

M3 N
(MIII) Differential Equations.

$$\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^{3x}$$

Put $\frac{d}{dx} = D$

$$\therefore D^2y + 3Dy + 2y = e^{3x}$$

$$\therefore (D^2 + 3D + 2)y = e^{3x}$$

$$\therefore \phi(D)y = f(x)$$

Solution of D.E :-

Solⁿ of D.E

Complimentary funⁿ
 $y_c/[C.F]$

Particular Integral.
 $y_p/[P.I]$

$$\therefore y = y_c + y_p$$

(1) Complimentary Function :-

$$\phi(D) = 0$$

Auxiliary Equation.

$$\phi(D) = 0$$



Roots

Real

Imaginary

Distinct

Repeated

Distinct

Repeated.

①

②

③

④

(MIII)

Type 1) :- Roots are Real & Distinct.

if, $D = m_1, D = m_2$

$$y_c = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

if, $D = m_1, D = m_2, D = m_3$

$$y_c = C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{m_3 x}$$

Type 2) :- Roots are Real & Repeated.

if, $D = m_1, D = m_1$

$$y_c = [C_1 + C_2 x] e^{m_1 x}$$

if, $D = m_1, D = m_1, D = m_1$

$$y_c = [C_1 + C_2 x + C_3 x^2] e^{m_1 x}$$

Type 3) :- Roots are Imaginary & Distinct.

if $D = \alpha \pm i\beta$

$$\therefore y_c = e^{\alpha x} [C_1 \cos \beta x + C_2 \sin \beta x]$$

We can find imaginary roots
by formula $\downarrow$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(MIII)

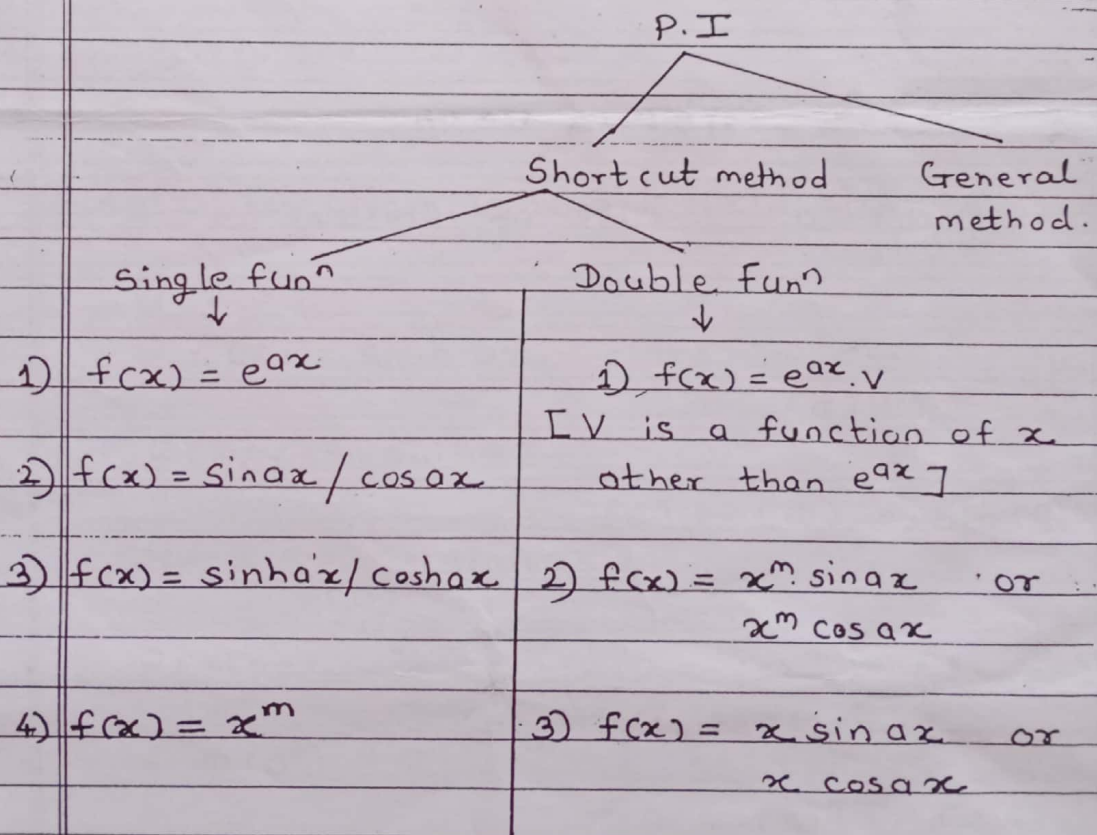
Type 4) :- Roots are Imaginary & Repeated

if $D = \alpha \pm i\beta$, $D = \alpha \pm i\beta$

$$y_c = e^{\alpha x} [(c_1 + c_2 x) \cos \beta x + (c_3 + c_4 x) \sin \beta x]$$

(2) Particular Integral [P.I./y_p] :-

$$y_p = \frac{1}{\phi(D)} \cdot f(x)$$



(MTH)

Single function Type 1) :- $f(x) = e^{ax}$

$$f(x) = e^{ax}$$

$$\phi(D)y = e^{ax}$$

$$y_p = \frac{1}{\phi(D)} e^{ax}$$

Replace $(D) \rightarrow (a)$

if $\phi(a) = 0$, case of failure.

$$y_p = \frac{x}{\phi'(D)} e^{ax}$$

$(D) \rightarrow (a)$ Replace.

if $\phi'(a) = 0$ case of failure

$$y_p = \frac{x^2}{\phi''(D)} e^{ax}$$

Replace $(D) \rightarrow (a)$

Corr 1) :- $f(x) = a^x$

Replace $D \rightarrow (\log a)$

Corr 2) :- $f(x) = a^{-x}$

Replace $D \rightarrow (-\log a)$

Corr 3) $f(x) = K$ [const]

Replace $D \rightarrow 0$

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Single function Type 2) :- $f(x) = \sin ax / \cos ax$

$$f(x) = \sin ax \text{ or } \cos ax$$

$$\phi(D)y = \sin ax$$

$$y_p = \frac{1}{\phi(D)} \sin ax$$

$$\text{Replace } D^2 \rightarrow (-a^2)$$

$$\text{if } \phi(-a^2) = 0 \quad \text{case of failure.}$$

$$= \frac{x}{\phi'(D)} \sin ax$$

$$\text{Replace } D^2 \rightarrow (-a^2)$$

Single Function Type 3) :- $f(x) = \sinh ax \text{ or } \cosh ax.$

$$f(x) = \sinh ax$$

$$\phi(D)y = \sinh ax.$$

$$y_p = \frac{1}{\phi(D)} \sinh ax$$

$$\text{Replace } D^2 \rightarrow (a^2)$$

$$\text{if } \phi(a^2) = 0 \quad \text{case of failure.}$$

$$y_p = \frac{x}{\phi'(D)} \sinh ax$$

$$\phi'(D^2) \rightarrow \phi'(a^2)$$

Single function Type 4) :- $f(x) = x^m$

$$\phi(D) y = x^m$$

$$y_p = \frac{1}{\phi(D)} x^m$$

$$= \frac{1}{\pm \text{min}^m \text{ term } [1 \pm (\psi D)]} x^m$$

$$= \frac{1}{\pm \text{min}^m \text{ term.}} [1 \pm \psi(D)]^{-1} \cdot x^m$$

Use this expansions:-

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \dots$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \dots$$

↓

e.g. $\Rightarrow y_p = \frac{1}{D^2 + 3D - 2} x^2$

$$= \frac{1}{-2 \left[1 - \left(\frac{D^2}{2} + \frac{3D}{2} \right) \right]} x^2$$

$$= \frac{1}{-2} \left[1 - \left(\frac{D^2}{2} + \frac{3D}{2} \right) \right]^{-1} \cdot x^2$$

$$= -\frac{1}{2} \left[1 + \left(\frac{D^2}{2} + \frac{3D}{2} \right) + \left(\frac{D^2}{2} + \frac{3D}{2} \right)^2 \right] \cdot x^2$$

Double Function Type 1) :- $f(x) = e^{ax} \cdot v$

[where v is a function of x , other than e^{ax}]

$$\phi(D)y = e^{ax} \cdot v$$

$$y_p = \frac{1}{\phi(D)} \cdot e^{ax} \cdot v$$

$$y_p = e^{ax} \cdot \frac{1}{\phi(D+a)} \cdot v \quad [D \rightarrow (D+a)]$$

Double Function Type 2) :- $f(x) = x^m \sin ax$
or
 $x^m \cos ax$

Use this :-	$e^{aix} = \cos ax + i \sin ax.$ $\Downarrow$ R.P $\Downarrow$ I.P.
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$$\phi(D) = x^m \cdot \sin ax$$

$$y_p = \frac{1}{\phi(D)} \cdot x^m \cdot \sin ax$$

$$= \text{I.P.} \cdot \frac{1}{\phi(D)} \cdot x^m \cdot e^{aix}$$

$$= \text{I.P.} \cdot e^{aix} \cdot \frac{1}{\phi(D+ai)} \cdot x^m$$

$$\phi(D) = x^m \cdot \cos ax$$

$$y_p = \frac{1}{\phi(D)} \cdot x^m \cdot \cos ax$$

$$y_p = \text{R.P.} \cdot \frac{1}{\phi(D)} \cdot x^m \cdot \cos ax$$

$$y_p = \text{R.P.} \cdot e^{aix} \cdot \frac{1}{\phi(D+ai)} \cdot x^m$$

Double Function Type 3) :- $f(x) = x \sin ax$ or $x \cos ax$

$$\phi(D)y = x \sin ax$$

$$y_p = \frac{1}{\phi(D)} \cdot x \cdot \sin ax$$

$$= \left[x - \frac{\phi'(D)}{\phi(D)} \right] \frac{1}{\phi(D)} \cdot \sin ax$$

$$D^2 \rightarrow (-a^2)$$

* Note :- 1) Not to rationalize outside the bracket.

2) In case of failure use earlier format.

General Method :-

$$\phi(D^2) = f(x)$$

$$y_p = \frac{1}{(D+m_1)(D+m_2)} f(x)$$

$$= \frac{1}{(D+m_2)} e^{-m_1 x} \int e^{m_1 x} f(x) \downarrow \phi(x)$$

$$= \frac{1}{(D+m_2)} e^{-m_1 x} \phi(x)$$

$$= e^{-m_2 x} \int e^{m_2 x} e^{-m_1 x} \phi(x)$$

* Examples On Single Function Type 1) :- $f(x) = e^{ax}$

1) $(D^2 - 5D + 6)y = 3 \cdot e^{5x}$

2) $(D^2 + 4D + 3)y = e^{-3x}$

3) $(D-1)^3 y = e^{2x} + 2^x - 3/2$

4) $(D-2)^2 (D+1)y = e^{2x} + 2^{-x}$

* Examples On Single function Type 2) :-
 $f(x) = \sin ax$ or $\cos ax$

1) $(D^2 + 2D + 1)y = 4 \cdot \sin 2x$

2) $(D^2 + 1)y = \sin x - \sin 2x$

3) $(D^3 + 4D)y = \sin 2x$

4) $(D^3 + 1)y = \sin(2x + 3)$

* Examples On Single function type 3) :-
 $f(x) = \sinh ax$ or $\cosh ax$

1) $(D^2 - 4D)y = 2 \cosh 2x$

* Examples On Single function type 4) :-
 $f(x) = x^m$

1) $(D^2 - D + 1)y = x^3 - 3x^2 + 1$

* Examples On Double function type 1) :-
 $f(x) = e^{ax} \cdot v$

1) $(D^2 - 4D + 3)y = x^3 \cdot e^{2x}$

2) $(D^2 - 6D + 13)y = 8 \cdot e^{3x} \sin 4x + 2^x$

* Examples On Double Function type 2) :-
 $f(x) = x^2 \sin ax$ or $x^2 \cos ax$

1) $(D^2 - 1)y = x^2 \sin 3x$

2) $(D^2 + 1)y = x^2 \cos 2x$

3) $(D^4 + 2D^2 + 1)y = x^2 \cos x$

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Examples On.

* Double Function Type 3):- $f(x) = x \sin x$ or $x \cos x$

- 1) $(D^2+4)y = x \sin x$
- 2) $(D^2+2D+1)y = x \cos x$
- 3) $(D^2+3D+2)y = x \sin 2x$
- 4) $(D^2-1)y = x \cdot e^x \cdot \sin x$
- 5) $(D^2+4)y = x \cdot \cos 2x$

Examples On General Method (3 types of Ex.)

I) Type 1) $f(x) = e^x$

- 1) $(D^2+3D+2)y = e^x$
- 2) $(D^2+3D+2)y = \sin(e^x)$
- 3) $(D^2+3D+2)y = e^x + \cos(e^x)$
- 4) $(D^2-3D+2)y = \sin(e^{-x})$
- 5) $(D^2-3D+2)y = \frac{1}{e^{e^{-x}}} + \cos \frac{1}{e^x}$

II) Type 2) $f(x) = \frac{1}{1+e^x}$

- 1) $(D^2+1)y = \frac{1}{1+e^x}$
- 2) $(D^2+3D+2)y = \frac{1}{1+e^x}$
- 3) $(D^2-1)y = (1+e^{-x})^{-2}$

III) Type 3) $\int e^x [f(x) + f'(x)] dx = e^x \cdot f(x)$

- 1) $(D^2-1)y = e^{-x} \sin(e^{-x}) + \cos(e^{-x})$
- 2) $(D^2-1)y = e^{-2x} \sec^2 x [1+2 \tan x]$
- 3) $(D^2-D-2)y = 2 \log x + \frac{1}{x} + \frac{1}{x^2}$
- 4) $(D^2-1)y = \log x + \frac{1}{x^2}$

Variation Of Parameter.

αx

$$\phi(D^2) = f(x)$$

$$y_c = C_1 y_1 + C_2 y_2$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}, \quad u = - \int \frac{y_2 f(x)}{D}, \quad v = \int \frac{y_1 f(x)}{D}$$

$$y_p = u \cdot y_1 + v y_2$$

* Examples On Variation Of Parameter :-

1) $(D^2 - 2D + 2)y = e^x \cdot \tan x$

2) $(D^2 + 4)y = \tan 2x$

3) $(D^2 + 4)y = 4 \cdot \sec 2x$

4) $(D^2 + 4)y = 4 \cdot \sec^2 2x$

5) $(D^2 - 4D + 4)y = e^{2x} \cdot \sec^2 x$

6) $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$

7) $(D^2 + 1)y = \operatorname{cosec} x$

8) $(D^2 + 1)y = x \cdot \sin x$

9) $(D^2 + 9)y = \frac{1}{1 + \sin 3x}$

10) $(D^2 - 1)y = e^{-x} \cdot \sin(e^x) + \cos(e^x)$

X11) $(D^2 - 1)y = 1/(1 + e^x)$

X12) $(D^2 - 1)y = 1/(1 + e^{-x})^2$

X13) $(D^3 + D)y = \operatorname{cosec} x$

(MIII)

Cauchy's And Legendre's Equations

1) Cauchy's Equations :-

$$x^3 \frac{d^3y}{dx^3} + x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = f(x)$$

$$\text{Put } \log x = z, \therefore x = e^z$$

$$x^3 \frac{d^3y}{dx^3} = D(D-1)(D-2)y$$

$$x^2 \frac{d^2y}{dx^2} = D(D-1)y$$

$$x \frac{dy}{dx} = Dy$$

2) Legendre's Equations :-

$$(ax+b)^3 \frac{d^3y}{dx^3} + (ax+b)^2 \frac{d^2y}{dx^2} + (ax+b) \frac{dy}{dx} + y = f(x)$$

$$\text{Put, } \log(ax+b) = z$$

$$\therefore ax+b = e^z$$

$$\therefore x = \frac{e^z - b}{a}$$

$$(ax+b)^3 \frac{d^3y}{dx^3} = a^3 D(D-1)(D-2)y$$

$$(ax+b)^2 \frac{d^2y}{dx^2} = a^2 D(D-1)y$$

$$(ax+b) \frac{dy}{dx} = a Dy$$

Examples On Cauchy's Equations.

