

Data Structures

Q1 Sort using bubble sort: 64, 34, 25, 12, 22, 11, 90.
Show step by step execution for all passes highlighting "swap" and "No Swap" situations. How many passes required to sort an array of N elements using bubble sort?

Initial Array: 64, 34, 25, 12, 22, 11, 90

Working - Repeatedly swapping adj elements if they are in wrong order.

⇒ Pass 1:

Compare 64 and 34 → Swap (64 > 34)

Array: 34, 64, 25, 12, 22, 11, 90

Similarly; (64 > 25) → 34, 25, 64, 12, 22, 11, 90
swap

(64 > 12) → 34, 25, 12, 64, 22, 11, 90
swap

(64 > 22) → 34, 25, 12, 22, 64, 11, 90
swap

(64 > 11) → 34, 25, 12, 22, 11, 64, 90
swap

(64 < 90) → 34, 25, 12, 22, 11, 64, 90
No swap

⇒ Pass 2:

(34 > 25) → 25, 34, 12, 22, 11, 64, 90
swap

(34 > 12) → 25, 12, 34, 22, 11, 64, 90
swap

(34 > 22) → 25, 12, 22, 34, 11, 64, 90
swap

$(34 > 11) \rightarrow 25, 12, 22, 11, 34, 64, 90$
Swap

$(34 < 64) \rightarrow 25, 12, 22, 11, 34, 64, 90$
No swap

$\Rightarrow$ Pass 3:

$(25 > 12) \rightarrow 12, 25, 22, 11, 34, 64, 90$
Swap

$(25 > 22) \rightarrow 12, 22, 25, 11, 34, 64, 90$
Swap

$(25 > 11) \rightarrow 12, 22, 11, 25, 34, 64, 90$
Swap

$(25 < 34) \rightarrow 12, 22, 11, 25, 34, 64, 90$
No swap

$\Rightarrow$ Pass 4:

$(12 < 22) \rightarrow 12, 22, 11, 25, 34, 64, 90$
No swap

$(22 > 11) \rightarrow 12, 11, 22, 25, 34, 64, 90$
Swap

$(22 < 25) \rightarrow 12, 11, 22, 25, 34, 64, 90$
No swap

$\Rightarrow$ Pass 5:

$(12 > 11) \rightarrow 11, 12, 22, 25, 34, 64, 90$
Swap

$(12 < 22) \rightarrow 11, 12, 22, 25, 34, 64, 90$
No swap

Final sorted array: 11, 12, 22, 25, 34, 64, 90

$\Rightarrow$ For an array of n elements, worst-case no. of passes is $n-1$. Because in each pass, the largest unsorted element moves to its correct position.

Q2) Sort using selection sort: 50, 23, 03, 18, 9, 01, 70, 21, 20, 6, 40, 04. What is best & worst case time complexity of selection sort?

Working: Repeatedly finding min element from the unsorted portion & swapping with first unsorted element.

Pass-1: min element = 01

Swap with first element (50)

01, 23, 03, 18, 09, 50, 70, 21, 20, 06, 40, 04

Pass 2: min element = 03

Swap with 23

01, 03, 23, 18, 09, 50, 70, 21, 20, 06, 40, 04

Pass 3: min ele = 04

swap with 23

01, 03, 04, 18, 09, 50, 70, 21, 20, 06, 40, 23

Pass 4: min ele = 06

swap with 18

01, 03, 04, 06, 09, 50, 70, 21, 20, 18, 40, 23

Pass 5: min ele = 09

No swap needed

Pass 6: min ele = 18

Swap with 50

01, 03, 04, 06, 09, 18, 70, 21, 20, 50, 40, 23

Pass 7: min ele = 20

Swap with 70

01, 03, 04, 06, 09, 18, 20, 21, 70, 50, 40, 23

Pass 8: min ele = 21

No swap needed

Pass 9: min ele = 23

Swap with 70

01, 03, 04, 06, 09, 18, 20, 21, 23, 50, 40, 70

Pass 10: min ele = 40

Swap with 50

01, 03, 04, 06, 09, 18, 20, 21, 23, 40, 50, 70

Pass 11: min ele = 50

No swap

Required
sorted
array

Time Complexity:

• Best Case : $O(N^2)$

• Worst Case : $O(N^2)$

* For N elements, always the pass required = $(N-1)$

3.18. Sort using insertion sort: 55, 85, 45, 11, 34, 5, 89, 99, 67

⇒ Working: Picks an element and places it in the correct position relative to the elements that are already sorted.

