$$I_A = \int_0^1 \frac{1}{1+x^2}\,dx$$

$$= \left[\tan^{-1}x\right]_0^1$$

$$= \tan^{-1}1 - \tan^{-1}0$$

$$= \frac{\pi}{4}$$

$$= 0.7853$$

Error = minute error

Euler's ~~th~~ theorm — Numerical sol ordinary

$\frac{dy}{dx} = f(x,y)$

where $x = x_0$, $y = y_0$

then G.S is

$$y_{n+1} = y_n + hf(x_{n-1}, y_{n-1})$$

1) $\frac{dy}{dx} = 1 + xy$ $\qquad x = 0,\ y = 1$

$x = 0(0.1)0.5$

$x_0 = 0$, $y_0 = 1$ $\qquad h = \frac{b-a}{n} = 0.1$

By euler's thm

$$y_{n+1} = y_n + hf(x_n, y_n) \quad \text{--- (1)}$$

x	$x_0 = 0$	$x_1 = 0.1$	$x_2 = 0.2$	$x_3 = 0.3$	$x_4 = 0.4$	$x_5 = 0.5$
y	$y_0 = 1$	$y_1 = \cancel{0.2}$ 1.21	$y_2 = \cancel{0.2}$ 1.211	$y_3 = \cancel{0.3}$ 1.331	$y_4 = 1.4$	$y_4 = 1.634$

1) Put $n=0$ in ①

$y_1 = y_0 + hf(x_0, y_0)$

$= y_0 + h(1 + x_0 y_0)$

$= 1 + (0.1)(1 + 0 \times 0.1)$

$= 1 + 0.1$

$y_1 = 1.1$

2) Put $n=1$ in ①

$y_2 = y_1 + hf(x_1, y_1)$

$= y_1 + h(1 + \cancel{\quad} xy)$

$y_2 = 1.1 + (0.1)(1 + 0.1 \times 1.1)$

$y_2 = 1.211$

3. Put $n=2$ in ①

$y_3 = 1.331$

4. $y_4 = 1.4$

5 $y_5 = 1.634$

Quick Work

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2. $\frac{dy}{dx} = \frac{y-x}{\sqrt{xy}}$ $y(1) = 2$

find y at $x = 1.5$

$x_0 = 1$ $y_0 = 2$

$x = 1.5$ $y = ?$

$$f(x,y) = \frac{y-x}{\sqrt{xy}}$$

Let $n = 5$

$$h = \frac{b-a}{n} = \frac{1.5-1}{5} = 0.1$$

$h = 0.1$

x	$x_0 = 1$	$x_1 = 1.1$	$x_2 = 1.2$	$x_3 = 1.3$	$x_4 = 1.4$	$x_5 = 1.5$
y	$y_0 = 2$	$y_1 = 2.070$	2.134	2.192	2.244	2.291

By Euler's thm

$$y_{n+1} = y_n + hf(x_n, y_n) \quad -(1)$$

1. Put $n=0$ in (1)

$$y_1 = y_0 + h\left(\frac{y_0 - x_0}{\sqrt{y_0 x_0}}\right)$$

$$y_1 = 2 + 0.1\left(\frac{2-1}{\sqrt{2}}\right)$$

$$y_1 = 2 + \frac{0.1}{\sqrt{2}}$$

$$y_1 = 2.070$$

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2. Put $n=1$ in eq 1

$$y_2 = y_1 + h\left[\frac{y_1 - x_1}{\sqrt{x_1 y_1}}\right]$$

$$= 2.070 + (0.1)\left[\frac{2.070 - 1.1}{\sqrt{1.1 \times 2.070}}\right]$$

$$y_2 = 2.131$$

3. Put $n=2$ in eq 1

$$y_3 = y_2 + hf(x_2, y_2)$$

$$y_3 = 2.192$$

4. $y_4 = 2.244$

5. $y_5 = 2.291$

Use euler's method to solve

$\frac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8.5$ from $x=0$ to $x=4$ with step size of 0.5

$y(0)=1$

$\frac{dy}{dx} = -2x^3+12x^2-20x+8.5$

$x_0=0$, $y_0=1$

$h = 0.5$

x	$x_0=0$	$x_1=0.5$	$x_2=1$	$x_3=1.5$	$x_4=2$	$x_5=2.5$	$x_6=3$	3.5	4
y	$y_0=1$	$y_1=5.25$	$y_2=5.875$	$y_3=5.125$	4.5	4.75	5.875	7.125	7