- 1) $x^3 \frac{d^3y}{dx^3} + 2x \frac{d^2y}{dx^2} + 3 \frac{dy}{dx} - 3y = 6x$
- 2) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 2 \sinh(\log x)$
- 3) $x^3 \frac{d^2y}{dx^2} + 3x^2 \frac{dy}{dx} + xy = \sin(\log x)$
- 4) $(x^3 D^3 + x^2 D^2 - 2) y = x^2 + x^{-3}$
- 5) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = \log \cos(\log x)$
- 6) $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \log x$
- 7) $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^2 + 2 \log x$

Examples On Legendre's Equations.

- 1) $(2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1)y \frac{dy}{dx} - 12y = 8x$
 $3n+2=e^2$
 $n=\frac{e^2-2}{3}$
- 2) $(3x+2) \frac{d^2y}{dx^2} + 3(3x+2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$
 $3x^2 + 2x + 1$
- 3) $(2x+1)^2 \frac{d^2y}{dx^2} + 2(2x+1) \frac{dy}{dx} + 4y = \left(\frac{e^2-2}{3}\right) \cdot e^2 + 1$
 $n(3n+2)+1$
 $4 \sin [\log (2x+1)^2]$
- 4) $(x+1)^2 \frac{d^2y}{dx^2} - (x+1) \frac{dy}{dx} - 3y = 6x$

Linear Diff. Eqⁿ

$$\frac{d^3 y}{dn^3} + 3 \frac{d^2 y}{dn^2} + 2 \frac{dy}{dn} + y = f(n)$$

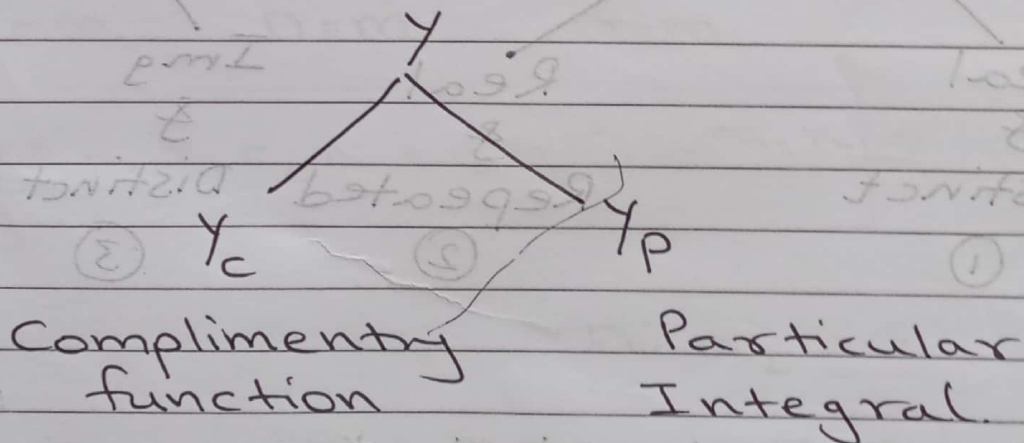
$$\text{put } \frac{d}{dn} = D \quad \frac{d^2}{dn^2} = D^2 \quad \frac{d^3}{dn^3} = D^3$$

$$D^3 y + 3D^2 y + 2Dy + y = f(n)$$

$$(D^3 + 3D^2 + 2D + 1)y = f(n)$$

$$\phi(D)y = f(n)$$

Solution of L.D.E.

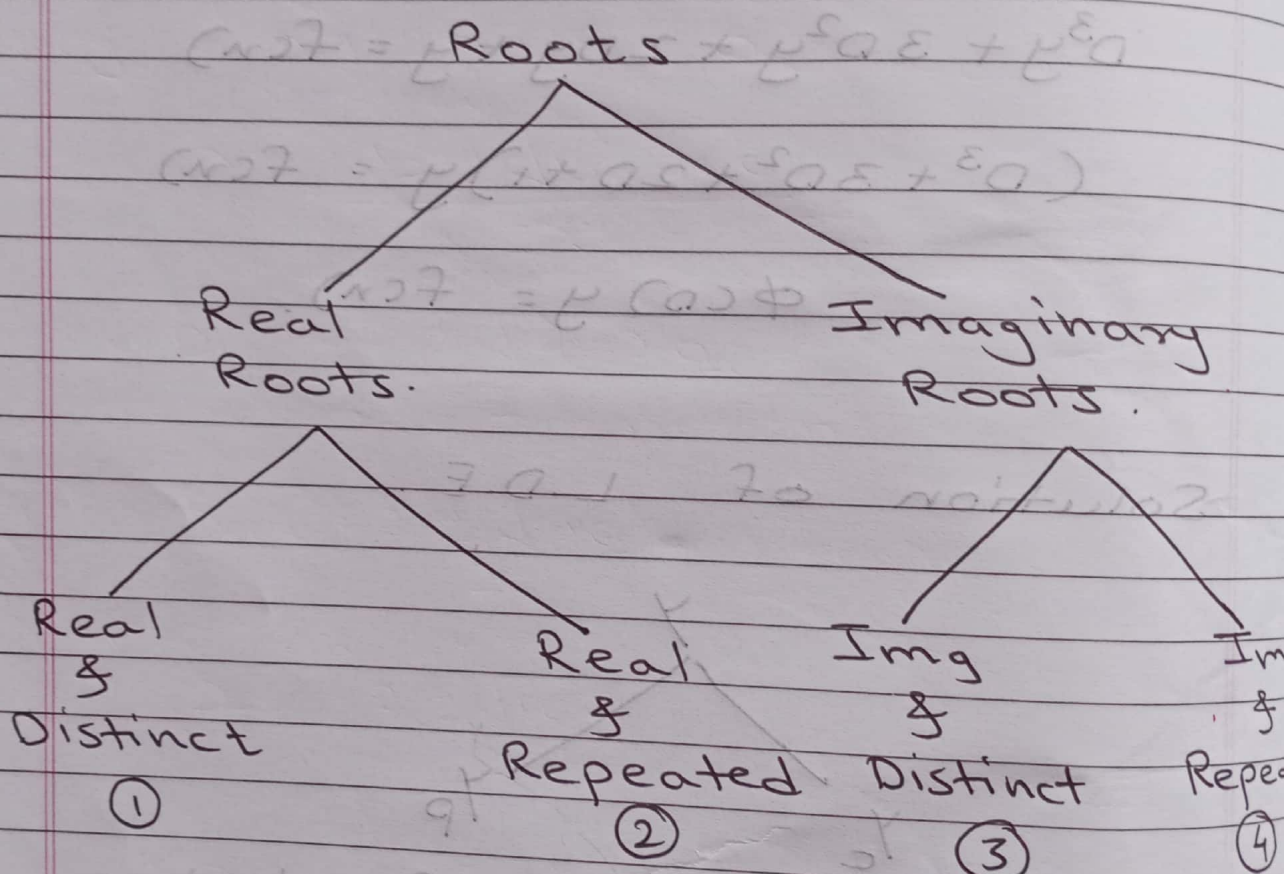


$$\therefore y = y_c + y_p$$

$y_c \rightarrow$ Complimentary function

$$\phi(\lambda) y = f(x)$$

$$\phi(\lambda) = 0 \text{ --- Auxillary eq}^n$$



1) Real and Distinct

$$D = m_1, \quad D = m_2$$

$$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$D = m_1, \quad D = m_2, \quad D = m_3$$

$$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$$

e.g. $(D^2 + 5D + 6) = 0$

$$(D+3)(D+2) = 0$$

$$D = -3 \quad D = -2$$

$$\therefore y_c = c_1 e^{-3x} + c_2 e^{-2x}$$

2) Real and Repeated

$$D = m_1 \quad D = m_1$$

$$y_c = (c_1 + c_2 x) e^{m_1 x}$$

$$D = m_1 \quad D = m_1 \quad D = m_1$$

$$y_c = (c_1 + c_2 x + c_3 x^2) e^{m_1 x}$$

$$D = m_1 \quad D = m_1 \quad D = m_2$$

$$y_c = (c_1 + c_2 x) e^{m_1 x} + c_3 e^{m_2 x}$$

e.g. $(D^2 + 4D + 4) = 0$

$$(D+2)(D+2) = 0$$

$$D = -2 \quad D = -2$$

$$y_c = (c_1 + c_2 x) e^{-2x}$$

3) Imaginary and distinct

$$D = \alpha \pm i\beta \quad (\alpha \neq 0) \quad (\beta \neq 0)$$

$$y_c = e^{\alpha x} [C_1 \cos \beta x + C_2 \sin \beta x]$$

eg. $D^2 + 2D + 5 = 0.$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-2 \pm \sqrt{4 - 20}}{2}$$

$$D = \frac{-2 \pm \sqrt{-16}}{2}$$

$$D = \frac{-2 \pm \sqrt{16}i}{2}$$

$$D = \frac{-2 \pm 4i}{2}$$

$$D = -1 \pm 2i$$

$$y_c = e^{-x} [C_1 \cos 2x + C_2 \sin 2x]$$

4) Imaginary and Repeated

$$D = \alpha \pm i\beta, \quad D = \alpha \pm i\beta$$

$$y_c = e^{\alpha x} [(c_1 + c_2 x) \cos \beta x + (c_3 + c_4 x) \sin \beta x]$$

e.g.

$$(D^2 + 3)^2 = 0$$

$$D^2 = -3$$

$$D^2 = -3$$

$$= 3x - 1$$

$$D^2 = 3i^2$$

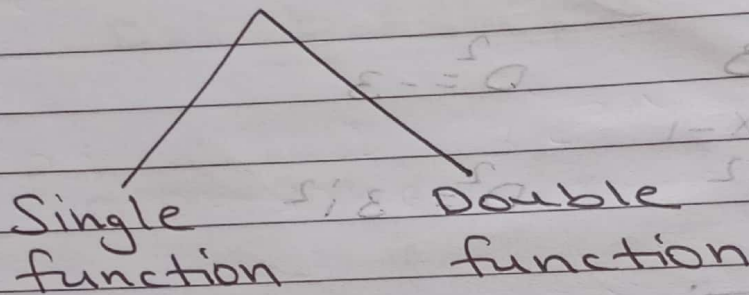
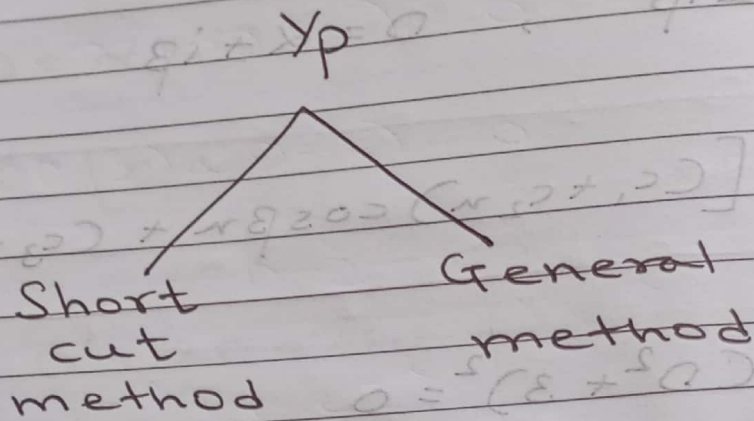
$$D^2 = 3i^2$$

$$D = \pm \sqrt{3}i$$

$$D = \pm \sqrt{3}i$$

$$y_c = e^{0x} [(c_1 + c_2 x) \cos \sqrt{3}x + (c_3 + c_4 x) \sin \sqrt{3}x]$$

$Y_p \rightarrow$ Particular Integral



1) $f(x) = e^{ax}$

1) $f(x) = e^{ax} \cdot v$

2) $f(x) = \frac{\sin ax}{\cos ax}$

2) $f(x) = \frac{x^2 \sin ax}{x^2 \cdot \cos ax}$

3) $f(x) = \frac{\sinh ax}{\cosh ax}$

3) $f(x) = \frac{x \cdot \sin ax}{x \cdot \cos ax}$

4) $f(x) = x^m$

Single fun - Type - 1 (a) (b)

$$f(n) = e^{an}$$

$$Y_p = \frac{1}{\phi(n)} e^{an}$$

$$D \rightarrow a$$

$$= \frac{1}{\phi(a)} e^{an}$$

$$\text{If } \phi(a) = 0$$

case of failure

$$= \frac{n}{\phi'(n)} e^{an}$$

$$D \rightarrow a$$

$$= \frac{n}{\phi'(a)} e^{an}$$

$$\text{If } \phi'(a) = 0$$

case of failure

$$= \frac{n^2}{\phi''(n)} e^{an}$$

$$D \rightarrow a$$

cor-1 $f(n) = a^n$ $D \rightarrow \log a$

cor-2 $f(n) = a^{-n}$ $D \rightarrow -\log a$

cor-3 $f(n) = k$ $D \rightarrow 0$

$$1) (D^2 - 5D + 6)y = 3e^{5x}$$

$$\text{A.E. } (D^2 - 5D + 6) = 0 \quad (x \neq 0)$$

$$(D-3)(D-2) = 0$$

$$D=3$$

$$D=2 \leftarrow \text{A}$$

$$y_c = c_1 e^{3x} + c_2 e^{2x}$$

$$y_p = \frac{1}{D^2 - 5D + 6} 3e^{5x}$$

$$= 3 \frac{1}{D^2 - 5D + 6} e^{5x}$$

$$= 3 \frac{1}{(5)^2 - (5)5 + 6} e^{5x}$$

$$= 3 \frac{1}{6} e^{5x}$$

$$y_p = \frac{1}{2} e^{5x}$$

$$y = y_c + y_p$$

$$2) (D^2 + 4D + 3)y = e^{-3x} = e^{x(-3)} \quad (E)$$

$$A.E. \quad D^2 + 4D + 3 = 0 \quad .E.A$$

$$D = -3 \quad D = -1$$

$$(D+3)(D+1) = 0$$

$$D = -3 \quad D = -1$$

$$D = -3 \quad D = -1$$

$$y_c = c_1 e^{-3x} + c_2 e^{-x}$$

$$y_p = \frac{1}{D^2 + 4D + 3} e^{-3x}$$

$$D \rightarrow -3 \quad \frac{1}{D^2 + 4D + 3} = \frac{1}{(-3)^2 + 4(-3) + 3} = \frac{1}{9 - 12 + 3} = \frac{1}{0}$$

$$= \frac{x}{2D + 4} e^{-3x}$$

$$D \rightarrow -3 \quad = \frac{x}{2(-3) + 4} e^{-3x} = \frac{x}{-2} e^{-3x}$$

$$y_p = \frac{x}{-2} e^{-3x}$$

$$y = y_c + y_p$$

3) $(D-1)^3 y = e^{2x} + 2^x - \frac{3}{2}$

A.E.

$$(D-1)^3 = 0$$

$$D=1 \quad D=1 \quad D=1$$

$$y_c = (C_1 + C_2 x + C_3 x^2) e^x$$

$$y_p = \frac{1}{(D-1)^3} \left[e^{2x} + 2^x - \frac{3}{2} \right]$$

$$= \frac{1}{(D-1)^3} e^{2x} + \frac{1}{(D-1)^3} 2^x - \frac{1}{(D-1)^3} \frac{3}{2}$$

