SY B.Tech-CSE

Question Bank
Unit - II

1. Let S be the sample space when a fair coin is tossed 3 times. Let X be the random variable which assigns to each point in S its largest number of successive heads. Find
   (a) The distribution of X.
   (b) Expectation of X.
2. Let S be the sample space when a fair coin is tossed 3 times. Suppose the coin is weighted so that P(H)=2/3 and P(T)=1/3. Let X be the random variable which assigns to each point in S its largest number of successive heads. Find
   (a) The distribution of X.
   (b) Expectation of X.
3. Let S be the sample space when a pair of fair dice is tossed. Let X and Y be the random variable on S namely X denotes the maximum of the numbers

appearing, i.e. X(a,b)=max(a,b) and Y denotes the sum of the numbers, i.e. Y(a,b)=a+b. Find

(a) The distribution f of X and the distribution g of Y.

(b) Expectation of X and Y.

(c) Variance of X and Y.

(d) Standard deviation of X and Y.

4. A player tosses a fair die. If a prime number 2,3 or 5 occurs the player wins that number of dollars but if a non prime number occurs the player loses that number of dollars. Find the value of the game and comment on whether its a fair game.

5. Let S be the sample space when a pair of fair dice is tossed. Let X and Y be the random variable on S namely X denotes the maximum of the numbers appearing, i.e. X(a,b)=max(a,b) and Y denotes the sum of the numbers, i.e. Y(a,b)=a+b. Find

   (a) The joint distribution of X and Y.

   (b) Covariance of X and Y.

   (c) Correlation of X and Y.

6. Let S be the sample space when a fair coin is tossed twice. Let X and Y be random variables on S. Suppose X=1 if the first toss is a head, X=0 otherwise and Y=1 if both tosses are heads, Y=0 otherwise.

   (a) Find the joint distribution of X and Y.

   (b) Prove whether its an independent random variables.

(c) Let random variable Z be Z=X+Y. Find E(Z) and Var(Z).

7. Let S be the sample space when a fair coin is tossed twice. Let X and Y be random variables on S. Suppose X=1 if the first toss is a head, X=0 otherwise and Y=1 if the second toss is a head, Y=0 otherwise.

   (a) Find the joint distribution of X and Y.
   (b) Prove whether its an independent random variables.
   (c) Let random variable Z be Z=X+Y. Find E(Z) and Var(Z).

8. Let X be a random variable with the following distribution function f:

$$f(x) = \begin{cases} \frac{1}{2}x & if \quad 0 \leq x \leq 2 \\ 0 & elsewhere \end{cases}$$

   Find

   (a) $P(1 \leq X \leq 1.5)$
   (b) E(X)
   (c) σ_x
   (d) Cumulative distribution function

9. Suppose a random variable X takes on the values -3,-1,2 and 5 with respective probabilities $\frac{2k-3}{10}, \frac{k+1}{10}, \frac{k-1}{10}, \frac{k-2}{10}$. Determine the distribution of X.

10. A fair coin is tossed 4 times. Let X denote the number of heads occurring. Find

    (a) The distribution f of X

(b) E(X)

11. A random sample with replacement of size n=2 is chosen from the set 1,2,3 yielding the 9 element equiprobable space
S={(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)}

    (a) Let X denote the sum of the two numbers. Find the distribution f of X and the expected value E(X).

    (b) Let Y denote the minimum of the two numbers. Find the distribution g of Y and the expected value E(Y).

12. A player tosses two fair coins. Player wins \$2 if two heads occur, \$1 if one head occurs and player looses \$3 if no head occurs. Find the value of the game and comment on whether it's a fair game.

