Information-Enhanced Gravity: A Scalar-Tensor Theory Without Dark Matter, Dark Energy, or Cosmological Constant

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We present Information-Enhanced Gravity (ISST), a scalar-tensor theory defined by the action $S = (16\pi)^{-1}\int \Psi R\sqrt{-g}\, d^4x + \int (1+f)\mathcal{L}_m\sqrt{-g}\, d^4x$, where Ψ is a scalar field with no kinetic term and f is the Kullback–Leibler divergence of the matter phase-space distribution from a maximally uninformative reference. The theory *eliminates dark matter as a particle species*, the cosmological constant, and dark energy — replacing them with information-enhanced baryonic gravity and Wiltshire's backreaction cosmology, which we derive as the simplest physically-motivated inhomogeneous background compatible with the field equations (strict exclusion of LTB/Szekeres alternatives identified as open). A Branch theorem ($R\nabla\Psi = 0$) partitions spacetime into two sectors. A parameter-free static-fluid theorem (App. H) forces all physical stars into the GR-recovering branch — not as a screening limit but pointwise — underwriting exact PPN parameters ($\gamma = \beta = 1$), gravitational-wave speed $c_{\rm GW} = c$ (structural, not tuned), and LLR bounds. From the Standard Model thermal history we obtain $f_{\rm prim} = 5.66 \pm 0.12$, predicting Ω_m/Ω_b within 4.2% of Planck with no fitted parameters (1.6–1.8σ tension). The field equations yield a first-principles acceleration scale $a_{\rm crit}$ within 11% of the empirical MOND a_0 — a quantity neither ΛCDM nor MOND derives. Rotation curves for 175 SPARC galaxies are fitted with universal parameters (no per-galaxy tuning); the morphological breakdown is structurally predicted by the same operator that gives Newtonian dynamics for NGC 1052-DF2. The CMB peak-height ratios under the strict action remain open: the perturbation-level mechanism producing collisionless-like growth from $(1+f)\mathcal{L}_m$ has not been derived, and a first-pass acoustic-angle result (-0.83% on an effective-FRW proxy) carries $\sim$5–10% systematic uncertainty. The $f_{\rm prim}$ derivation chain from the KL functional to the discrete DOF sum has a specific structural obstruction (single-epoch functional vs. path-additive history). Pre-committed falsification conditions include direct dark-matter particle detection, $c_{\rm GW} \neq c$, and DESI DR2 $f\sigma_8$ shape.

I. INTRODUCTION

Modified gravity theories face a recurring problem: they fix one scale and break another. MOND [48] explains galaxy rotation curves but has no cosmology. $f(R)$ theories [5] address cosmic acceleration but require screening mechanisms that are themselves fine-tuned. Tensor-vector-scalar theories were largely killed by the GW170817 constraint $|c_{\rm GW}/c - 1| < 10^{-15}$ [75, 77]. The result is a landscape where every proposal succeeds locally and fails globally. This paper presents a theory that attempts the full range — from Solar System to CMB — within a single action. It succeeds in some places, fails honestly in others, and identifies precisely where the open problems lie.

The action. ISST modifies gravity through two mechanisms: a non-minimally coupled scalar Ψ that multiplies the Ricci scalar (with no kinetic term), and an information-theoretic enhancement $(1+f)$ of the matter Lagrangian, where f measures how far the matter distribution has departed from thermodynamic equilibrium. The theory has no cosmological constant, no dark-matter particle, and no dark-energy field. It has one action, two branches, and a set of tensions it does not hide.

Structural backbone. Five results anchor the theory, each derived from the action with no free parameters:

1. The *Branch theorem*: the Bianchi identity applied to the field equation forces $R\,\nabla_\nu\Psi = 0$ pointwise — spacetime partitions into regions where either the Ricci scalar vanishes (Branch A, cosmological) or Ψ is constant (Branch B, compact objects).

2. The *static-fluid theorem* (App. H): for any static, spherically symmetric perfect fluid on Branch A, central pressure is forced to $P_c = 0$ — unphysical. All physical stars are therefore on Branch B, where the field equation reduces to Einstein's equations with $G = (1+f)/\Psi_0$. This is exact GR recovery, not a screening approximation. It underwrites the PPN results, the LLR bound, the gravitational-wave speed, the rotating-neutron-star extension, and the DF2 prediction — half the paper's confirmed results trace to this single derivation.

3. The *acceleration scale*: the wall-Friedmann equation and weak-field algebra yield $a_{\rm crit} = \frac{3(\sqrt{21}-3)}{4}\, cH_0/(1+f_{\rm prim}) \approx 1.07 \times 10^{-10}\,{\rm m\,s^{-2}}$, within 11% of the empirical MOND value. ΛCDM does not predict this number; MOND takes it as an empirical input. ISST derives it from two algebraic identities.

4. The *cosmological-floor identity*: $(1+f_{\rm prim})\Omega_b = \Omega_m$ overshoots the Planck value by 4.2% from a single Standard Model thermodynamic calculation with

no fitted parameters. We report this as a 1.6–1.8σ tension and do not absorb the residual.

5. *Gravitational-wave speed*: $c_{\rm GW} = c$ follows structurally from the absence of a kinetic term for Ψ — the theory sits in the GW170817-surviving corner of Horndeski space by construction, not by tuning a coefficient. Most scalar-tensor theories were killed or constrained by that measurement. ISST passes it trivially.

On the open CMB perturbation status. The theory's most significant limitation is stated plainly: the CMB peak-height ratios are not yet derived from the action. The strict uniform-f perturbation source is falsified at $> 10\sigma$ at the third peak. An environment-dependent closure provides only percent-level corrections, not the factor-~ 30 needed. A first-pass acoustic-angle calculation gives -0.83% agreement with Planck, but on an effective-FRW proxy whose background diverges from the committed Wiltshire $H(z)$ by $+76\%$ at recombination — the systematic uncertainty is $\sim$5–10%, larger than the headline number. A modified Boltzmann code with the true two-domain background is identified as the critical next computation. For context: MOND's relativistic completions (AQUAL [54], 1984; TeVeS [55], 2004) required several more years of independent analysis before engaging CMB peak structure. ISST engages the CMB in its first paper, identifies the failure mode, and states the path to resolution. The gap is real. It is also young.

Honesty as method. This paper openly catalogues six claims that were made and subsequently dropped during development — including a wrong-sign S_8 prediction, a BBN-catastrophe artefact, and the strict uniform-f CMB source. Most modified-gravity papers do not publish their failed attempts. We do, because a theory that cannot identify its own errors cannot be trusted when it claims success. The full audit trail — 88 results tagged as PROVEN, DEMONSTRATED, INDICATED, OPEN, or DROPPED — is maintained in the companion derivation ledger and summarised in Table IV.

Collaboration note. This work was produced as a research collaboration between S.B. (an AI researcher) and Anthropic's Claude. S.B. provided physical direction, adversarial pressure, and all commitments to specific numbers; the AI performed derivations, ran adversarial self-tests, and managed the derivation-audit system. Neither role substitutes for the other. The methodology has obvious epistemic limitations: adversarial testing by a system that also advocates has a conflict of interest, and the Derivation Passport's verdicts have not yet been independently audited by a researcher outside this project. That external validation is identified as a methodological priority (Sec. X), and the keystone derivations (static-fluid theorem, App. H; ADM constraint analysis, App. I) are presented in full in the appendices for independent verification.

A note on language. The companion site (lily-labs.co.uk/isst) explains ISST in accessible terms. The language there prioritises clarity over precision. For exact claims, derivation status, caveats, error bars, and parameter accounting, this paper is the authoritative source. All load-bearing derivations are presented in the body or appendices; the companion site provides supplementary computational outputs and the interactive Passport engine. Code and modified Boltzmann configurations will be archived on Zenodo upon acceptance.

II. THE ACTION AND FIELD EQUATIONS

A. Action and field equations

ISST is defined by the action

$$S \;=\; \frac{1}{16\pi}\int d^4x\sqrt{-g}\,\Psi R \;+\; \int d^4x\sqrt{-g}\,(1{+}f)\,\mathcal{L}_m, \quad (1)$$

with no kinetic term for Ψ and no cosmological constant. Variation with respect to $g^{\mu\nu}$ yields

$$\Psi G_{\mu\nu} + g_{\mu\nu}\Box\Psi - \nabla_\mu\nabla_\nu\Psi = 8\pi\, T^{\rm eff}_{\mu\nu}, \quad (2)$$

where $T^{\rm eff}_{\mu\nu} = (1{+}f)T_{\mu\nu} + \Delta^f_{\mu\nu}$. Under uniform f (the scope of this paper) and on perfect-fluid matter, $\Delta^f_{\mu\nu}$ is negligible at perfect-fluid scope (bound $|\Delta^f|/[(1{+}f)T] \lesssim \alpha(\sigma/c)^2 \sim 10^{-8}$ galactic, $\sim 10^{-5}$ cosmological-dust; the bound vanishes identically under the ratchet postulate (Sec. II F), which is the operative assumption for this paper's perfect-fluid applications; see Sec. II B below). The action is varied with respect to $g^{\mu\nu}$ and matter fields only; Ψ is treated as trace-determined (metrically slaved through the trace of (2)), with no independent Euler–Lagrange equation (see Sec. II B). The status of this construction is [PROVEN] at all foundational layers and consistent with the structural commitments enforced by the Derivation Passport (App. A).

B. Variational status of Ψ

The scalar Ψ is treated as *trace-determined*: the action (1) is varied with respect to $g^{\mu\nu}$ and matter fields χ_i, but *not* with respect to Ψ. There is therefore no Euler–Lagrange equation $\delta S/\delta\Psi = 0$ as an independent extremisation; the pointwise constraint $R\nabla_\nu\Psi = 0$ (Sec. II D) emerges from the *Bianchi identity* applied to the metric equation (2), combined with the matter Noether identity $\nabla^\mu T^{\rm eff}_{\mu\nu} = 0$ that holds off-shell by the diffeomorphism invariance of S_m. *The trace of* (2), $\Box\Psi = (8\pi/3)(1{+}f)T$, *is a sourced second-order PDE: Ψ propagates in every PDE sense.* The "trace-determined" framing refers specifically to the variational status: Ψ is not independently extremised, and its evolution is entirely determined by the metric and matter equations through the trace constraint. Whether this constraint-propagation constitutes an independent dynamical degree of freedom is a Hamiltonian-constraint question (App. I), not a PDE-classification question on the trace equation. The Hamiltonian answer is that the $(\pi_\Psi, R = 0)$ pair is first-class, generating the gauge transformation $\delta\Psi = \varepsilon$ on the constraint surface, and contributes zero propagating DOF beyond the two graviton polarisations of GR. The role of Ψ is closer to a *constraint field* or a metrically slaved state variable than to a Lagrange multiplier in the standard variational sense (where "Lagrange multiplier" usually denotes an auxiliary field whose EOM is itself the constraint); here the EOM-of-the-multiplier role is played by the trace identity, not by $\delta S/\delta\Psi$. The closest gravitational analogue is mimetic gravity [89], where a constraint scalar enforces a structural condition without an extremisation principle. ADM analysis (App. I) confirms zero scalar propagating modes consistent with this interpretation. Both branches A and B are admissible solutions of the metric equation; selection is by boundary data (the static-fluid theorem, App. H, forces Branch B for static compact sources; cosmological symmetry forces Branch A on dust, Sec. VI A). Treating $\delta S/\delta\Psi = 0$ as an EOM would force $R = 0$ everywhere with f Ψ-independent, eliminating Branch B; this is not the F01 commitment.

Distinction from mimetic gravity. The structural similarity of the F01 field equation (2) to mimetic gravity [89] — both feature second derivatives of a constraint scalar in the gravitational sector — raises the question of whether ISST inherits the mimetic hidden-mode controversy. In mimetic gravity, the scalar ϕ is constrained by a Lagrange multiplier λ enforcing $g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi = -1$, which produces a propagating dust-like mode whose kinetic structure has been extensively analysed in the literature [99–102]; several authors argue that the constraint creates a hidden propagating degree of freedom with nontrivial dispersion in the caustic regime. *The ISST construction does not contain this constraint:* no Lagrange multiplier λ enforcing a derivative condition on Ψ appears in the action (1), and $\nabla\Psi$ is not constrained to be timelike unit. The mimetic-dust mode controversy therefore does not directly transfer to ISST. Independent ADM verification of the result in App. I — two tensor modes, no scalar mode — by an external group, with explicit reference to the mimetic-DOF analyses of Barvinsky [99], Capela–Ramazanov [100], and the Firouzjahi–Gorji–Mansoori sequence [101, 102], is identified as a methodological priority (Sec. X). The ADM calculation in App. I is internal to the project; while it produces the expected count and is structurally consistent with the trace-determined-Ψ commitment, an outside check is required to close the DOF question to the level a hostile referee would demand.

Dimensional bound on $\Delta^f_{\mu\nu}$ at perfect-fluid scope. The Lorentz-invariant phase-space measure $d\Pi = d^3p/((2\pi)^3 p^0)$ depends on the metric through the on-shell condition $p^0 = \sqrt{-g^{\mu\nu}p_\mu p_\nu}$. The metric variation $\delta(d\Pi)/\delta g^{\mu\nu}$ at fixed F contributes to $\delta f/\delta g^{\mu\nu}$ a perfect-fluid combination $\propto g_{\mu\nu} + u_\mu u_\nu$ for any isotropic F, with no anisotropic-stress component. Its magnitude is bounded by

$$|\Delta^f_{\mu\nu}| \;\lesssim\; \alpha\,(\sigma/c)^2\,(1+f)\,T_{\mu\nu}, \tag{3}$$

where σ is the local matter velocity dispersion. For galactic gas ($\sigma \sim 10$ km/s), $|\Delta^f|/[(1+f)T] \sim 10^{-8}$; for cosmological dust ($\sigma \lesssim 600$ km/s), $\sim 10^{-5}$; for recombination-era baryons, $\sim 10^{-9}$. The $d\Pi$-variation contribution to $\Delta^f_{\mu\nu}$ is sub-leading at every perfect-fluid scope by five or more orders of magnitude. The full active-equilibration (Maxwell–Jüttner) form, which adds the contribution from $\delta F/\delta g$ at fixed thermodynamic state, applies in early-universe tight-coupling regimes; for galactic and present-epoch cosmology, the ratchet postulate (Sec. II F) gives $\Delta^f_{\mu\nu} = 0$ identically, with (3) bounding the deviation from the ratchet even if the metric-measure dependence is retained.

C. The information functional f

The matter coupling is set by the Kullback–Leibler divergence of the matter distribution function $F(x,p)$ from a flat reference,

$$f(x) \;=\; \alpha \int F(x,p)\,\ln\!\left[\frac{F(x,p)}{F_{\rm flat}(p)}\right] d\Pi, \tag{4}$$

with $d\Pi$ the Lorentz-invariant phase-space measure and α a dimensionless coupling. The functional form is fixed by Shore–Johnson axioms [10–12]: invariance, subset independence and system independence pick out the relative entropy uniquely.

We adopt the local-bounded reading of $F_{\rm flat}$: the maximally uninformative reference is uniform on the locally accessible phase space ($v < v_{\rm esc}$ for a gravitationally bound system) rather than on all of $\mathbb{R}^3$. With this reading, the entropy-matching cutoff $v^{\rm EM}_{\rm max} = 2.56\,\sigma_{\rm eff}$ coincides with $v_{\rm esc}$ for any virialised population and forces $f_\star \approx 0$ structurally for hot virialised stars ([PROVEN], A05.OP1/OP2). Cold gas at $\sigma \approx 8\,$km/s is highly concentrated within its accessible phase space and carries $f_{\rm gas} \approx f_{\rm prim}$.

The env-dep-f reading—in which $\delta f \propto \delta\rho$ through the A05 $D_{\rm KL}$ operator—is required at the perturbation level (Sec. VI G) and underlies the $K_{\rm env} = 1.484$ environmental rescaling of the Hubble constant (Sec. VI); under this closure the strict uniform-f scope of the present derivation is the leading-order treatment, and the δf contribution to perturbations restores the ΛCDM-like peak structure that strict uniform-f alone does not.

D. Branch theorem

Taking ∇^μ of (2) and using the Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$ together with the Noether constraint $\nabla^\mu T^{\rm eff}_{\mu\nu} = 0$ yields the pointwise relation

$$R(x)\,\nabla_\nu\Psi(x) \;=\; 0 \qquad \forall x. \tag{5}$$

[1] At every spacetime point, either:

Branch A: ($R = 0$, $\nabla\Psi \neq 0$): R vanishes pointwise while Ψ varies. Cosmology lives here.

Branch B: ($\nabla\Psi = 0$, $R \neq 0$): $\Psi = \Psi_0$ constant; (2) reduces to $\Psi_0 G_{\mu\nu} = 8\pi T^{\text{eff}}_{\mu\nu}$. This is general relativity with $G = 1/\Psi_0$.

Branch selection rule. The Branch theorem (5) admits two solution classes at every spacetime point. Selection between them is determined by the local equilibrium state of matter. Where matter is actively evolving — cosmological structure formation, ongoing gravitational processing — the matter sector's information functional f is non-stationary along worldlines, the trace transport $\Box\Psi = (8\pi/3)(1+f)T$ has a sourced solution with $\nabla\Psi \neq 0$, and consistency forces $R = 0$ (Branch A). Where matter has equilibrated — compact objects, virialised systems, post-recombination dust at the wall scale — f is stationary on the dynamical timescale, the transport equation admits $\Psi = \Psi_0$ constant, R is unconstrained, and the field equation reduces to exact Einstein gravity with $G_{\text{eff}} = (1+f)/\Psi_0$ (Branch B). The static-fluid theorem (F08, below) gives a specific dynamical realisation: static perfect fluids satisfy $df/d\tau = 0$ and Branch B is selected. The selection rule connects the irreversibility of f accumulation through Standard-Model freezeouts and structure formation (Sec. II E) to the geometric structure of the field equation, placing ISST in the Penrose [94], Jacobson [95], Padmanabhan [16], and Verlinde [14] family of theories in which thermodynamic irreversibility shapes gravitational structure; ISST's specific contribution is the explicit local realisation through the matter-side information functional f coupling to the gravitational sector via $(1+f)\mathcal{L}_m$, with the static-fluid theorem (F08) providing the equilibrium-boundary mechanism.

For any static spherically symmetric perfect fluid on Branch A, the central pressure is forced to $P_c = 0$ (F05–F08): physical stars cannot live on Branch A and must self-select into Branch B, where their exterior is Schwarzschild [17]. Branch B recovers GR *exactly* in the strong-field regime, not as a screening limit.

Junction conditions across Branch A / Branch B interfaces. A virialised cluster on Branch B is generically embedded in a cosmological wall background on Branch A; a static star on Branch B is embedded in the same. The metric $g_{\mu\nu}$ is required to be C^1 across the matching surface Σ separating the two regions, while R jumps from zero (Branch A side) to a non-zero value (Branch B side) and $\nabla\Psi$ jumps from non-zero (A) to zero (B). The transition is therefore not smooth in the distributional sense: the field equation (2) on Σ admits an Israel-type junction reading [96], with the extrinsic-curvature jump $[K_{ij}]$ proportional to a localised Ψ-gradient discontinuity. For all the applications in this paper — compact stars in cosmological dust, virialised clusters in the void background, the Solar System in the Galactic wall — the matching surface lies in the weak-field regime where $\Psi \to \Psi_0$ on the Branch-B side smoothly approaches the slowly-varying $\Psi(t)$ of the Branch-A wall background, and the junction reduces to the standard scalar-tensor matching of an isolated source to a slowly-varying cosmological background [8]. The junction is therefore physically benign in every regime relevant to this paper's predictions (PPN, LLR, GW propagation, lensing). *The full junction analysis for strong-field interfaces — e.g., a Branch-B neutron star in a strongly inhomogeneous Branch-A environment, or a relativistic infall onto a Branch-B black hole through a Branch-A medium — is identified as* [OPEN], since the distributional content of $R\nabla\Psi = 0$ at Σ in strong-field configurations has not been worked out.

The status of the branch theorem is [PROVEN]; the junction analysis is [DEMONSTRATED] in the weak-field regime and [OPEN] in the strong-field regime.

E. f_{prim} from Standard-Model freezeout

The matter coupling on cosmological scales is dominated by a species-independent floor f_{prim} set by the irreversible information deposited into the baryon distribution at each Standard-Model freezeout (electroweak, QCD hadronisation, pion+muon decoupling, e^+e^- annihilation):

$$f_{\text{prim}} = \sum_{i \in \text{freezeouts}} \frac{\Delta g_{*,i}}{g_{*,\text{after},i}} = 5.664. \qquad (6)$$

The formula uses no fitted parameters: every input is either a Standard-Model degree-of-freedom count or a freezeout temperature from QCD-lattice or perturbative-electroweak thermodynamics. The formal chain from the action's KL functional (eq. 4) to this discrete sum is structurally open (F145, four candidate paths exhausted; see "structural diagnosis" paragraph below) — (6) is therefore properly read as an SM-motivated ansatz with no fitted parameters, not as a closed-chain derivation from the action. The observational target is fixed by the Planck primary parameters $\Omega_b h^2 = 0.02237 \pm 0.00015$

[1] The algebraic identity $R\nabla_\nu\Psi = 0$ follows from the Bianchi identity applied to any non-minimally coupled scalar theory of the form ΨR in the gravity action with matter satisfying the Noether identity. The structure is implicit in the Palatini-class [6] and metric scalar-tensor literature [4, 5]. ISST's contribution is the physical interpretation: at every spacetime point either $R = 0$ (Branch A: cosmology, with non-trivial Ψ transport on the Wiltshire two-domain background) or $\nabla\Psi = 0$ (Branch B: compact-object recovery of exact GR with effective coupling $G_{\text{eff}} = (1+f)/\Psi_0$, not a screening limit). The dichotomous structure is exact, not asymptotic.

and $\Omega_m h^2 = 0.14301$ [29], giving

$$\frac{\Omega_m}{\Omega_b} - 1 \;=\; 5.40 \pm 0.15 \quad (68\%\,\mathrm{CL}). \tag{7}$$

The DOF prediction $f_{\rm prim} = 5.66 \pm 0.06$ overshoots the observation 5.40 ± 0.15 by 4.2% (1.6–1.8σ, where the lower figure propagates the ± 0.06 model uncertainty on $f_{\rm prim}$); equivalently, it predicts $\Omega_m = 0.329$ versus 0.3153 ± 0.0073. We commit to the DOF-counting value $f_{\rm prim} = 5.664$ as the single ISST output and report the residual as an honest limitation of the sharp-transition approximation in the DOF sum (real freezeouts are smooth crossovers; some species are partially decoupled at each stage), *not* as a fit to the observation. The alternative practice of quoting $f_{\rm prim} = 5.40$ would absorb the residual into $f_{\rm prim}$ and produce a misleading 0.1% "match"—we explicitly do not do this.

A smooth-crossover evaluation of the DOF sum (F139) using realistic transition widths (QCD: $\Delta T \sim 20$ MeV from lattice thermodynamics; e^+e^-: $\Delta T \sim 0.3$ MeV) returns $f_{\rm prim} = 5.78$, marginally *larger* than the sharp-transition value, not smaller. The smooth integral $\int (1/g_*)\,|dg_*/dT|\,dT$ is path-independent and gives $\ln(g_{*,\rm init}/g_{*,\rm final}) = 3.31$ regardless of crossover width; a per-transition evaluation that respects the discrete freezeout structure returns 5.78. The 1.6–1.8σ tension is therefore a genuine feature of the DOF-counting prediction, not an artefact of the sharp-transition approximation. An operator-level derivation from the full $(1+f)\mathcal{L}_m$ thermal history, which would track the continuous entropy injection without discretising into stages, remains [OPEN] (Sec. X). Status: [DEMONSTRATED] (SM-motivated ansatz with no fitted parameters and stated 1.6–1.8σ tension; smooth-crossover refinement does not close the gap, F139; the formal chain from the action's KL functional to the discrete sum is structurally open, F145).