$y_{p_1} \qquad y_{p_2} \qquad y_{p_3}$

$$y_{p_1} = \frac{1}{(D-1)^3} e^{2x}$$

$$D \rightarrow 2 \quad = \frac{1}{(2-1)^3} e^{2x}$$

$$y_{p_1} = e^{2x}$$

$$y_{p_2} = \frac{1}{(D-1)^3} 2^x$$

$$D \rightarrow \log 2$$

$$y_{p_2} = \frac{1}{(\log 2 - 1)^3} 2^x$$

$$Y_{P3} = \frac{1 + \frac{3}{2}}{(D-1)^3} = \frac{(1+Q)^2 (S-Q)}{(D-1)^3} \quad (A)$$

$$0 = (D \rightarrow 0)^2 (S-Q) \Rightarrow A$$

$$Y_{P3} = \frac{-3}{2} \quad D=0 \quad S=Q \quad S=0$$

$$Y_P = Y_{P1} + Y_{P2} - Y_{P3}$$

$$Y = Y_c + Y_p \quad \frac{[x^{-1} + x^5]}{(1+Q)^2 (S-Q)} = QX$$

$$\begin{aligned} & (D-1)^2 (1+Q)^2 (S-Q) \\ & (1+Q)(1+Q+Q^2) \\ & D^2 - 2D + 1 + QD^2 - 2QD + Q + Q^2 D^2 - 2Q^2 D + Q^2 \\ & D^2 - 2D + 1 + QD^2 - 2QD + Q + Q^2 D^2 - 2Q^2 D + Q^2 \end{aligned}$$

$$[x^{-1} + x^5] \frac{1}{D^2 - 2D + 1 + QD^2 - 2QD + Q + Q^2 D^2 - 2Q^2 D + Q^2} = QX$$

$$\begin{aligned} & D^2 - 2D + 1 + QD^2 - 2QD + Q + Q^2 D^2 - 2Q^2 D + Q^2 \\ & D^2 - 2D + 1 + QD^2 - 2QD + Q + Q^2 D^2 - 2Q^2 D + Q^2 \end{aligned}$$

$$4) (D-2)^2(D+1) = e^{2x} + e^{-x}$$

$$\text{A.E. } (D-2)^2(D+1) = 0$$

$$D=2 \quad D=2 \quad D=-1$$

$$y_c = (c_1 + c_2 x) e^{2x} + c_3 e^{-x}$$

$$y_p = \frac{1}{(D-2)^2(D+1)} [e^{2x} + e^{-x}]$$

$$(D-2)^2(D+1)$$

$$(D^2 - 4D + 4)(D+1)$$

$$D^3 + D^2 - 4D^2 - 4D + 4D + 4$$

$$D^3 - 3D^2 + 4$$

$$y_p = \frac{1}{D^3 - 3D^2 + 4} [e^{2x} + e^{-x}]$$

$$= \frac{1}{D^3 - 3D^2 + 4} e^{2x} + \frac{1}{D^3 - 3D^2 + 4} e^{-x}$$

$y_{p1} \qquad y_{p2}$

$$y_{p_1} = \frac{1}{D^3 - 3D^2 + 4} e^{2x}$$

$$D \rightarrow 2$$

C.F. $\cos x = \cos \phi$

$$= \frac{(x) - 5}{3D^2 - 6D} e^{2x}$$

$$D \rightarrow 2$$

C.F. $\cos x = \frac{1}{2}(1 + \cos 2x)$

$$= \frac{x^2}{6D - 6} e^{2x}$$

$$0 = D \rightarrow 2$$

$$= \frac{x^2}{6(2) - 6} e^{2x}$$

$$y_{p_1} = \frac{x^2}{6} e^{2x}$$

$$y_{p_2} = \frac{1}{D^3 - 3D^2 + 4} 2^{-x}$$

$$D \rightarrow -\log 2$$

$$y_{p_2} = \frac{1}{(-\log 2)^3 - 3(-\log 2)^2 + 4} 2^{-x}$$

$$y_p = y_{p_1} + y_{p_2}$$

$$y = y_c + y_p$$

Single function - Type-2

$$f(x) = \sin ax / \cos ax$$

$$y_p = \frac{1}{\phi(D)} \sin ax$$
$$D^2 \rightarrow -(a^2)$$

$$1) (D^2 + 2D + 1)y = 4 \cdot \sin 2x$$

$$\text{A.E. } D^2 + 2D + 1 = 0$$

$$(D+1)(D+1) = 0$$

$$D = -1$$

$$D = -1$$

$$y_c = (C_1 + C_2 x) e^{-x}$$

$$y_p = \frac{1}{D^2 + 2D + 1} 4 \cdot \sin 2x$$

$$= 4 \frac{1}{D^2 + 2D + 1} \sin 2x$$

$$D^2 \rightarrow -(4)$$

$$= 4 \frac{1}{-4 + 2D + 1} \sin 2x$$

$$= 4 \frac{1}{2D - 3} \sin 2x$$

$$= 4 \frac{(2D+3)}{(2D+3)(2D-3)} \sin 2x$$

$$= 4 \frac{(2D+3)}{4D^2-9} \sin 2x$$

$$1 \pm D^2 \rightarrow -4$$

$$= 4 \frac{(2D+3)}{4(-4)-9} \sin 2x = \frac{4}{-25} (2D+3) \sin 2x$$

$$= \frac{4}{-25} (2D+3) \sin 2x$$

$$(y+A) = \frac{4}{-25} [2A \cos 2x + 2 + 3 \sin 2x]$$

$$y_p = \frac{4}{-25} [4 \cos 2x + 3 \sin 2x]$$

$$y_p = \frac{4}{-25} [4 \cos 2x + 3 \sin 2x]$$

$$y_p = \frac{4}{-25} [4 \cos 2x + 3 \sin 2x]$$

$$y_p = \frac{4}{-25} [4 \cos 2x + 3 \sin 2x]$$

$$D^2 \rightarrow -1$$

$$2) (D^2+1)y = \sin x \cdot \sin 2x$$

$$A.E. \quad D^2+1=0$$

$$D^2 = -1$$

$$D = \pm i$$

$$y_c = C_1 \cos x + C_2 \sin x$$

$$y_p = \frac{\sin x \cdot \sin 2x}{D^2+1}$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$\sin 2x \sin x = \frac{1}{2} [\cos(2x-x) - \cos(2x+x)]$$

$$= \frac{1}{2} [\cos x - \cos 3x]$$

$$y_p = \frac{1}{D^2+1} \cdot \frac{1}{2} (\cos x - \cos 3x)$$

$$= \frac{1}{2} \left[\frac{1}{D^2+1} \cos x - \frac{1}{D^2+1} \cos 3x \right]$$

y_{p_1} y_{p_2}

$$y_{p_1} = \frac{1}{D^2+1} \cos x$$

$$D^2 \rightarrow -1$$

C.F.

$$= \frac{x}{20} \cos x = y(D^2 + 0) \quad (3)$$

$$y_{p1} = \frac{x}{2} \sin x$$

$$y_{p2} = \frac{1}{D^2 + 1} \cos 3x$$

$$D^2 \rightarrow -9$$

$$= \frac{1}{-9 + 1} \cos 3x$$

$$y_{p2} = \frac{\cos 3x}{-8}$$

$$y_p = \frac{1}{2} [y_{p1} - y_{p2}]$$

$$y = y_c + y_p$$

$$3) (D^3 + 4D)y = \sin 2x$$

$$\text{A.E. } D^3 + 4D = 0$$

$$D(D^2 + 4) = 0$$

$$D = 0 \quad D^2 = -4$$

$$D^2 = 4i^2$$

$$D = \pm 2i$$

$$y_c = C_1 + C_2 \cos 2x + C_3 \sin 2x$$

$$y_p = \frac{1}{D^3 + 4D} [\sin 2x]$$

$$D^2 \rightarrow -4$$

C.F.

$$= \frac{x}{3D^2 + 4} \sin 2x$$

$$D^2 \rightarrow -4$$

$$= \frac{x}{3(-4) + 4} \sin 2x$$

$$y_p = \frac{x}{-8} \sin 2x$$

$$4) (D^3+1)y = \sin(2x+3)$$

$$\text{A.E. } D^3+1=0$$

$$(D+1)(D^2-D+1)=0$$

$$D=-1$$

$$D = \frac{1 \pm \sqrt{1-4}}{2}$$

$$D = \frac{1 \pm \sqrt{3}i}{2}$$

$$D = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

$$y_c = c_1 e^{-x} + e^{\frac{1}{2}x} \left[c_2 \cos \frac{\sqrt{3}}{2}x + c_3 \sin \frac{\sqrt{3}}{2}x \right]$$

$$y_p = \frac{1}{D^3+1} \sin(2x+3)$$

$$D^2 \rightarrow -4$$

$$= \frac{1}{1-40} \sin(2x+3)$$

$$= \frac{(1+40)}{(1+40)(1-40)} \sin(2x+3)$$

$$= \frac{(1+40)}{1-160^2} \sin(2x+3)$$

$$D^2 \rightarrow -4$$

$$= \frac{(1+40)}{1-16(-4)} \sin(2x+3)$$

$$= \frac{1}{+65} (1+40) \sin(2x+3)$$

$$y_p = \frac{1}{65} [\sin(2x+3) + 4 \cos(2x+3)]$$

$$y_p = \frac{1}{65} [\sin(2x+3) + 8 \cos(2x+3)]$$

$$y = y_c + y_p$$

$$y = \frac{1}{65} [\sin(2x+3) + 8 \cos(2x+3)]$$

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$$y = \frac{1}{65} [\sin(2x+3) + 8 \cos(2x+3)]$$

Single function - Type-3

$$f(x) = \sinh x / \cosh x$$

$$y_p = \frac{1}{\phi(D)} \sinh x$$

$$D^2 \rightarrow a^2$$

$$1) (D^3 - 4D)y = 2 \cosh 2x$$

$$\text{A.E. } D^3 - 4D = 0$$

$$D(D^2 - 4) = 0$$

$$D = 0 \quad D^2 = 4$$

$$D = \pm 2$$

$$y_c = c_1 + c_2 e^{2x} + c_3 e^{-2x}$$

$$y_p = \frac{1}{D^3 - 4D} 2 \cosh 2x$$

$$D^2 \rightarrow 4$$

$$= \frac{2}{3} \text{ C.F.}$$

$$= 2 \frac{x}{3D^2 - 4} \cosh 2x$$

$$D^2 \rightarrow 4$$

$$= 2 \frac{x}{3(4) - 4} \cosh 2x$$

$$= 2 \frac{\pi}{8} \cosh 2x$$

$$y_p = \frac{\pi}{4} \cosh 2x$$

$$y = y_c + y_p$$

$$y_c = C_1 e^{2x} + C_2 e^{-2x}$$

$$y_p = \frac{\pi}{4} \cosh 2x$$

$$y = C_1 e^{2x} + C_2 e^{-2x} + \frac{\pi}{4} \cosh 2x$$

$$y = C_1 e^{2x} + C_2 e^{-2x} + \frac{\pi}{4} \cosh 2x$$

Single funⁿ - Type - 4 $\phi(z) = 1 + z + z^2 + \dots$

$$f(z) = z^m = 1 + z + z^2 + \dots$$

$$Y_p = \frac{1}{\phi(z)} \frac{z^m - 1}{z} = 0$$

$$= \frac{1}{\pm \text{min term } [1 \pm \psi(z)]} z^m$$

$$= \frac{1}{\pm \text{min term}} [1 \pm \psi(z)]^{-1} \cdot z^m$$

$$(1+z)^{-1} = 1 - z + z^2 - z^3 + z^4 - \dots$$

$$(1-z)^{-1} = 1 + z + z^2 + z^3 + z^4 + \dots$$

No Y_{p1}, Y_{p2} Split-up

$$(1+z)^{-1} \cdot [1 + (z^2 + z^3 + \dots)] =$$

$$[1 + (z^2 + z^3 + \dots) - 1] =$$

$$(1+z)^{-1} (z^2 + z^3 + \dots)$$

$$(1+z)^{-1} \cdot [z^2 + z^3 + z^4 + \dots] =$$

$$D (D^2 - D + 1) y = x^3 - 3x^2 + 1$$

$$\text{A.E. } D^2 - D + 1 = 0$$

$$D = \frac{1 \pm \sqrt{1-4}}{2}$$

$$D = \frac{1 \pm \sqrt{3}i}{2}$$

$$D = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$y_c = e^{\frac{1}{2}x} \left[c_1 \cos \frac{\sqrt{3}}{2} x + c_2 \sin \frac{\sqrt{3}}{2} x \right]$$

$$y_p = \frac{1}{D^2 - D + 1} (x^3 - 3x^2 + 1)$$

$$= \frac{1}{1 + (D^2 - D)} (x^3 - 3x^2 + 1)$$

$$= [1 + (D^2 - D)]^{-1} \cdot (x^3 - 3x^2 + 1)$$

$$= [1 - (D^2 - D) + (D^2 - D)^2 - (D^2 - D)^3] \cdot (x^3 - 3x^2 + 1)$$

$$= [1 - D^2 + D + D^4 - 2D^3 + D^2 + D^3] \cdot (x^3 - 3x^2 + 1)$$

$$= [1 + 0 - 0^3] \cdot (n^3 - 3n^2 + 1)$$

$$= n^3 - 3n^2 + 1 + 3n^2 - 6n - 6$$

$$y_p = n^3 - 6n - 5$$

$$y = y_c + y_p$$

$$(D^2 - 4D + 3)y = x^2 e^{2x} \quad (1)$$

$$D^2 - 4D + 3 = 0$$

$$(D-3)(D-1) = 0$$

$$D=3, D=1$$

$$y = c_1 e^{3x} + c_2 e^x$$

$$y = c_1 e^{3x} + c_2 e^x$$

$$D \rightarrow D+3$$

$$(D+3)^2 - 4(D+3) + 3 = 0$$

$$(D+3)^2 - 4(D+3) + 3 = 0$$

Double function - Type - 1

$$f(x) = e^{ax} \cdot y$$

$$y_p = \frac{1}{\phi(D)} e^{ax} \cdot y$$

$$D \rightarrow D+a$$

$$= e^{ax} \frac{1}{\phi(D+a)} \cdot y$$

$$1) (D^2 - 4D + 3) y = x^3 \cdot e^{2x}$$

$$\text{A.E. } D^2 - 4D + 3 = 0$$

$$(D-3)(D-1) = 0$$

$$D=3, D=1$$

$$y_c = c_1 e^{3x} + c_2 e^x$$

$$y_p = \frac{1}{D^2 - 4D + 3} e^{2x} \cdot x^3$$

$$D \rightarrow D+2$$

$$= e^{2x} \frac{1}{(D+2)^2 - 4(D+2) + 3} x^3$$

$$= e^{2x} \frac{1}{D^2 + 4D + 4 - 4D - 8 + 3} x^3$$

$$x_5 = \frac{e^{2x} \cdot 1}{0^2 - 1} = x^3 (0 + 0 + 0) \quad (5)$$

$$= e^{2x} \frac{1}{-1 [1 - D^2]} x^3$$

$$= -e^{2x} [1 - D^2]^{-1} \cdot x^3$$

$$= -e^{2x} [1 + (D^2) + (D^2)^2 + (D^2)^3] \cdot x^3$$

$$= -e^{2x} [1 + D^2] \cdot x^3$$

$$y_p = -e^{2x} [x^3 + 6x]$$

$$[x_5 \sin x + x_6 \cos x] e^{3x} = y$$

$$y = y_c + y_p$$

$$[x_5 + x_6 \sin x] e^{3x} = y$$

$$x_5 = \frac{1}{D^2 - 6D + 13} e^{3x} \sin x$$

$$x_5$$

$$x_6$$

$$x_6 = \frac{1}{D^2 - 6D + 13} e^{3x} \cos x$$

$$x_6 = \frac{1}{D^2 - 6D + 13} e^{3x} \cos x$$

$$2) (D^2 - 6D + 13)y = 8e^{3x} \sin 4x + 2^x$$

$$\text{A.E. } D^2 - 6D + 13 = 0$$

$$D = \frac{6 \pm \sqrt{36 - 52}}{2}$$

$$D = \frac{6 \pm \sqrt{-16}}{2}$$

$$D = \frac{6 \pm 4i}{2}$$

$$D = 3 \pm 2i$$

$$y_c = e^{3x} [C_1 \cos 2x + C_2 \sin 2x]$$

$$y_p = \frac{1}{D^2 - 6D + 13} [8e^{3x} \sin 4x + 2^x]$$

$$= \frac{1}{D^2 - 6D + 13} 8e^{3x} \sin 4x + \frac{1}{D^2 - 6D + 13} 2^x$$

$$= y_{p_1} + y_{p_2}$$

$$y_{p_1} = \frac{1}{D^2 - 6D + 13} 8e^{3x} \sin 4x$$

$$= 8 \frac{1}{D^2 - 6D + 13} e^{3x} \sin 4x$$

$$D \rightarrow D + 3$$

$$= \frac{8}{(0+3)^2 - 6(0+3) + 13} \sin 4x$$

$$= \frac{8}{0^2 + 6 \cdot 0 + 9 - 18 + 13} \sin 4x$$

$$= \frac{8}{0^2 + 4} \sin 4x$$

$$= \frac{8}{-16 + 4} \sin 4x$$

$$= \frac{8}{-12} \sin 4x$$

$$\therefore y_{p1} = -\frac{2}{3} \sin 4x$$

$$y_{p2} = \frac{1}{0^2 - 6 \cdot 0 + 13} 2^x$$

$$\therefore y_{p2} = \frac{1}{(1 \log 2)^2 - 6(1 \log 2) + 13} 2^x$$

$$\therefore y_p = y_{p1} + y_{p2}$$

$$y = y_c + y_p$$

Double function - Type - 3

$$f(x) = x^2 \sin ax \quad / \quad x^2 \cos ax$$

$$Y_p = \frac{1}{\phi(\omega)} x^2 \sin ax$$

$$= I.P. \frac{1}{\phi(\omega)} x^2 e^{aix}$$

$$D \rightarrow D + ai$$

$$= I.P. e^{aix} \frac{1}{\phi(D + ai)} \cdot x^2$$

$$Y_p = \frac{1}{\phi(\omega)} x^2 \cos ax$$

$$= R.P. \frac{1}{\phi(\omega)} x^2 e^{aix}$$

$$D \rightarrow D + ai$$

$$= R.P. e^{aix} \frac{1}{\phi(D + ai)} \cdot x^2$$

$$e^{aix} = \cos ax + i \sin ax$$

↓

R.P.