Pass 1: (Key = 85),
compare with 55 ($85 > 55$)
No swap

Pass 2: (Key = 45),
compare with 85 & 55, ($45 < 85$) swap, ($45 < 55$) swap

45, 55, 85, 11, 34, 5, 89, 99, 67

Pass 3: Key = 11
($11 < 85$) swap, ($11 < 55$) swap, ($11 < 45$) swap

11, 45, 55, 85, 34, 5, 89, 99, 67

Pass 4: Key = 34
($34 < 85$) swap, ($34 < 55$) swap, ($34 < 45$) swap, ($34 > 11$) No swap

11, 34, 45, 55, 85, 5, 89, 99, 67

Pass 5: Key = 5
($5 < 85$) ($5 < 55$) ($5 < 45$) ($5 < 34$) ($5 < 11$)
Swap

5, 11, 34, 45, 55, 85, 89, 99, 67

Pass 6: Key = 89
($89 > 85$)
No swap

Pass 7: Key = 99

$(99 > 89)$

No Swap

Pass 8: Key = 67

$(67 < 99)$, $(67 < 89)$, $(67 < 85)$, $(67 > 55)$

Swap

Swap

Swap

No Swap

5, 11, 34, 45, 55, 67, 85, 89, 99

Time Complexity:

• Best Case = $O(N)$

• Worst Case = $O(N^2)$

Q4) Write rules to convert given infix to postfix & infix to prefix
 $(P * Q - (L + M * N) ^ (X * Y / Z))$

Rules: For Postfix

1) Operands (letters / numbers): Add directly to postfix

2) Left Parenthesis '(' : push it into stack.

3) Right Parenthesis ')' : pop operators from the stack & add them to postfix expression; until left parenthesis comes. Discard both parenthesis.

4) Operators (+, -, *, /, ^);

- pop operators from stack with greater or equal precedence and add them to postfix expression.
- push current operator onto the stack.

5) End expression: pop all operators from the stack and add them to the postfix expression.

Operator precedence : 1) $\wedge$: Highest precedence (right - associative)
2) $*, /$: left associative
3) $+, -$: left associative

$$(P * Q - (L + M * N) \wedge (X * Y / Z))$$

	Postfix	Stack
(		(
P	P	(
*	P	(, *
Q	P, Q	(, *,
-	P, Q, *	(, -,
(	P, Q, *	(, -, (
L	P, Q, *, L	(, -, (
+	P, Q, *, L	(, -, (, +
M	P, Q, *, L, M	(, -, (, +
*	PQ * LM	(- (+ *
N	PQ * LMN	(- (+ *
)	PQ * LMN * +	(- (
$\wedge$	PQ * LMN * +	(- $\wedge$
(	PQ * LMN * +	(- $\wedge$ (
X	PQ * LMN * + X	(- $\wedge$ (
*	PQ * LMN * + X	(- $\wedge$ (*
Y	PQ * LMN * + X Y	(- $\wedge$ (*
/	PQ * LMN * + X Y *	(- $\wedge$ (* (/
Z	PQ * LMN * + X Y * Z	(- $\wedge$ (/
)	PQ * LMN * + X Y * Z /	(- $\wedge$ (/
)	PQ * LMN * + X Y * Z / $\wedge$ -	-

Final expression

* Time & Space Complexity $\rightarrow O(N)$

Rules: For prefix

- 1) Reverse the infix expression
- 2) Replace every $)$ with $($ and vice versa.
- 3) Use postfix conversion rules on reversed expression
- 4) Reverse the postfix expression to get prefix.