From KL functional to DOF sum: structural diagnosis. The cosmological-floor $f_{\rm prim}$ is intended as the value of f (eq. 4) at horizon scale on the post-recombination baryon distribution. The discrete formula (6) is *motivated* by the Aczél–Shore–Johnson uniqueness of $D_{\rm KL}$ applied to the Standard-Model thermal history: each freezeout that lowers $g_{*,s}$ injects an irreversible information increment $\sim \Delta g_*/g_{*,\rm after}$ into the photon-baryon plasma, which the ratchet postulate (Sec. II F) then preserves into the baryon distribution.

The chain from this motivation to the discrete formula does not close as a derivation. F145 tested four candidate paths from the action's KL functional to the discrete sum (Table I) and ruled them all out: each gives the wrong magnitude, the wrong scope, or the wrong information measure to recover (6) from the action with the galactic-calibrated $\alpha = 0.869$ (F119).

The smooth path-integral form

$$f^{\rm smooth}_{\rm prim} \;=\; \alpha_{\rm cosmo} \int_{T_{\rm EW}}^{T_{\rm rec}} \frac{1}{g_{*,s}} \left| \frac{dg_{*,s}}{dT} \right| dT \;=\; \alpha_{\rm cosmo} \ln\!\left(\frac{g_{*,s}(T_{\rm EW})}{g_{*,s}(T_{\rm rec})} \right) \;=\; 3.31\,\alpha_{\rm cosmo} \tag{8}$$

TABLE I. Four candidate paths from the action's KL functional $f = \alpha \int F \ln(F/F_{\rm flat})\, d\Pi$ to the discrete DOF sum $f_{\rm prim} = 5.664$, all ruled out (F145).

Path	Result	Verdict
$D_{\rm KL}$(Maxw. @$T_{\rm actual}$ vs. @$T_{\rm ref}$)	1.36	factor $\sim$4 short
$D_{\rm KL}$(Maxw. vs. uniform-on-$\lvert p\rvert < m_p c$)	$\sim$40–60	factor $\sim$8 too large; g
$\sum_i D^{(i)}_{\rm KL}$ (per-step transient, 4 freezeouts)	0.30	factor $\sim$19 short; SJ a
$\sum_i D^{(i)}_q$ Rényi at $q = 2$ (4 freezeouts)	5.82	accidental $\sim$3% matcl

between $g_{*,s}(T_{\rm EW}) = 106.75$ and $g_{*,s}(T_{\rm rec}) = 3.91$ gives $\ln(106.75/3.91) = 3.31$ path-independently (F139). Matching this to $f_{\rm prim} = 5.66$ requires $\alpha_{\rm cosmo} \approx 1.71$ — distinct from the galactic calibration $\alpha = 0.869$ from F119 used in the local-bounded reading (Sec. III). Whether the two α values share a common origin in a single bounds-dependent functional is a propagated open question (F145).

Staging dependence. The discrete sum (6) is a right-endpoint Riemann sum on a coarse partition. With N-step uniform-log sub-partitions of $[g_{*,\rm final},\, g_{*,\rm init}]$ the right-endpoint sum decreases monotonically: 26.3 (N=1), 5.14 (N=4, uniform-log), 3.31 ($N \to \infty$). *Refining the partition does not converge to* 5.664 *from above; it diverges away from* 5.664 *toward the integral limit* 3.31. The paper's non-uniform 4-event partition (the SM mass thresholds) gives 5.66 because the Standard Model has four freezeouts at those particular $g_{*,s}$ values; the integer 4 is a contingent feature of the SM mass spectrum, not a derived structural number. A beyond-Standard-Model extension that adds or removes mass thresholds would shift the prediction.

Structural obstruction. The action's f in eq. 4 is a *single-epoch* $D_{\rm KL}$ integral on a single distribution at a single time. The discrete sum (6) is a *path-additive* event-counting quantity that depends on the partition into freezeout events. These are different mathematical objects. Shore–Johnson uniqueness (additivity over independent components, not over time-separated events) gives the single-epoch form; it does not produce path additivity across cosmic history. *The discrete formula* (6) *is therefore not a Riemann-sum approximation of* (8), nor of any single-epoch $D_{\rm KL}$. Closing the chain requires either a path-integral extension of the action (which would re-check A05, the ratchet postulate F46, $(1+f)\mathcal{L}_m$, and the Shore–Johnson uniqueness derivation) or a demonstration that the discrete sum emerges from a different limit of $(1+f)\mathcal{L}_m$ than the single-epoch $D_{\rm KL}$. We commit to the discrete formula (6) as the working ansatz; its theoretical justification from the action remains [OPEN] as the operator-level derivation flagged in Sec. X (F145).

Uncertainty on $f_{\rm prim}$. The discrete-vs-smooth-crossover spread is $f^{\rm sharp}_{\rm prim} = 5.664$ vs. $f^{\rm smooth}_{\rm prim} = 5.78$, an $\approx 2\%$ method uncertainty. Lattice-QCD uncertainty on $g_{*,s}(T_{\rm QCD})$ in the crossover region [98] contributes a further $\sim$ 1–2%. Combining these in quadrature gives

$f_{\rm prim} = 5.66 \pm 0.06$ at minimum (method spread) and $f_{\rm prim} = 5.66 \pm 0.12$ if lattice systematics are propagated. The tension with the Planck value $\Omega_m/\Omega_b - 1 = 5.40 \pm 0.15$ is 1.6–1.7σ when method+lattice uncertainty is propagated (equivalent to 1.8σ if only the Planck error is used, ignoring model uncertainty). We continue to quote the more conservative 1.6–1.7σ figure as the honest tension level.

Neutrino sector contribution. Neutrino decoupling at $T \sim 1$ MeV does not by itself change $g_{*,s}$ in the integrand of (8): the decoupled neutrinos retain $T_\nu = T_\gamma$ until the next event. The neutrino sector enters $f_{\rm prim}$ through the subsequent e^+e^- annihilation at $T \sim 0.3$ MeV, at which the photon-coupled effective DOF drops sharply while the decoupled neutrinos are not reheated — yielding the famous $T_\gamma/T_\nu = (11/4)^{1/3}$ ratio. The e^+e^- entropy injection reaches the baryon distribution through the photon-baryon tight coupling that persists until recombination ($z \sim 1090$); the ratchet postulate (below) then ensures the information increase is permanently recorded in the baryon distribution and is not undone by subsequent dynamics. The DOF sum (6) therefore tracks the cumulative photon-coupled-plasma DOF history, not a direct neutrino-baryon interaction.

F. The ratchet postulate

Several results in this paper rely on the assumption that the information functional f is irreversibly accumulated through cosmic history. We state this explicitly:

Postulate (Ratchet). *Along any matter worldline, the information functional f is monotonically non-decreasing: $df/d\tau \geq 0$.*

Physical basis. The $D_{\rm KL}$ operator measures departure from maximum entropy (the flat reference $F_{\rm flat}$). Gravitational structure formation increases this departure ($df/d\tau > 0$ during collapse and virialisation), while local thermalisation in the matter sector reduces it locally but increases total entropy — the second law applied to the structured component of the matter distribution. The ratchet is the explicit statement that information *increments* produced by Standard-Model freezeouts (Sec. II E) and by structure formation are not erased by subsequent equilibration: once a baryon distribution carries f, that contribution persists.

Where the ratchet is invoked in this paper. (i) The DOF sum (6) is path-additive across SM freezeouts only because each increment is preserved through the next stage. (ii) The "lighthouse" Bullet/A520 mechanism (Sec. V C) distinguishes shock-thermalised gas (which loses its working memory f_s but retains $f_{\rm prim}$) from intact stars (which retain both): the ratchet allows local thermalisation to wipe the structure-formation contribution to f_s without unwinding $f_{\rm prim}$. (iii) The $\Delta^f_{\mu\nu} = 0$ identity used at perfect-fluid scope (Sec. II) is the ratchet's metric-variation consequence: under the ratchet, the metric-variation derivative of the Maxwell–Jüttner functional vanishes identically, leaving only the dimensional bound (3) from the $d\Pi$-measure.

The ratchet is a postulate, not a theorem of F01: it is consistent with the action but is not derivable from it without further specification of the matter sector dynamics under coarse-graining. A full operator-level derivation that closes the chain action → ratchet → $f_{\rm prim}$ remains open (Sec. X).

G. The galactic-vs-cosmological coupling: a factor-2 scale dependence

The KL coupling α in (4) is fitted at galactic scale to $\alpha_{\rm gal} = 0.869$ (the calibration procedure is described in Sec. IV B: minimise the median rotation-curve RMS over the 14-galaxy SPARC subsample). Recovering the cosmological floor $f_{\rm prim} = 5.664$ from the smooth path-integral form (8), however, requires $\alpha_{\rm cosmo} \approx 1.71$ — a factor-~ 2 difference between scales.

This running has not been derived from the action. It may reflect genuine scale-dependent physics: the $D_{\rm KL}$ operator probes different aspects of the phase-space distribution at galactic versus cosmological scales (the local-bounded reading at galactic scale uses $F_{\rm flat}$ uniform on $v < v_{\rm esc}$; the cosmological reading would use a horizon-bounded reference that has not been formally constructed). Alternatively, the factor-~ 2 may indicate that the discrete sum and the continuous integral measure different quantities — the discrete sum being a path-additive event count over freezeout stages, the continuous integral being a single-epoch $D_{\rm KL}$, with a scale-dependent normalisation between the two (see "Structural obstruction" above and Sec. II E).

Until the operator-level derivation of $f_{\rm prim}$ from $(1+f)\mathcal{L}_m$ on the SM thermal history is closed — including a unified treatment of α at galactic and cosmological scales — this factor-~ 2 scale dependence is an *acknowledged open tension in the theory.* It does not invalidate the galactic calibration (which is internally self-consistent for rotation-curve phenomenology) or the cosmological prediction (which is internally self-consistent for the SM thermal-history input), but it does mean that α is not, at present, a single parameter universally calibrated across all scales. Status: [OPEN].

H. Degree-of-freedom count

ADM 3+1 decomposition of (2) gives two propagating tensor degrees of freedom and no scalar mode (App. I; framework adm_decomposition [ALLOWED]). The scalar Ψ is a constraint field with first-class $(\pi_\Psi, R = 0)$ pair generating the gauge transformation $\delta\Psi = \varepsilon$ on the constraint surface; the theory sits in the surviving Horndeski corner that GW170817 left intact. Status: [PROVEN] internally; external verification against the mimetic-DOF literature ([99–102]) flagged in Sec. X.

I. Environment-dependent f and the perturbation closure

Headline. *The env-dep-f closure derived below (F138) is a few-percent multiplicative correction to the gravitational source, not a free-growing collisionless channel.* It does not structurally rescue the factor-~ 30 third-peak suppression of strict uniform-f that F134 documents (Sec. VI G); a first-pass linear-scaling estimate leaves a residual gap of ~ 30 relative to the Planck-observed H_3/H_1 in either of the two readings of $v_{\rm esc}(\delta)$ considered. The CAMB Path A treatment of $(1+f_{\rm prim})\Omega_b$ as collisionless CDM at the perturbation level is therefore an *empirical approximation* rather than a derived consequence of F01: F138 establishes that the env-dep-f closure does not derive the proxy, only that it bounds the ISST-internal correction at the $\pm 16\%$ level. Two resolution paths remain: a $V(\Psi)$ extension that decouples Ψ-mediated structure from photon dynamics (action-level extension beyond F01), or the honest-flag position adopted in Sec. VI G. The remainder of this subsection derives the closure form, expands it to linear order, gives the two readings of $v_{\rm esc}(\delta)$, and quotes the $\beta(z)$ values from F138.

The closure form. The information functional from A05, under the F119 local-bounded reading (with $F_{\rm flat}$ uniform on velocities $v < v_{\rm esc}(x)$ locally accessible from position x), evaluates for non-relativistic Maxwellian matter to

$$D_{\rm KL}/n \;=\; 3\ln(v_{\rm esc}/\sigma) - 2.825 + v_{\rm bulk}^2/(2\sigma^2), \tag{9}$$

with $\sigma^2 = T/m_p$ and the constant $-2.825 = \ln(4\pi/3) - (3/2)\ln(2\pi e)$ absorbing volume normalisations. At background ($v_{\rm bulk} = 0$), $\bar f = \alpha\cdot[3\ln(\bar v_{\rm esc}/\bar\sigma) - 2.825]$, calibrated to $f_{\rm prim} = 5.664$ by A04.

Linear perturbation expansion. To first order in $\delta_b \equiv \delta\rho_b/\bar\rho_b$, dropping $O(\delta^2)$ and noting that the linear-in-$v_{\rm bulk}$ contribution vanishes at background:

$$\delta f(x,t) \;=\; \beta(z)\,\delta_b(x,t) \;+\; O(\delta^2), \tag{10}$$

with the coefficient $\beta(z)$ determined by the linear response of $\ln v_{\rm esc}$ and $\ln\sigma$ to δ_b:

$$\beta(z) \;=\; \alpha\cdot[3\,\delta\ln v_{\rm esc} - 3\,\delta\ln\sigma]/\delta_b. \tag{11}$$

Two readings of $v_{\rm esc}(\delta)$. The local-bounded $v_{\rm esc}$ is set by the local gravitational potential, $v_{\rm esc}^2 = -2\Phi$. Its perturbative response depends on what scale sets $\bar v_{\rm esc}$:

- *Reading B (local-collapse).* For a perturbation at scale $1/k$, $\bar v_{\rm esc}^2 \propto G\bar\rho_b(1+\bar f)/k^2$; the self-consistent solve gives $\delta\ln v_{\rm esc} = (1/2)[1 + \beta/(1+\bar f)]\delta_b$. This is the convention under which the F119 galactic calibration $\alpha = 0.869$ was performed.

- *Reading A (horizon-scale, relativistic).* The maximal binding scale is the cosmic horizon, $\bar v_{\rm esc} \approx c$. For sub-horizon CMB modes ($k \gg H/c$), $\delta\ln v_{\rm esc}$ is suppressed by $(H/k)^2$ and is essentially zero.

Which reading applies at cosmological linear-perturbation scales is itself an open question; the F119 derivation directly calibrates only the galactic regime.

The σ response. For pre-decoupling photon-coupled baryons, $T_b = T_\gamma \propto \rho_\gamma^{1/4}$ adiabatically, $\delta\ln\sigma = (1/6)\delta_b$. For post-decoupling free baryons with adiabatic index $\gamma_b = 5/3$, $T_b \propto \rho_b^{2/3}$, $\delta\ln\sigma = (1/3)\delta_b$.

Numerical $\beta(z)$ at $\alpha = 0.869$, $\bar f = 5.664$ (F138 `F138_class_estimate.py`):

	$\beta_{\rm pre-dec}$	$\beta_{\rm post-dec}$
Reading B (local-collapse):	$+1.08$	$+0.54$
Reading A (horizon-scale):	-0.43	-0.87

(12)

The modified gravitational source. The Poisson source acquires

$$\nabla^2\delta\Phi \;=\; 4\pi G\,\bar\rho_b\,[(1+\bar f) + \beta(z)]\,\delta_b, \tag{13}$$

giving an enhancement of $+8\%$ to $+16\%$ over CAMB Path A's $(1+f_{\rm prim}) = 6.664$ baseline under Reading B, and a suppression of -6% to -13% under Reading A.

Why the closure does not rescue the third peak. δf is a linear functional of δ_b (and of $\delta\sigma$, itself a function of δ_b through the local thermodynamic state), so it inherits the photon-coupled oscillations of δ_b pre-decoupling exactly: there is no free-growing collisionless channel that decouples from photon-baryon dynamics during radiation era. The factor-33 third-peak suppression of strict uniform-f (F134, modified CLASS) is therefore not rescued within F01 by this closure alone. Status: [PROVEN] for the closure form (10)–(13) and the $\beta(z)$ values (12); [DEMONSTRATED] for the leading-order suppression of $\Delta^f_{\mu\nu}$ pending explicit evaluation of $\bar\Lambda$ (F138 §8); [OPEN] for a derivation of the CAMB Path A proxy from F01 alone, which would close the F134 falsification within the committed action.

III. SOLAR SYSTEM AND LOCAL TESTS

A. Post-Newtonian parameters

In the Solar System, matter is concentrated, static, and well described by the Branch-B reduction $\Psi = \Psi_0$, $G = 1/\Psi_0$. By the branch theorem, the field equation is pointwise GR. We therefore have

$$\gamma_{\rm PPN} = 1, \qquad \beta_{\rm PPN} = 1, \tag{14}$$

exactly, by two cross-checking routes that share the Branch-B selection (App. H) but verify the PPN coefficients independently: (i) directly from the Branch-B field equation $\Psi_0 G_{\mu\nu} = 8\pi(1+f)T_{\mu\nu}$ giving exact GR with $G_{\rm eff} = (1+f)/\Psi_0$, hence $\gamma = \beta = 1$; and (ii) from the Palatini-class PPN analysis (framework palatini_ppn [ALLOWED]). In the Palatini reading ([6, 7]), the ΨR

sector with no kinetic term gives $\gamma = \beta = 1$ identically because the metric-sector PPN parameters depend only on the matter coupling: the field equation in Palatini form is $G_{\mu\nu}[\hat{g}] = 8\pi T_{\mu\nu}/\Psi$ in the auxiliary metric $\hat{g}$, and the conformal factor relating $\hat{g}$ to g is determined algebraically by the trace, leaving the standard PPN expansion of GR with a coupling rescaling. The Branch-B route delivers the same answer by the different mechanism $\nabla\Psi = 0$ inside the source, which reduces (2) to GR. *Both routes presume Branch B inside the source, which presumes the static-fluid theorem; they are not independent of that selection, only of each other in the PPN evaluation.* ISST's structural non-propagation of Ψ distinguishes it from Damour–Esposito-Farese tensor-multi-scalar PPN, which requires a propagating scalar [8]. Cassini's $|\gamma - 1| < 2.3 \times 10^{-5}$ [78] is satisfied at leading order. Status: [PROVEN]. Sub-leading corrections of order $f_s^{\odot}$ (the local stellar information content) are unmeasured; this is identified as an [OPEN] refinement (Sec. X).

Extension to rotating compact sources. The Branch-B selection extends to stationary axisymmetric configurations. At first order in the rotation rate Ω (slow-rotation Hartle–Thorne expansion [97]), the (rr), $(\theta\theta)$, (tt), $(\phi\phi)$ projections of the F01 field equation are even in Ω and unchanged from the static case at $O(\Omega^0)$; the F08 algebra forces $\nabla\Psi = 0$ and $\Psi = \Psi_0$ throughout the fluid. The new $O(\Omega)$ projection is the $(t\phi)$ frame-dragging equation, $\Psi_0 G_{t\phi} = 8\pi(1+f_\star)T_{t\phi}$, which determines the standard Hartle–Thorne function $\omega(r,\theta)$ with $G_{\rm eff} = (1+f_\star)/\Psi_0$ and does not constrain Ψ. This covers all astrophysical neutron stars: even the fastest known pulsar PSR J1748−2446ad at 716 Hz has $\Omega/\Omega_K \approx 0.43$, with $O(\Omega^2)$ corrections bounded at $\sim 18\%$. For arbitrary rotation rate (including Kepler-frequency configurations) and for differentially rotating equilibrium, the branch-selection rule (Sec. II D) selects Branch B by stationary-equilibrium ($df/d\tau = 0$) of the matter in its rest frame, locally pointwise for differential rotation. Vacuum Kerr black holes satisfy $R_{\mu\nu} = 0 \Rightarrow R = 0$ and admit either branch trivially in the metric equation; the global $\Psi = \Psi_0$ value is set by junction continuity to the surrounding matter in Branch B. Branch B therefore holds for all astrophysically relevant compact-object configurations including rotating neutron stars, white dwarfs, and stellar-mass and supermassive black holes; the GR-recovery and PPN $\gamma = \beta = 1$ results extend without modification.

B. Fifth force and equivalence principle

There is no *propagating-scalar* fifth force in ISST. The scalar Ψ does not propagate (no kinetic term in (1); F128 ADM count gives zero scalar degrees of freedom). The Derivation Passport explicitly denies all standard screening mechanisms (chameleon, symmetron, Vainshtein, Yukawa) as inapplicable to F01: their structural predicate is a propagating scalar, which ISST does not have. The universal $(1+f)\mathcal{L}_m$ coupling implies the weak equivalence principle at the action level. The $(1+f)$ factor appearing in the modified Poisson equation (16) is a matter-side rescaling of the gravitational charge, not a fifth-force mediator: it has no propagator, no finite range, and does not introduce a new force-carrying particle. ISST therefore occupies neither the Yukawa-type fifth-force category nor the Damour–Esposito-Farese tensor-multi-scalar category; it is in the ΨR Palatini-class corner with non-propagating Ψ. Status: [PROVEN].

C. Lunar Laser Ranging $\dot{G}/G$

The wall expansion in the Wiltshire two-domain background drives a slow evolution of the dressed gravitational coupling. The calculation: in the Solar-System neighbourhood, the static-fluid theorem (App. H) selects Branch B, so $\Psi(x,t) = \Psi_0$ is constant throughout the local bound region to the precision of the Branch-B reduction; any time-dependence in $G_{\rm eff} = (1+f)/\Psi_0$ comes from boundary-condition drag of the cosmological wall background on the local Branch-B value. With the wall background's $\dot{\Psi}/(H\Psi)|_0 = (\sqrt{21} - 3)/2$ attractor (Sec. IV B, Eq. 25) and a geometric suppression factor $\varepsilon \sim 10^{-7}$ from the ratio of the Solar-System scale to the wall scale (the local boundary value $\Psi_{\rm wall}(t)$ leaks into the Solar System on the time-scale set by $L_{\rm wall}/c$, with the suppression controlled by the ratio of the bound-system size to the wall scale under the static-source Poisson kernel), the rate is

$$\left|\dot{G}/G\right|^{\rm ISST}_{\rm wall} \sim \frac{p}{n} H_{\rm wall}\,\varepsilon = 5.63 \times 10^{-18}\,{\rm yr}^{-1}, \quad (15)$$

five orders of magnitude below the Lunar Laser Ranging bound $7 \times 10^{-13}\,{\rm yr}^{-1}$ [79, 80]. An FRW evaluation, where Branch A propagates Ψ-evolution into the Solar-System region without the Branch-B switch, gives $\sim 5 \times 10^{-11}\,{\rm yr}^{-1}$, two orders *above* LLR; the difference is a direct measure of the Wiltshire background's necessity, not a free parameter, and a sharp discriminator for the static-fluid theorem's role in localising Ψ at compact-source scales. Status: [PROVEN].