↓

I.P.

$$1) (D^2 - 1)y = x^2 \sin 3x$$

$$AE \quad D^2 - 1 = 0$$

$$D^2 = 1$$

$$D = \pm 1$$

$$y_c = c_1 e^x + c_2 e^{-x}$$

$$y_p = \frac{1}{D^2 - 1} x^2 \sin 3x$$

$$= IP \frac{1}{D^2 - 1} x^2 e^{3ix}$$

$$D \rightarrow D + 3i$$

$$= IP \frac{e^{3ix}}{(D + 3i)^2 - 1} x^2$$

$$= IP e^{3ix} \frac{1}{D^2 + 6Di + 9i^2 - 1} x^2$$

$$= IP e^{3ix} \frac{1}{D^2 + 6Di - 10} x^2$$

$$= IP e^{3ix} \frac{1}{-10 \left[1 - \left(\frac{D^2}{10} + \frac{6Di}{10} \right) \right]} x^2$$

$$= IP \frac{e^{3ix}}{-10} \left[1 - \left(\frac{D^2}{10} + \frac{6Di}{10} \right) \right]^{-1} x^2$$

$$= IP \frac{e^{3ix}}{-10} \left[1 + \left(\frac{0^2}{10} + \frac{60i}{10} \right) + \left(\frac{0^2}{10} + \frac{60i}{10} \right) \right]$$

$$= IP \frac{e^{3ix}}{-10} \left[1 + \frac{0^2}{10} + \frac{60i}{10} + \frac{360^2 i^2}{100} \right]$$

$$= IP \frac{e^{3ix}}{-10} \left[1 - \frac{260^2}{100} + \frac{60i}{10} \right] x^2$$

$$= IP \frac{e^{3ix}}{-10} \left[x^2 - \frac{26}{100} \times 2 + \frac{6}{10} \times 2xi \right]$$

$$= IP \frac{e^{3ix}}{-10} \left[x^2 - \frac{26}{50} + \frac{6x}{5} i \right]$$

$$= IP \frac{1}{-10} (\cos 3x + i \sin 3x) \left[\left(x^2 - \frac{26}{50} \right) + \left(\frac{6x}{5} \right) i \right]$$

$$\therefore Y_p = \frac{1}{-10} \left[\left(\frac{6x}{5} \right) \cos 3x + \left(x^2 - \frac{26}{50} \right) \sin 3x \right]$$

$$Y = Y_c + Y_p$$

$$\left[\left(\frac{10i}{10} + \frac{50}{10} \right) - 1 \right] 01 -$$

$$\left[\left(\frac{10i}{10} + \frac{50}{10} \right) - 1 \right] 01 -$$

$$2) (D^2+1)y = x^2 \cos 2x$$

$$\text{A.E. } D^2+1=0$$

$$D^2 = -1 = i^2$$

$$D = \pm i$$

$$y_c = C_1 \cos x + C_2 \sin x$$

$$y_p = \frac{1}{D^2+1} \cos 2x \cdot x^2$$

$$= R.P. \frac{1}{D^2+1} x^2 e^{2ix} \quad D \rightarrow D+2i$$

$$= R.P. e^{2ix} \frac{1}{(D+2i)^2+1} x^2$$

$$= R.P. e^{2ix} \frac{1}{D^2+4Di+4i^2+1} x^2$$

$$= R.P. e^{2ix} \frac{1}{D^2+4Di-3} x^2$$

$$= R.P. e^{2ix} \frac{1}{-3 \left[1 - \left(\frac{D^2}{3} + \frac{4Di}{3} \right) \right]} x^2$$

$$= R.P. \frac{e^{2ix}}{-3} \left[1 - \left(\frac{D^2}{3} + \frac{4Di}{3} \right) \right]^{-1} x^2$$

$$= R.P. \frac{e^{2ix}}{-3} \left[1 + \left(\frac{D^2}{3} + \frac{4Di}{3} \right) + \left(\frac{D^2}{3} + \frac{4Di}{3} \right)^2 \right] x^2$$

$$= RP \frac{e^{2in}}{-3} \left[1 + \frac{0^2}{3} + \frac{40i}{3} + \frac{160^2 i^2}{9} \right] \cdot n^2$$

$$= RP \frac{e^{2in}}{-3} \left[1 - \frac{130^2}{9} + \frac{40i}{3} \right] \cdot n^2$$

$$= RP \frac{e^{2in}}{-3} \left[n^2 - \frac{13}{9} \times 2 + \frac{4}{3} \times 2ni \right]$$

$$= RP \frac{e^{2in}}{-3} \left[n^2 - \frac{26}{9} + \frac{8ni}{3} \right]$$

$$= RP \frac{1}{-3} (\cos 2n + i \sin 2n) \left[\left(n^2 - \frac{26}{9} \right) + \left(\frac{8ni}{3} \right) \right]$$

$$= \cancel{RP} \frac{\cancel{e^{2in}}}{\cancel{-3}}$$

$$\therefore y_p = \frac{1}{-3} \left[\left(n^2 - \frac{26}{9} \right) \cos 2n + \left(\frac{8n}{3} \right) \sin 2n \right]$$

$$\therefore y = y_c + y_p$$

$$3) (D^4 + 2D^2 + 1) y = x^2 \cos x$$

$$AE \quad D^4 + 2D^2 + 1 = 0$$

$$(D^2 + 1)^2 = 0$$

$$D^2 = -1 \quad D^2 = -1$$

$$D = \pm i \quad D = \pm i$$

$$y_c = (C_1 + C_2 x) \cos x + (C_3 + C_4 x) \sin x$$

$$y_p = \frac{1}{(D^2 + 1)^2} x^2 \cos x$$

$$= RP \frac{1}{(D^2 + 1)^2} x^2 e^{ix}$$

$$D \rightarrow D + i$$

$$= RP e^{ix} \frac{1}{(D^2 + 1)^2} x^2$$

$$[(D + i)^2 + 1]^2$$

$$= RP e^{ix} \frac{1}{(D^2 + 2Di + i^2 + 1)^2} x^2$$

$$= RP e^{ix} \frac{1}{(D^2 + 2Di)^2} x^2$$

$$= RP e^{ix} \frac{1}{D^4 + 4D^3i + 4D^2i^2} x^2$$

$$D^4 + 4D^3i + 4D^2i^2$$

$$= RP e^{ix} \frac{1}{0^4 + 40^3 i - 4D^2} x^2$$

$$= RP e^{ix} \frac{1}{-4D^2 \left[1 - \left(\frac{0^4}{40^2} + \frac{40^3 i}{40^2} \right) \right]}$$

$$= RP e^{ix} \frac{1}{-4D^2 \left[1 - \left(\frac{0^2}{4} + 0i \right) \right]}$$

$$= RP \frac{e^{ix}}{-4D^2} \left[1 - \left(\frac{D^2}{4} + 0i \right) \right] \cdot x^2$$

$$= RP \frac{e^{ix}}{-4D^2} \left[1 + \left(\frac{D^2}{4} + 0i \right) + \left(\frac{D^2}{4} + 0i \right)^2 \right]$$

$$= RP \frac{e^{ix}}{-4D^2} \left[1 + \frac{D^2}{4} + 0i - D^2 \right] \cdot x^2$$

$$= RP \frac{e^{ix}}{-4D^2} \left[1 - \frac{3}{4} D^2 + 0i \right] \cdot x^2$$

$$= RP \frac{e^{ix}}{-4D^2} \left[x^2 - \frac{3}{4} x^2 + 2xi \right]$$

$$= RP \frac{e^{ix}}{-4D^2} \left[x^2 - \frac{3}{4} + 2xi \right]$$

$$= RP \frac{e^{ix}}{-4} \left[\frac{x^4}{12} - \frac{3x^2}{4} + \frac{x^3 i}{3} \right]$$

$$= \frac{-1}{4} RP (\cos u + i \sin u) \left[\frac{u^4}{12} - \frac{3u^2}{4} + \frac{u^3}{3} \right]$$

$$\therefore y_p = \frac{-1}{4} RP \left[\left(\frac{u^4}{12} - \frac{3u^2}{4} \right) \cos u - \left(\frac{u^3}{3} \right) \sin u \right]$$

$$\therefore y = y_c + y_p$$

$$2) (D^2 + 2D + 1)y = x \cdot \cos x$$

$$\text{A.E. } D^2 + 2D + 1 = 0$$

$$(D+1)^2 = 0$$

$$D = -1 \quad D = -1$$

$$y_c = (C_1 + C_2 x) e^{-x}$$

$$y_p = \frac{1}{D^2 + 2D + 1} x \cdot \cos x$$

$$= \left[x - \frac{2D+2}{D^2+2D+1} \right] \frac{1}{D^2+2D+1} \cos x$$

$$= \left[x - \frac{2D+2}{-1+2D+1} \right] \frac{1}{-1+2D+1} \cos x$$

$$= \left[x - \frac{2D+2}{2D} \right] \frac{1}{2D} \cos x$$

$$= \left[x - \frac{D+1}{D} \right] \frac{1}{2D} \cos x$$

$$= \frac{1}{2} \left[\frac{x}{D} - \frac{D+1}{D^2} \right] \cos x$$

$$D^2 \rightarrow -1$$

$$x = \frac{1}{2} \left[\frac{x}{D} + \frac{D+1}{-1} \right] \cos x$$

$$= \frac{1}{2} \left[\frac{x}{D} + \frac{D+1}{-1} \right] \cos x$$

$$= \frac{1}{2} [x \cdot \sin x + (-\sin x) + \cos x]$$

$$\therefore y_p = \frac{1}{2} [x \cdot \sin x - \sin x + \cos x]$$

$$\therefore y = y_c + y_p$$

$$3) (D^2 + 3D + 2)y = x \cdot \sin 2x$$

$$\text{AE } D^2 + 3D + 2 = 0$$

$$(D+2)(D+1) = 0$$

$$D = -2 \quad D = -1$$

$$y_c = C_1 e^{-2x} + C_2 e^{-x}$$

$$y_p = \frac{1}{D^2 + 3D + 2} x \cdot \sin 2x$$

$$= \left[x - \frac{2D+3}{D^2 + 3D + 2} \right] \frac{1}{D^2 + 3D + 2} \sin 2x$$

$$= \left[x - \frac{2D+3}{-4+3D+2} \right] \frac{1}{-4+3D+2} \sin 2x$$

$$= \left[x - \frac{2D+3}{3D-2} \right] \frac{1}{3D-2} \sin 2x$$

$$= \left[\frac{x}{3D-2} - \frac{2D+3}{(3D-2)^2} \right] \sin 2x$$

$$= \left[\frac{x}{3D-2} - \frac{2D+3}{9D^2 - 12D + 4} \right] \sin 2x$$

$$= \left[\frac{x}{3D-2} - \frac{2D+3}{9(-4) - 12D + 4} \right] \sin 2x$$

$$= \left[\frac{x}{3D-2} - \frac{2D+3+0}{-36-12D+4} \right] \sin 2x$$

$$= \left[\frac{x}{3D-2} - \frac{(8-2D+3+0)}{8-12D-32} \right] \sin 2x$$

$$= \left[\frac{x}{3D-2} + \frac{2D+3+0}{4(3D+8)} \right] \sin 2x$$

$$= \left[\frac{x}{3D-2} \sin 2x + \frac{2D+3}{4(3D+8)} \sin 2x \right]$$

y_{p1} y_{p2}

$$y_{p1} = \frac{x}{3D-2} \sin 2x$$

$$= \frac{x(3D+2)}{(3D+2)(3D-2)} \sin 2x$$

$$= \frac{x(3D+2)}{9D^2-4} \sin 2x$$

$D^2 \rightarrow -4$

$$= \frac{x(3D+2)}{9(-4)-4} \sin 2x$$

$$= \frac{x(3D+2)}{-40} \sin 2x$$

$$= \frac{x}{-40} [3 \cos 2x \cdot 2 + 2 \sin 2x]$$

$$\therefore y_{p1} = \frac{x}{-40} [6 \cos 2x + 2 \sin 2x]$$

$$y_{p2} = \frac{20+3}{4(30+8)} \sin 2x$$

$$= \frac{(20+3)(30-8)}{4(30+8)(30-8)} \sin 2x$$

$$= \frac{60^2 - 160 + 90 - 24}{4(90^2 - 64)} \sin 2x$$

$$= \frac{-24 - 160 + 90 - 24}{4(90 - 64)} \sin 2x$$

$$= \frac{-70 - 48}{4(-100)} \sin 2x$$

$$= \frac{1}{400} (70 + 48) \sin 2x$$

$$= \frac{1}{400} [7 \cos 2x \cdot 2 + 48 \sin 2x]$$

$$\therefore y_{p2} = \frac{1}{400} [14 \cos 2x + 48 \sin 2x]$$

$$\therefore y_p = y_{p1} + y_{p2}$$

$$\therefore y = y_c + y_p$$

$$4) (D^2 - 1)y = x \cdot e^x \cdot \sin x$$

$$A.E. \quad D^2 - 1 = 0$$

$$D^2 = 1$$

$$D = \pm 1$$

$$y_c = c_1 e^x + c_2 e^{-x}$$

$$y_p = \frac{x \cdot e^x \cdot \sin x}{D^2 - 1}$$

$$D \rightarrow D+1$$

$$= e^x \frac{1}{(D+1)^2 - 1} x \cdot \sin x$$

$$= e^x \frac{1}{D^2 + 2D + 1 - 1} x \cdot \sin x$$

$$= e^x \frac{1}{D^2 + 2D} x \cdot \sin x$$

$$= e^x \left[x - \frac{2D+2}{D^2+2D} \right] \frac{1}{D^2+2D} \sin x$$

$$= e^x \left[x - \frac{2D+2}{2D-1} \right] \frac{1}{2D-1} \sin x$$

$$= e^x \left[\frac{x}{2D-1} - \frac{2D+2}{(2D-1)^2} \right] \sin x$$

$$= e^x \left[\frac{x}{2D-1} - \frac{2D+2}{4D^2-4D+1} \right] \sin x$$

$D^2 \rightarrow -1$

$$= e^x \left[\frac{x}{2D-1} - \frac{2D+2}{-4D-3} \right] \sin x$$

$$= e^x \left[\frac{x}{2D-1} + \frac{2D+2}{4D+3} \right] \sin x$$

$$= e^x \left[\frac{x}{2D-1} \sin x + \frac{2D+2}{4D+3} \sin x \right]$$

y_{p1} y_{p2}

$$y_{p1} = \frac{x}{2D-1} \sin x$$

$$= \frac{x(2D+1)}{(2D+1)(2D-1)} \sin x$$

$$= \frac{x(2D+1)}{4D^2-1} \sin x$$

$D^2 \rightarrow -1$

$$= \frac{x}{-5} (2D+1) \sin x$$

$$\therefore y_{p1} = \frac{x}{-5} (2 \cos x + \sin x)$$

$$y_{p2} = \frac{2D+2}{4D+3} \sin x$$

$$= \frac{(2D+2)(4D+3)}{(4D+3)(4D-3)} \sin x$$

$$= \frac{8D^2 + 6D + 8D + 6}{16D^2 - 9} \sin x$$

$$= \frac{-8 + 14D + 6}{-25} \sin x$$

$$= \frac{14D - 2}{-25} \sin x$$

$$\therefore Y_{P_2} = \frac{1}{-25} (14 \cos x - 2 \sin x)$$

$$\therefore Y_P = Y_{P_1} + Y_{P_2}$$

$$\therefore y = y_c + y_p$$

General Method.