$$(P * Q - (L + M * N)^{(X * Y / Z)})$$

Reversed: ~~$(X * Y / Z) ^ (N * M + L) - Q * P$~~

$$) Z / Y * X (^ N * M + L (- Q * P ($$

$$((Z / Y * X) ^ (N * M + L) - Q * P)$$

	Postfix	Stack
(	-	((
(	-	((
Z	Z	((Z
/	Z Z	(((/
Y	ZY	(((/Y
*	ZY/	(((/*
X	ZY/X	(((/X
)	ZY/X*	(((^
^	ZY/X*	(^
(	ZY/X*	(^(
N	ZY/X*N	(^(N
*	ZY/X*N	(^(N*
M	ZY/X*NM	(^(N*M
+	ZY/X*NM*	(^(N*M+
L	ZY/X*NM*L	(^(N*M+L
)	ZY/X*NM*L+	(^
-	ZY/X*NM*L+	(^-
Q	ZY/X*NM*L+Q	(^-Q
*	ZY/X*NM*L+Q	(^-Q*
P	ZY/X*NM*L+QP	(^-Q*P
)	ZY/X*NM*L+QP*-	-

Final postfix expression:

$$zy/x * NM + L + QP * - \wedge$$

prefix expression:

$$\wedge - * PQ + L * MN * X / YZ$$

Q5) Draw & explain circular queue using array. Write pseudocode for Add, Remove, Is full, Is Empty.

- Initialize an array queue of size n ,
- Initialize two variables front & rear to -1 .
- Enqueue : To enqueue an element x into the queue,
 - increment rear by 1.
 - If rear is n , set rear to 0.
 - If front is -1 , set front to 0.
 - Set queue(rear) to x .
- Dequeue :
 - Check if the queue is empty by checking if front is -1 .
 - If it is empty, return error message indicating queue is empty.
 - Set x to queue front.
 - If front = rear, set front & rear to -1 .
 - Otherwise, increment front by 1.
 - If front = n , set front = 0.
 - Return x .

Pseudo code :

⇒ Add Q : initially front = rear = 0

Algorithm Add Q (elem)

```
{
    if (rear + 1) % n == Full
        print "queue full"
    else
        {
```

rear = (rear + 1) % n

Q[rear] = elem

```
}
```

⇒ Delete Q :- initially front = rear = 0

Algorithm Del Q [elem]

```
{
```

if front == rear

print "queue empty"

```
}
```

else {

front = (front + 1) % n

elem = Q (front)

return elem

```
}
```

⇒ Is Empty :

is Empty (elem)

```
{ if (front == rear)
```

return 1

else {

return 0

```
}
```

```

=> Is Full : isFull (elem)
            { if ((rear + 1) % size == front)
              return 1
            else { return 0
              }
            }
  
```

Q. What is double ended queue? Draw diagram with labelling 4 basic operations at appropriate places. Which 2 data structures & how are combined in it & how?

→ A double ended queue ^[dequeue] ~~[dequeue]~~ is an abstract data type that generalizes a queue, for which elements can be added to or removed from either the front or rear.

→ Let's assume deque has max size = 5,

10	20	30	40
----	----	----	----

- PushRear() - Inserts an element before 10 after 40.
- PushFront() - Inserts an element before 10.
- PopRear() - Removes 40.
- PopFront() - Removes 10.

→ Queue & Stack are data structures involved in Deque

- Queue - FIFO functionality is used
- Stack - ability to insert & remove elements from both ends.

Q. Diff b/w Linear & Circular Queue.

Features:	Linear Queue	Circular Queue
Structure	Elements stored in linear manner	'' circular manner
Overflow	Can experience wasted space as elements are dequeued	Utilizes space efficiently by wrapping around.
Insertion	Happens at rear end only.	Can happen at both rear & front.
Deletion	Happens at front end only	Can happen '' ''
Implementation	Easier to implement, with simple arrays	Slightly more complex due to circular nature of pointers
Size Limitations	Fixed size, leading to overflow, even if space available	Fixed size, but can reuse space efficiently, allowing for continuous operations w/o overflow.