IV. GALAXY-SCALE PHENOMENOLOGY

A. Rotation curves

In the Branch-A weak-field regime, the modified Poisson equation takes the form

$$\nabla^2\Phi = \frac{4}{3}\cdot 4\pi G_N\,(1+f)\,\rho = \frac{16\pi G_N}{3}\,(1+f)\,\rho \quad \text{(rotation-supported} \quad (16)$$

where the prefactor 4/3 multiplies the standard Newtonian $4\pi G_N$ coupling and is the unique value compatible with the Branch-A constraint $R = 0$ on the rotation-supported background. The isotropic-gauge 2/3 used in an earlier draft was incompatible with Branch A and has been corrected; the explicit weak-field derivation reproducing 4/3 is given in Appendix B 1 alongside the lensing 2/3 and scalar-transport 2/3 coefficients. On top of (16) we apply a Newton-limit regulator $\mathcal{S}(a_{\rm bar}) = 1/[1 + (a_{\rm bar}/a_{\rm crit})^2]$ that damps the ISST correction at high acceleration:

$$V^2(R) = V_{\rm bar}^2(R)\left[\,1 + (E(R) - 1)\,\mathcal{S}(a_{\rm bar})\,\right], \tag{17}$$

with $E = \frac{4}{3}\left[1 + f_{\rm prim}(a_0/a_{\rm bar})^q\right]$.

B. Acceleration scales: one derived, one empirical

The two acceleration scales in the regulator have asymmetric status.

$a_{\rm crit}$ is derived in two stages, both from the committed action. A full derivation is given in Appendix B; we summarise the load-bearing algebra here.

Stage 1 (linearised perturbations on Minkowski). The two-potential weak-field metric $ds^2 = -(1{+}2\Phi)\,dt^2 + (1{-}2\Phi_N)\,\delta_{ij}\,dx^i dx^j$ with $\Psi = \Psi_0 + \delta\Psi$ and $T = -\rho$ (dust), substituted into (2) and linearised, yields three independent Laplacian equations and the Branch-A constraint:

$$2\Psi_0\nabla^2\Phi_N - \nabla^2\delta\Psi = 8\pi(1+f)\rho, \tag{18}$$

$$2\Psi_0\nabla^2\Phi - 4\Psi_0\nabla^2\Phi_N + 3\nabla^2\delta\Psi = -8\pi(1+f)\rho, \tag{19}$$

$$\Psi_0(\nabla^2\Phi_N - \nabla^2\Phi) = \nabla^2\delta\Psi, \tag{20}$$

$$R = -2\nabla^2\Phi + 4\nabla^2\Phi_N = 0. \tag{21}$$

Solving (18)–(21) simultaneously gives

$$\nabla^2\Phi = \tfrac{4}{3}\,4\pi G_N(1{+}f)\rho, \quad \nabla^2\Phi_N = \tfrac{2}{3}\,4\pi G_N(1{+}f)\rho, \quad \nabla^2\left(\tfrac{\delta\Psi}{\Psi_0}\right) = -\tfrac{2}{3}\,4\pi G_N(1{+}f)\rho \tag{22}$$

For a point mass M the gradient $|\nabla(\delta\Psi/\Psi_0)| = (2/3)(1{+}f)\,a_{\rm bar}/c^2$, so the *stability-gradient coefficient* is uniquely

$$C_{\rm grad} = \tfrac{2}{3}, \tag{23}$$

gauge-invariant (Bardeen) under the choice of slicing. The other candidate values 4/3 (rotation curves), 2 (lensing sum), and 1 all fail the off-diagonal constraint (20).

Stage 2 (matter-era power-law on the wall background). On the wall Friedmann (34) $H^2 = (16\pi/9)(1{+}f_{\rm prim})\rho/\Psi$ coupled to $\Box\Psi = -(8\pi/3)(1{+}f_{\rm prim})\rho$ with continuity $\dot\rho + 3H\rho = 0$, the power-law ansatz $a \propto t^n$, $\Psi \propto t^p$, $\rho \propto t^{-3n}$ yields a *forced* algebraic system. Power matching gives $p = 2 - 3n$; substituting into the Ψ-transport amplitude balance $p(p - 1 + 3n)\Psi_0 = (8\pi/3)(1{+}f_{\rm prim})\rho_0$ and using the Friedmann amplitude $n^2 = (16\pi/9)(1{+}f_{\rm prim})\rho_0/\Psi_0$ reduces the system to the quadratic

$$3n^2 + 6n - 4 = 0, \tag{24}$$

whose physical root is $n = (\sqrt{21} - 3)/3 \approx 0.528$, giving

$$\left.\frac{\dot\Psi}{H\Psi}\right|_0 = \frac{p}{n} = \frac{\sqrt{21} - 3}{2} \approx 0.7913. \tag{25}$$

Note on units: (25) is the dimensionless matter-era attractor of $\dot\Psi/(H\Psi)$, *not* the critical acceleration. The full $a_{\rm crit}$ formula (26) below combines this p/n with $C_{\rm grad}^{-1} = 3/2$ from (23) and the $(1 + f_{\rm prim})^{-1} \approx 0.150$ matter-coupling divisor; the algebraic prefactor on cH_0 is therefore $(3/2) \times (\sqrt{21} - 3)/2/(1 + f_{\rm prim}) = 3(\sqrt{21} - 3)/[4(1{+}f_{\rm prim})] \approx 0.178$, evaluating to 1.07×10^{-10} m/s^2 at $H_0 = 61.79$ km/s/Mpc. This is the unique attractor of the coupled system; the alternative forms B (cross-term retained) and B′ (BD $\omega = 0$) yield $\sqrt{3}$ and 1 respectively, both ruled out empirically against the SPARC critical scale (Appendix B).

Combining stages. The stability-gradient crossover equates the local Branch-A galactic gradient $C_{\rm grad}(1{+}f_{\rm prim})\,a_{\rm bar}/c^2$ to the cosmological floor $H_0^2 L_{\rm eff}/c^2$ with $L_{\rm eff} \equiv (p/n)\,c/H_0$, giving

$$\boxed{a_{\rm crit}^{\rm ISST} = \frac{1}{C_{\rm grad}}\,\frac{p}{n}\,\frac{cH_0}{1 + f_{\rm prim}} = \frac{3(\sqrt{21} - 3)}{4}\,\frac{cH_0}{1 + f_{\rm prim}} \approx 1.069 \times 1} \tag{26}$$

at $H_0 = 61.79$ km/s/Mpc and $f_{\rm prim} = 5.66$, sitting 10.9% below the MOND value $a_0 = 1.20 \times 10^{-10}$ m/s^2. The result is parameter-free at fixed H_0 and $f_{\rm prim}$; every factor on the RHS of (26) is ISST-committed ($C_{\rm grad}$ from (23); p/n from (25); $f_{\rm prim}$ from Sec. II E; H_0 from Sec. VI C). *Sensitivity to H_0 choice:* at Planck $H_0 = 67.4$ km/s/Mpc the prediction is 1.166×10^{-10} m/s^2 (-2.8% from MOND); at SH0ES $H_0 = 73.0$, 1.263×10^{-10} m/s^2 ($+5.3\%$). The "10.9% below MOND" figure is therefore specific to the ISST-committed dressed value $H_0 = 61.79$; a falsification claim against an a_0 measurement requires committing to the same H_0. Status: [PROVEN] at the committed $(H_0, f_{\rm prim})$; [DEMONSTRATED] as a single-number prediction independent of the H_0 choice.

a_0 (regulator internal scale) is empirical. The scale $a_0 = 0.01 \times 10^{-10}$ m/s^2 in the regulator is fitted on a 14-galaxy SPARC subsample; the regulator exponent $q \approx 0.45$ is also empirical at present. Their derivation from second-order Branch-A perturbation theory is identified as [OPEN].

C. Full SPARC sample

Evaluated on *every galaxy in the SPARC database* (175 galaxies, no per-galaxy retuning, no quality cuts):

$$\langle {\rm RMS}\rangle_{\rm ISST}^{N=175} = 13.8\,{\rm km/s},$$

$$\langle {\rm RMS}\rangle_{\rm MOND}^{N=175} = 12.7\,{\rm km/s},$$

with ISST winning per-galaxy on 86/175 (49%). The MOND comparison uses the simple interpolating function $\mu(x) = x/(1{+}x)$ [56] with $a_0 = 1.2 \times 10^{-10}$ m/s^2; the standard interpolating function $\mu(x) = x/\sqrt{1+x^2}$ shifts the MOND median by $\lesssim 1$ km/s and does not affect the qualitative comparison. On the high-quality Q=1 subsample of 99 galaxies (Lelli et al.'s top flag [50]), ISST wins the median (12.1 vs. 13.3 km/s, 52/99 per-galaxy wins).

Held-out cross-validation (F147). *The 14-galaxy subsample on which* $(a_0^{\rm reg}, q)$ *were originally tuned is contained within the 175-galaxy comparison set;* the headline median is therefore partially in-sample. With the regulator frozen at its 14-galaxy tuning values, the held-out 161-galaxy subset gives ISST median RMS 13.86 km/s vs. MOND 13.00 km/s, ISST winning 76/161 (47.2%). On the held-out Q=1 subsample ($N = 87$) the result is ISST 12.12 vs. MOND 13.34 km/s, ISST winning 44/87 (50.6%) — essentially indistinguishable from the in-sample-included Q=1 numbers (12.1 vs. 13.3 on 52/99). The central comparison axis (the high-quality Q=1 subsample) is therefore robust to the in-sample contamination; the full-sample headline qualifies modestly (the in-sample-included gap of 1.10 km/s tightens to 0.86 km/s on held-out, with MOND retaining a small lead). The morphological-class medians reported below should be read as the in-sample-included full-sample numbers. The baryonic Tully–Fisher slope is 1.15, consistent with the literature value 1.0 ± 0.1 [51, 59]. Status: [DEMONSTRATED]. ISST does not *derive* MOND: the internal a_0 is 120× smaller than Milgrom's, and the formula does not assume the deep-MOND scaling. The match on the full sample is a competitive fit, not a derivation, and is identified as such.

The morphological breakdown is informative and points to the next step. ISST matches MOND on dwarfs and irregulars (median 10.1 vs. 9.6 km/s, N=81), wins on late-type spirals (Sc–Sd, 10.1 vs. 11.5 km/s, N=48), and loses on early-type spirals (Sa–Sbc, 24.9 vs. 21.4 km/s, N=43). The Sa–Sbc loss-channel is star-dominated, which is exactly the regime where the local-bounded $F_{\rm flat}$ reading predicts $f_\star \approx 0$ and the $(1+f)$ enhancement does *not* act on the dominant component. The early-type loss is the operator-level prediction's signature, not its refutation.

D. Radial Acceleration Relation

The RAR [52, 53] emerges from the same regulator structure. The cross-over acceleration is the derived $a_{\rm crit}$; the deep-MOND-like asymptote follows from the $(a_0/a_{\rm bar})^q$ branch with empirical a_0 and q. The MDAR shape and BTFR slope are reproduced; the 11% offset of the cross-over scale from MOND's a_0 is a sharp ISST prediction that tightens with future a$_0$ precision measurements. Status: [DEMONSTRATED] (derived $a_{\rm crit}$; empirical a_0, q).

E. Dispersion-supported gas-poor dwarfs and tidal dwarf galaxies

The local-bounded $F_{\rm flat}$ reading (Sec. II C) makes a sharp prediction for galactic systems where matter is virialised in its own self-potential without a deeper external halo: $f \to 0$ for the dominant component, and ISST recovers Newtonian dynamics through Branch B selection (F08).

NGC 1052-DF2 [90, 91]. With $M_\star \approx 2 \times 10^8 M_\odot$, $r_{\rm eff} \approx 2.2$ kpc, and $M_{\rm gas}/M_\star \ll 0.01$, the system has $v_{\rm esc} \approx 12.5$ km/s, $v_{\rm max}^{\rm EM} = 2.56\,\sigma_\star \approx 21.8$ km/s. The OP2 condition $v_{\rm max}^{\rm EM} \geq v_{\rm esc}$ is satisfied: stellar matter fills its accessible phase space, $f_\star \approx 0$. Static spherical equilibrium forces Branch B (F08), $G_{\rm eff} = (1+f_\star)/\Psi_0 = G_N$. ISST predicts $\sigma_{\rm DF2}^{\rm ISST} = \sqrt{GM_\star/(5\,r_{\rm eff})} \approx 8.9$ km/s, consistent with the observation 8.5 ± 2.3 km/s within 1σ. *The local-bounded $F_{\rm flat}$ reading was introduced into the paper foundations prior to a quantitative DF2 audit, but the reading itself is motivated by the A05 Aczél–Shore–Johnson axioms and the entropy-matching cutoff at $v_{\rm max}^{\rm EM} = 2.56\sigma_{\rm eff}$ identified independently by F119, not constructed post-hoc to handle DF2.* DF2 is therefore a successful application of an independently-motivated reading, sharing the same operator-level mechanism (zero f for matter that fills its accessible phase space) that explains the Sa–Sbc loss-channel in the SPARC sample (Sec. IV). The σ prediction has no quoted model uncertainty, so "consistent with 1σ" is the honest framing rather than "predicted to 0.2σ".

Tidal dwarf galaxies (Lelli et al. 2015 [92], NGC 5291 system). For TDG gas at $\sigma_{\rm gas} \approx 8$ km/s in $v_{\rm esc} \approx 50$ km/s self-potential, the local-bounded reading gives $f_{\rm gas}^{\rm TDG} = \alpha[3\ln(v_{\rm esc}/\sigma_{\rm gas}) - 2.825] \approx 2.3$ (with $\alpha = 0.869$ from F119), predicting a moderate $(1+f) \approx 3.3$ enhancement. Lelli et al.'s reported $M_{\rm dyn}/M_{\rm bar} \approx 1$ is in mild tension with this prediction. The tension is sensitive to the assumed gas σ (turbulent + thermal): at $\sigma_{\rm gas} \approx 15$ km/s the prediction recovers Newton ($f \to 0$ via OP2). Improved $\sigma_{\rm gas}$ measurements would discriminate.

Discriminator vs. MOND. MOND with the external field effect predicts Newtonian dynamics for dwarfs in a host's deep external field (DF2 in NGC 1052) but full MOND for isolated dwarfs (TDGs in low-density environments). ISST predicts Newton for DF2 (same answer as MOND-with-EFE) but moderate enhancement for TDGs in low-density environments (different answer). Isolated low-density dispersion-supported dwarfs are therefore the cleanest ISST-vs.-MOND-with-EFE discriminator, identified as a future-work observational target.

Kill condition. Three or more dynamically-confirmed dark-matter-free galaxies in static equilibrium, with σ measured precisely enough to set $\sigma_{\rm bar}/v_{\rm esc} > 0.4$ (satisfying the OP2 condition), showing $M_{\rm dyn}/M_{\rm bar} > 1.5$, would falsify the local-bounded reading at A05.OP2. DF2 alone is consistent with the prediction; future systems (e.g. DF4, AGC 114905 [93]) sample the same regime.

V. CLUSTER-SCALE PHENOMENOLOGY

A. The cluster mass identity (A04)

The ISST identity at the action level is

$$\Omega_m \;=\; (1 + f_{\rm prim})\,\Omega_b. \tag{27}$$

This is a structural prediction: the same $(1+f)$ factor that enhances galactic rotation accounts for the cluster mass discrepancy. With the SM-derived $f_{\rm prim} = 5.66 \pm 0.06$ (Sec. II E) and Planck $\Omega_b = 0.0493$, the prediction is $\Omega_m = 0.329$, against the observation $\Omega_m = 0.3153 \pm 0.0073$ [29]: a 4.2% overshoot, 1.6–1.8σ depending on whether the model uncertainty is propagated. The structural identity is [PROVEN]; the numerical realisation is [DEMONSTRATED] at 1.6–1.8σ tension, with the residual stated openly as a limit of the sharp-transition approximation. The conventional practice of inverting (27) to extract $f_{\rm prim} = 5.40$ would absorb the residual—we do not.

B. Modified lensing and the lighthouse mechanism

In ISST, gravitational lensing measures the Ψ field where the photons travel, not the local mass at the source. The convergence kernel under F72 is

$$\kappa_{\rm ISST} = \frac{1+f}{\Psi_0}\,\kappa_{\rm GR}, \tag{28}$$

and the photon-trajectory parameter is $\eta = \Phi/\Phi_N = 2$ exactly. The framework verdict is modified_lensing_scalar_tensor [ALLOWED], with the GR Poisson source explicitly [DENIED] and replaced by $(1+f)\rho$. Status: [PROVEN] for the kernel form; the [ALLOWED] verdict descends from the slip predicate stamped by F84.

The information content of matter splits into a primordial floor $f_{\rm prim}$ frozen at QCD-era temperatures ($\sim 10^{12}$ K) and a post-formation working memory f_s accumulated during structure formation. A merger shock at $\sim 10^8$ K thermalises f_s—the gas's working memory, accumulated by violent relaxation during structure formation [49]—but cannot touch the QCD-frozen $f_{\rm prim}$. The gas's effective gravitational coupling drops from $(1+f_{\rm prim}+f_s)\rho$ to $(1+f_{\rm prim})\rho$. Stars, which pass through the collision essentially without interaction, retain their full working memory. Lensing follows the stronger source.

C. Bullet Cluster

The status of the Bullet Cluster in ISST is **mechanism-demonstrated, local-columns-close-bulk-of-gap** (F131). It is not *solved* or *proven*. The lighthouse mechanism predicts a per-mass contrast $(1+f_s^\star)/(1+f_s^{\rm gas,shock}) \approx 4.55/1.21 = 3.76$ between the surviving star-dominated lensing and the shock-erased gas. *The values $f_s^\star \approx 3.55$ and $f_s^{\rm gas,shock} \approx 0.21$ used in this contrast are fitted to the local-column convergence ratio inferred from Paraficz* et al. *(2016)[63]; their derivation from first principles — specifically, the violent-relaxation working memory accumulated by stars during structure formation versus the residual primordial-only floor of shock-thermalised gas — is identified as* [OPEN]. Counted honestly, they are empirical parameters of the lighthouse application, not derived consequences of the action. The Paraficz local-column densities themselves are inferred under a mass model that includes a dark-matter component; treating them as the total baryon column requires re-interpreting their model decomposition, an additional assumption that the quantitative contrast is sensitive to. With these caveats: at local columns, the per-mass contrast required for $\kappa_{\rm gal}/\kappa_{\rm gas} \geq 2$ is 1.6; the operator delivers 3.76, closing with margin throughout the Paraficz 1σ envelope under the empirical f_s values.

The full per-pixel κ ratio at FWHM-matched smoothing requires the local $\Sigma_{\rm baryon}$ contrast at the two peaks. Three recent works push directly on this observable. Cha *et al.* (2025) [64] present the highest-resolution Bullet mass reconstruction to date from JWST, combining 146 strong-lensing constraints from 37 systems with 398 sources/arcmin^2 of weak lensing; the resulting κ map resolves *at least three subclumps aligned with the brightest cluster galaxies*, and—critically for the lighthouse mechanism—the intracluster-light distribution is found to track the JWST mass map to a modified Hausdorff distance of 19.80 ± 12.46 kpc. The ICL traces stars by construction, so this ~ 20 kpc scale agreement is direct evidence that the κ peak tracks stellar mass at the resolution of the JWST data, supporting the lighthouse mechanism's central prediction. Rihtaršič *et al.* (2026) [65] provide an updated GR lensing κ map of the Bullet using the latest galaxy-member catalogue (219 spectroscopic members with new JWST photometry). Hernandez (2026) [66] computes the QUMOND prediction for the same baryonic distribution—the 219 stellar galaxies modelled as PIEMD profiles plus a β-model X-ray gas—and compares pixel-by-pixel against the Rihtaršič GR κ map. The QUMOND map matches GR to a residual $|\Delta\kappa| \lesssim 0.15$ throughout (in the standard units of $1.83 \times 10^9\,M_\odot/{\rm kpc}^2$), comparable to or smaller than the residuals between independent GR lensing analyses. The galaxies, 7% of the total baryonic mass, contribute 48% of the QUMOND phantom mass, because point-like sources concentrate the QUMOND signal where extended gas does not.

ISST's lighthouse operator predicts the same morphology (κ peaks at star-dominated stellar mass, not at the gas centroid) through a distinct mechanism: per-pixel f_s-thermalisation contrast at the action level (intact stars retain $f_s^\star \approx 4.55 - 1$, shocked gas has $f_s^{\rm gas,shock} \approx 0.21$).

The qualitative agreement between Cha's $\sim$ 20 kpc ICL/κ alignment and the lighthouse prediction is direct ([DEMONSTRATED]); extracting $\kappa_{\rm gal}/\kappa_{\rm gas}$ at the two main peaks from the Rihtaršič κ map and comparing to the operator $(1+f_s^\star)/(1+f_s^{\rm gas,shock}) \approx 3.76$ is identified as the next quantitative step ([OPEN]). The standard ΛCDM reading of the same data invokes a collisionless dark-matter component with a self-interaction cross-section bound below $\sigma/m \lesssim 1\,{\rm cm}^2/{\rm g}$ [61, 62]; ISST does not introduce such a particle.

The Bullet does not discriminate ISST from MOND. Hernandez [66] demonstrates that QUMOND reproduces the Bullet κ map without dark matter through compactness (point-like galaxies → peaked phantom signal; diffuse gas → diffuse phantom signal). ISST reproduces it through processing history (f_s stripped from shocked gas; f_s retained in stars). Both predictions agree at the Bullet because the gas is shocked *and* diffuse, while stars are intact *and* compact—the two mechanisms are observationally degenerate at this system. The discriminator between ISST and MOND is therefore not the Bullet itself but systems where the two mechanisms predict differently: $a_{\rm crit}$ vs. a_0 (Sec. IV B, 11% offset predicted), native cosmology (Sec. VI, MOND has none), θ_* (Sec. VI G, −0.83% ISST, MOND silent), and *ram-pressure-stripped jellyfish galaxies* where gas and stars share the same Newtonian acceleration but have different processing histories: MOND predicts equal enhancement of both components in the low-acceleration regime; ISST predicts suppressed enhancement of the stripped gas tail ($f_s \to 0$ from ram-pressure shock heating), unsuppressed enhancement of the intact stellar disk. This system-class is identified in Sec. X as the live falsifier between the two theories.

Earlier ISST attempts that imported a halo ontology (F65–F71) violated F01's pointwise source structure and have been retracted (Sec. VIII D).

D. Abell 520 (pile-up geometry)

The lighthouse mechanism makes a second, independent prediction in multi-body pile-up mergers (F132). Dynamical friction [73, 74] concentrates massive bulges (high $f_s^\star$, surviving the merger crossing $\sim 0.5\,{\rm Gyr}$ [72]) at the merger centre, where shocked gas also accumulates centrally. Convergence tracks the surviving bulge population, which spatially overlaps the gas, with

(i) stellar-population prediction: passive, red, old populations dominating the merger core, with rapid quenching and no starburst;

(ii) mass-to-light prediction: $M/L_R \in [38, 80]$ in solar units at the core.

Prediction (i) is independently confirmed by Deshev *et al.* (2017) [71] on 400+ galaxies. Prediction (ii) is consistent with Clowe *et al.* (2012) [69]; the alternative Jee *et al.* (2014) reading of the same HST/ACS data [70] would put F01 in tension. The Clowe–Jee disagreement is unresolved in the literature; ISST predicts the Clowe reading. Status: [INDICATED] (consistent-or-indicated).

E. Differential void-lensing prediction

The same kernel makes a sharp differential prediction at the void scale: the wall-versus-void $(1+f)/\Psi_0$ ratio implies a +2.5% enhancement of the dressed coupling in voids relative to local laboratory measurements,

$$G_{\rm void}/G_{\rm local} = 1.025 \quad \text{(F72, +2.5\%)}. \tag{29}$$

This is a discriminator versus chameleon and metric $f(R)$ scenarios, both of which screen *toward* GR in low-density environments. Status: [INDICATED]. Euclid void-lensing tomography is the live test (Sec. VIII).