13. Find the Mean, Variance and Standard deviation of each distribution given below.

    (a)

x	3	8	12
f(x)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

    (b)

x_i	-6	-4	3	12
p_i	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

    (c)

x_i	2	3	5	8
p_i	0.3	0.1	0.4	0.2

    (d)

x	2	3	8
f(x)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

    (e)

x	-2	-1	7
f(x)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

(f)

x_i	-1	0	1	2	3
p_i	0.3	0.1	0.1	0.3	0.2

(g)

x_i	1	2	3	6	7
f(x)	0.2	0.1	0.3	0.1	0.3

14. A fair die is tossed. Let X denote twice the number appearing and let Y be 1 or 3 according as an odd or even number appears. Find the distribution and expectation of:

    (a) X

    (b) Y

    (c) Let Z and W be random variables defined by Z(s)=X(s)+Y(s) and W(s)=X(s) Y(s).

        i. Find the distribution and expectation of Z.

        ii. Find the distribution and expectation of W.

        iii. Verify whether E(X+Y)=E(X)+E(Y)

15. Let X be a random variable with distribution:

x	1	2	3
P(X=x)	0.3	0.5	0.2

Find the Mean,Variance and Standard deviation of X. Then find the Distribution, Mean, Variance and Standard deviation of the random variable $Y = \Phi(X)$, where

(a) $\Phi(x) = x^3$

(b) $\Phi(x) = 2^x$

(c) $\Phi(x) = x^2 + 3x + 4$

16. Let X and Y be random variables with the joint distribution given below:

$X\backslash Y$	-3	2	4	Sum
1	0.1	0.2	0.2	0.5
3	0.3	0.1	0.1	0.5
Sum	0.4	0.3	0.3	

Find:

(a) The distributions of X and Y

(b) Cov(X,Y)

(c) $\rho(X, Y)$

(d) Are X and Y independent random variables?

17. Let X and Y be independent random variables with the following distributions. Find the joint distribution of X and Y.

x_i	1	2
$f(x_i)$	0.6	0.4

Table 1: Distribution of X

y_j	5	10	15
$g(y_j)$	0.2	0.5	0.3

Table 2: Distribution of Y

18. A fair coin is tossed three times. Let X equal 0 or 1 according as a head or a tail occurs on the first toss and let Y equal the total number of heads that occur.

(a) Find the distributions of X and Y

(b) Find the joint distribution h of X and Y

(c) Determine whether X and Y are independent

(d) Find Cov(X,Y)

19. Let X be the random variable with the following distribution and let $Y = X^2$.

x	-2	-1	1	2
f(x)	1/4	1/4	1/4	1/4

Find:

(a) The distribution of Y

(b) The joint distribution of X and Y

(c) Cov(X,Y)

(d) $\rho(X, Y)$

(e) Determine whether X and Y are independent.

20. Let X be a random variable with the following distribution function f:

$$f(x) = \begin{cases} \frac{1}{6}x + k & if \quad 0 \leq x \leq 3 \\ 0 & elsewhere \end{cases}$$

Find

(a) Evaluate k

(b) $P(1 \leq X \leq 2)$

21. Let X be a continuous random variable whose distribution f is constant on an interval, say $I = \{a \leq x \leq b\}$, and 0 elsewhere; namely,

$$f(x) = \begin{cases} k & if \quad a \leq x \leq b \\ 0 & elsewhere \end{cases}$$

(a) Determine k

(b) Find the mean μ of X

(c) Determine the cumulative distribution function F of X.

22. Suppose a random variable X takes on the values -3,2,4,7 with respective probabilities $\frac{k+1}{10}$, $\frac{2k-2}{10}$, $\frac{3k-5}{10}$, $\frac{k+2}{10}$. Find

    (a) The distribution of X

    (b) Expected value of X

    (c) Standard deviation of X

23. A pair of dice is thrown. Let X denote the minimum of two numbers which occur. Find the probability distribution and expectation of X.

24. A fair coin is tossed four times. Let X denote the number of heads occurring and Y denote the longest string of heads occurring. Find

    (a) The distribution of X

    (b) The distribution of Y

    (c) The joint distribution of X and Y

    (d) Cov(X,Y) and $\rho(X, Y)$

25. A player tosses three fair coins. He wins \$5 if three heads occur, \$3 if two heads occur and \$1 if only one head occurs. On the other hand, he loses \$15 if three tails occur. Find the value of the game to the player.

26. Let X be a random variable with the following distribution:

x	1	3	4	5
f(x)	0.4	0.1	0.2	0.3

(a) Find the Mean, Variance and Standard deviation of X.

(b) Find the Distribution, Mean, Variance and Standard deviation of $Y = X^2 + 2$.