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$$Y_p = \frac{1}{(D+m_1)(D+m_2)} f(x)$$

$$= \frac{1}{(D+m_2)} e^{-m_1 x} \int e^{m_1 x} f(x) dx$$

$$= \frac{1}{(D+m_2)} e^{-m_1 x} \phi(x)$$

$$= \frac{1}{e^{-m_2 x}} \int e^{m_2 x} \frac{1}{e^{-m_1 x}} \phi(x) dx$$

$$y_1 + y_2 = y \therefore$$

$$y_1 + y_2 = y \therefore$$

Type - 1 $f(x) = e^x$.

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$$1) (D^2 + 3D + 2)y = e^x$$

AE

$$D^2 + 3D + 2 = 0$$

$$(D+2)(D+1) = 0$$

$$D = -2$$

$$D = -1$$

$$y_c = c_1 e^{-2x} + c_2 e^{-x}$$

$$y_p = \frac{1}{D^2 + 3D + 2} e^x$$

$$= \frac{1}{(D+2)(D+1)} e^x$$

$$= \frac{1}{(D+2)} e^x \int e^x e^x dx$$

$$I = \int e^x e^x dx$$

$$\text{put } e^x = t$$

$$e^x dx = dt$$

$$\int e^t dt$$

$$e^t = e^x$$

$$= \frac{1}{(D+2)} e^x \cdot e^x$$

$$= \frac{-2\pi}{e} \int e^{2u} \cdot \frac{e^u}{e^u} \cdot e^u du$$

$$= \frac{-2\pi}{e} \int e^u \cdot e^u du$$

$$\therefore y_p = \frac{-2\pi}{e} \cdot e^u$$

$$y = y_c + y_p$$

$$2) (D^2 + 3D + 2)y = \sin e^x \quad (x > 0)$$

AE $D^2 + 3D + 2 = 0$

$$(D+2)(D+1) = 0$$

$$D = -2 \text{ or } D = -1$$

$$y_c = c_1 e^{-2x} + c_2 e^{-x}$$

$$y_p = \frac{1}{D^2 + 3D + 2} \sin e^x$$

$$= \frac{1}{(D+2)(D+1)} \sin e^x$$

$$= \frac{1}{(D+2)} e^x \int e^x \sin e^x dx$$

$$I = \int e^x \sin e^x dx$$

put $e^x = t$

$$e^x dx = dt$$

$$\int \sin t dt$$

$$= -\cos t$$

$$= -\cos e^x$$

$$= \frac{1}{(D+2)} e^x (-\cos e^x)$$

$$= -\frac{1}{(0+2)} e^{-n} \cdot \cos e^n$$

$$= -e^{-2n} \int e^{2n} \cdot e^{-n} \cdot \cos e^n \, dn$$

$$= -e^{-2n} \int e^n \cos e^n \, dn$$

I

$$I = \int e^n \cos e^n \, dn$$

$$\text{put } e^n = t$$

$$e^n \, dn = dt$$

$$\int \cos t \, dt$$

$$\sin t$$

$$\sin e^n$$

$$\therefore y_p = -e^{-2n} \sin e^n$$

$$y = y_c + y_p$$

$$3) (D^2 + 3D + 2)y = e^{e^x} + \cos e^x$$

$$\text{AE } D^2 + 3D + 2 = 0$$

$$(D+2)(D+1) = 0$$

$$D = -2 \quad D = -1$$

$$y_c = c_1 e^{-2x} + c_2 e^{-x}$$

$$y_p = \frac{1}{(D+2)(D+1)} e^{e^x} + \cos e^x$$

$$= \frac{1}{(D+2)} \int e^{e^x} (e^{e^x} + \cos e^x) dx$$

$$I = \int e^u (e^{e^u} + \cos e^u) du$$

$$\text{put } e^u = t$$

$$e^u du = dt$$

$$\int e^t + \cos t \, dt$$

$$e^t + \sin t$$

$$e^{e^u} + \sin e^u$$

$$= \frac{1}{(D+2)} e^{e^x} (e^{e^x} + \sin e^x)$$

$$= e^{-2x} \int e^{2x} \cdot e^{e^x} (e^{e^x} + \sin e^x) dx$$

$$= e^{-2n} \int e^n (e^n + \sin e^n) dn$$

$$I = \int e^n + \sin e^n dn$$

$$\text{put } e^n = t$$

$$e^n dn = dt$$

$$\int e^t + \sin t dt$$

$$e^t - \cos t$$

$$e e^n - \cos e^n$$

$$\therefore y_p = e^{-2n} (e e^n - \cos e^n)$$

$$y = y_c + y_p$$

$$4) (D^2 - 3D + 2)y = \sin(e^{-x})$$

$$AE \quad D^2 - 3D + 2 = 0$$

$$(D-2)(D-1) = 0$$

$$D=2, D=1$$

$$y_c = c_1 e^{2x} + c_2 e^x$$

$$y_p = \frac{1}{(D-2)(D-1)} \sin(e^{-x})$$

$$= \frac{1}{(D-2)} e^x \int e^{-x} \sin e^{-x} dx$$

$$I = \int e^{-x} \sin e^{-x} dx$$

$$\text{put } e^{-x} = t$$

$$-e^{-x} dx = dt$$

$$e^{-x} dx = -dt$$

$$-\int \sin t dt$$

$$\cos t$$

$$\cos e^{-x}$$

$$= \frac{1}{(D-2)} e^x \cdot \cos e^{-x}$$

$$= e^{2x} \int e^{-2x} \cdot e^x \cdot \cos e^{-x} dx$$

$$= e^{2n} \int_I \bar{e}^n \cos \bar{e}^n dn$$

$$I = \int \bar{e}^n \cos \bar{e}^n dn$$

$$\text{put } \bar{e}^n = t$$

$$-\bar{e}^n dn = dt$$

$$\bar{e}^n dn = -dt$$

$$-\int \cos t dt$$

$$-\sin t$$

$$-\sin \bar{e}^n$$

$$\therefore y_p = e^{2n} (-\sin \bar{e}^n)$$

$$\therefore y = y_c + y_p$$

$$5) (D^2 - 3D + 2)y = \frac{1}{e^{-n}} + \cos \frac{1}{e^n}$$

AE.

$$D^2 - 3D + 2 = 0$$

$$(D-2)(D-1) = 0$$

$$D=2, D=1$$

$$y_c = c_1 e^{2n} + c_2 e^n$$

$$y_p = \frac{1}{(D-2)(D-1)} \left(\frac{1}{e^{-n}} + \cos \frac{1}{e^n} \right)$$

$$= \frac{1}{(D-2)(D-1)} \left(e^{-n} + \cos e^{-n} \right)$$

$$= \frac{1}{(D-2)} \int e^{-n} (e^{-n} + \cos e^{-n}) dn$$

$$I = \int e^{-n} (e^{-n} + \cos e^{-n}) dn$$

$$\text{put } e^{-n} = t$$

$$e^{-n} dn = -dt$$

$$\int e^t + \cos t dt$$

$$= [-e^t + \sin t]$$

$$= [-e^{-n} + \sin e^{-n}]$$

$$= -\frac{1}{(0-2)} e^n [\bar{e} e^{-n} + \sin \bar{e}^{-n}]$$

$$= -e^{2n} \int \bar{e}^{-2n} \cdot e^n [\bar{e} e^{-n} + \sin \bar{e}^{-n}]$$

$$= -e^{2n} \int \bar{e}^{-n} [\bar{e} e^{-n} + \sin \bar{e}^{-n}] dn$$

$$I = \int \bar{e}^{-n} [\bar{e} e^{-n} + \sin \bar{e}^{-n}] dn$$

$$\text{put } \bar{e}^{-n} = t$$

$$\bar{e}^{-n} dn = -dt$$

$$= \int e^t + \sin t dt$$

$$= [-e^t - \cos t]$$

$$= -e^t - \cos t$$

$$= -\bar{e}^{-n} - \cos \bar{e}^{-n}$$

$$y_p = -e^{2n} [\bar{e} e^{-n} + \cos \bar{e}^{-n}]$$

$$y = y_c + y_p$$

Type-2. $f(n) = \frac{1}{(1-0)(1+e^n)} = \frac{1}{1+e^n}$

$\int \frac{f'(n)}{f(n)} dn = \log f(n).$

1) $(D^2 - 1)y = \frac{1}{1+e^n}$

AE $D^2 - 1 = 0$

$D^2 = -1 = i^2$

$D = \pm i$

$y_c = c_1 \cos n + c_2 \sin n$

$y_p = \frac{1}{1+e^n}$

1) $(D^2 - 1)y = \frac{1}{1+e^n}$

AE $D^2 - 1 = 0$

$D^2 = 1$

$D = \pm 1$

$y_c = c_1 e^n + c_2 e^{-n}$

$$y_p = \frac{1}{(0+1)(0-1)} \frac{1}{1+e^n}$$

$$= \frac{1}{(0-1)} e^n \int \frac{e^n}{1+e^n} dn$$

$$= \frac{1}{(0-1)} e^n \cdot \log(1+e^n)$$

$$= e^n \int \cancel{e^n} \cdot \cancel{e^n} \cdot \log(1+e^n) dn$$

$$= e^n \int \log(1+e^n) \underbrace{e^n}_{(u)} \cdot \underbrace{e^{-2n}}_{(v)} dn$$

$$= e^n \left[\log(1+e^n) \int e^{-2n} dn - \right.$$

$$\left. \int \frac{d}{dn} \log(1+e^n) \int e^{-2n} dn \right]$$

$$= e^n \left[\log(1+e^n) \cdot \frac{e^{-2n}}{-2} - \int \frac{1}{1+e^n} e^n \cdot e^{-2n} dn \right]$$

$$= e^n \left[-\log(1+e^n) \frac{e^{-2n}}{2} + \int \frac{e^n}{1+e^n} dn \right]$$

I

$$I = \int \frac{e^{-x}}{1+e^x} dx$$

$$= \int \frac{e^{-x}}{1+\frac{1}{e^{-x}}} dx$$

$$= \int \frac{e^{-x} \cdot e^{-x}}{e^{-x} + 1} dx$$

put $e^{-x} + 1 = t \quad \therefore e^{-x} = t - 1$

$$e^{-x} dx = -dt$$

$$I = - \int \frac{t-1}{t} dt$$

$$= - \int \left(1 - \frac{1}{t} \right) dt$$

$$= - [t - \log t]$$

$$= - (e^{-x} + 1) + \log (e^{-x} + 1)$$

— in (I)

$$\therefore Y_p = e^x \left[-\log (1+e^x) \cdot \frac{e^{2x}}{2} - (e^{-x} + 1) + \log (e^{-x} + 1) \right]$$

$$Y = Y_c + Y_p$$

$$2) (D^2 + 3D + 2)y = \frac{1}{1+e^n}$$

$$AE \quad D^2 + 3D + 2 = 0$$

$$(D+2)(D+1) = 0$$

$$D = -2 \quad D = -1$$

$$y_c = c_1 e^{-2n} + c_2 e^{-n}$$

$$y_p = \frac{1}{(D+2)(D+1)} \frac{1}{1+e^n}$$

$$= \frac{1}{(D+2)} e^{-n} \int e^n \frac{1}{1+e^n} dn$$

$$= \frac{1}{(D+2)} e^{-n} \cdot \log(1+e^n)$$

$$= e^{-2n} \int e^{2n} \cdot e^{-n} \cdot \log(1+e^n) dn$$

$$= e^{-2n} \int e^n \cdot \log(1+e^n) dn$$

$$= e^{-2n} \left[\log(1+e^n) \int e^n dn - \int \frac{d}{dn} \log(1+e^n) \int e^n dn \right]$$

$$= e^{-2x} \left[\log(1+e^x) \cdot e^x - \int \frac{1}{1+e^x} \cdot e^x dx \right]$$

$$y_p = y_c [C_1' + C_2'] \quad \text{--- (I)}$$

$$I = \int \frac{e^x \cdot e^x}{1+e^x} dx$$

$$\text{put } 1+e^x = t \quad \therefore e^x = t-1$$

$$e^x dx = dt$$

$$= \int \frac{t-1}{t} dt \quad 1+e^x = 0$$

$$= \int \left(1 - \frac{1}{t} \right) dt$$

$$t - \log t$$

$$(e^x + 1) - \log(e^x + 1) \quad \text{--- in (I)}$$

$$\therefore y_p = e^{-2x} \left[\log(1+e^x) \cdot e^x - (e^x + 1) + \log(e^x + 1) \right]$$

$$y = y_c + y_p$$

Type-3.