F. Missing mass: no particle

If (27) accounts for the full cluster mass discrepancy through baryons amplified by $(1+f_{\rm prim})$, no dark-matter particle is needed. ISST predicts that direct WIMP, axion, and sterile-neutrino detection at CDM density continue to return null. This is a hard-falsifier prediction (Sec. VIII), not a post-hoc consistency check.

VI. COSMOLOGY

A. Inhomogeneous background: Wiltshire as minimal realisation

Branch A combined with dust matter is observationally incompatible with a spatially flat homogeneous FRW background, and the minimum geometric structure that recovers the observed matter-era expansion is a domain split into Milne-like voids and walls with a spatial Ψ profile. The full algebra is in Appendix C; we summarise here.

Claim 1 (geometric). On a spatially flat homogeneous FRW metric $ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$, the Ricci scalar is $R = 6(\ddot a/a + H^2) = 6(\dot H + 2H^2)$. The pointwise Branch-A constraint $R = 0$ therefore reduces to

$$\dot H + 2H^2 = 0 \quad \Longrightarrow \quad H(t) = \frac{1}{2t}, \quad a(t) \propto t^{1/2}. \tag{30}$$

This is a purely *geometric* consequence of $R = 0$: no matter input was used. The expansion law is forced.

Claim 2 (observational). The (tt) component of (2) on flat FRW with $\Psi = \Psi(t)$ and perfect-fluid dust $T^t{}_t = -\rho$ reads

$$3\Psi H^2 + 3H\dot\Psi = 8\pi(1+f)\rho. \tag{31}$$

Substituting $H = 1/(2t)$ from Claim 1 and dust scaling $\rho = \rho_0(t_0/t)^{3/2}$ (from $\rho a^3 =$ const and $a \propto t^{1/2}$), this becomes the first-order linear ODE

$$\dot{\Psi}(t) + \frac{\Psi(t)}{2t} \;=\; \frac{16\pi}{3}(1{+}f)\rho_0\, t_0^{3/2}\, t^{-1/2}, \qquad (32)$$

whose general solution (with integrating factor $t^{1/2}$) is

$$\Psi(t) \;=\; A\, t^{1/2} + K\, t^{-1/2}, \qquad A = \frac{16\pi}{3}(1{+}f)\rho_0\, t_0^{3/2}, \qquad (33)$$

with K a free integration constant. The system is *algebraically consistent*: $\Psi(t_0) = At_0^{1/2} + Kt_0^{-1/2}$ is finite, $G_{\rm eff} = (1{+}f)/\Psi(t_0)$ is finite, and the trace transport $\Box\Psi = -(8\pi/3)(1{+}f)\rho$ is satisfied identically by both modes (the $Kt^{-1/2}$ mode is the homogeneous solution; the $At^{1/2}$ mode is the particular solution sourced by dust).

The incompatibility is observational, not algebraic. Claim 1 forces $a(t) \propto t^{1/2}$, which is the radiation-era expansion law. The matter-era expansion observed at intermediate redshift through SN-Ia luminosity distances, BAO scales, and CMB acoustic-peak positions follows $a(t) \propto t^{2/3}$ at the relevant epochs in any matter-dominated dust cosmology. Branch A on flat FRW therefore predicts the wrong expansion law during the matter era, regardless of the integration constant K. The mismatch is at the level of $\mathcal{O}(1)$ in the deceleration parameter ($q = 1$ for $a \propto t^{1/2}$ vs. $q = 1/2$ for $a \propto t^{2/3}$), not a subleading correction.

The same algebra shows traceless matter ($T = -\rho + 3P = 0$, i.e. $P = \rho/3$, radiation) is consistent with Branch-A on flat FRW because the transport equation $\Box\Psi = (8\pi/3)(1+f)T$ then admits $\Psi = \Psi_0 =$ const, and (31) reduces to a single algebraic identity — consistent with the radiation-era a $\propto t^{1/2}$ scaling. Branch-A flat FRW is the correct radiation-era cosmology; it is inconsistent with matter-era observations.

Consequence. Reconciling Branch A (geometric, $R = 0$ forces $a \propto t^{1/2}$) with matter-era observations ($a \propto t^{2/3}$ at intermediate z) requires a background that is not flat FRW. The minimum geometric structure satisfying the constraint is a *two-domain split*:

- *Voids*, with $\rho \to 0$ and $T \to 0$, admit Milne expansion ($a_V \propto t$, $H_V = 1/t$, $R = 0$ trivially in the traceless limit);
- *Walls*, with dust matter, admit $R = 0$ pointwise via a non-FRW metric: the planar ansatz $ds^2 = -dt^2 + a(t)^2(dx^2{+}dy^2){+}b(t,z)^2\, dz^2$ with $\Psi = \Psi_0(t){+}\psi(z)$ satisfies $R = 0$ pointwise iff $a(t) \propto t^{2/3}$ and $\partial_z^2\psi = -(8\pi/3)(1+f)\rho$, with the ($z$-derivative) terms in $b(t,z)$ cancelling identically between R_{tt} and R_{zz} at the bound-wall fixed point (Appendix C §C 3).

Spatial averaging the wall solution recovers (34) with the $16\pi/9$ coefficient verified independently (see Sec. VI B).

Wiltshire selection. The two-domain split is the *minimal* geometric realisation that reconciles Claim 1 (geometric: $R = 0 \Rightarrow a \propto t^{1/2}$ on flat FRW) with the observed matter-era expansion (Claim 2: $a \propto t^{2/3}$ at intermediate z). It is not the unique such realisation; strict exclusion of LTB/Szekeres alternatives is identified below. The selection of *Wiltshire's* specific two-domain timescape geometry over alternative inhomogeneous constructions (Lemaître–Tolman–Bondi, Szekeres) rests on three criteria, given in decreasing strength: (i) minimal domain count—two domains (voids + walls) is the smallest count for which the void/wall split satisfies $R = 0$ separately in each domain without continuous tuning of a bang-time function; (ii) explicit pointwise $R = 0$ realisation—the planar wall metric above is constructed; for LTB or Szekeres, $R = 0$ pointwise on dust requires solving a coupled system $E(r)$, $t_B(r)$ with no closed-form solution we have verified; (iii) empirical match to the apparent-acceleration phenomenology parameterised by $f_{v0} \approx 0.762$ from SN-Ia [23, 25]. Strict *exclusion* of LTB/Szekeres alternatives is identified as a residual classification problem ([OPEN]); the constructive realisation of *some* inhomogeneous background reconciling Branch A + dust + matter-era observations is [PROVEN] here. Frameworks buchert_averaging and wiltshire_two_domain are [ALLOWED] on the passport; LTB and Szekeres remain unstamped.

B. Wall Friedmann equation

The wall Friedmann equation derived from (2) is

$$H_w^2 \;=\; \frac{16\pi}{9}\,\frac{(1+f)\rho_b}{\Psi}, \qquad (34)$$

which under $(1+f)\rho_b \equiv \rho_m$ reproduces the Einstein–de Sitter tracker for matter-era walls up to an overall rescaling of cosmic time. The Milne void is automatically Branch-A-compatible. With spatial $f(x)$, f is replaced by its mass-weighted wall average; the uniform-f case is the zero-correlation special case. The $16\pi/9$ coefficient is *derived pointwise* from the planar wall ansatz via the Branch-A field equations (Appendix D, F141), with explicit verification that energy-momentum conservation holds at every step of the anisotropic-to-isotropic transition. The $2/3$ factor relative to standard Buchert dust is identified as the fraction of the gravitational source absorbed into the spatial Ψ profile by the trace transport (F01 transport equation): the spatial Poisson $\partial_z^2\psi = -(8\pi/3)(1{+}f)\rho$ absorbs $1/3$ of the source, leaving $(16\pi/3)(1{+}f)\rho$ on the (tt) RHS, divided by the 3Ψ prefactor of $G^t{}_t$ to give $16\pi/9$. Coordinate-independence is verified in synchronous, conformal, and harmonic gauges by general covariance of the field equation. Status: [PROVEN].

Note on coefficients across geometries. The wall Friedmann $16\pi/9$ is *not* obtained by scaling the standard $8\pi G/3$ by the rotation-curve enhancement $4/3$ (which would give $32\pi/9$). It is derived structurally from the (tt) projection of (2) on the wall slab, where the static spatial Ψ profile absorbs $(8\pi/3)(1+f)\rho$ via the Branch-A transport, so that the cosmological source reduces from $8\pi(1+f)\rho$ to $(16\pi/3)(1+f)\rho$ on the RHS, then divides by 3Ψ for H^2. The three structural coefficients in this paper—4/3 (rotation, $\nabla^2\Phi$, Eq. 16), 2/3 (lensing, $\nabla^2\Phi_N$, Appendix B 1), and $16\pi/9$ (cosmological wall, Eq. 34)—arise from three distinct field-equation projections in three distinct geometries (axisymmetric disk, weak-field point mass, planar wall slab) and are independently consistent with the same F01 EOM.

C. H_0 and the apparent acceleration

The two-domain background admits three distinct Hubble rates whose relations are explicit, not phenomenological. The full pipeline is in Appendix E; we summarise the algebra here.

The three frames. (i) The *bare* frame is the Buchert volume average, with scale factor $\bar{a}(t)$ and Hubble rate $\bar{H} = \dot{\bar{a}}/\bar{a}$ defined on the bare time coordinate t. (ii) The *wall* frame is the proper rest frame of a galaxy observer in a wall, with proper time τ related by $dt = \gamma_w(\tau)\, d\tau$. (iii) The *dressed* frame is what a wall observer infers when fitting their distance-redshift data with a homogeneous FLRW template; it is what Planck and SH0ES report.

Wiltshire dressing relations. On the matter-dominated two-domain tracker [22, 23]:

$$H_0^{\rm dressed} = \mathcal{R}(f_{v0})\, H_0^{\rm bare}, \qquad \mathcal{R}(f_v) = \frac{4f_v^2 + f_v + 4}{2(2+f_v)}, \tag{35}$$

which evaluates at $f_{v0} = 0.762$ to $\mathcal{R} = 1.2825$. The present-epoch lapse $\gamma_0 = 1.348$ controls density dressing $\Omega_m^{\rm dressed} = \gamma_0^3\, \Omega_m^{\rm bare}$, distinct from the Hubble dressing $\mathcal{R}$.

ISST-native bare H_0. A model-independent input from Planck (the comoving angular-diameter distance $D_A^{\rm com}(z_*) = 14130.8$ Mpc derived from r_s and θ_*, no ΛCDM assumption) feeds the ISST Branch-A two-domain tracker. With ISST's parameter commitments ($f_{\rm prim} = 5.664$, $f_{v0} = 0.762$, $\Omega_b h^2 = 0.02237$) integrated through the Buchert background (F26, F26a), the tracker outputs

$$H_0^{\rm ISST,bare} = 57.56\, {\rm km\, s^{-1}\, Mpc^{-1}}. \tag{36}$$

Applying (35) gives the corresponding dressed

$$H_0^{\rm dressed,uniform-f} = 1.2825\times 57.56 = 73.82\, {\rm km\, s^{-1}\, Mpc^{-1}}, \tag{37}$$

the uniform-f tracker prediction. The committed paper value

$$H_0^{\rm ISST,dressed} = 61.79\, {\rm km\, s^{-1}\, Mpc^{-1}} \tag{38}$$

arises from the $K_{\rm env}$-corrected closure (Sec. VI D) which incorporates an environmental rescaling that the uniform-f run lacks; the 73.82 uniform-f result is reported here transparently as the diagnostic input to that closure, not as the prediction.

The Planck and SH0ES recoveries. A wall observer fitting ISST's two-domain $d_L(z)$ with a ΛCDM template recovers different H_0 values depending on *which* template parameters are anchored; the F85 forward pipeline computes both:

$$H_0^{\rm Planck-template} = 62.8, \qquad H_0^{\rm SH0ES-ladder} = 70.6\, {\rm km\, s^{-1}\, Mpc^{-1}} \tag{39}$$

where the Planck-template recovery uses the CMB-anchored $(\Omega_m, \Omega_b h^2)$ and the SH0ES-ladder recovery uses ladder-anchored M_B on $z \in [0.023, 0.15]$. Observed values are 67.4 and 73.0 respectively. The ISST-internal spread $70.6 - 62.8 = 7.8$ km/s/Mpc reproduces *the structure* of the Hubble tension; the residual 2.5%–5% offset of each pipeline output from the corresponding observed value reflects the gap between the effective-FRW Wiltshire proxy and a full two-domain photon-path integration with $K_{\rm env}$ inheritance and is reported honestly as such (see Sec. VI D and Appendix E). Status: [DEMONSTRATED] (structure of the tension reproduced; literal recovery of 73.0 and 67.4 [OPEN]).

D. Coincidence and void fraction tracker

A three-parameter $\chi_w(z)$ sigmoid closes the CMB and SN anchors [25, 26]. The amplitude $A_{\rm thermo} = f_{\rm prim}/N_{\rm stages} = 1.416$ from the arithmetic mean of four SM freezeout-stage increments matches the phenomenological best-fit to 1.1%. The pivot $z_{\rm tr} = 1.555$ from the Sheth–van de Weygaert two-barrier (void-in-cloud) abundance [28] with $\sigma_{8,\rm eff} = 4.92$ matches to 3.7%. The environmental rescaling $K_{\rm env} = 1.484$ is recovered as the squared ratio of two independent Hubble outputs. The full closure gives $(A, z_{\rm tr}, n, H_0) = (-0.688, 1.555, 4.83, 61.79)$ against phenomenological $(-0.65, 1.50, 2.00, 61.80)$. The shape index n is a sharper-than-phenomenological prediction and is the one residual. Status: [DEMONSTRATED] (amplitude, pivot, H_0); [OPEN] (n).

E. Growth of perturbations: $f\sigma_8$

The wall-growth equation derived from (2) on the Wiltshire two-domain background, with $(1+f)$ source enhancement, Ψ friction, and AP retemplating, gives

$$f\sigma_8(z = 0.57)^{\rm ISST} = 0.430 \pm 0.028, \tag{40}$$

versus ΛCDM 0.472 (F89, framework wiltshire_two_domain [ALLOWED] and ISST-native wall growth). On 8 RSD surveys [40–45] the joint $\chi^2 = 8.6$–9.5 ($p \approx 0.30$–0.45). The shape distinguishability from ΛCDM is forecast at 13σ at $z = 1.1$ with full DESI DR2 [33, 34] + Euclid Y1 [38, 39] binning. DESI DR2 BAO+RSD results are now published [34]; a quantitative ISST-vs-ΛCDM comparison on the released DR2 $f\sigma_8(z)$ points (specifically the high-z bins at $z \gtrsim 0.7$

where the ISST shape diverges most from ΛCDM) is identified as the immediate next step, with the kill condition specified in Sec. VIII. Status: [PROVEN] on the eight current RSD surveys at the joint-χ^2 level; [OPEN] for the DR2-released $f\sigma_8(z)$ comparison. The earlier templating "crisis" on the Wiltshire background was an artefact resolved by the AP retemplating used here.

F. Integrated Sachs–Wolfe: structural null

The integrated Sachs–Wolfe contribution from individual line-of-sight voids on Branch A gives a per-photon RMS of order $\sqrt{N}\langle A\rangle$ with $N \approx 140$ voids, but the ensemble mean is identically zero (F106/F107/F108): the line integral of $\dot{\Psi}$ along a void is structurally cancelling under the F01 transport equation across the photon flight time $\tau_{\rm flight}/\tau_{\rm cycle} \leq 1.7\%$. ISST therefore does not predict a coherent ISW signal at the linear level beyond the LCDM baseline. Status: [PROVEN] (ISST-NATIVE; the latent v2 isst_void_isw pattern, identified for explicit framework registration).

G. The CMB acoustic angle: first-pass Boltzmann result

The theory's reach across scales narrows here. The preceding subsections established cosmological successes (wall-Friedmann derivation, H_0 frame artefact, $f\sigma_8$ agreement, structural ISW null) on the Wiltshire-effective background. The CMB perturbation level is where the single-action programme runs into its first hard limitation: the strict uniform-f source is falsified at the third peak by $> 10\sigma$, and the env-dep-f closure provides only percent-level corrections (Sec. III). The first-pass acoustic-angle result reported here uses an empirical proxy at the perturbation level; the proxy's justification, its systematic uncertainty, and the open question it leaves are stated explicitly. This is the most honestly open result in the paper.

A first-pass run of the CAMB Boltzmann hierarchy [27] on the Wiltshire-effective background [22, 23, 25] gives, at the ISST-committed parameters, a sub-percent residual on the CMB acoustic angle. The Wiltshire dressing of the late-time expansion is captured by an effective dark-energy parameterisation $(w_0, w_a) = (-0.674, -1.241)$ extracted from low-z supernova matching; the matter density is set by the A04 identity $(1 + f_{\rm prim})\Omega_b h^2 = \Omega_m h^2$ at the SM-prediction $f_{\rm prim} = 5.664$ (giving $\Omega_m h^2 = 0.149$, the same 4.2% overshoot vs. Planck reported in Sec. II E); the Hubble constant is the dressed value $H_0 = 61.79$ km/s/Mpc.

Boltzmann configuration (reproducible inputs). The CAMB run uses the Path A treatment: $(1 + f_{\rm prim})\Omega_b$ is modelled as the total matter component at the perturbation level, with the information-enhanced extra contribution carried by an effectively collisionless CDM component (the empirical proxy whose derivation from F01 alone is the open problem of Sec. III). Specifically: $\Omega_b h^2 = 0.02237$ (Planck); $\Omega_c h^2 = \Omega_m h^2 - \Omega_b h^2 = 0.149 - 0.02237 = 0.127$ enters as the CDM proxy carrying $f_{\rm prim}\Omega_b$; $H_0 = 61.79$ km/s/Mpc; $w_0 = -0.674$, $w_a = -1.241$; remaining cosmological parameters (n_s, A_s, τ) at Planck values. The modified CLASS source patch implementing the strict uniform-f test (below) and the exact CAMB `params.ini` configuration files will be archived as supplementary material on acceptance. Four configurations were run:

Config	$100\,\theta_*$	description
C0	1.04118	ΛCDM Planck baseline
C1	1.03107	drop-Λ, ISST H_0
C2	1.07125	Wilt DE, uniform-f $H_0 = 75.28$
C3	1.03258	ISST committed (Wilt DE + dressed H_0)

The headline result is

$$\theta_*^{\rm ISST}({\rm C3})/\theta_*^{\rm Planck} = 0.99174 \quad (-0.83\%), \tag{41}$$

a 14× reduction from the earlier −12% analytical estimate (superseded as a recipe artefact; see Sec. VIII D).

F95 retraction. The F95 mixed-$\alpha(z)$ recipe is withdrawn as the committed value. F95 Scenario B (wall $\alpha = 2/3$ pre-recombination, returning −0.77%), which F95 itself dismissed as "not derivationally justified," is independently confirmed by the Boltzmann hierarchy at −0.83%. Two sympy audits underpin the result. Step 5: the photon–baryon sound speed $c_s^2 = c^2/[3(1 + R)]$ with $R = 3\rho_b/(4\rho_\gamma)$ is unchanged by the $(1 + f)$ coupling, which enters only the Poisson source, not the fluid EOM, since $T^{\rm eff}_{\mu\nu} = (1 + f)T_{\mu\nu}$ under uniform f implies $\nabla_\mu T_{\mu\nu} = 0$ identically. Step 6: Ψ is frozen on perturbed radiation at all orders in linear perturbation theory, because $T = -\rho+3P$ vanishes exactly for $P = \rho/3$ at background, linear, and nonlinear order; the recombination redshift $z_* \approx 1090$ is therefore inherited from RECFAST unmodified.

Independent code cross-check. CLASS v3.x via classy [88], built locally with WinLibs MinGW gcc 15.2 (toolchain notes in F134), returns $100\theta_s = 1.03188$ on the same C3 configuration—agreeing with CAMB's 1.03258 to **0.07%**. Two independent Boltzmann codes confirm the result; the first-pass −0.83% to −0.92% residual is robust at the level of code precision *relative to the proxy configuration*, but not relative to the committed Wiltshire $H(z)$ (see Proxy systematic below).

Proxy systematic. The (w_0, w_a) parameterisation of the late-time expansion is calibrated against the committed Wiltshire two-domain $H(z)$ at low redshift (where SN-Ia residuals fix the dressing) but diverges from the Wiltshire expansion by +76% at $z = z_* \approx 1090$ (F140). A back-of-envelope propagation of this divergence through $\theta_* = r_s/D_A = \int_0^{z_*} c_s\,dz/H(z) \,/\, \int_0^{z_*} c\,dz/H(z)$ indicates that r_s is sensitive to H near z_* (because c_s peaks just before recombination), while D_A is dominated by late-time integration. The two integrals partially cancel under a common

H-rescaling but not exactly: a +76% enhancement of H near z_* shifts the ratio at the $\sim$ 5–10% level. The sub-percent headline −0.83% residual therefore sits inside the proxy systematic, not above it. *The committed ISST prediction for θ_* at the precision of the Planck measurement requires a Boltzmann code with the two-domain background substituted directly for $H(z)$;* this computation is identified as the next critical step (Sec. X). Until completed, θ_* should be reported as "consistent with Planck within $\sim$ 5–10% proxy systematic, sub-percent at the level of the proxy itself."

Strict uniform-f at the perturbation level: falsified. A surgical modification of CLASS `source/perturbations.c` implements the literal action-level uniform-f prediction: the $(1 + f_{\rm prim})$ extra-matter contribution to $\delta\rho$ tracks the *baryon* perturbation δ_b rather than evolving as a collisionless component. This is the direct consequence of $\delta((1+f)\rho_b) = (1+f)\delta\rho_b$ under uniform f. The result is dramatic and unfavourable:

$$H_3/H_1 = 0.013 \quad (\text{Planck}: 0.443), \quad \sigma_8 = 0.31 \quad (\text{Planck}: 0.811), \tag{42}$$

with $\ell_1 = 266$ (vs. Planck ~ 220). The third-peak suppression is a factor ~ 33, far outside any observational allowance. The mechanism is transparent: photon-coupled perturbations oscillate during radiation era rather than growing, so the gravitational wells the photon-baryon fluid oscillates in at recombination are too shallow to drive the third compression correctly. **Strict uniform-f ISST at the perturbation level is ruled out at $> 10\sigma$ by the Planck third peak.**

Environment-dependent f: perturbation closure. The A05 operator's linear response to density perturbations has been derived (Sec. III I). The perturbation $\delta f = \beta(z)\,\delta_b$ arises from the response of the local escape velocity to the gravitational potential. Two readings of the operator give $\beta = +0.54$ to $+1.08$ (Reading B, local-collapse F119 calibration) and $\beta = -0.43$ to -0.87 (Reading A, horizon-scale, relativistically correct). Neither generates a free-growing collisionless channel: the f-enhancement inherits photon-coupling from the baryon perturbation pre-decoupling. The strict uniform-f source is falsified at the third peak ($> 10\sigma$, Config 2 above); the linearised env-dep-f closure does *not* structurally rescue the peak-height ratios. A first-pass linear-scaling estimate shifts H_3/H_1 from F134's 0.013 (strict uniform-f) to ≈ 0.015 (Reading B) or ≈ 0.012 (Reading A); the factor-~ 30 gap to the Planck-observed ~ 0.44 persists.

The CAMB Path A treatment—modelling $(1+f_{\rm prim})\Omega_b$ as collisionless matter in the perturbation equations—reproduces sub-percent θ_* and ΛCDM-consistent peak ratios. This is an empirical proxy whose physical justification within F01 is not yet established. A candidate resolution path is a $V(\Psi)$ potential extension (F126, Path γ).

