Day 2 Exercises: Problem Solving Sessions 1 and 2

This handout combines both Day 2 problem-solving sessions.

Session 1

This session develops fluency with vector operations, matrix-vector products, and dimension checks. Focus on correct setup before arithmetic.

1. Compute $\begin{bmatrix} 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$.

2. Compute $2\begin{bmatrix} 1 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

3. For $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$, compute $A\mathbf{x}$.

4. Let $M = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 3 & 5 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$. Compute $M\mathbf{u}$.

5. Is AB defined if A is 2×3 and B is 3×2? What is the size of AB?

6. Is BA defined for the same matrices? If yes, what size? If no, explain quickly.

7. Snack stand model (set up only): Let x be the number of granola bars ($3 each) and y be the number of fruit cups ($5 each). If 120 total items were sold for $480 total revenue, write (i) the two equations in x, y, and (ii) the matrix form $A\mathbf{x} = \mathbf{b}$.

8. Write the matrix form $A\mathbf{x} = \mathbf{b}$ for your system in Problem 7.

9. Give one real meaning of a 2-entry vector in school life.

10. Explain why a dimension check should happen before arithmetic.

Session 2

This session practices determinant, inverse (2x2 only), and Gaussian elimination in both equation and matrix form. Focus on method choice, arithmetic accuracy, and checking solutions.

1. Compute $\det\left(\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}\right)$ and state whether the matrix is invertible.
2. Compute $\det\left(\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}\right)$ and state whether it is invertible.
3. Find the inverse of $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.
4. Find the inverse of $\begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$.
5. Solve by elimination in equation form: $\begin{cases} x + 2y = 11 \\ 3x - y = 4 \end{cases}$.
6. Solve by elimination in equation form: $\begin{cases} 2x + y = 7 \\ x - y = 2 \end{cases}$.
7. Solve in matrix form by row operations: $\left[\begin{array}{cc|c} 1 & 1 & 6 \\ 2 & -1 & 3 \end{array}\right]$.
8. Solve in matrix form by row operations: $\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 3 & 1 & 7 \end{array}\right]$.
9. Reflection: in one sentence, explain why $\det(A) = 0$ means the 2x2 system may fail to have a unique solution.