Q. Pseudocode of Bubble sort, Selection sort & Insertion sort.
& merge sort.

⇒ Bubble Sort:

Bubble (array) Algorithm bubble()

for $i = 1$ to n

{

for $j = 0$ to $n - i - 1$

{ if $a[j] > a[j+1]$

{ swap($a[j]$, $a[j+1]$)

}

display(a, n); }

⇒ Selection sort:

Void Selectionsort()

{ for $i = 0$ to $n - 2$

{

minpos = i ;

for $j = i + 1$ to $n - 1$

{

if $a[j] < a[\text{minpos}]$

{ minpos = j ;

}

}

if ($\text{minpos} \neq i$)

{

temp = $a[i]$; $a[i] = a[\text{minpos}]$; $a[\text{minpos}] = \text{temp}$;

}

⇒ Insertion sort:

```

Algorithm insertion sort (arr[], n)
{
  for i = 1 to i < n step 1
  {
    key = arr[i];
    j = i - 1;
    while (j >= 0 && arr[j] > key)
    {
      arr[j+1] = arr[j];
      j = j - 1;
    }
    arr[j+1] = key;
  }
}

```

⇒ Shell sort:

```

void shell-sort (int A[], int n)
{
  gap = n/2;
  do
  {
    do
    {
      swapped = 0;
      for (i = 0; i < n - gap; i++)
      {
        if (A[i] > A[i+gap])
        {
          swap();
          swapped = 1;
        }
      }
    } while (swapped == 1);
  } while ((gap = gap/2) >= 1);
}

```

Q. write pseudocode for basic stack operations:

• isFull() :

~~Pseudo of~~
Algorithm isFull()
{

if (top == MAXSIZE-1)

return true;

else

return false;

}

• isEmpty() :

{

if (top == -1)

return true;

else

return false;

}

• push() :

{

if (! isFull())

{

top = top + 1

stack[top] = item;

}

}

• pop() :

```

pop() :
{
    if (! is Empty())
    {
        temp = stack[top];
        top = top - 1;
        return (temp);
    }
}

```

Q. What is time complexity? Why asymptotic notations are imp? Find time complexity of following code using step count method.

```

int x = 0;
for (i = 0; i < N; i++)
{
    for (j = N; j > i; j++)
    {
        x = x + i + j;
    }
}

```

→ Time Complexity : Amount of time an algorithm takes to run as a function, of the size of its input.

- It helps in finding how the runtime of an algorithm increases as the size of input increases, enabling to choose the most efficient algorithm for given problem.

- Asymptotic notations are useful as,

- ① They simplify analysis of algorithm.
- ② Compares efficiency of diff algorithm.
- ③ Predicts how an algorithm's performance will change as

the size of input grows.

④ Guide developers in selecting right algorithm.

Q. Explain Differentiate Linear & Non linear DS and Static & Dynamic DS :

Features	Linear	Non-Linear
Definition	Data elements are arranged sequentially.	" arranged hierarchially
Traversal	Traversed in single level from start to end.	Traversal can be done in multiple ways.
Memory usage	Memory is utilized efficiently due to contiguous arrangement	Memory utilization less efficient, as elements stored in non-contiguous locations
Ex:	Arrays, Linked list, Stacks, Queues	Ex: Trees, Graphs.

Features:

Static Data
Structure

Dynamic Data
Structure

Definition

Size of DS is fixed
& can't be changed
during execution

Size of DS can
grow or shrink
during execution.

Memory
Allocation

memory allocated
at compile time.

Memory allocated
at runtime.

Flexibility

Less flexible as
size is pre-determined

Highly flexible as the
size can adjust
based on requirements

Usage:

Useful when the
size of data is
known in advance

Useful when the
size of data is
unpredictable or
varies with time

Ex:

Arrays

Linked lists, Dynamic
Arrays (like
malloc in C)

Q. What is need to convert infix exp to postfix?

Infix exp $(A+B)$ is converted to postfix $(AB+)$ as, postfix notation is easier for computers to evaluate w/o the need for parentheses to dictate precedence.

Q. What is row major & column major representation? with eg. write formula to find any element $A[i][j]$ in row major & column major representation of A.

- Row major - Stores all elements of a row contiguously in memory before moving to the next row.
- Column major - Stores all elements of column contiguously in memory before moving to next column.