Caveat on $V(\Psi)$ as a structural change. Adding a potential $V(\Psi)$ to the action gives Ψ a non-trivial Euler–Lagrange equation, $R/16\pi + V'(\Psi)/16\pi = 0$ at fixed f, which in general *makes Ψ propagate*: ADM analysis would then count an additional scalar mode, breaking the F128 "zero scalar DOF" result that supports the no-fifth-force claim (Sec. III B). It would also alter the Branch theorem (the RHS of $R\nabla\Psi = 0$ acquires a V'-dependent term from the modified Bianchi projection), the $c_{\rm GW} = c$ argument (a propagating Ψ can mix with the gravitational sector at non-trivial V''), the PPN computation (the propagating mode generically contributes $\gamma \neq 1$ unless screened, and ISST denies all standard screening mechanisms in its Passport), and the LLR $\dot{G}/G$ bound (a V-driven slow evolution of Ψ contributes directly). *We therefore do not claim a CMB resolution via $V(\Psi)$ in this paper.* Path γ is identified as a separate-theory direction whose claim to "preserving the F01 results" has to be earned property-by-property; until that is done, ISST's commitment is to the F01 action without $V(\Psi)$, with the third-peak gap reported as a live falsifier (Sec. VIII). The honest position is that ISST either fits the CMB through a yet-undiscovered F01-internal mechanism (the δf closure does not provide it) or the third peak is a genuine falsification of strict-F01.

Stage G updates from F134. Five framework verdicts change. boltzmann_hierarchy_cmb (FRW preconditions) remains [DENIED] on F01. isst_boltzmann_two_domain_w0wa_proxy (NEW) is registered [ALLOWED] as the published-prediction framework. isst_boltzmann_full_uniform_f (NEW) is registered [DENIED], falsified by the third peak. isst_boltzmann_env_dep_f (NEW) is registered [CONDITIONAL] pending implementation. The remaining caveat on Path A is the high-z extrapolation: the effective (w_0, w_a) proxy, extracted from low-z Wiltshire matching, diverges from the committed Wiltshire dressed $H(z)$ by +76% at $z = 1090$ (F140). Moreover, the committed ISST matter density $\Omega_m = (1+f_{\rm prim})\Omega_b = 0.329$ exceeds the Wiltshire bare matter density $\Omega_{m,\rm bare} = 0.125$ by a factor 2.6; this mismatch causes the effective dark-energy density inferred from Friedmann inversion to become *negative* at $z > 2$, precluding any tabulated $w(z)$ input to standard Boltzmann codes at the committed parameters. A true Wiltshire+ISST Boltzmann computation requires modifying the background solver to accept the two-domain volume-averaged expansion directly, not through a Friedmann proxy. The −0.83% result is therefore a proxy-level estimate; the systematic uncertainty from the +76% $H(z)$ divergence at recombination is unquantified and could shift θ_* by several percent in either direction.

Status: [DEMONSTRATED] at proxy level for the acoustic angle (F134, CAMB+CLASS), with a +76% $H(z)$ proxy systematic at $z = 1090$ (F140) that is unquantified at the observable level. The third-peak height H_3/H_1 remains [OPEN] (factor-~ 30 residual gap to Planck; F138 confirmed the linear env-dep-f closure does not rescue it). Env-dep-f closure form [PROVEN] (F138, Sec. III I); env-dep-f *rescue* of the third-peak gap [OPEN] within F01; resolution escalates to a $V(\Psi)$ extension (F126 Path γ, under investigation) or to the honest-flag position the paper takes. A Boltzmann code with the true Wiltshire two-domain background is identified as the most important remaining computation (Sec. X).

VII. EARLY UNIVERSE AND GRAVITATIONAL WAVES

A. BBN

Big-Bang Nucleosynthesis is inherited intact. The Branch-A radiation closure (F95) requires $\Pi_{\rm rad} = -\rho_r/2$, a sympy-derived consistency condition; physical tight-coupling delivers $\leq 10^{-3}$, satisfying it within numerical noise. Crucially, Ψ is frozen on a radiation background ($T = 0$, traceless source) so G does not run during nucleosynthesis. Standard BBN abundances (η_B, $Y_{\rm He}$, lithium) are inherited. Status: [PROVEN]. The earlier F113 BBN catastrophe ($G_{\rm BBN}/G_{\rm today} \approx 3000$) was a wrong-background artefact: the Schutz/Hawking convention on a naively-FRW background gave the catastrophe; the F95 closure on the correct radiation-dominated Branch-A removes it.

B. Gravitational waves

Gravitational waves propagate at c. The action (1) is linear in Ψ in the gravitational sector, so tensor perturbations $h_{\mu\nu}$ decouple from Ψ fluctuations at linear order on any background:

$$\Box h_{\mu\nu} = -\frac{16\pi}{\Psi}\tau_{\mu\nu}, \quad c_{\rm GW} = c \text{ structurally.} \tag{43}$$

The constraint $|c_{\rm GW}/c - 1| < 10^{-15}$ from GW170817 [75] is satisfied trivially. F128 ADM analysis confirms two tensor degrees of freedom and no scalar mode: LIGO measures $+$ and $\times$ polarisations only; no breathing mode. Status: [PROVEN].

C. No scalar dipole radiation

Because Ψ does not propagate, binary inspirals do not radiate into a scalar channel. Hulse–Taylor quadrupole-only behaviour is inherited; the framework scalar_dipole_radiation_binary is [DENIED] as inapplicable to F01. Status: [PROVEN].

VIII. FALSIFICATION COMMITMENTS

This section is the most important section in the paper. ISST is science only to the extent that it commits to falsifiers in advance, and those commitments include the predictions ISST has *retracted*.

A. Hard falsifiers

Each of the following observations would, by itself, kill ISST as formulated here.

- **$c_{\rm GW} \neq c$.** ISST sits in the Horndeski corner that GW170817 left intact, structurally not by tuning. Any future $|c_{\rm GW}/c - 1| > 10^{-15}$ at high-confidence kills the theory.
- **Fifth-force detection.** F01 has no propagating scalar and no fifth force to screen. *Specifically:* a composition-dependent acceleration anomaly inconsistent with WEP at the level the universal $(1+f)\mathcal{L}_m$ coupling permits, or a Yukawa-type force on test particles with finite range distinguishable from $1/r^2$ gravity at any laboratory or solar-system precision, would falsify ISST. An observed spatial variation of $G_{\rm eff}$ correlating with local matter content would by contrast be *consistent with* the $(1+f)$ mechanism (cf. the big-G speculation in Sec. X) and would not constitute a fifth force in the ISST sense; it would be a detection of the structural matter coupling, not a refutation of its absence.
- **$\gamma_{\rm PPN} \neq 1$ at $\gtrsim 10^{-5}$.** Tightening the Cassini bound below $\sim 10^{-5}$ would force a path commitment to a propagating-scalar variant or falsify F01.
- **BBN abundances inconsistent with SBBN.** ISST inherits SBBN intact; any confirmed deviation at the level of an evolving G during nucleosynthesis falsifies the F95 closure.
- **Direct dark-matter particle detection.** ISST identifies $(1+f_{\rm prim})\Omega_b$ as the entirety of Ω_m, predicting persistent nulls in dark-matter direct-detection experiments at the CDM density across all currently searched parameter spaces. Specifically: WIMP detection at $\sigma_{\rm SI} \gtrsim 10^{-47}\,{\rm cm}^2$ over 1–1000 GeV (within current XENONnT/LZ reach to the neutrino floor); QCD-axion detection at $g_{a\gamma\gamma} \gtrsim 10^{-15}\,{\rm GeV}^{-1}$ over 1–100 μeV (ADMX/HAYSTAC reach); or keV sterile-neutrino detection inconsistent with all-baryon $(1 + f_{\rm prim})\Omega_b = \Omega_m$ at $> 3\sigma$ would each falsify (27). Current nulls [82–84] are consistent with the prediction; the kill condition is a $> 5\sigma$ confirmed detection at any of these specifications.

B. Cumulative tensions and live falsifiers

Each of the following is a multi-observation falsifier with an explicit kill condition:

- **Bullet/merging-cluster lensing under Euclid.** The lighthouse mechanism predicts κ tracks bulge-dominated stellar mass, not gas mass. If Euclid cluster tomography shows the κ peak tracking gas mass (not stars) at $> 3\sigma$ across multiple mergers, the mechanism is falsified.

- **$f\sigma_8$ shape under DESI DR2.** If DR2+Euclid Y1 find $f\sigma_8(z)$ within 1σ of ΛCDM and ISST misses by $> 2\sigma$ in > 3 bins, wall-dynamical $f\sigma_8$ is falsified.

- **Void-lensing G-ratio under Euclid.** ISST predicts $G_{\rm void}/G_{\rm local} = 1.025$ (a +2.5% enhancement). If Euclid/LSST measure $G_{\rm void}/G_{\rm local} = 1.000 \pm 0.005$ (GR-consistent), the differential prediction is falsified, forcing either a $V(\Psi)$ chameleon path or retirement.

- **MOND a_0 tightening.** ISST predicts an 11% offset of $a_{\rm crit}$ from MOND a_0. If a_0 is measured to better than 5% with a central value within 2% of 1.20×10^{-10}, the F80 prediction is falsified.

- **Jellyfish-galaxy lensing (ISST vs. MOND discriminator).** In ram-pressure-stripped jellyfish galaxies the stellar disk and the stripped gas tail experience the same Newtonian acceleration but have different processing histories: ISST predicts $f_s \to 0$ in the shocked gas tail (suppressed enhancement, $\kappa \to (1 + f_{\rm prim})\,\Sigma_{\rm gas}$ only) and f_s ∼few in the disk (full enhancement). MOND/QUMOND predicts equal phantom-density contribution from both components at the local acceleration, modulated only by compactness. Stacked weak-lensing of jellyfish samples in clusters with kinematic disk-versus-tail separation (Euclid + ground-based spectroscopy) discriminates: if the disk-to-tail κ ratio per unit baryonic surface density agrees with the QUMOND compactness-only prediction within 1σ, ISST's processing-history claim is falsified.

- **Peak-height ratios under env-dep-f full C_ℓ.** F134 surgically modified CLASS to test strict uniform-f at the perturbation level—the literal action prediction with $\delta((1 + f)\rho_b) = (1 + f_{\rm prim})\delta\rho_b$, photon-coupled. The result is incompatible with Planck: $H_3/H_1 = 0.013$ vs observed ~ 0.44 (factor 33), $\sigma_8 = 0.31$ vs 0.81. Strict uniform-f at the perturbation level is therefore **ruled out at $> 10\sigma$** and is retracted.

  F138 update. F138 derives the env-dep-f linear closure $\delta f = \beta(z)\delta_b$ explicitly from A05 (Sec. III). The closure is a *few-percent correction to the gravitational source*, not a free-growing collisionless channel: δf is a functional of δ_b through the local $D_{\rm KL}$ response and inherits photon-coupling predecoupling. A first-pass linear-scaling estimate shifts H_3/H_1 from 0.013 (strict uniform-f) to ≈ 0.015 (Reading B) or ≈ 0.012 (Reading A); the factor-∼30 residual gap to Planck persists. The previous draft's claim that the env-dep-f closure "restores collisionless-like growth" is downgraded.

  Live falsifier (revised). The third-peak factor-∼30 gap is [OPEN] within F01: the F138 closure correction is bounded at ±16% of the CAMB Path A baseline, so a full direct CLASS implementation of the $\beta(z)\delta_b$ source modification cannot close more than a percent or so of the gap. The kill condition shifts as follows. (a) If a direct CLASS implementation of F138's $\beta(z)\delta_b$ source confirms a peak shift in the F138-predicted ±16% window, ISST's commitment is that the residual factor-∼30 gap is genuine and the resolution escalates beyond F01: either $V(\Psi)$ extension (F126 Path γ) or the honest-flag position of Sec. VI G. (b) If the direct implementation produces a much larger correction ($\gtrsim$ factor of order 10), the F138 derivation is incomplete and the closure has more content than identified—this would itself be a substantial finding. (c) If the $V(\Psi)$ extension is pursued and fails to recover Planck-consistent peak heights at percent level, the extension is falsified and the honest-flag position becomes the only path. ETA on the F138 direct CLASS implementation: 1–2 weeks (toolchain in F134 reproducible); working on. The status tag for the third-peak gap is therefore [OPEN].

C. Falsifier table: timeline, precision, observatory

The falsifiers are summarised in Table II with the timescale for the relevant data release, the target precision required to trigger the falsification, and the observatory or analysis pipeline through which the test arrives. Entries are ordered by approximate time-to-decision under nominal mission schedules; "current" indicates that the test can in principle be run on already-released data.

D. Results ISST has dropped

ISST has dropped its own claims in three categories during the course of this work. We list them here for transparency, because a theory that drops its failures is more credible than one that hides them.

- **S_8 tension: [dropped].** F127 audited the ISST S_8 prediction and found the wrong sign: $G_{\rm wall} < G_{\rm void}$ at the relevant scales weakens late-time lensing, predicting a wider S_8 than observed. The ISST prediction $S_8 \approx 0.824$ is in tension with the KiDS-1000 [35] reading ~ 0.760 (with consistent DES Y3 [36] and HSC Y3 [37] preferences). We therefore drop S_8 as an ISST claim. (The S_8 tension itself is fading as KiDS Legacy revises downward, independently of ISST.)

- **Halo-ontology Bullet attempts: [dropped].** F65, F66, and F68 attempted to resolve the Bullet Cluster by assigning per-galaxy halo masses,

TABLE II. ISST falsifier inventory. "Hard" falsifiers are single-observation kill conditions; "Live" falsifiers are multi-observation kill conditions with explicit thresholds. ETA is the approximate window from this paper's writing within which the relevant data become available at the target precision.

Falsifier	Class	ETA	Target precision	Observatory / pipe
Bullet $\kappa_{\rm gal}/\kappa_{\rm gas}$ ratio	Live	current	per-pixel match to 3.76 at $> 3\sigma$	Cha 2025 / Rihtar
$f\sigma_8$ shape at $z \sim 1.1$	Live	1–2 yr	13σ ISST-vs-ΛCDM at DR2 high-z bins	DESI DR2 + Eucl
Void-lensing $G_{\rm void}/G_{\rm local}$	Live	2–3 yr	±0.5% at $r > 50$ Mpc voids	Euclid + LSST/Ru
a_0 central value vs. $a_{\rm crit}$	Live	3–5 yr	a_0 to ±5% with central value ±2%	High-precision RA
Third-peak ratios under env-dep-f	Live	1–2 wk (compute)	H_3/H_1 within F138 ±16% window or beyond	Modified CLASS d
θ_* on committed two-domain background	Live	2–4 mo (compute)	sub-percent agreement with Planck	Modified Boltzman
Jellyfish disk-vs-tail κ	Live	3–5 yr	disk-to-tail ratio per unit $\Sigma_{\rm bar}$ vs. QUMOND	Euclid + ground-b
$c_{\rm GW} \neq c$	Hard	current	$\lvert c_{\rm GW}/c - 1\rvert > 10^{-15}$ at high confidence	LIGO/Virgo/KAG
$\gamma_{\rm PPN} - 1$ tightening	Hard	5–10 yr	Cassini bound below $\sim 10^{-5}$	Mars/asteroid radi
Direct dark-matter detection	Hard	3–7 yr	WIMP at $\sigma_{\rm SI} \gtrsim 10^{-47}$ cm^2	XENONnT / LZ F
			QCD-axion at $g_{a\gamma\gamma} \gtrsim 10^{-15}$ GeV^{-1}	ADMX / HAYSTA
			keV sterile-ν inconsistent with all-baryon Ω_m	X-ray line searches
BBN abundance deviation	Hard	current	any confirmed $\Delta G_{\rm BBN}/G$ at SBBN level	Light-element abur
Composition-dependent EP violation	Hard	current–5 yr	any anomaly inconsistent with universal $(1+f)\mathcal{L}_m$	MICROSCOPE fol

importing scalar-field stress-energy halos, or fitting halo-attached directional KL coefficients. All three violated F01's pointwise $(1 + f)\rho$ source structure. The Derivation Passport explicitly denies dark_matter_halo_ontology; these attempts have been retracted, and the F131 lighthouse closure (Sec. V C) is the F01-native treatment.

- **F113 BBN catastrophe: [dropped].** F113 reported $G_{\rm BBN}/G_{\rm today} \approx 3000$ on a naively-FRW background, violating BBN by $\sim 10^4$. F112a/F113a audits identified the error as wrong-background contamination; the F95 Branch-A radiation closure resolves it. The earlier finding has been retracted.

- **F95 -12% θ_* residual: [dropped].** F95 reported a mixed-$\alpha(z)$ analytical estimate of $\theta_*/\theta_*^{\rm Planck} = 0.916$. F134 ran a first-pass Boltzmann hierarchy in CAMB and CLASS on the Wiltshire-effective background and returned 0.992 / 0.991 (-0.83% / -0.92%), 14× smaller, with the two independent codes agreeing to 0.07%. The F95 analytical recipe is artefactual and the larger number is retracted. F95 Scenario B (-0.77%), which F95 itself dismissed as "not derivationally justified," is independently confirmed by both Boltzmann integrators (Sec. VI G). The Branch-A radiation closure $\Pi_{\rm rad} = -\rho_r/2$ that F95 derived for the BBN and recombination arguments is independent of the retracted analytical recipe and remains load-bearing.

- **Strict uniform-f at perturbation level: [dropped].** F134 modified CLASS `source/perturbations.c` to implement the literal action-level prediction $\delta((1+f)\rho_b) = (1+f_{\rm prim})\delta\rho_b$ photon-coupled. Result: $H_3/H_1 = 0.013$ vs Planck's 0.44 (factor 33 mismatch), $\sigma_8 = 0.31$ vs 0.81, $\ell_1 = 266$ vs 220. Photon-coupled matter does not grow during radiation domination, so the gravitational wells at recombination are too shallow to drive the third compression. The strict uniform-f reading at the perturbation level is incompatible with Planck at $> 10\sigma$ and is retracted. The committed Path A prediction continues to use a collisionless-CDM proxy for $f_{\rm prim}\Omega_b$ at the perturbation level; F138 has shown the linear env-dep-f closure provides only a few-percent correction to that proxy and does not derive it from F01 (Sec. VI G).

- **Smooth-crossover $f_{\rm prim}$ reduction: [dropped].** The speculation that replacing sharp-transition step functions with smooth Standard-Model crossovers would reduce $f_{\rm prim}$ from 5.664 toward the observational target 5.40 has been tested (F139). A per-transition smooth evaluation returns $f_{\rm prim} = 5.78$ (*larger*, not smaller); the smooth path integral $\int(1/g_*)\,|dg_*/dT|\,dT$ is path-independent and gives 3.31 regardless of crossover width. The 1.82σ residual is a genuine feature of the DOF sum, not an approximation artefact. The operator-level derivation from $(1+f)\mathcal{L}_m$ on the full thermal history remains [OPEN] as a separate programme (Sec. X).

IX. COMPARISON WITH EXISTING FRAMEWORKS

Table III summarises ISST against the principal alternatives. The vertical axis lists capability, the horizontal axis lists frameworks; entries are derived (D), assumed (A), absent (–), or excluded (×).

Honest parameter accounting. ISST's full ISST-specific parameter set, with the lighthouse working memories $f_s^{\star}, f_s^{\text{gas,shock}}$ now counted explicitly:

Parameter-free predictions from action + SM (2):

- $a_{\text{crit}} = 1.07 \times 10^{-10}$ m/s^2 — derived from the wall-Friedmann plus Branch-A weak-field algebra (Sec. IV B); depends on the committed $H_0 = 61.79$ and $f_{\text{prim}} = 5.66$.
- $f_{\text{prim}} = 5.66 \pm 0.12$ — discrete-sum ansatz over SM freezeouts (Sec. II E); 1.6–1.8σ tension with Planck. The formal chain from the action's KL functional to the discrete sum is structurally open (four candidate paths ruled out); f_{prim} is therefore properly framed as an SM-motivated ansatz, not as a closed-chain derivation.

Empirical (fitted to data) (7):

- $a_0^{\text{reg}} = 0.01 \times 10^{-10}$ m/s^2 — rotation-curve regulator internal scale, fitted on a 14-galaxy SPARC subsample (Sec. IV).
- $q \approx 0.45$ — regulator exponent, same subsample.
- $\alpha = 0.869$ — KL coupling, galactic calibration on the 14-galaxy subsample (the calibration procedure: for each trial α, compute $V_{\text{ISST}}(r)$ for all 14 galaxies via (16) with the observed baryon distribution and minimise the median RMS velocity residual). Note that $\alpha_{\text{cosmo}} \approx 1.71$ is required to match the discrete-sum f_{prim} from the smooth path-integral form (8), a factor-~ 2 scale dependence acknowledged as an open tension (Sec. II E).
- $n = 4.83$ — void-fraction sigmoid shape parameter in the $\chi_w(z)$ closure (Sec. VI C); residual, not derived.
- $K_{\text{env}} = 1.484$ — environmental rescaling recovered as the squared ratio of two independent Hubble outputs in the F61 closure on the Sheth–van de Weygaert two-barrier abundance (Sec. VI D, App. E); propagates into the committed $H_0 = 73.82/\sqrt{K_{\text{env}}} = 60.61$ km/s/Mpc dressed value and thence into a_{crit}. This is an empirical input rather than an action-derived prediction; deriving K_{env} from earlier ISST-committed quantities is identified as [OPEN].
- $f_s^{\star} \approx 3.55$ — lighthouse stellar working memory, fitted to the Paraficz *et al.* (2016) local-column convergence ratio at the Bullet (Sec. V C). Derivation from violent-relaxation working memory open.
- $f_s^{\text{gas,shock}} \approx 0.21$ — lighthouse shocked-gas working memory, fitted to the same Paraficz decomposition.

Inherited (1):

- $f_{v0} = 0.762$ — void fraction today, from the Wiltshire two-domain framework [23].

Initial conditions (shared with all theories) (2): $\Omega_b h^2 = 0.02237$ from BBN/Planck and $\Psi_0 = 1/G_N$ from matching to laboratory G.

Total ISST-specific count: 10 (2 derived/ansatz + 7 empirical + 1 inherited), plus 2 initial conditions. *This is more total parameters than ΛCDM's 6.* The honest framing is that ISST derives *two* of these from first principles (one fully from the action, one from SM thermal-history input with the chain to the action open) while ΛCDM fits all six of $(\Omega_b, \Omega_c, H_0, \tau, A_s, n_s)$ from CMB data. The epistemic status differs even when the count does not favour ISST on raw count alone — ISST's two parameter-free predictions are *predictions*, in advance of observational fitting; ΛCDM's six are *posteriors*.

The derivation of the regulator exponent q from second-order Branch-A perturbation theory, a single-α scale-bridging calibration that closes the factor-~ 2 galactic-vs-cosmological gap, the derivation of K_{env} from earlier ISST-committed quantities (it currently enters as an empirical squared-Hubble-ratio input rather than as an action-derived prediction), the lighthouse f_s values from violent-relaxation working memory, and the f_{prim} chain closure are identified as [OPEN].

vs. ΛCDM. ISST's empirical regulator parameters (a_0^{reg}, q) are fitted on rotation curves only and do not propagate into the cosmological prediction. The framing differs: ΛCDM *assumes* the dark sector and asks "what density?"; ISST proposes $(1+f_{\text{prim}})\Omega_b = \Omega_m$ as the cosmological-floor identity and asks "what observation could break it?".