$$\int e^n [f(n) + f'(n)] dn = e^n \cdot f(n)$$

$$1) (D^2 - 1)y = e^n \sin e^n + \cos e^n$$

$$\text{AE } D^2 - 1 = 0$$

$$D^2 = 1$$

$$D = \pm 1$$

$$y_c = c_1 e^n + c_2 e^{-n}$$

$$y_p = \frac{1}{(D+1)(D-1)} [e^n \sin e^n + \cos e^n]$$

$$\frac{d}{dn} \cos e^n = -\sin e^n (e^n) (-1)$$

$$= e^n \cdot \sin e^n$$

$$y_p = \frac{1}{(D-1)} e^n \int e^n [\cos e^n + e^n \sin e^n]$$

$$= \frac{1}{(D-1)} \cancel{e^n} \cdot \cancel{e^n} \cos e^n$$

$$(s+1) = \int_0^{\infty} e^{-st} \int_0^{\infty} \bar{e}^{-u} \cos \bar{e}^{-u} du dt$$

$$I = \int_0^{\infty} \bar{e}^{-u} \cos \bar{e}^{-u} du$$

$$\text{put } \bar{e}^{-u} = t$$

$$\bar{e}^{-u} du = -dt$$

$$= - \int \cos t dt$$

$$= - \sin t$$

$$= - \sin \bar{e}^{-u}$$

$$\therefore y_p = -e^{-u} \sin \bar{e}^{-u}$$

$$\therefore y = y_c + y_p$$

2) $(D^2 + 5D + 6)y = e^{-2x} \sec^2 x (1 + 2 \tan x)$

AE $D^2 + 5D + 6 = 0$

$$(D+3)(D+2) = 0$$

$$D = -3 \quad D = -2$$

$$y_c = c_1 e^{-3x} + c_2 e^{-2x}$$

$$y_p = \frac{1}{(D+2)(D+3)} e^{-2x} \sec^2 x (1 + 2 \tan x)$$

$$= \frac{1}{(D+2)(D+3)} e^{-2x} [\sec^2 x + 2 \sec^2 x \tan x]$$

$$\frac{d}{dx} \sec^2 x = 2 \sec x \cdot \sec x \cdot \tan x$$

$$= 2 \sec^2 x \cdot \tan x$$

$$y_p = \frac{1}{(D+2)} e^{-3x} \int e^{3x} e^{2x} (\sec^2 x + 2 \sec^2 x \tan x) dx$$

$$= \frac{1}{(D+2)} e^{-3x} \cdot e^x \cdot \sec^2 x$$

$$= \frac{1}{(D+2)} e^{-2x} \cdot \sec^2 x$$

$$= e^{-2x} \cdot \int e^{2x} \cdot e^{-2x} \cdot (\sec^2 x) dx$$

$$= e^{-2x} \int \sec^2 x dx = e^{-2x} \tan x + C$$

$$\therefore y_p = e^{-2x} \cdot \tan x$$

$$\therefore y = y_c + y_p$$

$$\frac{1}{x} + \frac{1}{x} + \frac{1}{x} = \frac{3}{x}$$

$$\frac{1}{x} + \frac{1}{x} + \frac{1}{x} = \frac{3}{x}$$

$$\left[\left(\frac{1}{x} + \frac{1}{x} + \frac{1}{x} \right) + \left(\frac{1}{x} - \frac{1}{x} \right) \right] = \frac{3}{x}$$

$$\left[\left(\frac{1}{x} + \frac{1}{x} + \frac{1}{x} \right) + \left(\frac{1}{x} - \frac{1}{x} \right) \right] = \frac{3}{x}$$

$$\left[\left(\frac{1}{x} + \frac{1}{x} + \frac{1}{x} \right) + \left(\frac{1}{x} - \frac{1}{x} \right) \right] = \frac{3}{x}$$

$$\left(\frac{1}{x} + \frac{1}{x} + \frac{1}{x} \right) = \frac{3}{x}$$

$$3) (D^2 - D - 2)y = 2 \log x + \frac{1}{x} + \frac{1}{x^2}$$

$$\text{AE } D^2 - D - 2 = 0$$

$$(D-2)(D+1) = 0$$

$$D = 2 \quad D = -1$$

$$y_c = c_1 e^{2x} + c_2 e^{-x}$$

$$y_p = \frac{1}{D^2 - D - 2} \left(2 \log x + \frac{1}{x} + \frac{1}{x^2} \right)$$

$$= \frac{1}{D^2 - D - 2} \left(2 \log x - \frac{1}{x} + \frac{2}{x} + \frac{1}{x^2} \right)$$

$$= \frac{1}{D^2 - D - 2} \left[\left(2 \log x - \frac{1}{x} \right) + \left(\frac{2}{x} + \frac{1}{x^2} \right) \right]$$

$$= \frac{1}{(D-2)(D+1)} \left[\underbrace{\left(2 \log x - \frac{1}{x} \right)}_{f(x)} + \underbrace{\left(\frac{2}{x} + \frac{1}{x^2} \right)}_{f(x)} \right]$$

$$= \frac{1}{(D-2)} \bar{e}^x \int e^x \left[\underbrace{\left(2 \log x - \frac{1}{x} \right)}_{f(x)} + \underbrace{\left(\frac{2}{x} + \frac{1}{x^2} \right)}_{f(x)} \right] dx$$

$$= \frac{1}{(D-2)} \bar{e}^x \cdot e^x \left(2 \log x - \frac{1}{x} \right)$$

$$\begin{aligned}
 &= e^{2n} \int e^{-2n} \left(2 \log n - \frac{1}{n} \right) dn \\
 &= e^{2n} \left[\int e^{-2n} \cdot 2 \log n \, dn - \int e^{-2n} \frac{1}{n} \, dn \right] \\
 &= e^{2n} \left[2 \log n \int e^{-2n} \, dn - \int \frac{d}{dn} 2 \log n \cdot \int e^{-2n} \, dn - \int e^{-2n} \frac{1}{n} \, dn \right] \\
 &= e^{2n} \left[2 \log n \cdot \frac{e^{-2n}}{-2} - \int \frac{2}{n} \cdot \frac{e^{-2n}}{-2} \, dn - \int \frac{e^{-2n}}{n} \, dn \right]
 \end{aligned}$$

$$\begin{aligned}
 &= e^{2n} \left[-\log n \cdot e^{-2n} + \int \frac{e^{-2n}}{n} \, dn - \int \frac{e^{-2n}}{n} \, dn \right] \\
 &= e^{2n} \left[-\log n \cdot e^{-2n} \right]
 \end{aligned}$$

$$\therefore y_p = -\log n$$

$$\therefore \frac{1}{n} y = y_c + y_p$$

$$\left[\log n \cdot \frac{1}{n} - \log n \cdot \frac{1}{n} \right] = 0$$

$$\left[\log n \cdot \frac{1}{n} \right] = 0$$

$$\left[\log n \cdot \frac{1}{n} - \log n \cdot \frac{1}{n} \right] = 0$$

$$4) (D^2 - 1)y = \log x + \frac{1}{x^2}$$

AE

$$D^2 - 1 = 0$$

$$(D+1)(D-1) = 0$$

$$D = 1, D = -1$$

$$y_c = c_1 e^x + c_2 e^{-x}$$

$$y_p = \frac{1}{(D+1)(D-1)} \left(\log x + \frac{1}{x^2} \right)$$

$$= \frac{1}{(D-1)} \bar{e}^x \int e^x \left[\left(\log x - \frac{1}{x} \right) + \left(\frac{1}{x} + \frac{1}{x^2} \right) \right] dx$$

$f(x) \qquad f'(x)$

$$= \frac{1}{(D-1)} \bar{e}^x \cdot e^x \cdot \left(\log x - \frac{1}{x} \right)$$

$$= e^x \int \bar{e}^x \left(\log x - \frac{1}{x} \right) dx$$

$$= e^x \left[\int \bar{e}^x \log x \, dx - \int \bar{e}^x \frac{1}{x} \, dx \right]$$

(v) (u)

$$= e^x \left[\log x \int \bar{e}^x \, dx - \int \frac{d}{dx} \log x \int \bar{e}^x \, dx \, dx - \int \frac{\bar{e}^x}{x} \, dx \right]$$

$$= e^x \left[\log x \cdot \frac{\bar{e}^x}{-1} - \int \frac{1}{x} \cdot \frac{\bar{e}^x}{-1} \, dx - \int \frac{\bar{e}^x}{x} \, dx \right]$$

$$= e^n \left[-\log n \cdot \bar{e}^n + \int \frac{\bar{e}^n}{n} dn - \int \frac{\bar{e}^n}{n} dn \right]$$

$$= \cancel{e^n} \left[-\log n \cdot \cancel{\bar{e}^n} \right]$$

$$\therefore y_p = -\log n$$

$$\therefore y = y_c + y_p$$

$$\begin{vmatrix} y & 1 \\ y' & 1 \end{vmatrix} = 0$$

$$y' - y = 0 \quad \therefore$$

Variation of Parameter

$$y_c = c_1 y_1 + c_2 y_2$$

$$y_p = u y_1 + v y_2$$

$$u = - \int \frac{y_2 f(x)}{D} dx$$

$$v = \int \frac{y_1 f(x)}{D} dx$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$\therefore D = y_1 y_2' - y_2 y_1'$$

$$\therefore D = y_1 y_2' - y_2 y_1'$$

$$1) (D^2 - 2D + 2)y = e^x \tan x$$

$$AE: D^2 - 2D + 2 = 0$$

$$D = \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = \frac{2 \pm 2i}{2}$$

$$D = 1 \pm i$$

$$y_c = e^x [c_1 \cos x + c_2 \sin x]$$

$$y_c = c_1 \frac{e^x \cos x}{y_1} + c_2 \frac{e^x \sin x}{y_2}$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$D = \begin{vmatrix} e^x \cos x & e^x \sin x \\ -e^x \sin x + \cos x \cdot e^x & e^x \cos x + \sin x \cdot e^x \end{vmatrix}$$

$$D = e^{2x} \cos^2 x + e^{2x} \sin^2 x - e^{2x} \sin x \cos x + e^{2x} \sin x \cos x$$

$$D = e^{2x}$$

$$u = - \int \frac{y_2 f(x)}{D} dx$$

$$= - \int \frac{e^x \sin x \cdot e^x \cdot \tan x}{e^{2x}} dx$$

$$= - \int \sin x \cdot \frac{\sin x}{\cos x} dx$$

$$= - \int \frac{\sin^2 x}{\cos x} dx$$

$$= - \int \frac{1 - \cos^2 x}{\cos x} dx$$

$$= - \int \frac{1}{\cos x} - \cos x dx$$

$$= - \int \sec x - \cos x dx$$

$$u = - [\log(\sec x + \tan x) - \sin x]$$

$$v = \int \frac{y_1 f(x)}{D} dx$$

$$= \int \frac{e^x \cdot \cos x \cdot e^x \cdot \tan x}{e^{2x}} dx$$

$$= \int e \cos x \cdot \frac{\sin x}{\cos x} dx$$

$$y = \int \sin nx \, dx = -\frac{1}{n} \cos nx + C$$

$$\therefore v = -\cos nx \cdot \frac{1}{n} = -\frac{1}{n} \cos nx$$

$$\therefore y_p = u y_1 + v y_2$$

$$\therefore y = y_c + y_p$$

$$\frac{y}{x} = \frac{C \cos nx + \frac{1}{n} \sin nx}{x}$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} x & x \\ 1 & 1 \end{vmatrix} = 0$$

$$= \begin{vmatrix} -5 \sin nx & -5 \cos nx \\ 5 \cos nx & 5 \sin nx \end{vmatrix}$$

$$= 5 \cos^2 nx + 5 \sin^2 nx$$

$$D = 5 \neq 0$$

$$u = \frac{y_2' D_1 - y_1' D_2}{D}$$

$$= \frac{5 \sin nx \cdot \cos nx - 5 \cos nx \cdot \sin nx}{5}$$

$$= \frac{5 \cos^2 nx - 5 \sin^2 nx}{5}$$

$$2) (D^2 + 4)y = \tan 2x$$

$$AE \quad D^2 + 4 = 0$$

$$D^2 = -4$$

$$D^2 = 4i^2$$

$$\therefore D = \pm 2i$$

$$y_c = \frac{C_1 \cos 2x}{y_1} + \frac{C_2 \sin 2x}{y_2}$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2 \sin 2x & 2 \cos 2x \end{vmatrix}$$

$$= 2 \cos^2 2x + 2 \sin^2 2x$$

$$\therefore D = 2$$

$$u = - \int \frac{y_2 f(x)}{D} dx$$

$$= - \int \frac{\sin 2x \cdot \tan 2x}{2} dx$$

$$= - \frac{1}{2} \int \sin 2x \cdot \frac{\sin 2x}{\cos 2x} dx$$

$$= \frac{-1}{2} \int \frac{\sin^2 2x}{\cos 2x} dx \quad (1+50) \quad (E)$$

$$= \frac{-1}{2} \int \frac{1 - \cos^2 2x}{\cos 2x} dx$$

$$= \frac{-1}{2} \int \sec 2x - \cos 2x dx$$

$$\therefore u = \frac{-1}{2} \left[\log \frac{\sec 2x + \tan 2x}{2} - \frac{\sin 2x}{2} \right]$$

$$v = \int \frac{y_1 f(x)}{D} dx$$

$$= \int \frac{\cos 2x \cdot \tan 2x}{\cos 2x} dx =$$

$$= \frac{1}{2} \int \frac{\cos 2x \cdot \sin 2x}{\cos 2x} dx =$$

$$= \frac{1}{2} \int \sin 2x dx$$

$$\therefore v = \frac{1}{2} \left[\frac{-\cos 2x}{2} \right]$$

$$y_p = u y_1 + v y_2$$

$$\therefore y = y_c + y_p$$

$$3) (D^2 + 4)y = 4 \sec 2x$$

$$\text{A.E } D^2 + 4 = 0$$

$$D^2 = -4$$

$$D = \pm 2i$$

$$y_c = C_1 \cos 2x + C_2 \sin 2x$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2 \sin 2x & 2 \cos 2x \end{vmatrix}$$

$$= 2 \cos^2 2x + 2 \sin^2 2x$$

$$\therefore D = 2$$

$$u = - \int \frac{y_2 f(x)}{D} dx$$

$$= - \int \frac{\sin 2x \cdot 4 \sec 2x}{2} dx$$

$$= -2 \int \sin 2x \cdot \frac{1}{\cos 2x} dx$$

$$= -2 \int \tan 2x dx$$

$$\therefore u = -2 \log (\sec 2x)$$

$$V = \int \frac{y_1 f(x)}{D} dx$$

$$= \int \frac{\cos 2x \cdot 4 \sec 2x}{2} dx$$

$$= 2 \int \cos 2x \cdot \frac{1}{\cos 2x} dx$$

$$= 2 \int 1 dx$$

$$\therefore V = 2x$$

$$y_p = u y_1 + v y_2$$

$$y_{\text{total}} = y_c + y_p$$

4) ~~$(D^2+4)y = 4 \sec^2 2x$~~

AE $D^2+4=0$

$$D^2 = -4$$

$$D = \pm 2i$$

$$\therefore y_c = \frac{C_1 \cos 2x}{y_1} + \frac{C_2 \sin 2x}{y_2}$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2 \sin 2x & 2 \cos 2x \end{vmatrix}$$

$$= 2 \cos^2 2x + 2 \sin^2 2x$$

$$\therefore D = 2$$

$$u = - \int \frac{y_2 f(x)}{D} dx$$

$$= - \int \frac{\sin 2x \cdot 4 \sec^2 2x}{2} dx$$

$$= -2 \int \sin 2x \cdot \frac{\sec 2x}{\cos 2x} dx$$

$$= -2 \int \sec 2x \cdot \tan 2x dx$$

$$\therefore u = -2 \sec 2x$$

$$\therefore u = -\sec 2x \cdot y \quad (1)$$

$$v = \int \frac{y \cdot f(x)}{D} dx = \int \frac{y \cdot \cos 2x}{\cos 2x} dx$$

$$= \int \cos 2x \cdot \frac{4 \sec 2x}{\cos 2x} dx$$

$$= 2 \int \sec 2x \cdot dx$$

$$= 2 \log (\sec 2x + \tan 2x)$$

$$= \log (\sec 2x + \tan 2x)$$

$$\therefore y_p = u y_1 + v y_2$$

$$\therefore y = y_c + y_p$$

$$5) (D^2 - 4D + 4) y = e^{2x} \sec^2 x$$

$$AE \quad D^2 - 4D + 4 = 0$$

$$(D - 2)^2 = 0$$

$$D = 2, D = 2$$

$$y_c = e^{2x} (C_1 + C_2 x)$$

$$\therefore y_c = \frac{C_1 e^{2x}}{y_1} + \frac{C_2 x e^{2x}}{y_2}$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} e^{2x} & x e^{2x} \\ 2e^{2x} & x 2e^{2x} + e^{2x} \end{vmatrix}$$

$$= e^{4x} + 2x e^{4x} - 2x e^{4x}$$

$$\therefore D = e^{4x}$$

$$u = - \int \frac{y_2 f(x)}{D} dx$$

$$= - \int \frac{x e^{2x} \cdot e^{2x} \sec^2 x}{e^{4x}} dx$$

$$= - \int \frac{x \cdot \sec^2 x}{u \quad v} dx$$

$$\therefore u = - [x \cdot \tan x - (1) \log (\sec x)]$$

$$V = \int \frac{y_1 f(x)}{S_1 D} (dx + 02 - 0) \quad (2)$$

$$= \int \frac{e^{2x} \cdot e^{2x} \cdot \sec^2 x}{e^{4x} + 02 - 0} dx$$

$$= \int \sec^2 x dx \quad (6-0)$$

$$\therefore V = \tan x \quad \delta=0 \quad \delta=0$$

$$\therefore y_p = u y_1 + v y_2 + \dots = y \therefore$$

$$\therefore y = \frac{y_c}{x} + \frac{y_p}{x} = y \therefore$$

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = 0$$

$$\begin{vmatrix} e^{2x} & e^{2x} \\ 2e^{2x} & 2e^{2x} \end{vmatrix} = 0$$

$$2e^{2x} + 2e^{2x} - 2e^{2x} - 2e^{2x} = 0$$

$$4e^{2x} = 0 \therefore$$

$$u = - \frac{1}{2} \frac{y_2 f(x)}{D}$$

$$v = \frac{1}{2} \frac{y_1 f(x)}{D}$$

$$= \frac{1}{2} \frac{y_1 f(x)}{D}$$

$$6) (D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$$

A.E.