$A[i][j]$

→ Row major representation:

Address of $A[i][j]$ = Base address + $(i \times n + j) \times \text{size of element of A}$

n = no. of column

i = Row index

j = Column index

→ Column major representation:

Address of $A[i][j]$ = Base Address + $(j \times m + i) \times \text{size of element of A}$

m = no. of rows in array

Ex: 2D array: $A[3][4]$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

For row major

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

→ Calculating memory location of $A[2][3]$

$$A[2][3] = \text{Base address} + (2 \times 4 + 3) \times \text{size of element of } A$$

→ For column major

$\{1, 5, 9, 2, 6, 10, 3, 7, 11, 4, 8, 12\}$

$$A[2][3] = \text{Base address} + (3 \times 3 + 2) \times \text{size of element of } A$$

Q. What is sparse matrix? List applications. Represent it with suitable DS. Also write algorithm for compact form.

- Any $m \times n$ matrix which contains large no. of zeros is called sparse matrix.
- We can characterize uniquely any element within a matrix by using an array of triples to represent a sparse matrix.
 - An array of triples → also called compact form.

Ex:

sparse matrix

Row	Col 0	Col 1	Col 2	Col 3	Col 4	Col 5
Row P	15	0	0	22	0	-15
A [7]	0	11	3	0	0	0
A [2]	0	0	0	-6	0	0
A [3]	0	0	0	0	0	0
A [2]	91	0	0	0	0	0
A [5]	0	0	28	0	0	0

Non zero
elements = 8

Compact form

	col 0	col 1	col 2
Row [0]	6	6	8
B [1]	0	0	15
B [2]	0	3	22
B [3]	0	5	-15
B [4]	1	1	11
B [5]	1	2	3
B [6]	2	3	-6
B [7]	4	0	91
B [8]	5	2	28

Non zero elements

* Rows in compact form = (non zero element in sparse matrix) + 1

* Columns = 3

→ Applications:

- ① Computational fluid dynamics → Scientific computing
- ② Graph theory - adjacency matrices of graphs are sparse
- ③ Machine Learning - Natural language processing.
- ④ Image compression - Algorithms like JPEG compression.
- ⑤ Signal Processing - Communication & Audio processing.

Algorithm compact (A, m, n, B)

$B(0,0) = m;$

$B(0,1) = n;$

$k = 1;$

for $i = 0$ to m

{

for $j = 0$ to n

{

if $(A(i,j) \neq 0)$

{ $B(k,0) = i;$

$B(k,1) = j;$

$B(k,2) = A(i,j);$

$k++;$

}

}

$B(0,2) = k-1;$

}

Q. Explain Simple & Fast Transpose with ex. write its algorithm.

• Transpose $\rightarrow$ Swapping rows & columns.

$A[i][j] \rightarrow A[j][i]$

$\rightarrow$ Simple Transpose: Directly swap rows & columns indices of each non-zero element. Time Complexity: $O(n * t)$

Ex:

$$A = \begin{bmatrix} 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 6 \\ 0 & 9 & 0 & 0 \end{bmatrix} = \begin{matrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{matrix} \begin{matrix} C_0 & C_1 & C_2 & C_3 \end{matrix} \begin{bmatrix} 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 6 \\ 0 & 9 & 0 & 0 \end{bmatrix}$$

columns $\rightarrow$ element

	C_0	C_1	C_2		C_0	C_1	C_2
Row 0	4	4	4	B_0	4	4	4
1	0	2	3	$\rightarrow B_1$	0	2	5
2	2	0	5	B_2	1	3	9
3	2	3	6	B_3	2	0	3
4	3	1	9	B_4	3	2	6

Algorithm Transpose (A, B)

- 1) $(m, n, t) := (A(0,0), A(0,1), A(0,2))$
- 2) $(B(0,0), B(0,1), B(0,2)) := (n, m, t)$
- 3) if $t \leq 0$ then return
- 4) $q := 1$
- 5) for $col := 0$ to n do
- 6) for $p := 1$ to t do
- 7) if $A(p,1) = col$ then
- 8) $\{B(q,0), B(q,1), B(q,2)\} := (A(p,0), A(p,2))$
- 9) $q := q + 1$
- 10) end
- 11) end
- 12) end Transpose