ISST is observationally indistinguishable from ΛCDM at the CMB level under the env-dep-f closure. F134 established that the strict uniform-f perturbation prediction is ruled out by the Planck third peak, and the env-dep-δf contribution that ISST already requires for the H_0 tracker restores LCDM-like clustering at recombination (Sec. VI G). The CMB therefore neither favours nor disfavours ISST relative to ΛCDM at first-pass precision. The distinguishing tests live entirely in the late-time structure-formation sector: the differential a_{crit} prediction at galactic scale (−11% vs. MOND a_0; Sec. IV B); the $f\sigma_8$ shape prediction at DESI DR2 + Euclid Y1 precision (13σ distinguishability at $z = 1.1$; Sec. VI); the +2.5% void-lensing G ratio versus GR (Sec. V); and the lighthouse-mechanism predictions for cluster-merger geometry (Bullet, Abell 520, and the jellyfish-galaxy ISST-vs.-MOND discriminator; Sec. VIII). The paper's claim that ISST "replaces three components of the standard model" therefore reduces, at the CMB level, to a reinterpretation of what ΛCDM's Ω_m and Ω_Λ *are*, not to a numerically different prediction; the substantive distinction is at late times.

vs. MOND/TeVeS [48, 54–58]. MOND's a_0 is empirical; ISST's a_{crit} is derived. ISST has cosmology native to the action; MOND does not. ISST's a_{crit} sits 11% below a_0, a sharp discriminator.

TABLE III. ISST vs. existing frameworks. D = derived, A = assumed parameter or input, – = no native treatment, × = excluded.

Capability	ISST	ΛCDM	MOND/TeVeS	metric $f(R)$
$\Omega_m/\Omega_b \approx 6$	D	A	–	A
Late-time acceleration	D (geometric)	A (Λ)	–	D (f'')
Rotation curves	D ($a_{\rm crit}$)	D (NFW fit)	D (a_0)	D
Cluster lensing	D (lighthouse)	D (NFW DM)	×	A
CMB peak structure	∼ (proxy sub-%, peak ratios open; $V(\Psi)$ under inv.)	D (6-param fit)	–	D
PPN $\gamma = \beta = 1$	D (exact)	D	D	A (screening)
Fifth-force screening	not needed	not needed	not needed	required
$c_{\rm GW} = c$	D (linearity)	D	D	D
DOF count	2	2	2+2	2+1
Parameter count: derived from action	1 ($a_{\rm crit}$)	0	0	0
Parameter count: SM-input ansatz	1 ($f_{\rm prim}$, chain open)	0	0	0
Parameter count: empirical	7 (see below)	6 (ΛCDM)	1 (a_0)	1 (f)
Parameter count: inherited	1 (f_{v0}, Wiltshire)	—	—	—
Initial conditions	2 ($\Omega_b h^2, \Psi_0$)	2	yes	yes

vs. metric $f(R)$. Both modify gravity; metric $f(R)$ introduces a propagating scalar that requires chameleon screening. ISST's Palatini-class structure has a Lagrange-multiplier Ψ that does not propagate; no screening is needed because there is no fifth force. The framework f_R_metric_equivalence is [DENIED] for F01.

vs. Horndeski [3, 75–77]. ISST sits in the Horndeski corner that GW170817 spared. It is not a generic Horndeski theory; the specific ΨR form with no kinetic term is a structural choice that survives every cosmological-tensor constraint applied to the broader class.

vs. backreaction. Wiltshire's two-domain framework [22, 23] is a special case of Buchert averaging [20, 21] that ISST adopts as the inhomogeneous background; ISST adds the action principle that requires *some* inhomogeneous background as the only Branch-A-compatible dust cosmology, with Wiltshire's two-domain form the minimal physically motivated realisation (Sec. VI A).

vs. Brans–Dicke. BD [1, 2, 4] gives Ψ a kinetic term and a propagating equation of motion; its $\omega_{\rm BD}$ parameter is constrained by Cassini to $\omega_{\rm BD} > 4 \times 10^4$. ISST has no kinetic term: Ψ is a Lagrange-multiplier and the BD comparison is structurally not applicable. The thermodynamic-gravity programme of Verlinde [14, 15] and Padmanabhan [16] shares ISST's information-as-source intuition but does not specify a covariant action; F01 (1) is the local realisation.

vs. small-scale ΛCDM stresses. JWST early-galaxy abundance [86] and small-scale halo challenges [85, 87] are problems for ΛCDM. ISST's structure-formation history through Wiltshire backreaction may address them but the quantitative work (full C_ℓ, $P(k)$) is identified as [OPEN]. The cosmic SFR history [60] provides the f_s-accumulation timeline that the lighthouse mechanism uses implicitly; deriving SFR from F01 itself is not in scope here.

X. DISCUSSION AND FUTURE WORK

A. Open items

The largest open items, with kill conditions where applicable, are:

- **Modified Boltzmann hierarchy.** The most important remaining computation is a Boltzmann code whose background solver accepts the Wiltshire two-domain volume-averaged expansion directly. The (w_0, w_a) proxy used in F134 diverges from the committed Wiltshire $H(z)$ by +76% at $z = 1090$ (F140); the committed ISST $\Omega_m = 0.329$ exceeds the Wiltshire bare $\Omega_m = 0.125$ by a factor 2.6, causing the equivalent dark-energy density under Friedmann inversion to become negative at $z > 2$ and precluding any Friedmann-template proxy at high redshift. A modified CLASS with (i) the two-domain background solver, (ii) the $V(\Psi)$ perturbation extension, and (iii) $(1+f)\rho_b$ as the gravitational source would provide the definitive CMB test. The framework isst_boltzmann_two_domain_tabulated is identified for passport stamping under (i); isst_boltzmann_env_dep_f remains [CONDITIONAL] pending the $V(\Psi)$ derivation under (ii). Estimated scope: 2–4 months of code development. [OPEN].
- **Operator-level $f_{\rm prim}$ chain.** The DOF-counting $f_{\rm prim} = 5.66 \pm 0.06$ overshoots the CMB target 5.40 ± 0.15 at 1.6–1.8σ. Smooth-crossover evaluation does not close this gap (F139: PER_TRANSITION returns 5.78, the path-independent integrand returns 3.31, neither reaches 5.40). *F145 sharpens the OPEN status from*

"derivation not yet done" to *"structurally obstructed under the four canonical paths."* The action's f in eq. 4 is a single-epoch D_{KL} integral; the discrete sum is a path-additive event-counting quantity over the four SM freezeouts. Single-epoch D_{KL} of post-thermal-history Maxwellians gives 1.36 (factor $\sim$ 4 short); D_{KL} vs. local-bounded uniform gives $\sim$ 40–60 (factor $\sim$ 8 too large; reading is galactic-specific); per-step transient D_{KL} summed gives 0.30 (factor $\sim$ 19 short; Shore–Johnson additivity does not extend to time-separated events); Rényi-2 sum at the 4-step partition gives 5.82 accidentally but Shore–Johnson forces $q = 1$ (Sec. II E, F145). Closing the chain therefore requires either (a) a path-integral extension of the action (re-checks A05, F46 ratchet, $(1+f)\mathcal{L}_m$, the Shore–Johnson uniqueness derivation), or (b) a demonstration that the discrete sum emerges from a different limit of $(1+f)\mathcal{L}_m$ than the single-epoch D_{KL}, or (c) a more abstract information-theoretic measure for which the discrete sum is the unique additive monotone functional. The $\sim$ 2–3σ two-α structural question ($\alpha_{\mathrm{gal}} = 0.869$ vs. $\alpha_{\mathrm{cosmo}} \approx 1.71$ for the smooth-integral form to recover 5.66) is identified as a propagated sub-problem. [OPEN].

- **Matter power spectrum $P(k)$ and BAO.** Same methodological obstacle as C_ℓ. [OPEN].

- **Newton-limit regulator exponent q.** Empirically $\sim$ 0.45; a derivation from second-order Branch-A expansion is identified. [OPEN].

- **$f_s^\odot$ and Cassini sub-leading.** Currently unmeasured; a kill condition is $f_s^\odot > 0.05$. [OPEN].

- **Path commitment (ζ vs. γ vs. ε).** Three Reading-4-class extensions remain on the table; the choice has consequences for void-lensing and screening. Held off pending Lily review of the F130b path catalogue. [OPEN].

- **Per-pixel Bullet κ map.** The local Σ_{baryon} contrast at the two peaks is now resolved by JWST [64] (modified Hausdorff 20 kpc ICL/κ alignment) and the GR κ map is published in Rihtaršič *et al.* (2026) [65], with QUMOND modelling residual $|\Delta\kappa| \lesssim 0.15$ in Hernandez (2026) [66]. Extracting $\kappa_{\mathrm{gal}}/\kappa_{\mathrm{gas}}$ from these maps and comparing to the operator $(1 + f_s^\star)/(1 + f_s^{\mathrm{gas,shock}}) \approx 3.76$ is identified as the next quantitative step (Sec. V C). [OPEN] (numerical extraction; observable now published).

- **BBN under alternative matter conventions.** The F95 closure has been verified under Brown trace and partially under Schutz/Hawking conventions; full coverage is identified. [OPEN].

B. The Derivation Passport as methodology

The Derivation Passport (Appendix A) is the machinery that has made the audit-and-retraction discipline of this paper possible. It is a deterministic gate-engine whose state records every property ISST has stamped (e.g. `scalar_propagates=false` from F128) and whose output is a verdict on every external formula one might reach for. We invite others to use it, criticise it, and extend it.

C. Speculative directions (parking lot)

The following are structurally consistent with the postulate but not yet derived. They are listed here so the record is complete and not in the results sections.

- *GW energy partition between NS–NS and BH–BH mergers* as a measure of information processing rate. Kill condition: if Ψ_{rad} is not derivable from the action as a conserved or source quantity.

- *Decoherence as gravitational information processing.* Highly speculative; no calculation; energy scale likely unfalsifiable.

- *Big-G laboratory scatter as f-correlated.* If $G = 1/\Psi$ with environment-dependent f, the persistent scatter in big-G measurements at national metrology institutes may correlate with local geoid/Bouguer anomalies.

- *Cyclic universe (Axiom VI) as a topological closure.* The horizon and monopole problems are addressed by topological inheritance from the previous cycle, not by intra-cycle inflation.

These are flagged as speculative throughout this paper's `predictions_pending.md` and carry their own kill conditions.

XI. CONCLUSION

ISST commits to a single action with one matter-side coupling and no kinetic term for the scalar. From that action: the Branch theorem partitions spacetime into a GR-recovering compact-object sector and an inhomogeneous cosmological sector; the static-fluid theorem (App. H) routes every physical star into Branch B, underwriting exact PPN ($\gamma = \beta = 1$), $c_{\mathrm{GW}} = c$ structurally rather than by tuning, $|\dot{G}/G|$ five orders below LLR, and inherited SBBN; the acceleration scale $a_{\mathrm{crit}} \simeq 1.07 \times 10^{-10}$ m/s^2 falls out of two algebraic identities, sitting 11% from MOND a_0; the cosmological-floor identity $(1+f_{\mathrm{prim}})\Omega_b = \Omega_m$ predicts Ω_m/Ω_b within 4.2% of Planck from a single SM thermodynamic input (1.6–1.8σ tension reported as honest residual). Rotation curves fit competitively on the full SPARC sample with the central Q=1 comparison robust under held-out cross-validation (F147); the lighthouse mechanism closes the bulk of the Bullet κ contrast under empirical f_s values; the H_0 tension structure is reproduced as a frame artefact (literal recovery open).

ISST *eliminates dark matter as a particle species*; the CMB perturbation structure under the strict F01 action remains open (Sec. VI G). The first-pass acoustic angle uses a (w_0, w_a) proxy carrying $\sim$5–10% systematic uncertainty; the strict uniform-f perturbation source is falsified at the third peak ($> 10\sigma$); the env-dep-f closure provides percent-level corrections, not the factor-$\sim$30 needed (Sec. III). The distinguishing predictions live in late-time structure formation: the lighthouse mechanism in cluster mergers, the $f\sigma_8$ shape on the Wiltshire background (DESI DR2), the +2.5% void-lensing G-ratio (Euclid), and the processing-history-dependent f_s in jellyfish galaxies. The full inventory of open items is in Sec. X; the dropped predictions in Sec. VIII D; the timeline-and-precision falsifier table in Sec. VIII C.

Self-containment and reproducibility. Every load-bearing derivation is presented in the body or appendices (App. A–I). The companion site lily-labs.co.uk/isst provides supplementary computational outputs and the interactive Derivation Passport engine; CAMB `params.ini`, the modified CLASS `source/perturbations.c` patch, and the SPARC analysis scripts will be archived on Zenodo upon acceptance. The invitation is open: here are our kill conditions; try to trigger them.

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Appendix A: The Derivation Passport

The passport is a JSON state file that records every structural property ISST has stamped, together with a property graph and an implication engine. Every external framework (e.g. chameleon_screening, palatini_ppn) carries preconditions; the gate evaluates the conjunction of preconditions against the passport state and returns one of [ALLOWED], [WARNING], [CONDITIONAL], [DENIED]. Every verdict names the F-task or axiom that stamped each contributing property, so that any verdict can be traced back through the derivation chain.

As of this paper, the passport has 70 properties stamped (75 live after implications), with 7 frameworks [ALLOWED], 56 [CONDITIONAL], and 25 [DENIED]. The [ALLOWED] list comprises buchert_averaging, szekeres_model, continuity_equation, palatini_ppn, horndeski_gravity (with corner note), adm_decomposition, and wiltshire_two_domain. The [DENIED] list is dominated by FRW-presuming or propagating-scalar frameworks (friedmann_equations, chameleon_screening, vainshtein_screening, linear_growth_factor, halo_model, convergence_poisson, boltzmann_hierarchy_cmb, etc.) plus damour_esposito_farese_ppn (requires propagating scalar) and birkhoff_theorem_gr (requires Ricci-flat exterior plus standard scalar coupling).

The hard rule is that any F-task using a [DENIED] framework without first updating the passport is structurally invalid. Six results in this paper's audit history triggered this rule and were retracted.

Appendix B: Critical-acceleration derivation

This appendix gives the full algebra behind Eqs. (18)–(26) of Sec. IV B. It assembles the linearised perturbation analysis of F84 and the matter-era power-law analysis of F80 into a single derivation chain.

1. Linearised Branch A: $C_{\rm grad} = 2/3$

Take the conformal-Newtonian (longitudinal) gauge metric

$$ds^2 = -(1+2\Phi)\,dt^2 + (1-2\Phi_N)\,\delta_{ij}\,dx^i dx^j, \tag{B1}$$

with $\Psi = \Psi_0 + \delta\Psi$, $\Psi_0 \equiv 1/G_N$, and dust stress-energy $T^\mu{}_\nu = \rho\, u^\mu u_\nu$ with $u^\mu = (1, \vec{0})$ and $T = -\rho$. Linearising the field equation (2) on Minkowski + Ψ_0, retaining all terms at first order in $(\Phi, \Phi_N, \delta\Psi/\Psi_0, \rho/\Psi_0)$ and reducing to Laplacian form, the four independent equations are (18)–(21) of the main text. From (21) substitute $\nabla^2\Phi = 2\nabla^2\Phi_N$ into (19):

$$4\Psi_0\nabla^2\Phi_N - 4\Psi_0\nabla^2\Phi_N + 3\nabla^2\delta\Psi = -8\pi(1+f)\rho \implies \nabla^2\delta\Psi = -\tfrac{8\pi}{3}(1+ \tag{B2}$$

Inserting back into (18):

$$2\Psi_0\nabla^2\Phi_N = 8\pi(1+f)\rho + \nabla^2\delta\Psi = 8\pi(1+f)\rho - \tfrac{8\pi}{3}(1+f)\rho = \tfrac{16\pi}{3}(1+f) \tag{B3}$$

giving $\nabla^2\Phi_N = (8\pi/3)(1+f)\rho/\Psi_0 = (2/3)\,4\pi G_N(1+f)\rho$, and (21) then gives $\nabla^2\Phi = (4/3)\,4\pi G_N(1+f)\rho$.

For a point mass M at the origin, integrating $\nabla^2\delta\Psi = -(8\pi/3)(1+f)M\,\delta^3(\vec{r})$ gives

$$\delta\Psi(r) = \tfrac{2}{3}(1+f)\,G_N M/r, \quad \left|\nabla(\delta\Psi/\Psi_0)\right| = \tfrac{2}{3}(1+f)\,\frac{G_N M}{r^2c^2} = \tfrac{2}{3}(1+f) \tag{B4}$$

The Bardeen gauge-invariant quantities Φ, Φ_N and the spacetime-scalar $\delta\Psi$ are insensitive to the longitudinal slicing, so this coefficient holds in any choice of slicing. The candidate alternatives (4/3, 2, 1) violate one of (20) or (21) (cross-checked in F84) and are ruled out structurally, not numerically.

2. Matter-era power-law on the wall background: $p/n = (\sqrt{21}-3)/2$

Take $a(t) \propto t^n$, $\Psi(t) \propto t^p$, $\rho(t) \propto t^{-3n}$ on the leading-order wall Friedmann

$$H^2 \;=\; \tfrac{16\pi}{9}\,(1{+}f_{\rm prim})\,\rho/\Psi, \tag{B5}$$

the Branch-A transport

$$\Box\Psi = -\ddot\Psi - 3H\dot\Psi \;=\; -\tfrac{8\pi}{3}\,(1{+}f_{\rm prim})\,\rho, \tag{B6}$$

and continuity $\dot\rho{+}3H\rho = 0$ (automatic for the power-law ansatz). Computing each term:

$$H = n/t, \quad H^2 = n^2/t^2, \tag{B7}$$

$$\ddot\Psi + 3H\dot\Psi = [p(p-1) + 3np]\,\Psi_0\,t^{p-2} = p(p-1+3n)\,\Psi_0\,t^{p-2}. \tag{B8}$$

Power matching the Friedmann equation gives $-2 = -3n - p$, hence $p = 2 - 3n$.

Friedmann amplitude: $n^2 = (16\pi/9)(1{+}f_{\rm prim})\,\rho_0/\Psi_0$, equivalently $(8\pi/3)(1{+}f_{\rm prim})\,\rho_0/\Psi_0 = (3/2)\,n^2$.

Transport amplitude: $p(p-1+3n) = (8\pi/3)(1{+}f_{\rm prim})\,\rho_0/\Psi_0 = (3/2)\,n^2$.

With $p = 2-3n$ the LHS becomes $(2-3n)\,(2-3n-1+3n) = (2-3n)\cdot 1 = 2-3n$. Therefore

$$2 - 3n = \tfrac{3}{2}\,n^2 \iff 3n^2 + 6n - 4 = 0, \tag{B9}$$

which is (24) of the main text. The roots are $n = (-3 \pm \sqrt{21})/3$; the physical (expanding) root is $n = (\sqrt{21}-3)/3 \approx 0.5275$, giving $p = 5 - \sqrt{21} \approx 0.4174$ and

$$\frac{\dot\Psi}{H\Psi}\bigg|_0 = \frac{p}{n} = \frac{3(5-\sqrt{21})}{\sqrt{21}-3} = \frac{3(5-\sqrt{21})(\sqrt{21}+3)}{(\sqrt{21}-3)(\sqrt{21}+3)} = \frac{3(2\sqrt{21}-6)}{12} = \frac{\sqrt{21}-3}{2}, \tag{B10}$$

the 0.7913 of (25).

This is a unique attractor of the coupled system: numerical Radau integration with rtol $= 10^{-11}$ from initial conditions $10^{-2}/H_0$ in the deep matter era to $1/H_0$ recovers the analytical asymptote to $|\Delta(p/n)| = 7.5\times10^{-13}$, with the Friedmann constraint residual at $2.1\times10^{-12}H^2$ peak (F80, `isst_f80_void_scale.py`). The alternative full Friedmann (cross-term retained) gives $p/n = \sqrt{3}$; the Brans–Dicke $\omega = 0$ form (no wall (2/3) subtraction) gives $p/n = 1$. Both alternatives predict $a_{\rm crit}$ values 50–100% off MOND a_0 at any $H_0 \in [60, 75]$ km/s/Mpc and are ruled out empirically (F84 Table 5).

3. Stability-gradient crossover

The Branch-A constraint $R = 0$ holds pointwise in the cosmological background; the field equation (2) with linear Ψ admits no propagating mode (Sec. II D). At galactic scales, the local $|\nabla(\delta\Psi/\Psi_0)|$ scales as $(2/3)(1{+}f_{\rm prim})\,a_{\rm bar}/c^2$ (Eq. B 1); the cosmological floor is set by the matter-era $\dot\Psi/(H\Psi)$ on the wall background giving a gradient scale $L_{\rm eff} = (p/n)\,c/H_0$ and a corresponding floor $H_0^2 L_{\rm eff}/c^2$. Equating:

$$\tfrac{2}{3}(1{+}f_{\rm prim})\,\frac{a_{\rm crit}}{c^2} \;=\; \frac{H_0^2}{c^2}\cdot\frac{p}{n}\,\frac{c}{H_0} \;=\; \frac{p}{n}\,\frac{H_0}{c}, \tag{B11}$$

which rearranges to

$$a_{\rm crit} \;=\; \tfrac{3}{2}\,\frac{p}{n}\,\frac{cH_0}{1+f_{\rm prim}} \;=\; \frac{3(\sqrt{21}-3)}{4}\,\frac{cH_0}{1+f_{\rm prim}}, \tag{B12}$$

the result (26). Numerically, with $cH_0 = 6.003\times10^{-10}$ m/s^2 at $H_0 = 61.79$ km/s/Mpc and $(1{+}f_{\rm prim}) = 6.664$:

$$a_{\rm crit}^{\rm ISST} \;=\; \frac{1.1869\times6.003}{6.664}\times10^{-10}\,{\rm m/s^2} \;=\; 1.069\times10^{-10}\,{\rm m/s^2}. \tag{B13}$$

At Planck $H_0 = 67.4$ the value is 1.166×10^{-10} (-2.8% vs MOND); at SH0ES $H_0 = 73.0$ it is 1.263×10^{-10} (+5.3%). The 11% offset at the committed ISST H_0 is the falsifiable differential prediction.

Appendix C: Wiltshire-forcing proof

This appendix expands the proofs of Claims 1 and 2 from Sec. VI A and gives the explicit pointwise $R = 0$ wall-metric construction.

1. Claim 1 in detail

The flat-FRW Christoffels are $\Gamma^0_{ij} = a\dot a\,\delta_{ij}$ and $\Gamma^i_{0j} = (\dot a/a)\,\delta^i_j$, giving the non-zero Ricci components

$$R_{00} = -3\,\frac{\ddot a}{a}, \qquad R_{ij} = (a\ddot a + 2\dot a^2)\,\delta_{ij}. \tag{C1}$$

The Ricci scalar is therefore

$$R = g^{\mu\nu}R_{\mu\nu} = 3\,\frac{\ddot a}{a} + \frac{3}{a^2}(a\ddot a + 2\dot a^2) = 6\big(\ddot a/a + H^2\big) = 6(\dot H + 2H^2). \tag{C2}$$

Imposing $R = 0$ gives the autonomous ODE $\dot H = -2H^2$. Integrating, $-1/H = -2t + C$; choosing the origin of time $t_0 = 0$ gives $H(t) = 1/(2t)$ and $a(t) = a_0(t/t_0)^{1/2}$. *No matter content was used.* The constraint is purely geometric.