$$D^2 - 6D + 9 = 0$$

$$(D-3)^2 = 0$$

$$D=3 \quad D=3$$

$$\therefore y_c = (C_1 + C_2 x) e^{3x}$$

$$\therefore y_c = \frac{C_1 e^{3x}}{y_1} + \frac{C_2 x e^{3x}}{y_2}$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} e^{3x} & x e^{3x} \\ 3e^{3x} & x 3e^{3x} + e^{3x} \end{vmatrix}$$

$$= e^{6x} + 3x e^{6x} - 3x e^{6x} - e^{6x}$$

$$\therefore D = e^{6x}$$

$$u = - \int \frac{y_2 f(x)}{D} dx$$

$$= - \int \frac{x e^{3x} \cdot e^{3x}}{e^{6x} x^2} dx$$

$$= - \int \frac{1}{x} dx$$

$$\therefore u = -\log x = y(1+0) \quad (*)$$

$$V = \int \frac{y_1 \cdot f(x)}{0} dx \quad \text{at } x=0 \quad \text{at } x=1$$

$$= \int \frac{e^{3x} \cdot e^{3x}}{e^{3x} x^2} dx = 0 \quad \therefore$$

$$= \int \frac{1}{x^2} dx \quad \frac{x^{n+1}}{n+1} = \frac{x^{-1}}{-1} = -\frac{1}{x}$$

$$V = -\frac{1}{x}$$

$$\left| \begin{matrix} x & 1 \\ x & x \end{matrix} \right| = 0$$

$$y_p = u y_1 + v y_2$$

$$\therefore y = y_c + y_p = \cos x + x^2 \sin x = \therefore$$

$$1 = 0 \quad \therefore$$

$$u = \int \frac{1}{x^2} dx = -\frac{1}{x}$$

$$v = \int \frac{1}{x^2} dx = -\frac{1}{x}$$

$$u = -\frac{1}{x}$$

$$v = -\frac{1}{x}$$

$$y = \cos x + x^2 \sin x$$

$$7) (D^2+1)y = \operatorname{cosec} x$$

$$\text{AE } D^2+1=0$$

$$D^2 = -1 = i^2$$

$$\therefore D = \pm i$$

$$y_c = \frac{C_1 \cos x}{y_1} + \frac{C_2 \sin x}{y_2}$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$\therefore = \cos^2 x + \sin^2 x = 1$$

$$\therefore D = 1$$

$$u = - \int \frac{y_2 f(x)}{D} dx$$

$$= - \int \frac{\sin x \cdot \operatorname{cosec} x}{1} dx$$

$$= - \int 1 dx$$

$$\therefore u = -x$$

$$v = \int \frac{y_1 f(x)}{D} dx$$

$$= \int \frac{\cos n \cdot \operatorname{cosec} n}{1} (dn) \quad (2)$$

$$= \int \cos n \cdot \frac{1}{\sin n} dn \quad \begin{matrix} 0 = 1 + 0 \\ 3A \end{matrix}$$

$$= \int \cot n \, dn \quad \begin{matrix} 0 = 1 \\ 0 = 0 \end{matrix}$$

$$\frac{x \sin 2 + x \cos 2}{x} = y$$

$$\therefore V = \log(\sin n) \quad \begin{matrix} y \\ y \\ y \end{matrix} \quad \begin{matrix} y \\ y \\ y \end{matrix} = 0$$

$$\therefore y_p = u y_1 + v y_2$$

$$\therefore y = y_c + y_p$$

$$x^2 \sin 2 + x^2 \cos 2 =$$

$$0 = 1 \therefore$$

$$x^2 \sin 2 + x^2 \cos 2 = -x$$

$$x^2 \sin 2 + x^2 \cos 2 = -x$$

$$x^2 \sin 2 + x^2 \cos 2 = -x$$

$$x^2 \sin 2 + x^2 \cos 2 = -x$$

$$x^2 \sin 2 + x^2 \cos 2 = -x$$

$$8) (D^2+1)y = x \cdot \sin x$$

$$\text{AE } D^2+1=0$$

$$D^2 = -1 = i^2$$

$$D = \pm i$$

$$y_c = C_1 \frac{\cos x}{y_1} + C_2 \frac{\sin x}{y_2}$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$= \cos^2 x + \sin^2 x$$

$$\therefore D = 1$$

$$u = - \int \frac{y_2 f(x)}{D} dx$$

$$= - \int \frac{\sin x \cdot x \cdot \sin x}{1} dx$$

$$= - \int x \cdot \sin^2 x dx$$

$$= - \int x \frac{(1 - \cos 2x)}{2} dx$$

$$= -\frac{1}{2} \int x - x \cdot \cos 2x dx$$

$$= -\frac{1}{2} \left[\frac{x^2}{2} - \left(x \cdot \frac{\sin 2x}{2} - (-1) \left(-\frac{\cos 2x}{4} \right) \right) \right]$$

$$\therefore u = -\frac{1}{2} \left[\frac{x^2}{2} - \frac{x \cdot \sin 2x}{2} - \frac{\cos 2x}{4} \right]$$

$$y = \int \frac{y_1 f(x)}{D} dx \quad \text{if } t = 0$$

$$= \int \cos x \cdot x \cdot \sin x \, dx$$

$$= \frac{1}{2} \int x \cdot 2 \sin x \cos x \, dx$$

$$= \frac{1}{2} \int x \sin 2x \, dx$$

$$= \frac{1}{2} \left[x \left(-\frac{\cos 2x}{2} \right) - (-1) \left(-\frac{\sin 2x}{4} \right) \right]$$

$$\therefore y = \frac{1}{2} \left[\frac{-x \cos 2x}{2} + \frac{\sin 2x}{4} \right]$$

$$\therefore y_p = u y_1 + v y_2$$

$$\therefore y = y_c + y_p$$

$$y = \frac{1}{2} \left[\frac{-x \cos 2x}{2} + \frac{\sin 2x}{4} \right] + y_c$$

$$9) (D^2 + 9)y = \frac{1}{1 + \sin 3x}$$

$$AE: D^2 + 9 = 0$$

$$D^2 = -9$$

$$D = \pm 3i$$

$$y_c = \frac{c_1 \cos 3x + c_2 \sin 3x}{y_1 \cdot y_2 \cdot y_2}$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 3x & \sin 3x \\ -3 \sin 3x & 3 \cos 3x \end{vmatrix}$$

$$= 3 \cos^2 3x + 3 \sin^2 3x$$

$$\therefore D = 3$$

$$u = - \int \frac{y_2 f(x)}{D} dx$$

$$= - \int \frac{\sin 3x}{3(1 + \sin 3x)} dx$$

$$= -\frac{1}{3} \int \frac{\sin 3x (1 - \sin 3x)}{(1 + \sin 3x)(1 - \sin 3x)} dx$$

$$= -\frac{1}{3} \int \frac{\sin 3x - \sin^2 3x}{1 - \sin^2 3x} dx$$

$$= -\frac{1}{3} \int \frac{\sin 3u - \sin^2 3u}{\cos^2 3u} du$$

$$= -\frac{1}{3} \int \sec 3u \tan 3u - \tan^2 3u du$$

$$= -\frac{1}{3} \int \sec 3u \tan 3u - (\sec^2 3u - 1) du$$

$$u = -\frac{1}{3} \left[\log(\sec 3u) - \frac{\tan 3u}{3} + u \right]$$

$$v = \int \frac{y_1 f(x)}{0} dx$$

$$= \int \frac{\cos 3u}{3(1 + \sin 3u)} du$$

$$= \frac{1}{3} \times \frac{1}{3} \int \frac{3 \cos 3u - 1}{1 + \sin 3u} du$$

$$v = \frac{1}{9} \log(1 + \sin 3u)$$

$$y_p = u y_1 + v y_2$$

$$y = y_c + y_p$$

$$10) (D^2 - 1)y = e^x \sin(e^x) + \cos(e^x)$$

AE

$$D^2 - 1 = 0$$

$$D^2 = 1$$

$$D = \pm 1$$

$$y_c = c_1 \frac{e^x}{y_1} + c_2 \frac{e^{-x}}{y_2}$$

$$D = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}$$

$$= -e^0 - e^0$$

$$= -1 - 1$$

$$\therefore D = -2$$

$$u = - \int \frac{y_2 f(x)}{D} dx$$

$$= - \int e^x [e^x \sin(e^x) + \cos(e^x)] dx$$

$$\text{put } e^x = t$$

$$e^x dx = dt$$

$$= + \int (t \sin t + \cos t) dt$$

$$= [t(-\cos t) - (1)(-\sin t) + \sin t]$$

$$= [-\cos t + \sin t + \sin t]$$

$$\therefore u = -\cos e^{-x} + 2 \sin e^{-x}$$

$$v = \int \frac{y_1 f(x)}{D} dx$$

$$= \int e^x [e^x \sin e^x + \cos e^x] dx$$

$$\therefore v = e^x \cdot \cos e^x$$

$$\therefore y_p = u y_1 + v y_2$$

$$\therefore y = y_c + y_p$$

Cauchy's Equation.

$$x^3 \frac{d^3 y}{dx^3} + x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = f(x)$$

put $\log x = z \therefore x = e^z$

$$x^3 \frac{d^3 y}{dx^3} = 0(0-1)(0-2)y = 0$$

$$x^2 \frac{d^2 y}{dx^2} = 0(0-1)y = 0$$

$$x \frac{dy}{dx} = 0y$$

$$x^2 \frac{d^2 y}{dx^2} = 0y$$

$$x \frac{dy}{dx} = 0y$$

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} - 3y = 6x$$

put $\log x = z \therefore x = e^z$

$$x^3 \frac{d^3 y}{dx^3} = D(D-1)(D-2)y$$

$$x^2 \frac{d^2 y}{dx^2} = D(D-1)y$$

$$x \frac{dy}{dx} = Dy$$

$$D(D-1)(D-2)y + 2D(D-1)y + 3Dy - 3y = 6e^z$$

$$(D^2 - D)(D-2)y + (2D^2 - 2D)y + 3Dy - 3y = 6e^z$$

$$(D^3 - 2D^2 - D^2 + 2D + 2D^2 - 2D + 3D - 3)y = 6e^z$$

$$(D^3 - D^2 + 3D - 3)y = 6e^z$$

AE $D^3 - D^2 + 3D - 3 = 0$

$$D^2(D-1) + 3(D-1) = 0$$

$$(D^2 + 3)(D-1) = 0$$

$$D^2 = -3 \quad D = 1$$

$$D = \pm \sqrt{3}i, \quad D = -1$$

$$y_c = c_1 e^z + c_2 \cos \sqrt{3}z + c_3 \sin \sqrt{3}z$$

$$\therefore y_c = c_1 x^{-1} + c_2 \cos \sqrt{3} \log x + c_3 \sin \sqrt{3} \log x$$

$$y_p = \frac{1}{0^3 - 0^2 + 3 \cdot 0 - 3} 6e^z$$

$$= 6 \frac{1}{(1)^3 - (1)^2 + 3(1) - 3} e^z$$

C.F.

$$= 6 \frac{z}{30^2 - 20 + 3} e^z$$

$$= 6 \frac{z}{3(1)^2 - 2(1) + 3} e^z$$

$$= 6 \frac{z}{4} e^z$$

$$= \frac{3}{2} z e^z$$

$$\therefore y_p = \frac{3}{2} \log x \cdot x$$

$$\therefore y = y_c + y_p$$

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = 2 \sinh(\log x)$$

put $\log x = z$ $\therefore x = e^z$

$$x^2 \frac{d^2 y}{dx^2} = D(D-1)y$$

$$x \frac{dy}{dx} = D y$$

$$D(D-1)y + Dy - y = 2 \sinh z$$

$$(D^2 - \cancel{D} + \cancel{D} - 1)y = 2 \sinh z$$

$$(D^2 - 1)y = 2 \sinh z$$

AE $D^2 - 1 = 0$

$$D^2 = 1$$

$$D = \pm 1$$

$$y_c = C_1 e^z + C_2 e^{-z}$$

$$\therefore y_c = C_1 x + C_2 x^{-1}$$

$$y_p = \frac{1}{D^2 - 1} 2 \sinh z$$

$D^2 \Rightarrow +1$

= C.F.

$$\therefore y_p = 2 \cdot \frac{z}{2D} \sinh z$$

$$= \frac{z}{D} \sinh z$$

$$= z \cdot \cosh z$$

$$\therefore y_p = \log u \cdot \cosh(\log u)$$

$$\therefore y = y_c + y_p$$

$$y = y_c + y_p$$

$$D^2 - 1 = 0$$

$$D^2 = 1$$

$$D = \pm 1$$

$$y_1 = e^x + e^{-x}$$

$$y_2 = e^x - e^{-x}$$

$$y_3 = \frac{1}{D^2 - 1} = \frac{1}{D^2 - 1}$$

$$C.F.$$

$$3) \quad x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = \cos(\log x)$$

$$\text{put } \log x = z \therefore x = e^z$$

$$\frac{x^2 d^2 y}{dx^2} = D(D-1)y$$

$$x \frac{dy}{dx} = Dy$$

$$D(D-1)y + Dy - y = \cos z$$

$$(D^2 - \cancel{D} + \cancel{D} - 1)y = \cos z$$

$$(D^2 - 1)y = \cos z$$

$$\text{AE } D^2 - 1 = 0$$

$$D^2 = 1$$

$$D = \pm 1$$

$$y_c = c_1 e^z + c_2 e^{-z}$$

$$\therefore y_c = c_1 x + c_2 x^{-1}$$

$$y_p = \frac{1}{D^2 - 1} \cos z$$

$$D^2 \rightarrow -1$$

$$= \frac{1}{-1-1} \cos z$$

$$\text{Compare } \frac{1}{-2} \cos 2 \log x \text{ with } \frac{1}{2} \cos 2 \log x$$

$$\therefore y_p = \frac{-1}{2} \cos(2 \log x) \text{ (particular solution)}$$

$$\therefore y = y_c + y_p$$

$$y_c = \frac{1}{2} \cos 2 \log x$$

$$y_c = \frac{1}{2} \cos 2 \log x + \frac{1}{2} \cos 2 \log x$$

$$y_c = \cos 2 \log x$$

$$y_c = \cos 2 \log x$$

$$y_c = \cos 2 \log x$$

$$y_c = \cos 2 \log x$$

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$$y_c = \cos 2 \log x$$

$$y_c = \cos 2 \log x$$

$$x^3 \frac{d^3 y}{dx^3} + x^2 \frac{d^2 y}{dx^2} - 2y = x^2 + x^3$$

put $\log x = z \therefore x = e^z$

$$x^3 \frac{d^3 y}{dx^3} = D(D-1)(D-2)y$$

$$x^2 \frac{d^2 y}{dx^2} = D(D-1)y$$

$$D(D-1)(D-2)y + D(D-1)y - 2y = e^{2z} + e^{-3z}$$

$$(D^2 - D)(D-2)y + (D^2 - D)y - 2y = e^{2z} + e^{-3z}$$

$$(D^3 - 2D^2 - D^2 + 2D + D^2 - D - 2)y = e^{2z} + e^{-3z}$$

$$(D^3 - 2D^2 + D - 2)y = e^{2z} + e^{-3z}$$

AE $D^3 - 2D^2 + D - 2 = 0$

$$D^2(D-2) + 1(D-2) = 0$$

$$(D-2)(D^2+1) = 0$$

$$D = 2 \quad D^2 = -1 \Rightarrow D = \pm i$$

$$D = 2 \quad D = \pm i$$

$$y_c = c_1 e^{2z} + c_2 \cos z + c_3 \sin z$$

$$\therefore y_c = c_1 x^2 + c_2 \cos(\log x) + c_3 \sin(\log x)$$

$$y_p = \frac{1}{D^3 - D^2 + D - 2} (e^{2x} + e^{-3x})$$

$$y_p = \frac{1}{D^3 - D^2 + D - 2} e^{2x} + \frac{1}{D^3 - D^2 + D - 2} e^{-3x}$$

$$y_{p_1} \quad y_{p_2}$$

$$y_{p_1} = \frac{1}{D^3 - D^2 + D - 2} e^{2x}$$

$$D \rightarrow 2$$

C.F.