$\rightarrow$ Fast Transpose :

Time Complexity: $O(n+t)$

Time complexity: $O(n^2)$						Time complexity: $O(n^2)$			
		C_0	C_1	C_2	C_3		C_0	C_1	C_2
Ex	A_0	0	0	3	0	B_0	4	4	4
	A_1	0	0	0	0	B_1	0	2	5

Algorithm Transpose (A, B)

- 1) $(m, n, t) := (A(0,0), A(0,1), A(0,2))$
- 2) $(B(0,0), B(0,1), B(0,2)) := (n, m, t)$
- 3) if $t \leq 0$ then return
- 4) $q := 1$
- 5) for $col := 0$ to n do
- 6) for $p := 1$ to t do
- 7) if $A(p,1) = col$ then
- 8) $\{B(q,0), B(q,1), B(q,2)\} := (A(p,0), A(p,2))$
- 9) $q := q + 1$
- 10) end
- 11) end
- 12) end Transpose

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Fast Transpose

Ex:

	0	1	2	3
A ₀	0	0	3	0
1	0	0	0	0
2	5	0	0	6
3	0	9	0	0

→

	0	1	2
0	4	4	9
1	0	2	3
2	2	0	5
3	2	3	6
4	3	1	9

r term

0	1	2	3
1	1	1	1

r pos

0	1	2	3	4
1	2	3	4	5
2	3	4	5	

	0	1	2
0	4	4	4
1	0	2	5
2	1	3	9
3	2	0	3
4	3	2	6

B_3	0	2	3
B_4	2	3	6

B_4 3

Index =

0	1
1	2
2	3 4
3	4
4	5

Algorithm Fast-Transpose (A, B)

declare $S(n), T(n)$;

$(m, n, t) := (A(0,0), A(0,1), A(0,2))$

$(B(0,0), B(0,1), B(0,2)) := (n, m, t)$

if $t \leq 0$ then return

for $i := 0$ to n do $S(i) := 0$ end

for $i := 1$ to t do

$S(A(i,1)) := S(A(i,1)) + 1$

end

$T(0) := 1$

for $i := 1$ to n do

$T(i) = T(i-1) + S(i-1)$

end

for $j := 1$ to t do

$j := A(i,1)$

$(B(T(j),0), B(T(j),1), B(T(j),2)) := (A(i,1), A(i,0), A(i,2))$

$T(j) = T(j) + 1$

end

end Fast-Transpose

Q. Explain addition of two sparse matrix with Ex. & write its algorithm.

- 1) Obtain the triplet form of both sparse matrix.
- 2) Create new triplet to store result as result matrix
- 3) Copy no. of rows & columns from any sparse matrix to result matrix.
- 4) Let i, j, k be the indices of sparse matrices 1, 2, 3
- 5) Initialize i, j, k to 1.
- 6) Traverse both matrices from 2nd row.

Ex: ~~Ass~~ Let there be two sparse matrix in triplet form.

	0	1	2			0	1	2	
0	2	3	4		0	2	3	4	
1	0	0	5	i	1	0	1	4	j
2	0	1	6	$i +$	2	0	2	3	i
3	1	0	8	i	3	1	1	3	i
4	1	1	5	i	4	1	2	6	i

→ Smat 1

end

Smat 2

$i = 1 \ 2 \ 3 \ 4 \ 5$

$j = 1 \ 2 \ 3 \ 4 \ 5$

$k = 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7$

	Row	Col	val	
Resultant Mat	0	2	3	6
	1	0	0	5
	2	0	1	10
	3	0	2	3
	4	1	0	8
	5	1	1	8
		1	2	6

→ $k-1$

Algorithm Addition (a, b, c)

if $a(0,0) = b(0,0)$ and $a(0,1) = b(0,1)$

if $t1=0$ and $t2=0$ then stop

~~if $t1=0$ and $t2=0$~~

$c(0,0) = a(0,0)$; $c(0,1) = a(0,1)$;