By euler's

$y_{n+1} = y_n + hf(x_n, y_n)$ — ①

1. Put $n=0$ in ①

$y_1 = y_0 + 0.5(-2x_0^3 + 12x_0^2 - 20x_0 + 8.5)$

$= 1 + 0.5(8.5)$

$y_1 = 5.25$

2. $n=1$ in ①

$y_2 = 5.875$

3. $n=2$ in ①

$y_3 = 5.125$

4. $n=3$ in ①

$y_4 = 4.5$

5. $n=4$ in ①

$y_5 = 4.75$

6. $n=5$ in ①

$y_6 = 5.875$

7. $n=6$ in ①

$y_7 = 7.125$

8. $n=7$ in ①

$y_8 = 7$

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Solve Diff eq using euler's method

$\frac{dy}{dx} = \frac{y-x}{y+x}$ with condition $y(0) = 1$

$y = ?$ at $x = 0.1$

$x_0 = 0, y_0 = 1$ $h = \frac{b-a}{n} = \frac{0.1-0}{5} = 0.02$

$x = 0.1, y = ?$

x	$x_0 = 0$	$x_1 = 0.02$	$x_2 = 0.04$	$x_3 = 0.06$	$x_4 = 0.08$	$x_5 = 0.1$
y	$y_0 = 1$	$y_1 = 1.02$	$y_2 = 1.039$	$y_3 = 1.057$	$y_4 = 1.075$	$y_5 = 1.092$

By eulers

$y_{n+1} = y_n + hf(x_n, y_n)$ — ①

1. Put $n = 0$

$y_1 = y_0 + 0.02\left[\frac{y_0 - x_0}{y_0 + x_0}\right]$

$= 1 + 0.02\left[\frac{1-0}{1+0}\right]$

$y_1 = 1.02$

2. $n = 1$ in ①

$y_2 = 1.039$

3. $n = 2$ in ①

$y_3 = 1.057$

4. $n = 3$ in ①

$y_4 = 1.075$

5. $n = 4$ in ①

$y_5 = 1.092$

$$y_{n+1} = y_n + h[f(x_n, y_n)]$$

$$y_1 = y_0 + h[f(x_0, y_0)]$$

$$y_2 = y_1 + h[f(x_1, y_1)]$$

iterations:

$$y_1 = y_0 + h[f(x_0, y_0)]$$

$$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1)]$$

$$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$$

$$y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})]$$

$$y_2 = y_1 + h[f(x_1, y_1)]$$

$$y_2^{(1)} = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2)]$$

$$y_2^{(2)} = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^{(1)})]$$

$$y_2^{(3)} = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^{(2)})]$$

Q1. $y(0.1) = ?$ $\frac{dy}{dx} = y - \frac{2x}{y}$

$y(0) = 1$, $h = 0.1$

→ $x_0 = 0$, $y_0 = 1$
$x_1 = 0.1$, $y_1 = ?$

$f(x,y) = y - \frac{2x}{y}$

$h = 0.1$
By euler's
$y_{n+1} = y_n + h[f(x_n, y_n)]$

Put $n = 0$
$y_1 = y_0 + h[f(x_0, y_0)]$

$y_1 = 1 + 0.1\left[1 - \frac{2(0)}{1}\right]$

$y_1 = 1.1$

MEM
$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1)\right]$

$= 1 + \frac{0.1}{2}\left[(1) + \left(1.1 - \frac{2 \times 0.1}{1.1}\right)\right]$

$y_1^{(1)} = 1.0$

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$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1 + \frac{0.1}{2}$$

2) Solve eq

$\frac{dy}{dx} = 1 + xy$ by using modified euler's version

and calculate $x = 0.1$ & $x = 0.2$

$y(0) = 1$

$h = 0.1$

$x_0 = 0, \quad y_0 = 1$

$x_1 = 0.1, \quad y_1 = ?$

$x_2 = 0.2 \quad y_2 = ?$

$f(x, y) = 1 + xy$

By euler's

$y_{n+1} = y_n + h[f(x_n, y_n)]$

$n = 0$

$y_1 = y_0 + h[f(x_0, y_0)]$

$= 1 + 0.1[1 + (0)(1)]$

$y_1 = 1.1$

MEM

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1)]$

$= 1 + \frac{0.1}{2}[1 + (1 + 0.1 \times 1.1)]$

$y_1^{(1)} = 1.1055$

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$= 1 + \frac{0.1}{2}[1 + (0.1)(1.1055) + 1]$

$y_1^{(2)} = 1.1055$

$x_1 = 0.1, \quad y_1 = 1.1055$

$1 + (0.1)(1.1055)$

$1+$ $\quad 1+$

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NOW lets find y_2 at $x_2 = 0.2$

