u(x,0) = \pi x - x^2 \; ; \; 0 \le x \le \pi$.

iv) u is finite $\forall \; t$.

4) i) $u(x,t)$ is bounded

ii) $u(0,t)=0 \quad \forall\, t$

iii) $u(l,t)=0 \quad \forall\, t$

iv) $u(x,0)=\frac{u_0 x}{l}$; $0 \le x \le l$.

5) i) $u(x,t)$ is bounded

ii) $u(0,t)=0 \quad \forall\, t$.

iii) $u(1,t)=0 \quad \forall\, t$

iv) $u(x,0)=x(1-x)$; $0 \le x \le 1$.

6) A homogenous rod of conducting material of length 4 m has its ends kept at zero temperature and the temperature initially is,

$u(x,0)=2x$; $0 \le x \le 4$.

Find the temperature $u(x,t)$ at any time.

7) Solve the equation $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$,

subject to the cond^{ns}:

i) $u(x,0)=1$; $0<x<\pi$.

ii) $u(0,t)=u(\pi,t)=0 \quad \forall\, t$.

iii) $u(x,t)$ is finite.

find $u(x,t)$ at any time t.

8) Solve the eq^n $\frac{\partial u}{\partial t} = 1 \frac{\partial^2 u}{\partial x^2}$ with

boundary cond^{ns}.

$u(x,0) = 3 \sin n\pi x, \quad 0 \le x \le 1.$

$u(0,t) = 0 \quad \forall\ t$

$u(1,t) = 0 \quad \forall\ t$

where $u(x,t)$ is finite.

Find $u(x,t)$ as any time t.

9) Solve the eq^n $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$;

subject to following cond^{ns}.

i) $u(x,0) = u_0 \sin^3\left(\frac{\pi x}{l}\right) \quad \forall x \in (0,l)$.

ii) $u(0,t) = 0 \quad \forall t$

iii) $u(l,t) = 0 \quad \forall t$

iv) $u(x,\infty)$ is bounded.

Find $u(x,t)$ at any time t.

10) A homogenous rod of conducting material of length "l" has its ends kept at zero temperature and temperature of the rod initially is $u(x,0) = a \sin\frac{\pi x}{l}$.

Find the temperature $u(x,t)$ at any time t.

Practice Questions Unit-2

Examples on One-dimensional Wave Eqⁿ:-
(Partial Differential Equations).

1) A flexible string of length π is tightly stretched between $x=0$ and $x=\pi$, on x-axis, its ends being fixed at these points. When set into small transverse vibration, the displacement $y(x,t)$ from x-axis of any point x at time t is given by

$$\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2}$$

. Find the solution of the equation which satisfies

i) $y(0,t)=0$ ii) $y(\pi,t)=0$

iii) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$ ~~iv) y(x,0) = 0.1 sin x + 0.01~~

iv) $y(x,0) = 0.1\sin x + 0.01\sin 4x;\ x\in[0,\pi]$

2) If $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, represents the vibrations of a string of length $l=\pi$ fixed at both ends find the solution with boundary conditions,

i) $u(0,t)=0\ \forall t$; ii) $u(\pi,t)=0\ \forall t$.

and initial conditions,

iii) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$

iv) $u(x,0) = 0.01(\pi - x)$.

3) A string is stretched and fastened to two points "l" apart. Motion is started by displacing the string in the form $u = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t=0$. Find the displacement $u(x,t)$ from one end. (wave eqn: $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$)

4) A tightly stretched string with fixed end points $x=0$ and $x=l$. is initially in a position given by $u(x,0) = u_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this poistion, find the displacement u at any distance x from one end at any time t.

5) A string is stretched and fastened to two points distance l apart is displaced into the form $y(x,0) = 40x - x^2$; $0 \leq x \leq 40$. from which it is released at $t=0$. Find the displacement of the string at a distance x from one end. take length of string $l = 40$ units.

6) Solve the following p.d.e.
$\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$ subject to the initial and boundary conditions
i) $u(0,t) = 0 \ \forall t$ ii) $u(4,t) = 0 \ \forall t$
iii) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0 \ \forall t$
iv) $u(x,0) = 2x \ ; \ 0 \leq x \leq 4.$

7) If $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ represents the vibrations of a string of length $l=1$ fixed at both ends. Find the solution with boundary and initial condns
i) $u(0,t) = 0 \ \forall t$ ii) $u(1,t) = 0 \ \forall t$
iii) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0 \ \forall x$
iv) $u(x,0) = u_0 x(1-x) \ ; \ 0 \leq x \leq 1.$

8) If $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ represents the vibrations of a string of length "π" fixed at both ends. Find the solution with boundary and initial condns. i) $u(0,t) = 0 \ \forall t$ ii) $u(\pi,t) = 0 \ \forall t$
iii) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0 \ \forall x$
iv) $u(x,0) = \pi x - x^2 \ \forall x \in [0,\pi].$