27. Let X be a random variable with distribution:

x	-1	1	2
f(x)	0.2	0.5	0.3

(a) Find the Mean,Variance and Standard deviation of X.

(b) Find the Distribution, Mean, Variance and Standard deviation of the random variable $Y = \Phi(X)$, where

i. $\Phi(x) = x^4$

ii. $\Phi(x) = 3^x$

iii. $\Phi(x) = 2^{x+1}$

28. Two cards are selected from a box which contains five cards numbered 1,1,2,2 and 3. Let X denote the sum and Y the maximum of the two numbers drawn.

(a) Find the Distribution, Mean, Variance and Standard deviation of the random variables: X and Y

(b) Determine the Joint distribution of X and Y

(c) Find Cov(X,Y) and $\rho(X, Y)$

(d) Z=X+Y

(e) W=X.Y

29. Consider the joint distribution of X and Y given below. Find E(X), E(Y), Cov(X,Y), $\rho(X,Y)$,σ_x and σ_y.

(a)

$X \backslash Y$	-4	2	7	Sum
1	1/8	1/4	1/8	1/2
5	1/4	1/8	1/8	1/2
Sum	3/8	3/8	1/4	

(b)

$X \backslash Y$	-2	-1	4	5	Sum
1	0.1	0.2	0	0.3	0.6
2	0.2	0.1	0.1	0	0.4
Sum	0.3	0.3	0.1	0.3	

30. Let X and Y be independent random variables with the following distributions. Find the joint distribution of X and Y. Also verify that Cov(X,Y)=0.

x_i	1	2
$f(x_i)$	0.7	0.3

Table 3: Distribution of X

y_j	-2	5	8
$g(y_j)$	0.3	0.5	0.2

Table 4: Distribution of Y

31. Consider the joint distribution of X and Y.

$X \backslash Y$	2	3	4	Sum
1	0.06	0.15	0.09	0.30
2	0.14	0.35	0.21	0.70
Sum	0.20	0.50	0.30	

(a) Find E(X) and E(Y)

(b) Determine whether X and Y are independent

(c) Find Cov(X,Y)

32. Consider the joint distribution of X and Y.

$X\backslash Y$	-2	-1	0	1	2	3	Sum
0	0.05	0.05	0.10	0	0.05	0.05	0.30
1	0.10	0.05	0.05	0.10	0	0.05	0.35
2	0.03	0.12	0.07	0.06	0.03	0.04	0.35
Sum	0.18	0.22	0.22	0.16	0.08	0.14	

(a) Find E(X) and E(Y)

(b) Determine whether X and Y are independent

(c) Find the Distribution, Mean and Standard deviation of the random variable Z=X+Y.

33. A random sample with replacement of size n=2 is chosen from the set {1,2,3,4,5}. Let X=0 if the first number is even and X=1 otherwise. Let Y=1 if the second number is odd and Y=0 otherwise.

(a) Show that the distribution for X and Y are identical.

(b) Find the Joint distribution of X and Y.

(c) Are X and Y independent?

34. Let X be a continuous random variable with the following distribution:

$$f(x) = \begin{cases} \frac{1}{8} & if \quad 0 \leq x \leq 8 \\ 0 & elsewhere \end{cases}$$

Find

a. $P\left(2 \leq X \leq 5\right)$

b. $P\left(3 \leq X \leq 7\right)$

c. $P\left(X \geq 6\right)$

35. Let X be a continuous random variable with the following distribution

$$f(x) = \begin{cases} kx & if \quad 0 \leq x \leq 5 \\ 0 & elsewhere \end{cases}$$

i Evaluate k.

ii Find

a. $P\left(1 \leq X \leq 3\right)$

b. $P\left(2 \leq X \leq 4\right)$

c. $P\left(X > 4\right)$

d. $P\left(X \leq 3\right)$

36. A coin weighted so that $P(H) = \frac{3}{4}$ and $P(T) = \frac{1}{4}$,is tossed three times. Let X denote the number of heads that appear.
Find

(a) The distribution of X

(b) E(X)

37. A coin weighted so that $P(H) = \frac{1}{3}$ and $P(T) = \frac{2}{3}$, is tossed until a head or five tails occur. Find the expected number E of tosses of the coin.