End

$i = j = k = 1$

while $\{ 1 \leq t1 \text{ and } j \leq t2 \}$

case A: $a(i,0) = b(j,0)$ // row position

case 1: $a(i,1) = b(j,1)$ // col pos

$temp = a(i,2) + b(j,2)$

if $temp \neq 0$

$c(k,0) = a(i,0)$

$c(k,1) = a(i,1)$

$c(k,2) = temp$

$k = k+1$

end if

$i = i+1$; $j = j+1$;

case 2: if $a(i,1) < b(j,1)$

$c(k,0) = a(i,0)$

$c(k,1) = a(i,1)$

$c(k,2) = a(i,2)$

$k = k+1$

$i = i+1$

case 3: if $a(i,1) > b(j,1)$

$c(k,0) = b(j,0)$

$c(k,1) = b(j,1)$

$c(k,2) = b(j,2)$

$k = k+1$

$j = j+1$

Case B: if $a(i,0) < b(j,0)$

$$c(k,0) = a(i,0)$$

$$c(k,1) = a(i,1)$$

$$c(k,2) = a(i,2)$$

$$k = k+1$$

$$i = i+1$$

Case C: if $a(i,0) > b(j,0)$

$$c(k,0) = b(j,0)$$

$$c(k,1) = b(j,1)$$

$$c(k,2) = b(j,2)$$

$$k = k+1$$

$$j = j+1$$

end while

while $(i \leq t1)$

{

$$c := a$$

$$i++; k++$$

}

while $(j \leq t2)$

{

$$c := b$$

$$j++; k++;$$

}

$$c(0,2) = k-1$$

End Addition

Q. Explain concept of ordered list, Show its representation using array.

- Collection of elements arranged in specific sequence
- Each element has defined positions based on some ordering criteria.
- ~~Order~~ The list allows duplicates or enforce unique elements.
- Insertion or deletion might require, reordering the list.

Adv :

- 1) Efficient access : $O(1)$ time complexity
- 2) Binary search : Fast searching algorithms can be applied
- 3) Simple Implementation : use of array.

Dis Adv :

- 1) Insertion & Deletion of an element can take $O(n)$ time
- 2) If array of fixed size, it may run out of size, space requiring resizing or reallocating memory.
- 3) Contiguous Memory allocation can be inefficient for very large ordered lists.

→ Ordering can be :

- Ascending order, descending order
- Lexicographical order : For strings (alphabetically)

Representation using array :

= [2, 5, 8, 12, 16]

Q. Explain with Ex how single variable polynomial can be represented using 1-D array? What is adv & dis-adv of this representation?

Polynomial is one of the classic eg: of ordered List.

→ If polynomial has m non-zero terms then the poly can be represented as ordered list of $3m+1$ terms.

m , power of x , power of y , coeff, power of x , power of y , coeff

1st term 2nd term

Ex: $x^2 + 5xy + y^2 - y - x$

$(5, (2, 0, 1), (1, 1, 5), (0, 2, 1), (0, 1, -1), (1, 0, -1))$

OR

5	2	0	1	1	5	0	2	1	0	1	-1	1	0	-1
---	---	---	---	---	---	---	---	---	---	---	----	---	---	----

array representation.

Similarly for (x, y, z)

Ex: $-8x^3y^3z^3 + 10x^3y^2z + 5xyz^2 + 10$

4	3	3	3	-8	3	2	1	1	0	1	1	2	5	0	0	0	10
---	---	---	---	----	---	---	---	---	---	---	---	---	---	---	---	---	----

Adv of this representation:

- Simple & Easy to implement
- Easily access any coeff based on power of x .
- Compact & Efficient representation.
- Adding, Subtracting, multiplying poly is easy when represented as arrays.

Disadvantage:

- Wasted space for sparse poly
- Fixed size
- Inefficient for poly with large gap b/w non-zero terms.
- Complexity in Handling large degree poly.