By EM

$y_2 = y_1 + h[f(x_1, y_1)]$

$= 1.055 + 0.1[1 + (0.1)(1.1055)]$

$y_2 = 1.2165$

By MEM

$y_2^{(1)} = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2)]$

$= 1.1055 + \frac{0.1}{2}[1.1105 + (1 + 0.2 \times 1.2165]$

$y_2^{(1)} = 1.2231$

$y_2^{(2)} = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^{(1)})]$

$= 1.055 + \frac{0.1}{2}[1.1105 + (1 + 0.2 \times 1.2231]$

$y_2^{(2)} = 1.2232$

$x_2 = 0.2, \; y_2 = 1.2232$

$\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ & $h = 0.1$

$y_1 = ?$ $x_1 = 0.1$

$y_2 = ?$ $x_2 = 0.2$

→ $x_0 = 0$, $y_0 = 1$

$x_1 = 0.1$ $y_1 = ?$

$x_2 = 0.2$ $y_2 = ?$

$f(x, y) = x^2 + y$

By EM

$y_{n+1} = y_n + h[f(x_n, y_n)]$

Put $n = 0$

$y_1 = y_0 + h[f(x_0, y_0)]$

$= 1 + 0.1[0^2 + 1]$

$= 1 + 0.1(1)$

$= 1 + 0.1$

$y_1 = 1.1$

By MEM:

$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1)]$

$= 1 + \frac{0.1}{2}[1 + (0.1^2 + 1.1)]$

$y_1^{(1)} = 1.1055$

$0 \to 1$

$1 + \frac{0.1}{2}[2.11]$

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$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$= 1 + \frac{0.1}{2}[1 + (0.1^2 + 1.1055)]$

$1 + \frac{0.1}{2}[1 +$

$y_1^{(2)} = 1.105775$

$x_1 = 0.1, \quad y_1 = 1.105$

$y_2 = y_1 + h[f(x_1, y_1)]$

$= 1.105 + 0.1[0.1^2 + 1.105]$

$y_2 = 1.2165$

$y_2^{(1)} = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2)]$

$= 1.105 + \frac{0.1}{2}[(0.1^2 + 1.105) + (0.2^2 + 1.2165)]$

$= 1.105 + \frac{0.1}{2}[2.3715]$

$1.105 + 0.118575$

$y_2^{(1)} = 1.223$

$y_2^{(2)} =$

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Runge Kutta Method of 4^{th} order - Method 3

$k_1 = hf(x_0, y_0)$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$y = y_0 + k$

1) $\frac{dy}{dx} = \sqrt{x+y}$, $x_0 = 0$, $y_0 = 1$
$x = 0.2$, $y = ?$
$h = 0.2$

① $k_1 = hf(x_0, y_0)$
$= 0.2(\sqrt{x_0 + y_0})$
$= 0.2(\sqrt{0+1})$
$k_1 = 0.2$

② $k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$
$= 0.2\sqrt{(x_0 + h/2) + (y_0 + k_1/2)}$
$= 0.2\sqrt{(0 + 0.2/2) + (1 + 0.2/2)}$
$k_2 = 0.2190$

③ $k_3 = hf(x_0 + h/2, y_0 + k_2/2)$
$= 0.2\sqrt{(0 + 0.2/2) + (1 + 0.2190/2)}$
$k_3 = 0.2199$

1.1710

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4. $k_4 = hf(x_0+h, y+k_3)$
$= 0.2(\sqrt{x_0+h+y+k_3}$
$= 0.2(\sqrt{0+0.2+1+0.2194})$
$k_4 = 0.2383$

5. $k = \frac{1}{6}[k_1+2k_2+2k_3+k_4]$

$= 0.2193$

$y_1 = y_0 + k$
$y_1 = 1 + 0.2193$
$y_1 = 1.2193$

Q Solve $\frac{dy}{dx} = x^2+y^2$, at condition $x=0, y=1$ $h=0.1$
$x=0.2, y=?$

$f(x,y) = x^2+y^2$

$x_0 = 0, y_0 = 1, h = 0.1$
$x_1 = 0.1, y_1 =$
$x_2 = 0.2, y_2 =$

1. $k_1 = hf(x_0, y_0)$
$= 0.1(0^2+1^2)$
$k_1 = 0.1$

2. $k_2 = hf(x_0+\frac{h}{2}, y_0+\frac{k_1}{2})$

$= 0.1\left[\left(0+\frac{0.1}{2}\right)^2 + \left(1+\frac{0.1}{2}\right)^2\right]$

$k_2 = 0.1105$

3. $k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.1\left[\left(0 + \frac{0.1}{2}\right)^2 + \left(1 + \frac{0.1105}{2}\right)^2\right]$

$k_3 = 0.1116$

4. $k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.1\left[(0 + 0.1)^2 + (1 + 0.1116)^2\right]$