2. Claim 2 in detail

The trace of (2) with $g^{\mu\nu}G_{\mu\nu} = -R$ and $g^{\mu\nu}g_{\mu\nu} = 4$ is

$$-\Psi R + 3\Box\Psi = 8\pi(1+f)T. \tag{C3}$$

Branch A ($R = 0$) reduces this to the transport equation $\Box\Psi = (8\pi/3)(1+f)T$. The (t,t) mixed-index field equation

$$\Psi G^t{}_t + \Box\Psi - \nabla^t\nabla_t\Psi = 8\pi(1+f)T^t{}_t, \tag{C4}$$

on flat FRW with $\Psi = \Psi(t)$ and dust $T^t{}_t = -\rho$, with $G^t{}_t = -3H^2$, $\nabla^t\nabla_t\Psi = -\ddot{\Psi}$, and $\Box\Psi = -\ddot{\Psi} - 3H\dot{\Psi}$, becomes

$$3\Psi H^2 + 3H\dot{\Psi} = 8\pi(1+f)\rho. \tag{C5}$$

Substituting Claim 1's $H = 1/(2t)$ and dust scaling $\rho(t) = \rho_0(t_0/t)^{3/2}$ into (31):

$$\frac{3\Psi(t)}{4t^2} + \frac{3\dot{\Psi}(t)}{2t} \;=\; 8\pi(1+f)\rho_0\, t_0^{3/2}\, t^{-3/2}. \tag{C6}$$

Multiplying by $2t/3$ rearranges this as the first-order linear ODE

$$\dot{\Psi} + \frac{\Psi}{2t} \;=\; \frac{16\pi}{3}(1+f)\rho_0\, t_0^{3/2}\, t^{-1/2}. \tag{C7}$$

The integrating factor is $\mu(t) = \exp\int dt/(2t) = t^{1/2}$. Multiplying both sides by μ gives $d/dt\big(t^{1/2}\Psi\big) = (16\pi/3)(1+f)\rho_0 t_0^{3/2}$, which integrates to

$$\Psi(t) \;=\; A\, t^{1/2} + K\, t^{-1/2}, \qquad A \equiv \frac{16\pi}{3}(1+f)\rho_0\, t_0^{3/2}, \tag{C8}$$

with K a free integration constant. We verify both modes by direct substitution: for $\Psi = At^{1/2}$, $\dot{\Psi} = (A/2)t^{-1/2}$, the LHS of (C6) is $3At^{-3/2}/4 + 3At^{-3/2}/4 = (3A/2)t^{-3/2}$, matching the RHS on $A = (16\pi/3)(1+f)\rho_0 t_0^{3/2}$. For $\Psi = Kt^{-1/2}$, $\dot{\Psi} = -(K/2)t^{-3/2}$, the LHS is $3Kt^{-5/2}/4 - 3Kt^{-5/2}/4 = 0$, satisfying the homogeneous equation. Both modes are therefore valid; the system is algebraically consistent. The trace transport $\Box\Psi = -(8\pi/3)(1+f)\rho$ is satisfied identically by both modes under the same A matching.

The incompatibility is observational. Claim 1 forces $a(t) \propto t^{1/2}$, which is the radiation-era expansion law ($q = 1$). Matter-era observations through SN-Ia, BAO, and CMB acoustic-peak positions require an effective expansion law significantly closer to $a(t) \propto t^{2/3}$ ($q = 1/2$) at intermediate redshift, regardless of the value of K in (C8) (the integration constant rescales Ψ but does not change the functional form of $a(t)$, which is fixed by the geometric condition $R = 0$ in Claim 1). Branch A on flat FRW therefore predicts the wrong cosmological deceleration parameter at the matter era. The mismatch is $\mathcal{O}(1)$ in q, not a subleading correction.

For radiation matter ($T = -\rho + 3P$ with $P = \rho/3$), the trace vanishes identically: $T = 0$. The transport equation reduces to $\Box\Psi = 0$, which admits the static solution $\Psi = \Psi_0 = \text{const}$. Substituting $\dot{\Psi} = 0$, $\ddot{\Psi} = 0$ into (C6) (with $\rho_{\rm rad} \propto a^{-4} \propto t^{-2}$ on $a \propto t^{1/2}$) gives $3\Psi_0/(4t^2) = 8\pi(1+f)\rho_{\rm rad,0}\, t^{-2}$, both sides $\propto t^{-2}$. Consistent. Branch A flat FRW is the correct radiation-era cosmology; the matter-era expansion law it predicts is incompatible with observation.

3. Pointwise $R = 0$ wall metric

The minimum geometric structure compatible with Branch A + dust uses the planar anisotropic ansatz (F12)

$$ds^2 = -dt^2 + a(t)^2(dx^2+dy^2) + b(t,z)^2\, dz^2, \qquad \Psi(t,z) = \Psi_0(t) + \psi(z) \tag{C9}$$

Computing the Ricci scalar of (C9) symbolically:

$$R = R_{\rm temporal}(t) + R_{\rm spatial}(z) + R_{\rm cross}(t,z;\partial_z b), \tag{C10}$$

where the cross terms involving $\partial_z b$ enter R_{tt} and R_{zz} with opposite signs at the bound-wall fixed point $K \equiv \dot{b}/b - \dot{a}/a = 0$. With $a(t) \propto t^{2/3}$ (matter-era wall expansion), $R_{\rm temporal} = 0$ identically. The remaining condition is the spatial balance

$$\partial_z^2\psi = -\tfrac{8\pi}{3}(1+f)\rho, \tag{C11}$$

which is the spatial form of the Branch-A transport equation. The z-derivatives of $b(t,z)$ cancel exactly between R_{tt} and R_{zz} at the bound-wall fixed point, so $R = 0$ pointwise reduces to a 1-parameter ODE in t at each z, with $K = 0$ as its stable fixed point. Numerical verification on a 200×200 grid (F12, working/wall_metric_pointwise_R0.md) confirms: (i) spatial balance (C11) satisfied pointwise to 4×10^{-10}; (ii) spatial average $\langle\Delta\psi\rangle_w = -(8\pi/3)(1+f)\langle\rho\rangle_w$ recovering F12's (34) coefficient $16\pi/9$ at six-digit agreement; (iii) pointwise $|R|$ decays from 6×10^{-5} (initial finite-difference noise) to 3×10^{-9} as K relaxes to zero.

This is the explicit constructive proof that the wall-side of the two-domain split admits Branch A + dust pointwise. Combined with the Milne void (where $T \to 0$ trivially admits $R = 0$ and $\Psi = \text{const}$), the two-domain split is constructively realised.

4. Selection between two-domain alternatives

Claim 2 motivates an inhomogeneous background (the matter-era observational mismatch on flat FRW); Sec. C 3 constructs the planar wall metric. Whether the planar wall-pointwise construction is *unique* among all inhomogeneous Branch A + dust solutions reduces to the classification problem: *which dust spacetimes satisfy $R = 0$ pointwise?* For the two metric classes most studied in inhomogeneous cosmology:

- **Lemaître–Tolman–Bondi (LTB)** dust spacetimes $ds^2 = -dt^2 + R'^2/(1+2E)\,dr^2 + R^2 d\Omega^2$ have Ricci scalar that depends on $E(r)$ and the bang-time $t_B(r)$; setting $R = 0$ pointwise requires solving a coupled PDE system for these arbitrary functions. We have not constructed an explicit LTB solution with $R = 0 +$ dust, nor proved one cannot exist; the classification is identified as [OPEN].

- **Szekeres** spacetimes generalise LTB and admit broader inhomogeneity. The $R = 0 +$ dust classification is similarly unresolved.

Wiltshire's specific timescape geometry [22, 23] is the simplest two-domain Buchert-averaged backreaction model that: (i) realises $\langle R\rangle = 0$ on the volume average; (ii) admits the pointwise $R = 0$ wall metric of Sec. C 3 as its wall-interior limit; (iii) matches the SN-Ia luminosity–redshift relation at $f_{v0} = 0.762$ [23, 25]. The selection over LTB/Szekeres is therefore on the basis of minimality (criterion i, two domains rather than continuous $E(r)/t_B(r)$), constructive realisation (ii), and observational match (iii). *Strict exclusion of all LTB/Szekeres alternatives is identified as a separate classification problem,* [OPEN].

5. What this proves

Within the closed-form analytic results above:

- Branch A on flat FRW with dust is algebraically consistent (general solution (C8)) but observationally incompatible with matter-era expansion: the geometric constraint $R = 0$ forces $a \propto t^{1/2}$ (radiation-era scaling), whereas matter-era observations require $a \propto t^{2/3}$ ([PROVEN] for the geometric obstruction; observational mismatch is at $\mathcal{O}(1)$ in the deceleration parameter).

- The two-domain split (Milne voids + planar-anisotropic walls) *constructively* realises Branch A + dust + observed matter-era cosmology ([PROVEN]).

- Wiltshire's timescape phenomenology arises as the volume-average of this two-domain construction at $f_{v0} = 0.762$ ([DEMONSTRATED]; matches Wiltshire 2009 to 0.09% on the dressing ratio $\mathcal{R}$).

- Strict exclusion of LTB/Szekeres alternative inhomogeneous backgrounds is *not* proved; that classification is [OPEN].

Appendix D: Wall Friedmann derivation: anisotropic slab to isotropic limit

This appendix gives the full derivation of (34) from F01 on the planar wall slab, with explicit verification that energy-momentum is conserved at every step of the anisotropic-to-isotropic transition, providing a clear bridge between the anisotropic wall slab and the averaged Friedmann equation that ensures no energy-momentum is lost in the averaging process. The full derivation, including symbolic verification with SYMPY, is provided in the supplementary material accompanying this paper.

The pointwise existence of the wall metric is established in Appendix C 3; F141 (this appendix) proves the $16\pi/9$ coefficient *from the field equations* on that metric and verifies energy-momentum conservation in the isotropisation limit.

1. The planar wall ansatz and Hubble rates

We work on the planar-symmetric metric

$$ds^2 = -dt^2 + a(t)^2(dx^2+dy^2) + b(t,z)^2\,dz^2, \qquad \Psi = \Psi_0(t) + \psi(t,z), \tag{D1}$$

already used in Appendix C 3. Define $H_\perp \equiv \dot a/a$ (z-independent perpendicular Hubble), $K(t,z) \equiv \partial_t b/b$ (z-dependent parallel expansion), and $\mu(t,z) \equiv \partial_z b/b$ (anisotropy gradient). The bound-wall fixed point has $K = 0$ (frozen thickness) and $b \to 1$.

2. Branch-A constraint and dynamical isotropisation

Direct Christoffel computation on (D1) (verified in `F141_em_conservation_check.py`) gives

$$R = -4\dot H_\perp - 6H_\perp^2 - 2\partial_t K - 2K^2 - 6H_\perp K. \tag{D2}$$

The μ-dependent terms (spatial derivatives of b) cancel identically between R_{tt} and R_{zz}, so the Branch-A constraint $R = 0$ reduces to a one-parameter ODE in t at each z:

$$2\dot H_\perp + \partial_t K + 3H_\perp^2 + K^2 + 2H_\perp K = 0. \tag{D3}$$

This admits two fixed points in K: the bound-wall $K = 0$ branch, which gives $a \propto t^{2/3}$ (standard EdS matter-era), and the unstable $K = -2H_\perp$ branch corresponding to collapsing thickness. Linear stability analysis around $K = 0$ gives a decay rate $\sim H_\perp$ to the bound branch; physical bound walls relax to this attractor, and the wall therefore isotropises dynamically. This is *not* a Wiltshire import: the isotropisation is a stable consequence of (D3).

3. Trace transport and the spatial Ψ profile

The trace of (2) on Branch A gives

$$\Box\Psi = -\tfrac{8\pi}{3}(1+f)\rho \quad (\text{dust}, T = -\rho). \tag{D4}$$

On (D1),

$$\Box\Psi = -\ddot{\Psi}_0 - \ddot{\psi} - (2H_\perp + K)(\dot{\Psi}_0 + \dot{\psi}) + (1/b^2)(\partial_z^2\psi - \mu\,\partial_z\psi). \tag{D5}$$

The fast (spatially-varying) part of (D4), in the bound-wall limit $b \to 1$, $\mu \to 0$, gives the spatial Poisson equation

$$\partial_z^2\psi \;\approx\; -\tfrac{8\pi}{3}(1+f)\rho_w \quad \text{(inside the wall, } \rho \approx \rho_w \text{ uniform)}, \tag{D6}$$

which derives the spatial-balance assumption made in F12 rather than imposing it.

4. The isotropisation limit and the $16\pi/9$ coefficient

Substituting the spatially-balanced ψ profile into the (tt) component of (2) on the isotropic wall ($K \to 0$, $b \to 1$, $H_\perp \to H_w$, $a \to a_w$):

$$\Psi_0(-3H_w^2) - 3H_w\dot{\Psi}_0 - \tfrac{8\pi}{3}(1+f)\rho \;=\; -8\pi(1+f)\rho, \tag{D7}$$

$$\therefore \quad 3\Psi_0 H_w^2 + 3H_w\dot{\Psi}_0 \;=\; \tfrac{16\pi}{3}(1+f)\rho. \tag{D8}$$

Dividing by $3\Psi_0$ and dropping the $\dot{\Psi}_0/\Psi_0$ term (bounded at $\sim 10^{-7}H_0$ per LLR; F121),

$$\boxed{H_w^2 \;=\; \frac{16\pi}{9}\,\frac{(1+f)\rho_b}{\Psi_0}} \quad \equiv (34). \tag{D9}$$

Bookkeeping of the coefficient. The $16\pi/9$ traces to the partition

$$\underbrace{8\pi(1+f)\rho}_{\text{full }(tt)\text{ source}} \;-\; \underbrace{\tfrac{8\pi}{3}(1+f)\rho}_{\text{absorbed by }\partial_z^2\psi\text{ via (D6)}} \;=\; \underbrace{\tfrac{16\pi}{3}(1+f)\rho}_{\text{residual on (tt) RHS}}, \tag{D10}$$

divided by the $3\Psi_0$ prefactor of $G^t{}_t$ in the FRW limit. The spatial Ψ profile $\psi(z)$ absorbs 1/3 of the gravitational source pointwise; the residual 2/3 drives H_w^2 expansion.

5. Energy-momentum conservation: explicit no-leak check

The Noether identity for the diffeomorphism-invariant matter action $S_m = \int\sqrt{-g}\,(1+f)\mathcal{L}_m$ gives

$$\nabla^\mu T^{\text{eff}}_{\mu\nu} = 0 \quad \text{identically} \tag{D11}$$

on any matter configuration. For the uniform-f scope of the wall Friedmann derivation ($\Delta^f_{\mu\nu} = 0$, Sec. II), (D11) reduces to $\nabla^\mu[(1+f)T_{\mu\nu}] = 0$, which on (D1) for dust gives the continuity equation

$$\dot{\rho} + (2H_\perp + K)\rho = 0. \tag{D12}$$

In the isotropic limit $K \to H_\perp \to H_w$, this deforms continuously to

$$\dot{\rho} + 3H_w\rho = 0 \quad \Longrightarrow \quad \rho \propto a_w^{-3}. \tag{D13}$$

The dilution-rate coefficient $(2H_\perp + K) \to 3H_w$ smoothly as the wall isotropises; *no terms are dropped or absorbed in the limit*, no leak terms appear, and the coefficient of $\dot{\rho}$ remains exactly 1 throughout. The Noether identity (D11) holds at every K in the transient. The full symbolic verification in `F141_em_conservation_check.py` confirms (D12) as the divergence of $(1+f)T_{\mu\nu}$ on (D1) and (D13) as its $K \to H_\perp$ limit.

6. Buchert cross-check: the 2/3 factor's origin

Standard Buchert averaging [20] for dust on a generic spatially-foliated spacetime gives

$$3\,(\dot{a}_D/a_D)^2 = 8\pi G\langle\rho\rangle_D - \tfrac{1}{2}\langle R\rangle_D - \tfrac{1}{2}Q_D, \tag{D14}$$

with Q_D the kinematic backreaction and R the spatial Ricci. For ISST Branch A with bound-wall fixed point, $\langle R\rangle_D = 0$ (Branch A pointwise) and $Q_D = 0$ (uniform expansion at the fixed point), so (D14) reduces to $3H_w^2 = 8\pi G\langle\rho\rangle_w$, giving the standard $8\pi/3 = 24\pi/9$ coefficient.

ISST's $16\pi/9 = (8\pi/3) \times (2/3)$ differs by exactly the 2/3 factor identified in (D10). The standard Buchert framework *does not include* the Ψ-Lagrange-multiplier sector—it assumes Einstein gravity $G_{\mu\nu} = 8\pi T_{\mu\nu}$. ISST's modification is the additional Ψ transport equation (D4), which absorbs 1/3 of the gravitational source into the spatial profile $\psi(z)$. The effective Buchert coefficient becomes $8\pi/3 \times 2/3 = 16\pi/9$.

This is not a coincidence with the Wiltshire two-domain parameterisation. Wiltshire's wall Friedmann uses standard GR and identifies the dust density as the cosmological matter density ρ_m. Identifying $(1+f)\rho_b \equiv \rho_m$ via A04, ISST's $H_w^{\text{ISST}} = \sqrt{2/3}\,H_w^{\text{WI}}$. The 2/3 rescaling propagates through the Wiltshire dressing $\mathcal{R}(f_{v0}) = 1.2825$ to give the H_0 predictions of Sec. VI C self-consistently; the coefficient invariance result of F12.INV confirms $\mathcal{R}$ depends only on f_{v0}, not on the wall coefficient.

7. Coordinate-independence

The $16\pi/9$ coefficient relates two observer-frame scalars (H_w^2 and ρ_b). The derivation in §C.4 used the synchronous gauge (D1). Conformal-gauge ($ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2)$) and harmonic-gauge ($\Box x^\mu = 0$) re-derivations give the same coefficient *by construction*: the F01 field equation (2) is generally covariant, so its (tt)-projection on the wall solution yields a gauge-invariant scalar relation between H_w^2 and ρ_b/Ψ_0. The bookkeeping (D10) is gauge-independent because each step uses only the trace of (2) and the spatial Poisson (D6), both of which are scalar equations.

8. Status

The $16\pi/9$ coefficient in (34) is upgraded from [DEMONSTRATED] to [PROVEN] by this appendix:

- The pointwise $\mathrm{R} = 0$ wall metric admits the bound-wall fixed point as a stable attractor of (D3) (no Wiltshire isotropisation import).
- The trace transport (D4) on Branch A gives the spatial Ψ profile (D6) that absorbs 1/3 of the source.
- The (tt) projection in the isotropic limit gives (D9) with coefficient $16\pi/9$.
- Noether identity (D11) guarantees energy-momentum conservation at every K; the continuity equation (D12)–(D13) deforms continuously without leak terms.
- The 2/3 factor relative to standard Buchert is the explicit fraction of the source absorbed by Ψ transport, traced through (D10).
- Coordinate-independence holds by general covariance of (2).

Appendix E: H_0 frame artefact: bare-to-dressed pipeline

This appendix expands Sec. VI C with the full algebraic chain from the bare two-domain background to the values a wall observer fitting a ΛCDM template recovers under the SH0ES local-distance-ladder and Planck CMB-template anchors.

1. The three frames and their relations

On the matter-dominated two-domain Buchert average [22, 23], the bare scale factor $\bar{a}(t)$ and the wall scale factor $a_w(\tau)$ are related through the lapse $\gamma_w(\tau) \equiv dt/d\tau$ by the volume constraint

$$\bar{a}^3 = f_w\, a_w^3 + f_v\, a_v^3, \tag{E1}$$

where f_w, $f_v = 1 - f_w$ are the wall and void volume fractions. The void scale factor is Milne, $a_v(\tau) \propto \tau$. Differentiating and re-expressing in the wall proper time, the present-epoch tracker solution gives the relation

$$\mathcal{R}(f_v) \;\equiv\; \frac{H_0^{\rm dressed}}{H_0^{\rm bare}} \;=\; \frac{4f_v^2 + f_v + 4}{2(2+f_v)}, \tag{E2}$$

derived as Eq. (25) of Wiltshire [24] (and verified independently in F26). The lapse itself satisfies

$$\gamma_0 \;=\; \frac{1}{2}\Big[1 + \tfrac{1}{2} f_{v0} + \sqrt{1 + f_{v0} + \tfrac{9}{4} f_{v0}^2}\Big], \tag{E3}$$

reading $\gamma_0 = 1.348$ at $f_{v0} = 0.695$ (Duley et al. best fit) and $\gamma_0 = 1.380$ at $f_{v0} = 0.762$ (ISST commit). The density dressing relation is

$$\Omega_m^{\rm dressed} \;=\; \gamma_0^3\, \Omega_m^{\rm bare}, \tag{E4}$$

distinct from $\mathcal{R}$ and not equal to $\mathcal{R}^3$. Conflating γ_0 with $\mathcal{R}$ has caused order-unity errors in earlier ISST drafts and is flagged explicitly here.

2. ISST-native bare H_0

A model-independent input from the CMB acoustic angle and sound horizon gives the comoving angular-diameter distance to the last-scattering surface

$$D_A^{\rm com}(z_*) = r_s/\theta_* = 14130.8 \text{ Mpc}, \tag{E5}$$

using $r_s = 147.09$ Mpc and $\theta_* = 1.04092 \times 10^{-2}$ (Planck [29] primaries). This is independent of ΛCDM. Integrating the Wiltshire two-domain tracker forward from $z_* = 1089$ to $z = 0$ with the ISST-committed parameters $f_{v0} = 0.762$ and $\Omega_b h^2 = 0.02237$, requiring the integrated comoving distance to match $D_A^{\rm com}(z_*)$, fixes

$$H_0^{\rm ISST,bare} = 57.56 \text{ km s}^{-1}\text{ Mpc}^{-1}. \tag{E6}$$

This is $\sim$ 15% above the Wiltshire-implied bare value ($\sim$ 50.1 km/s/Mpc; Duley et al. [25]), reflecting ISST's larger f_{v0} commitment and the no-Λ closure. No phenomenological fit was performed; the Planck primaries plus the ISST tracker plus f_{v0} uniquely determine (E6) (script `working/isst_bare_h0_from_cmb.py`).

3. Uniform-f dressed value at $f_{v0} = 0.762$

Applying (E2) at $f_{v0} = 0.762$:

$$\mathcal{R}(0.762) = \frac{4\,(0.762)^2 + 0.762 + 4}{2\,(2.762)} = \frac{2.323 + 0.762 + 4}{5.524} = 1.2825. \tag{E7}$$

The corresponding dressed value is

$$H_0^{\rm dressed,uniform-f} = 1.2825{\times}57.56 = 73.82 \text{ km s}^{-1}\text{ Mpc}^{-1}. \tag{E8}$$

This number is the diagnostic uniform-f tracker output, not the ISST commit. It evaluates close to but not equal to the SH0ES central value 73.04 km/s/Mpc. The ISST commit replaces this with the $K_{\rm env}$-corrected closure described next.

4. $K_{\rm env}$-corrected dressed value: 61.79

The uniform-f tracker neglects the environment-dependent re-weighting that ISST's $\chi_w(z)$ sigmoid

(Sec. VI D) introduces. Defining the environmental rescaling

$$K_{\rm env} = \left(H_{\rm uniform}/H_{\rm phen}\right)^2, \tag{E9}$$

the F61 closure on the Sheth–van de Weygaert two-barrier abundance gives $K_{\rm env} = 1.484$. The corrected dressed value is [25, 26]

$$H_0^{\rm ISST,dressed} = H_0^{\rm dressed,uniform-f}/\sqrt{K_{\rm env}} = 73.82/\sqrt{1.484} = 60.61, \tag{E10}$$

and a small additional retemplating from the wall $\dot{\Psi}/(H\Psi)$ correction shifts this to the committed 61.79 km/s/Mpc reported in (38). The full T3k closure is reproduced numerically (F26a, F61) and matches phenomenological best fit [26] to within 0.02 km/s/Mpc.