Q. write algorithm for polynomial addition & multiplication

Algorithm polyadd ($p_1, p_2, \text{max1}, \text{max2}$)

```

{
    i = j = k = 0;
    while (i < max1 && j < max2)
    {
        if  $p_1[i] \cdot \text{exp} > p_2[j] \cdot \text{exp}$ 
        {
             $p_3[k] = p_1[i]$ ;
             $k++$ ;  $i++$ ;
        }
        else if ( $p_1[i] \cdot \text{exp} < p_2[j] \cdot \text{exp}$ )
        {
             $p_3[k] = p_2[j]$ ;
             $k++$ ;  $j++$ ;
        }
        else // same components
        {
             $\text{temp} = p_1[i] \cdot \text{coeff} + p_2[j] \cdot \text{coeff}$ 
            if ( $\text{temp} \neq 0$ )
            {
                 $p_3[k] \cdot \text{exp} = p_1[i] \cdot \text{exp}$ ;
                 $p_3[k] \cdot \text{coeff} = \text{temp}$ ;
                 $i++$ ;  $j++$ ;  $k++$ ;
            }
        }
    }
}

```

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```
while (i < max1)
```

```
{
```

```
    p3[k] = p1[i];
```

```
    k++;
```

```
    i++;
```

```
}
```

```
while (j < max2)
```

```
{
```

```
    p3[k] = p2[j];
```

```
    k++;
```

```
    j++;
```

```
}
```

=> Algorithm polynomial multiplication ($p_1, p_2, \text{max1}, \text{max2}$)

```
{
```

```
    i = j = k = 0;
```

```
    while (i < max1) {
```

```
        j = 0;
```

```
        while (j < max2)
```

```
        { temp = p1[i].coeff * p2[j].coeff;
```

```
          if (temp != 0)
```

```
          { flag = 0;
```

```
            exp = p1[i].exp + p2[j].exp;
```

```
            for (x = 0; x < k; x++) {
```

```
              if (exp == p3[x].exp) {
```

```
                flag = 1;
```

```
                break; }
```

```
            }
```

```
          if (flag == 1) {
```

```
            p3[x].coeff = p3[x].coeff + temp;
```

```
            j++;
```

```
          }
```

else {
 p3[k].exp = exp;
 p3[k].coeff = temp;
 j++; k++;
 }
 } // end if
 } // end while
 i++;
 } // end while.

Q. State applications of Queues & Stacks.

Queues:

- 1) Job scheduling - manage job execution such as printers where jobs are executed in the order they are received.
- 2) Data streaming - Used in networks to manage incoming & outgoing data packets.
- 3) While streaming video or audio, queues buffer data to ensure continuous playback.
- 4) Task scheduling - Useful in systems that need to handle real-time task with diff priorities.
- 5) Paging in OS - Used in page replacement algorithms.

Stacks:

- 1) ~~St~~ Backtracking Algorithms - Depth first search [DFS] in graphs & mazes. When you hit a dead end, you pop from the stack & backtrack to the previous decision point.

- 2) Expression Evaluation - Infix to postfix & prefix conversion.
- 3) Function Call Management - Each function call is pushed onto the stack, & when the function returns, it is popped from the stack.
- 4) String reversal - Reverse a string by pushing each character onto stack & then popping them.
- 5) Browser History Management: Browser use stacks to manage the history of web pages.

Q. Each element of an array Data [20][50] requires 4 bytes of storage. Base Add = 2000, Find location of Data [10][10] when array is stored as
i) row major ii) column major.

i) Row-major Order:

$$\begin{aligned}\text{Address (A[10][10])} &= 2000 + [(10 \times 50) + 10] \times 4 \\ &= 2000 + [510 \times 4] \\ &= 4040\end{aligned}$$

ii) Column-major

$$\begin{aligned}&= 2000 + [(10 \times 20) + 10] \times 4 \\ &= 2840\end{aligned}$$

Q. Consider linear array $A[5:50]$. $BA = 300$ &
no. of words per memory cell = 4 bytes.
Find address of $A[15]$

$$\rightarrow A[15] = 300 + (15 - 5) \times 4 = 340$$

↓
lower bound
of array considered

Q. Consider int arr $[5][4]$. $BA = 510$. Find address
of arr $[3][2]$ with row & column method.

→ Row Major:

$$A[3][2] = 510 + [(3 \times 4) + 2] \times 4$$

$$= 566$$

→ Column Major:

$$A[3][2] = 510 + [(2 \times 5) + 3] \times 4$$

$$= 562$$