$k_4 = 0.1245$

5. $k = \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

$= \frac{1}{6}\left[0.1 + 2(0.1105) + 2(0.1116) + 0.1245\right]$

$k = 0.1114$

$y_1 = y_0 + k$

$= 1 + 0.1114$

$y_1 = 1.1114$

Now we will calculate y at $x = 0.2$

Let, $x_0 = 0.1$, $y_0 = 1.114$, $h = 0.1$

$x = 0.2$, $y = ?$

$f(x, y) = x^2 + y^2$

① $k_1 = hf(x_0, y_0)$

$= 0.1(0.1^2 + 1.114^2)$

$k_1 = 0.12345$

② $k_2 = hf(x_0 + h/2, y_0 + k_1/2)$

$= 0.1\left[\left(0.1 + \frac{0.1}{2}\right)^2 + \left(1.114 + \frac{0.1245}{2}\right)^2\right]$

$k_2 =$ ~~[illegible]~~ 0.1399

③ $k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.1418$

④ $k_4 = 0.1610$

⑤ $k = 0.1414$

$y_1 = y_0 + k$
$= 1.114 + 0.1414$
$y_1 = 1.2528$

$\therefore$ at $x = 0.2$, $y = 1.2528$

Method : 4 – Milne's Predictor & corrector method

Consider an initial value problem $\frac{dy}{dx} = f(x, y)$

$y(x_0) = y_0$, $y(x_1) = y_1$, $y(x_2) = y_2$ . . .

Where $x_0, x_1, x_2, \ldots$ are equidistance value of x with step size h

then Milne's Predictor formula is :

$$y^p_{n+1} = y^c_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

Milnes corrector for :

$$y^c_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

Where $y' = \frac{dy}{dx} = f(x, y)$

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① use MPCM

find y when $x=0.8$

$$\frac{dy}{dx} = x-y^2$$

$y(0)=0,\ y(0.2)=0.02,\ y(0.4)=0.0795$

$y(0.6)=0.1762$

$y' = f(x,y) = x-y^2$

x		
$x_0=0$	$y_0=0$	$y'=f(x,y)=x-y^2$
$x_1=0.2$	$y_1=0.02$	$y_0'=0$
$x_2=0.4$	$y_2=0.0795$	$y_1'=0.2-0.02^2=0.1996$
$x_3=0.6$	$y_3=0.1762$	$y_2'=0.3936$
$x_4=0.8$	$y_4=?$	$y_3'=0.5689$
$x_4=0.8$	$y_4^p=0.3049$	$y_4'=0.8-0.3049^2=0.7070$

* MPF :-

$$y_{n+1}^p = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

Put $n=3$

$$y_4^p = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$y_4^p = 0 + \frac{4(0.2)}{3}\left[2(0.1996) - (0.3936) + 2(0.5689)\right]$$

$$y_4^p = 0.3049$$

MCF :

$$y_{n+1}^c = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right] \quad \text{... Put } n=3$$

$$y_4^c = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$\boxed{y_4^c = 0.3043}$$

$$\frac{0.13992 \times 0.4}{3}$$

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Solve eq

$5x\frac{dy}{dx} + y^2 - 2 = 0$ find y when $x = 4.4$ & $x = 4.5$

$$\frac{dy}{dx} = \frac{2-y^2}{5x}$$

x	y	$y' = f(x,y) = \frac{2-y^2}{5x}$
$x_0 = 4$	$y_0 = 1$	$y_0' = 0.05$
$x_1 = 4.1$	$y_1 = 1.0049$	$y_1' = 0.0483$
$x_2 = 4.2$	$y_2 = 1.0097$	$y_2' = 0.04688$
$x_3 = 4.3$	$y_3 = 1.0143$	$y_3' = 0.0451$
$x_4 = 4.4$	$y_4^p = 0.0686$	y_4'
$x_5 = 4.5$	$y_5^p = 1.0355$	$y_5' = 0.0412$

MPF

$$y^p_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

Put $n = 3$

$$y_4^p = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$= \cancel{0.05}\ 1 + \frac{4(0.1)}{3}\left[2(0.0483) - 0.04688 + 2(0.0451)\right]$$

$$y_4^p = 0.0686$$

MCF :

$$y^c_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right] \quad \ldots \text{Put } n = 3$$

$$y_4^c = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$\left[y_4^c = 1.018\right]$ ✓

Now let's find y_5^p

$\therefore n=4$

$y_5^p = y_1 + \frac{4h}{3}[2y_2' - y_3' + 2y_4']$

$y_5^p = 1.0049 + \frac{4(0.1)}{3}[2(0.0468) - 0.0451 + 2y_4']$

$y_5^p = 1.0355$

MCM

$y_5^c = y_3 + \frac{h}{3}[y_3' + 4y_4' + y_5']$

$\boxed{y_5^c = 1.029}$ ✓