Day 6 Exercises: Problem Solving Sessions 1 and 2

This handout combines both Day 6 problem-solving sessions.

Session 1

Compute exact probabilities, interpret expected value, and summarize data with basic statistics.

1. Roll one die: find $P(\text{even})$.

2. Flip 3 fair coins: find $P(\text{exactly 2 heads})$.

3. For a fair die, find $P(\text{not even})$ using complement rule.

4. Game pays 10 with probability 0.2 and 0 otherwise. Find expected payout.

5. A raffle has 1 winning ticket out of 200 tickets and pays \$500. What is expected value of one ticket from payout only?

6. Explain in one or two sentences why expected value may not be an actual outcome.

7. For data $\{4, 6, 8, 12\}$, compute the mean.

8. For data $\{2, 2, 6, 6\}$, compute the population variance and population standard deviation.

9. For sample data $\{5, 7, 9\}$, compute the sample variance and sample standard deviation.

Session 2

Design simulation plans, evaluate simulation quality, and connect to linear least squares.

1. Design a simulation to estimate $P(\text{at least one head in 4 flips})$ and list the trial steps clearly.

2. You run 200 trials and observe event count 78. What is your estimate?

3. You run 5000 trials and observe event count 1970. What is your estimate? Compare with Problem 2.

4. Briefly explain why two simulation runs (for the same event) can give 0.49 and 0.52.

5. Give one process where simulation is better than hand counting, and explain why.

6. For points $(1, 2), (2, 3), (3, 5)$, verify that $a = 1.5$ and $b = 1/3$ satisfy both normal equations.

7. For line $\hat{y} = 1.5x + 1/3$, interpret slope and intercept in context of these data.

8. Compare total squared residuals for lines $\hat{y} = x+1$ and $\hat{y} = 1.5x+1/3$ on points $(1, 2), (2, 3), (3, 5)$. Which fits better?