5. Wall-observer template recoveries

A wall observer fitting ISST's two-domain $d_L(z)$ with a ΛCDM template recovers different H_0 depending on the anchoring. The F85 forward pipeline computes both:

Planck-template recovery. Anchor $(\Omega_m, \Omega_b h^2, \theta_*)$ to Planck values; minimise χ^2 on a synthetic ISST C_ℓ via the effective (w_0, w_a) proxy of Sec. VI G. The recovered H_0 is

$$H_0^{\rm Planck-template} = 62.8\ {\rm km\,s^{-1}\,Mpc^{-1}}. \tag{E11}$$

Observed Planck value: 67.4. Pipeline residual −7%.

SH0ES-ladder recovery. Anchor M_B on a Cepheid+SN distance ladder calibrated by ISST $d_L(z)$ in $z \in [0.023, 0.15]$, then fit ΛCDM($\Omega_m = 0.315$) to the ladder-output d_L. The recovered H_0 is

$$H_0^{\rm SH0ES-ladder} = 70.6\ {\rm km\,s^{-1}\,Mpc^{-1}}. \tag{E12}$$

Observed SH0ES value: 73.04. Pipeline residual −3.4%.

Pipeline-internal spread. The ISST pipeline produces (E11) and (E12) from a *single underlying cosmology* (the same two-domain background with the same $f_{v0} = 0.762$ commit). The spread

$$\Delta H_0^{\rm pipeline} = 70.6 - 62.8 = 7.8\ {\rm km\,s^{-1}\,Mpc^{-1}} \tag{E13}$$

reproduces the observed Hubble tension structure $\Delta H_0^{\rm obs} = 73.04 - 67.4 = 5.6$ km/s/Mpc with the correct sign and within $\sim 40\%$ on magnitude. The literal recovery of 73.0 and 67.4 is *not* achieved: the SH0ES-side pipeline output is −3.4% low, the Planck-side −7% low. These residuals reflect (a) the effective (w_0, w_a) proxy used in the Boltzmann run instead of full Wiltshire $H(z)$ at recombination (a +76% divergence at $z = 1090$, F140; see Sec. VI G); and (b) the absence of photon-path averaging in the SH0ES ladder pipeline (an ISST-specific correction not yet built; F85 §6).

Conclusion. The Hubble tension is reproduced as a pipeline artefact between Planck-template and SH0ES-ladder recoveries of the same ISST cosmology, not as a tension between data sets. Literal closure of both observed values within 1% requires the photon-path-corrected SH0ES pipeline and the full Wiltshire C_ℓ, both [OPEN]; the structure is reproduced already.

Appendix F: Audit summary and result inventory

The retroactive audit (2026-04-25) classifies all 88 audited results:

TABLE IV. Audit classification.

Class	Count	Action
GREEN	60	no action; load-bearing
YELLOW	19	note context predicate in writeup
ORANGE	0	all 9 re-derived clean; promoted GREEN
RED	6	retracted or superseded
BLACK	3	retracted; halo-ontology imports

The RED list (Sec. VIII D): F32–F34 (used FRW background; superseded by F82/F89), F69 (GR Poisson source), F70 (Shore-Johnson additivity fails), F76 (f(R) metric equivalence inapplicable), F77 (chameleon screening requires V(Ψ); F01 has none), F112/F113/F114 (FRW contamination; superseded by F95).

The BLACK list (Sec. VIII D): F65, F66, F68; all halo ontology imports.

The ORANGE→GREEN promotions (F02, F09, F82, F87, F106, F110, F112a, F113a, and the legacy `isst_growth.py`) re-derived each result through an [ALLOWED] or ISST-native framework in `working/rederivations/`.

Appendix G: Suggested figures

We list, but do not include here, the figures we recommend for the referee version:

1. Branch-A vs. Branch-B map: which scales live in which branch, with the surface Σ between them.

2. Rotation-curve grid: ISST v8 vs. MOND vs. Newton on a representative SPARC galaxy each from dwarf, LSB, late-type spiral, and early-type spiral classes.

3. BTFR scatter: ISST v8 prediction versus observed slope 1.0 ± 0.1.

4. $f\sigma_8(z)$ comparison: ISST F89 band, ΛCDM band, and the eight current RSD measurements with errorbars.

5. Bullet κ schematic: $\Sigma_\star$, $\Sigma_{\rm gas}$, $(1+f_s)$ enhancement at the main and sub-cluster peaks.

6. Parameter-vs-derived table: ΛCDM 6 free parameters versus ISST $f_{\rm prim}$ + initial conditions.

7. Passport compatibility matrix: ALLOWED / CONDITIONAL / DENIED verdicts for the 12 frameworks the paper's results depend on.

Appendix H: Static-fluid theorem: physical inadmissibility of Branch A perfect fluids

This appendix gives the explicit derivation of the static-fluid theorem (informally, "F08") invoked throughout the body for PPN $\gamma = \beta = 1$ (Sec. III), $c_{\rm GW} = c$ via Branch B selection of compact binaries (Sec. VII), the LLR $\dot{G}/G$ bound (Sec. III), the rotating-NS extension (Sec. III), and the DF2 / TDG predictions (Sec. IV E). The result: any static spherically symmetric isotropic perfect fluid on Branch A is forced to have $P_c = 0$ and $P''(0) < 0$, hence $P(r) < 0$ at small radius, *regardless of the equation of state*. No physical fluid with $P \geq 0$ and $dP/d\rho > 0$ admits this profile; physical static sources are forced into Branch B, where the field equation reduces to exact GR with $G_{\rm eff} = (1 + f)/\Psi_0$.

1. Setup

Work in natural units $8\pi G = 1$. Take a static spherically symmetric metric

$$ds^2 = -e^{2\Phi(r)}dt^2 + A(r)^{-1}dr^2 + r^2 d\Omega^2, \qquad A = e^{-2\Lambda}, \tag{H1}$$

with perfect-fluid matter of density ρ_0 at the centre, pressure $P(r)$, and constant interior $f = f_\star \geq 0$. Regularity at $r = 0$ requires $A(0) = 1$, $\Phi'(0) = 0$, $\Psi'(0) = 0$, $f'(0) = 0$. Expand

$$A = 1 + a_2 r^2 + a_4 r^4 + O(r^6), \tag{H2}$$
$$\Phi = \Phi_c + \tfrac{1}{2}\Phi_2 r^2 + \tfrac{1}{24}\Phi_4 r^4 + O(r^6), \tag{H3}$$
$$\Psi = \Psi_c + \tfrac{1}{2}\psi_2 r^2 + \tfrac{1}{24}\psi_4 r^4 + O(r^6), \tag{H4}$$
$$P = P_c + \tfrac{1}{2}P_2 r^2 + O(r^4). \tag{H5}$$

2. Leading-order coefficients

Imposing the Branch-A constraint $R = 0$, the trace transport $\Box\Psi = \frac{1}{3}(1 + f_\star)T$ with $T = -\rho_0 + 3P$, the (tt) field equation, and the origin-consistency of (rr) and $(\theta\theta)$ yields

$$\Phi_2 = -a_2 = \tfrac{2(1+f_\star)\rho_0}{9\,\Psi_c} \quad \text{(from } R = 0 \text{ and } (tt)\text{)}, \tag{H6}$$
$$\psi_2 = -\tfrac{(1+f_\star)\rho_0}{9} \quad \text{(transport)}, \tag{H7}$$
$$P_c = 0 \quad \text{(origin consistency)}. \tag{H8}$$

The vanishing of P_c is not assumed; it is forced uniquely by the requirement that the algebraic pressure relation

$$P(r) = \Psi\,\frac{A - 1}{r^2} + \frac{2A}{r}\big(\Psi\,\Phi' + \Psi'\big) + A\,\Phi'\,\Psi' \tag{H9}$$

gives a finite consistent value at $r = 0$. The transport equation combined with the (rr) field equation collapses to (H9) with no Φ'' or Ψ'' appearing; the $(\theta\theta)$ equation, after use of the same transport and the constraint $R = 0$, reduces to the same relation. Pressure is therefore algebraically determined by the metric state.

Explicit derivation of $P_c = 0$. Substituting the regular-centre expansions into (H9) and collecting $O(1)$ terms,

$$P_c = \Psi_c\,a_2 + 2\,\Psi_c\,\Phi_2 + 2\,\psi_2. \tag{H10}$$

With $\Phi_2 = -a_2$ ($R = 0$), $a_2 = -2(1+f_\star)\rho_0/(9\Psi_c)$ ((tt)), and $\psi_2 = -(1 + f_\star)\rho_0/9$ (transport),

$$P_c = \Psi_c a_2 - 2\Psi_c a_2 + 2\psi_2 = \tfrac{2(1+f_\star)\rho_0}{9} - \tfrac{2(1+f_\star)\rho_0}{9} = 0. \tag{H11}$$

The cancellation is algebraic: the (tt) a_2 and the transport ψ_2 enter with opposite signs and the $R = 0$ identity $\Phi_2 = -a_2$ doubles the a_2 contribution to produce the cancellation. The $(1 + f_\star)$ factor enters every term uniformly and does not affect the cancellation; the same algebra with $f_\star = 0$ (ordinary GR with Branch-A $R = 0$) equally gives $P_c = 0$.

3. Next-order coefficients and physical inadmissibility

At $O(r^2)$, self-consistency of (rr) gives

$$P_2 = -\frac{2(1 + f_\star)\rho_0^2}{9\,\Psi_c} < 0, \tag{H12}$$

and substitution confirms $(\theta\theta)$ yields the same value. Hydrostatic equilibrium $P' = -(\rho_0 + P)\Phi'$ at leading order requires $P_2 = \rho_0 a_2$, which with $a_2 = -2(1+f_\star)\rho_0/(9\Psi_c)$ reproduces (H12) exactly — equilibrium is automatic by the Bianchi identity.

Theorem (physical inadmissibility). For any static spherically symmetric isotropic perfect-fluid source in Branch A with regular centre, constant interior $f = f_\star \geq 0$, and $\rho_0 = \rho(0) > 0$:

$$P(0) = 0, \qquad P''(0) = P_2 = -\frac{2(1 + f_\star)\rho_0^2}{9\,\Psi_c} < 0. \tag{H13}$$

The pressure therefore takes negative values at all sufficiently small $r > 0$. No matter obeying a thermodynamic equation of state $P = P(\rho)$ with $P \geq 0$ and $dP/d\rho > 0$ admits this profile. Branch A has no static spherically symmetric isotropic-fluid solution describing ordinary self-gravitating matter, regardless of the density profile $\rho(r)$ (extension to $\rho(r)$ with ρ_2 varying gives the same P_c, P_2 by Bianchi-redundancy in the higher-order coefficients).

Corollary (the static-fluid theorem). Every static spherically symmetric source of ordinary matter (baryons obeying a physical EOS) is in Branch B, with $\Psi = \Psi_c$ constant throughout the interior. Junction continuity at the stellar surface with the vacuum exterior $\Psi = \Psi_0 + D/r$ forces $D = 0$: $\Psi = \Psi_0$ identically in the exterior. The PPN parameters satisfy $\gamma = \beta = 1$ exactly by the Branch-B reduction $\Psi_0 G_{\mu\nu} = 8\pi(1 + f)T_{\mu\nu}$, which is exact GR with $G_{\text{eff}} = (1 + f)/\Psi_0$.

4. Scope

The theorem assumes (i) static spherical symmetry, (ii) regular centre, (iii) isotropic perfect-fluid matter with a single scalar pressure $P(\rho)$. Anisotropic stress (e.g. a strange-quark crust with $P_r \neq P_t$) falls outside the origin-consistency forcing of $P_c = 0$ and is not covered here. Slowly rotating configurations are reduced to the static case at $O(\Omega^0)$ (Sec. III); rapidly rotating equilibria invoke the generalised stationary-equilibrium branch-selection rule ($df/d\tau = 0$ in the matter rest frame), not this theorem directly. Constant-$f_\star$ extension to radially varying $f(r)$ with $f'(0) = 0$ is conjectured to follow from the same Bianchi-redundancy structure; the extension is not load-bearing for the body's PPN, c_{GW}, LLR, and rotating-NS results, which use only the $f = f_\star$ statement of the theorem.

5. Numerical confirmation

A full-GR shooter (`paper/branch_a_shooter_v3.py`) integrates the Branch-A system with state $\{\Phi, \Phi', A, \Psi, \Psi'\}$ from $r_0 = 10^{-4}$ to $r_{\max} = 3$ in natural units, with DOP853 at relative tolerance 10^{-11}. A density scan over $\rho_0 \in \{0.1, 0.3, 1, 3, 10\}$, a Ψ_c scan over $\{0.3, 1, 3, 10\}$, and an $f_\star$ scan over $\{0, 1, 5, 5.4, 20\}$ reproduce the parametric prediction $P_2 = -2(1+f_\star)\rho_0^2/(9\Psi_c)$ to 0.7% at $f_\star = 20$ and 0.2% at $f_\star = 5.4$. Consistency residuals $|P_{rr} - P_{\theta\theta}|/|P_{rr}| \sim 10^{-12}$ and emergent $|R| \sim 10^{-10}$ hold across the full scan. Numerical confirmation is therefore complete at the level of the constant-$f_\star$ ODE system; the analytical theorem above is the load-bearing statement.

Appendix I: ADM constraint analysis: degree-of-freedom count

This appendix gives the explicit ADM (Arnowitt–Deser–Misner) constraint analysis of (1) that underpins the body's "two tensor modes, no propagating scalar" claim and the resulting predictions $c_{\text{GW}} = c$ structurally (Sec. VII), no fifth force at the propagating-scalar level (Sec. III B), no scalar dipole radiation in binary inspirals (Sec. VII), and the Branch-theorem's identification of Ψ as a Lagrange multiplier (Sec. II B). The result is internal to the project; external verification, with explicit reference to the mimetic-DOF analyses of Barvinsky, Capela–Ramazanov, and the Firouzjahi–Gorji–Mansoori sequence ([99–102]), is identified as a methodological priority (Sec. X).

1. ADM variables and Lagrangian density

Standard $3+1$ decomposition gives lapse N, shift N^i, spatial metric h_{ij}, with conjugate momenta π_N, π_i, π^{ij}; plus the scalar pair (Ψ, π_Ψ). Per spatial point, 11 configuration variables and 22 phase-space variables.

Using the Gauss–Codazzi decomposition $R = R^{(3)} + (K_{ij}K^{ij} - K^2) +$ total derivatives with $K_{ij} = (2N)^{-1}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$, the F01 gravity Lagrangian density is

$$\mathcal{L}_{\text{grav}} = \frac{1}{16\pi}\, N\sqrt{h}\,\Psi\left[R^{(3)} + (K_{ij}K^{ij} - K^2)\right] + (\text{total derivatives}). \tag{I1}$$

Key observation. Ψ appears *linearly* multiplying the ADM density; $\dot{\Psi}$ does not appear anywhere in $\mathcal{L}$, because $R^{(3)}$ contains only h_{ij} and spatial derivatives, $(K_{ij}K^{ij} - K^2)$ contains $\dot{h}_{ij}$ but not $\dot{\Psi}$, and the total-derivative term $\partial_t(\sqrt{h}\,\Psi K)$ is dropped from the bulk Lagrangian. The matter Lagrangian $(1 + f)\mathcal{L}_m$ contains no Ψ at all (f is a matter-only functional of the distribution function $F(x, p)$). Hence the full F01 Lagrangian density has *no* $\dot{\Psi}$ *dependence.*

2. Primary constraints

The momentum conjugate to Ψ is therefore

$$\pi_\Psi \;\equiv\; \partial\mathcal{L}/\partial\dot{\Psi} \;=\; 0 \quad \text{identically}, \tag{I2}$$

a primary constraint in the Dirac sense and the hallmark signature of a Lagrange multiplier. Together with the standard GR primaries $\pi_N = 0$, $\pi_i = 0$, the F01 system has 5 primary constraints.

Cross-check vs. Brans–Dicke $\omega = 0$. BD gravity with $\omega = 0$ has the action $S_{\text{BD}} = (16\pi)^{-1}\int\sqrt{-g}\,\Psi R$ (no kinetic term in the $\omega \to 0$ limit). The F01 gravity sector *is* this case (modulo the $(1 + f)$ matter coupling); the matter-side coupling is what distinguishes ISST. The Chiba $f(R)$ ↔BD equivalence specialised to $f(R) = R$ is the degenerate Einstein-Hilbert-with-prescribed-coupling case, which is exactly F01 without $V(\Psi)$.

3. Hamiltonian and secondary constraints

The total Hamiltonian (modulo boundary terms) is

$$H = \int d^3x\,[\,NH_\perp + N^i H_i + u_N\pi_N + u^i\pi_i + u_\Psi\pi_\Psi\,], \tag{I3}$$

with $H_\perp = h^{-1/2}\pi^{ij}(\pi_{ij} - \frac{1}{2}\pi h_{ij})/\Psi + (\sqrt{h}/16\pi)\Psi R^{(3)} +$ matter and $H_i = -2\nabla_j \pi^j{}_i +$ matter flux.

Consistency of the primary constraints generates

$$\dot{\pi}_N = \{\pi_N, H\} \approx -H_\perp \;\Rightarrow\; H_\perp = 0 \quad \text{(secondary, GR scalar)} \tag{I4}$$

$$\dot{\pi}_i = \{\pi_i, H\} \approx -H_i \;\Rightarrow\; H_i = 0 \quad \text{(secondary, GR vector)} \tag{I5}$$

$$\dot{\pi}_\Psi = \{\pi_\Psi, H\} \approx -\delta H/\delta\Psi. \tag{I6}$$

Collecting all Ψ-derivative terms of H and demanding they vanish is algebraically equivalent to the trace of the F01 metric field equation,

$$-\Psi R + 3\Box\Psi = 8\pi T_{\text{eff}}, \tag{I7}$$

which on Branch A ($R = 0$, with $\nabla\Psi \neq 0$) reduces to

$$\Box\Psi = \tfrac{8\pi}{3}(1+f)T. \tag{I8}$$

This is the trace-determined evolution of Ψ: Ψ *propagates as a sourced second-order PDE* but does so as a constraint determined by matter, not as an independent dynamical field with its own variational equation. In Hamiltonian language, the secondary constraint from $\dot{\pi}_\Psi = 0$ is $R = 0$ (Branch A); Ψ's evolution is then fixed point-by-point by the trace identity.

4. Constraint classification and DOF count

The pair $(\pi_\Psi, R = 0)$ is first-class: $\{\pi_\Psi(x), R(y)\} = -\partial R(y)/\partial\Psi(x) = 0$ because R contains no Ψ (the gravity action is ΨR, so R itself is a metric-only functional). With $R = R[g, \pi^g]$ and $\mathcal{L}_m = \mathcal{L}_m[g, \chi_i]$ both Ψ-independent, $\delta R/\delta\Psi = 0$ identically, hence the Poisson bracket vanishes weakly.

The first-class pair $(\pi_\Psi, R = 0)$ generates the gauge transformation $\delta\Psi = \varepsilon(x)$, $\delta g_{ij} = 0$, $\delta\pi^{ij} = 0$, $\delta(\text{matter}) = 0$ on the constraint surface. This is the formal statement that Ψ is pure gauge modulo the $R = 0$ slice — only its gradient appears in observables, through $\Box\Psi$ and $\nabla_\mu\nabla_\nu\Psi$ in the metric field equation.

The full constraint count is:

- First-class: 8 (GR diffeomorphism: $\pi_N = 0, H_\perp = 0, \pi_i = 0, H_i = 0$, $\times 2$) plus 1 from $(\pi_\Psi, R = 0)$ taken as a first-class pair — total 9.
- Second-class: 0.

By the Dirac formula $\text{DOF}_{\text{config}} = (N_{\text{phase}} - 2N_{1\text{C}} - N_{2\text{C}})/2 = (22 - 18 - 0)/2 = \mathbf{2}$.

F01 has two propagating degrees of freedom: the two graviton polarisations of GR, no propagating scalar mode.

5. Distinction from mimetic gravity

The mimetic-gravity DOF controversy is the natural concern raised by F01's structural similarity to mimetic gravity [89]. In mimetic gravity, a Lagrange multiplier λ enforces $g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi = -1$ on the scalar ϕ. This constraint forces ϕ to play the role of a clock-like coordinate and produces a propagating dust-like mode whose kinetic structure has been analysed by Barvinsky [99], Capela–Ramazanov [100], and the Firouzjahi–Gorji–Mansoori sequence [101, 102]; several authors argue that the constraint creates a hidden propagating degree of freedom with non-trivial dispersion in the caustic regime.

The F01 action does not contain this constraint. No Lagrange multiplier λ enforcing a derivative condition on Ψ appears in (1), and $\nabla\Psi$ is not constrained to be timelike unit. The trace identity $\Box\Psi = (8\pi/3)(1+f)T$ that emerges from $\dot{\pi}_\Psi = 0$ does not impose a kinematic condition on $\nabla\Psi$ analogous to mimetic's $g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi = -1$; it is a Poisson-type sourced equation determining Ψ's gradient in terms of matter, with no preferred-frame structure on Ψ. The mimetic-dust hidden-mode mechanism therefore does not directly transfer to F01.

Open: external verification. The internal ADM count above is consistent with the no-propagating-scalar reading of the F01 action and structurally distinct from the mimetic constraint. However, given the mimetic-DOF literature's history of subtle hidden modes in apparently constrained systems, an independent ADM/Hamiltonian-constraint analysis by an external group — with explicit cross-comparison against the cited mimetic literature — is required to close the DOF question to the level a hostile referee would demand. This external verification is identified as a methodological priority (Sec. X).

6. What the trace-determined evolution does and does not mean

The trace equation $\Box\Psi = (8\pi/3)(1+f)T$ is a second-order PDE in Ψ. It is sourced by matter. In the PDE sense, Ψ *propagates*: hyperbolic equations on a Lorentzian background admit characteristic propagation along null cones. The phrase "non-dynamical Ψ" is misleading at the PDE level and is replaced throughout the paper by "trace-determined," "metrically slaved," or "constraint-propagated" (Sec. II B).

What is true is the variational statement: Ψ is not independently extremised in the action — there is no $\delta S/\delta\Psi = 0$ EOM. Its evolution is entirely determined by the metric and matter equations through the trace constraint. Whether this constraint-propagation constitutes an independent dynamical degree of freedom is a Hamiltonian-constraint question (this appendix), not a PDE-classification question on the trace equation. The Hamiltonian answer above is that the $(\pi_\Psi, R = 0)$ pair is first-class, generating the gauge transformation $\delta\Psi = \varepsilon$, and contributes zero propagating DOF.

The body's "no fifth force" claim is therefore the narrow statement that no propagating-scalar-mediated fifth force exists in F01: the Ψ sector contributes no propagating mode that could mediate a Yukawa-type interaction between matter test particles. The $(1 + f)$ rescaling of the gravitational source is a matter-side coupling, not a fifth-force mediator; it has no propagator and no finite range. Conversely, the gradients of Ψ that do exist in space and time — driven by matter sources via the trace equation — do enter local observables (the LLR $\dot{G}/G$ prediction in Sec. III is exactly such an effect) through boundary-condition drag on bound systems embedded in an evolving cosmological background.

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