$$= \frac{2}{3D^2 - 2D + 1} e^{2x}$$

$$D \rightarrow 2$$

$$= \frac{2}{3(2)^2 - 2(2) + 1} e^{2x}$$

$$= \frac{2}{9} e^{2x}$$

$$\therefore y_{p_1} = \frac{\log x}{9} \cdot x^2$$

$$y_{p_2} = \frac{1}{D^3 - D^2 + 2D - 2} e^{-3x}$$

$$D \rightarrow -3$$

$$= \frac{1}{(-3)^3 + (-3)^2 - 2(-3) - 2} e^{3x}$$

$$= \frac{1}{-27 + 9 + 6 - 2} e^{3x}$$

$$= \frac{1}{-14} e^{3x}$$

$$\therefore y_{p2} = \frac{-1}{14} x^3$$

$$\therefore y_p = y_{p1} + y_{p2}$$

$$\therefore y = y_c + y_p$$

$$0 = (1 + 0)(x - 0)$$

$$0 = x - 1 = 0, x = 1$$

$$x = 1$$

$$x = 1$$

$$(x = 1)$$

5) $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^2 + 2 \log x$

put $\log x = z \quad \therefore x = e^z$

$x^2 \frac{d^2 y}{dx^2} = D(D-1)y$

$x \frac{dy}{dx} = Dy$

$D(D-1)y - 2Dy - 4y = e^{2z} + 2z$

$(D^2 - D - 2D - 4)y = e^{2z} + 2z$

$(D^2 - 3D - 4)y = e^{2z} + 2z$

AE $D^2 - 3D - 4 = 0$

$(D-4)(D+1) = 0$

$D = 4, D = -1$

$y_c = c_1 e^{4z} + c_2 e^{-z}$

$\therefore y_c = c_1 x^4 + c_2 x^{-1}$

$y_p = \frac{1}{D^2 - 3D - 4} (e^{2z} + 2z)$

$$\therefore Y_p = \frac{108}{D^2 - 3D - 4} e^{2z} + \frac{1}{D^2 - 3D - 4} z^2$$

 Y_{p1} Y_{p2}

$$Y_{p1} = \frac{1}{D^2 - 3D - 4} e^{2z}$$

 $D \rightarrow 2$

$$= \frac{1}{(2)^2 - 3(2) - 4} e^{2z}$$

$$= \frac{1}{4 - 6 - 4} e^{2z}$$

$$= \frac{-1}{6} e^{2z}$$

$$\therefore Y_{p1} = \frac{-1}{6} z^2$$

$$Y_{p2} = \frac{1}{D^2 - 3D - 4} z^2$$

$$= z \cdot \frac{1}{-4 \left[1 - \left(\frac{D^2}{4} - \frac{3D}{4} \right) \right]} z$$

$$= \frac{-1}{2} \left[1 - \left(\frac{D^2}{4} - \frac{3D}{4} \right) \right]^{-1} \cdot z$$

$$= \frac{-1}{2} \left[1 + \left(\frac{D^2}{4} - \frac{3D}{4} \right) \right] \cdot z$$

$$= \frac{-1}{2} \left[1 - \frac{D^2}{4} + \frac{3D}{4} \right] \cdot z$$

$$= \frac{-1}{2} \left[1 + \frac{3D}{4} \right] \cdot z$$

$$\therefore y_{p_2} = \frac{-1}{2} \left[z + \frac{3}{4} \right]$$

$$\therefore y_p = y_{p_1} + y_{p_2}$$

$$\therefore y = y_c + y_p$$

Legendre's Equation

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$$(ax+b)^3 \frac{d^3 y}{dx^3} + (ax+b)^2 \frac{d^2 y}{dx^2} +$$

$$(ax+b) \frac{dy}{dx} + y = f(x)$$

put $\log(ax+b) = z$

$$\therefore (ax+b) \leq e^z \quad \text{if } b^2(1+x^2)$$

$$(ax+b)^3 \frac{d^3 y}{dx^3} = a^3 D(D-1)(D-2)y$$

$$(ax+b)^2 \frac{d^2 y}{dx^2} = a^2 D(D-1)y$$

$$(1-x^2) \frac{dy}{dx} = a \cdot D \cdot y$$

$$(1-x^2) \frac{dy}{dx} = a \cdot D \cdot y$$

$$1-x^2 = a \cdot D \cdot y$$

$$0 = (D+1)(D-3)$$

$$0 = (D+1)(D-3)$$

$$D = -1 \quad D = 3$$

$$y_1 = x^{-1} \quad y_2 = x^3$$

$$1) (2x+1)^2 \frac{d^2 y}{dx^2} - 2(2x+1) \frac{dy}{dx} - 12y = 8x$$

put $\log(2x+1) = z \quad \therefore 2x+1 = e^z$

$$\therefore x = \frac{e^z - 1}{2}$$

$$(2x+1)^2 \frac{d^2 y}{dx^2} = 2^2 \cdot D(D-1)y$$

$$(2x+1) \frac{dy}{dx} = 2 \cdot D \cdot y$$

$$4D(D-1)y - 2 \times 2 Dy - 12y = 8 \left(\frac{e^z - 1}{2} \right)$$

$$(4D^2 - 4D - 4D - 12)y = 8 \left(\frac{e^z - 1}{2} \right)$$

$$(4D^2 - 8D - 12)y = 4(e^z - 1)$$

$$4(D^2 - 2D - 3)y = 4(e^z - 1)$$

$$(D^2 - 2D - 3)y = e^z - 1$$

AE $D^2 - 2D - 3 = 0$

$$(D-3)(D+1) = 0$$

$$D = 3 \quad D = -1$$

$$y_c = c_1 e^{3z} + c_2 e^{-z}$$

$$\therefore y_c = c_1 (2n+1)^3 + c_2 (2n+1)^1 \quad (1)$$

$$y_p = \frac{1}{D^2 - 2D - 3} e^z$$

$$y_p = \frac{1}{D^2 - 2D - 3} e^z = \frac{1}{D^2 - 2D - 3} \cdot 1$$

y_{p1}

y_{p2}

$$y_{p1} = \frac{1}{D^2 - 2D - 3} e^z$$

$$D \rightarrow 1$$

$$= \frac{1}{(1)^2 - 2(1) - 3} e^z$$

$$y_{p1} = \frac{e^z}{-4}$$

$$\therefore y_{p1} = \frac{2n+1}{-4}$$

$$y_{p2} = \frac{1}{D^2 - 2D - 3} \quad D \rightarrow 0$$

$$y_{p2} = \frac{1}{-3}$$

$$\therefore y_p = y_{p1} - y_{p2}$$

$$\therefore y = y_c + y_p$$

$$2) (3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2) \frac{dy}{dx} - 36y = 0$$

$$(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2) \frac{dy}{dx} - 36y = 0$$

$$\text{put } \log(3x+2) = z \quad \therefore 3x+2 = e^z$$

$$(3x+2)^2 \frac{d^2y}{dx^2} = 3^2 D(D-1)y$$

$$(3x+2) \frac{dy}{dx} = 3 \cdot D \cdot y$$

$$9D(D-1)y + 3 \times 3(Dy) - 36y = 0 \left(\frac{e^z - 3}{3} \right)^2$$

$$(9D^2 - 9D + 9D - 36)y = \frac{e^{2z}}{3} - \frac{2e^z}{3} + 1$$

$$(9D^2 - 36)y = \frac{e^{2z}}{3} - \frac{2e^z}{3} + 1$$

$$\text{AE} \quad 9D^2 - 36 = 0$$

$$D^2 - 4 = 0$$

$$D^2 = 4$$

$$D = \pm 2$$

$$y_c = c_1 e^{2z} + c_2 e^{-2z}$$

$$\therefore y_c = c_1 (3x+2)^2 + c_2 (3x+2)^{-2}$$

$$Y_p = \frac{1}{9D^2 - 36} \left[\frac{e^{2z}}{3} + \frac{2e^z}{1-3} + 1 \right]$$

$$Y_p = \frac{1}{9D^2 - 36} \frac{e^{2z}}{3} + \frac{1}{9D^2 - 36} \frac{2e^z}{3} + \frac{1}{9D^2 - 36}$$

Y_{p1}

Y_{p2}

Y_{p3}

$$Y_{p1} = \frac{1}{9D^2 - 36} \frac{e^{2z}}{3}$$

= C.F.

$$= \frac{2}{18D} \frac{e^{2z}}{3}$$

$$= \frac{1}{54} \frac{2}{D} e^{2z}$$

$$= \frac{1}{54} \frac{2}{2} \frac{e^{2z}}{2}$$

$$= \frac{1}{108} 2 \cdot e^{2z}$$

$$Y_{p1} = \frac{1}{108} \log(3n+2) \cdot (3n+2)^2$$

$$Y_{p2} = \frac{1}{9D^2 - 36} \frac{2e^z}{3}$$

$D \rightarrow 1$

$$= \frac{2}{3} \frac{1}{9(1)^2 - 36} e^z$$

$$= \frac{2}{3} \frac{1}{-27} e^z$$

$$= -\frac{2}{81} e^z$$

$$\therefore y_p = \frac{-2}{81} (3x+2)$$

$$y_p = \frac{1}{90^2 - 36}$$

$$D \rightarrow 0$$

$$\therefore y_p = \frac{1}{-36}$$

$$\therefore y_p = y_{p1} + y_{p2} + y_{p3}$$

$$\therefore y = y_c + y_p$$

$$(s+x)^2 \cdot (s+x)^2$$

$$\frac{1}{801}$$

$$\frac{1}{801}$$

$$(2x+1)^2 \frac{d^2 y}{dx^2} + 2(2x+1) \frac{dy}{dx} + 4y = 0$$

$$4 \sin [\log (2x+1)^2]$$

$$4 \sin [2 \log (2x+1)]$$

put $\log (2x+1) = z \quad \therefore (2x+1) = e^z$

$$(2x+1)^2 \frac{d^2 y}{dx^2} = 2^2 D(D-1)y$$

$$(2x+1) \frac{dy}{dx} = 2 D y$$

$$2^2 D(D-1)y + 2 \times 2 D y + 4y = 4 \sin 2z$$

$$(4D^2 - 4D + 4D + 4)y = 4 \sin 2z$$

$$(4D^2 + 4)y = 4 \sin 2z$$

$$4(D^2 + 1)y = 4 \sin 2z$$

$$(D^2 + 1)y = \sin 2z$$

AE $D^2 + 1 = 0$

$$D^2 = -1 = i^2$$

$$D = \pm i$$

$$y_c = C_1 \cos z + C_2 \sin z$$

$$y_c = C_1 \cos [\log (2x+1)] + C_2 \sin [\log (2x+1)]$$

$$Y_p = \frac{1}{D^2 + 1} \sin 2z \quad \text{where } D^2 \rightarrow -4$$

$$[(1+Ks) \text{ pol}] \sin 2z$$

$$[(1+Ks) \text{ pol}] \sin 2z = \frac{1}{-4+1} \sin 2z$$

$$Y_p = (1+Ks) \sin 2z = (1+Ks) \text{ pol} \sin 2z$$

$$= \frac{-1}{3} \sin 2z$$

$$\therefore Y_p = \frac{-1}{3} \sin 2 \log(2x+1)$$

$$\therefore Y_p = \frac{-1}{3} \sin [\log(2x+1)^2]$$

$$\therefore Y = Y_c + Y_p$$

$$Y_c = C_1 e^{t/2} + C_2 e^{-t/2}$$

$$Y_c = C_1 (1 + D^2) Y$$

$$Y_c = C_1 (1 + D^2) Y$$

$$D^2 + 1 = 0$$

$$D^2 = -1$$

$$D = \pm i$$

$$Y_c = C_1 \cos z + C_2 \sin z$$

$$Y = Y_c + Y_p$$

$$4) (x+1)^2 \frac{d^2 y}{dx^2} - (x+1) \frac{dy}{dx} - 3y = 6x$$

put $\log(x+1) = z \quad \therefore (x+1) = e^z$

$\therefore x = e^z - 1$

$$(x+1)^2 \frac{d^2 y}{dx^2} = 1 \cdot 0 \cdot (0-1)y$$

$$(x+1) \frac{dy}{dx} = 1 \cdot 0 \cdot y - (1) \cdot 1 = -1$$

$$0(0-1)y - 0y - 3y = 6(e^z - 1)$$

$$(0^2 - 0 - 0 - 3)y = 6(e^z - 1)$$

$$(0^2 - 2 \cdot 0 - 3)y = 6(e^z - 1)$$

AE $0^2 - 2 \cdot 0 - 3 = 0$

$$(0-3)(0+1) = 0$$

$$0 = 3 \quad 0 = -1$$

$$y_c = c_1 e^{3z} + c_2 e^{-z}$$

$$\therefore y_c = c_1 (x+1)^3 + c_2 (x+1)^{-1}$$

$$y_p = \frac{1}{0^2 - 2 \cdot 0 - 3} 6(e^z - 1)$$

$$\therefore y_p = 6 \left[\frac{1 \cdot e^z}{D^2 - 2D - 3} - \frac{1 \cdot (1+x)}{D^2 - 2D - 3} \right]$$

$$y_{p_1} \quad y_{p_2}$$

$$\therefore y_{p_1} = \frac{1}{D^2 - 2D - 3} e^z$$

$$D \rightarrow 1 \quad \frac{1 \cdot e^z}{1^2 - 2(1) - 3} = \frac{1 \cdot e^z}{-4}$$

$$= \frac{1}{(1)^2 - 2(1) - 3} e^z = \frac{1}{-4} e^z$$

$$= \frac{1}{-4} e^z$$

$$\therefore y_{p_1} = \frac{1}{-4} (x+1) \cdot (\delta - 0 - 0 - \delta)$$

$$(1 - \delta) \delta = \delta (\delta - 0 - \delta - 0)$$

$$y_{p_2} = \frac{1}{D^2 - 2D - 3} \cdot 1$$

$$D \rightarrow 0$$

$$0 = (1+0)(\delta - 0)$$

$$\therefore y_{p_2} = \frac{1}{-3}$$

$$1 - \delta = 0 \quad \delta = 0$$

$$\therefore y_p = y_{p_1} - y_{p_2}$$

$$(1+x) \delta + (1+x) \delta = \delta$$

$$\therefore y = y_c + y_p$$

$$(1 - \delta) \delta = \delta (\delta - 0 - \delta - 0)$$