LDE: Solve

Q.1 $(D^2-2D+1)y = x.e^x.\sin x$

Q.2. Solve by the method of variation of parameters. $(D^2-6D+9)y = \frac{e^{3x}}{x^2}$

Q.3. $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(logx) + xsin(logx)$

Q.4. $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2+4x+1$

5. (D^2-2D+5)y = $25x^2$
6. $(D^3+3D^2-4)y = 6\,e^{-2x}$
7. $(2x+1)^2\frac{d^2y}{dx^2}$ - 2 (2x +1)$\frac{dy}{dx}$ - 12y = 6x
8. $(D^2+1)y = cosx$
9. (D^2+3D+2)y $= x\ sin\ 2x$
10. $x^2\frac{d^2y}{dx^2}$ - 2x $\frac{dy}{dx}$ - 4y = x^2+ log x
11. $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = 32\cos x\ sin\ x$
12. $\frac{d^2y}{dx^2} - 4y = e^{-2x}x^2$
13. $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{1}{(1+e^x)}$
14. $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x^4\sin x$
15. $\frac{d^2y}{dx^2} + 4y = sec^2 2x$ by method of variation of parameters.
16. $(2x+1)^2\frac{d^2y}{dx^2} - 6(2x+1)\frac{dy}{dx} + 16y = 8(2x+1)^2$
17. (D^2+3D+2)y = $e^{2x}+1$
18. $(D^4+D^2+1)y = e^{-x}.cos\sqrt{3}\,x$
19. $(2x-3)^2\frac{d^2y}{dx^2}$ - 2 (2x-3)$\frac{dy}{dx}$ - 12y = 2x
20. (D^3-3D-2)y = $540x^3.e^{-x}$

21. $(D^4 + 6D^2 + 8)y = sin^2 x . \cos 2x$

Laplace Transforms:

1. Find the Laplace Transform of $f(t) = \int_0^t \frac{e^{-2t} \sin 2t}{t} dt$

2. Find the Laplace Transform of $f(t) = [e^{-2t} t\ (2 \cosh 3t - 4 \sinh 2t)]$

3. Find the Inverse Laplace Transform of $F(s) = \frac{1}{((s-2)^2(s+3))}$

4. Find the Inverse Laplace Transform of $F(s) = [\cot^{-1}(s)]$

5. Find the solution of the differential equation with given conditions using Laplace Transform

$y'' + 4\,y' + 3y = 10 \sin t\ ; y(0) = y'(0) = 0$

6. Find the Laplace transform of $f(t) = te^{-t} sin\ 2t$

7. Find the Inverse Laplace transform of $F(s) = log\ \left(1 + \frac{1}{s}\right)$

8. Using Laplace transform, find the solution of differential equation

$y'' - 6y' + 9y = 0$ where $y(0) = 2$ & $y'(0)=9$

9. Find the Inverse Laplace transform of $F(s) = \frac{14s+10}{49s^2+28s+13}$

10. Using Laplace transform, find the solution of differential equation

$\frac{dy}{dt} - y = e^{3t}$ where $y(0) = 2$

Fourier Transforms:

1. Find Fourier cosine transform of the function $f(x) = e^{-x} + e^{-2x}$ and show that the Fourier integral representation

$$e^{-x} + e^{-2x} = \frac{2}{\pi}\int_0^{\infty} \frac{6 + 3\lambda^2}{\lambda^4 + 5\lambda^2 + 4} \cos \lambda u \, d\lambda$$

2. Find Fourier sine transform of the function $f(x) = e^{-x} \cos x$ and show that

$$e^{-x} \cos x = \frac{2}{\pi}\int_0^{\infty} \frac{\lambda^3}{\lambda^4 + 4} \sin \lambda u \, d\lambda$$

3. Solve the integral equation

$$\int_0^{\infty} f(x) \cos \lambda x \, dx = \begin{cases} \sin \pi\lambda, & 0 < \lambda < 2 \\ 0, & \lambda > 2 \end{cases}$$

4. Solve the integral equation

$$\int_0^{\infty} f(x) \cos \lambda x \, dx = \begin{cases} 4 - \lambda^2, & 0 < \lambda < 2 \\ 0, & \lambda > 2 \end{cases}$$

5. Find Fourier transform of the function $f(x) = \begin{cases} \sin x, & 0 < \lambda < \pi \\ 0, & \lambda > \pi \end{cases}$ and write in the Fourier integral representation.
6. Find Fourier transform of the function $f(x) = \begin{cases} x \cos x, & 0 < \lambda < \pi \\ 0, & \lambda > \pi \end{cases}$ and write in the Fourier integral representation.
7. Find Fourier transform of the function $f(x) = \begin{cases} 9 - \lambda^2, & 0 < \lambda < 3 \\ 0, & \lambda > 3 \end{cases}$ and write in the Fourier integral representation.

PDE: Heat Equations:

1. Solve the one dimensional heat flow equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ for the function u(x,t) subject to following conditions.

   A homogeneous rod of conducting material of length 4m has its ends kept at zero temperature and initially temperature is $u(x, 0) = 2x,\ 0 \le x \le 4$.

   Find the temperature $u(x, t)$ at any time t.

2. Solve the equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ with given conditions.

- $u(x,0) = 1, \quad 0 < x < \pi$
- $u(0,t) = u(\pi,t) = 0, \quad \forall t$
- $u(x,t)$ is finite

Find $u(x,t)$ at any time t.

3. Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$

- $u(x,0) = 10, \ 0 < x < 1$
- $u(0,t) = u(1,t) = 0, \ \forall t$
- $u(x,t)$ is finite

Find $u(x,t)$ at any time t.

4. Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$

- $u(x,0) = x, \ 0 < x < 5$
- $u(0,t) = u(5,t) = 0, \ \forall t$
- $u(x,t)$ is finite

Find $u(x,t)$ at any time t.

5. Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$

- $u(x,0) = 5, \ 0 < x < 5$
- $u(0,t) = u(5,t) = 0, \ \forall t$
- $u(x,t)$ is finite

Find $u(x,t)$ at any time t.

Z Transforms:

1. The Z-transform of $f(k) = 4^k, k < 0$ is
2. Inverse Z- transform of $F(z) = \frac{z}{z-2}$, $|z| < 2$ is
3. Solve the difference equation
   $f(k+2) + 3f(k+1) + 2\,f(k) = 0\,,\ f(0) = 0\,, f(1) = 1$
4. Find the Z-transform of $f(k) = \left(\frac{1}{2}\right)^{|k|}, for\ all\ k$
5. The Z-transform of $f(k) = \frac{3^k}{k!}, k \geq 0$

6. Find the inverse Z-transform of $F(z) = \frac{z^3}{(z-3)(z-2)}, |z| > 3$
7. Find inverse Z-transforms by inversion integral method $F(z) = \frac{1}{(z-1)(z-2)}$
8. Solve the difference equation: $f(k+1) + \frac{1}{2}f(k) = \left(\frac{1}{2}\right)^k$ $, f(0) = 0, k \geq 0$
9. The Z-transform of $Z\left[\frac{2^k}{k!}\right], k \geq 0$
10. Inverse Z-transform of $F(z) = \frac{z^3}{(z-3)^3}$; $|z| > 3$ is
11. Solve the difference equation $f\ (k+1) + \ f\ (k) = 1, f\ (0) = \ 0$
12. Find the inverse Z-transform of $F(Z) = \frac{Z^2}{\left(Z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}$ $; \frac{1}{5} < |z| < \frac{1}{4}$