Lecture 5

Basics of mass conservation

-In this lecture we assume that the mass is well mixed and we are not paying attention to the spatial aspect of transfer

Conservation Laws:

Conservation laws are foundational principles in physics that state certain physical quantities remain constant throughout the process of any physical interaction or transformation. These laws are directly tied to the symmetries of nature.

Mass: The total mass of an isolated system remains constant. (chapter 1 and 2)

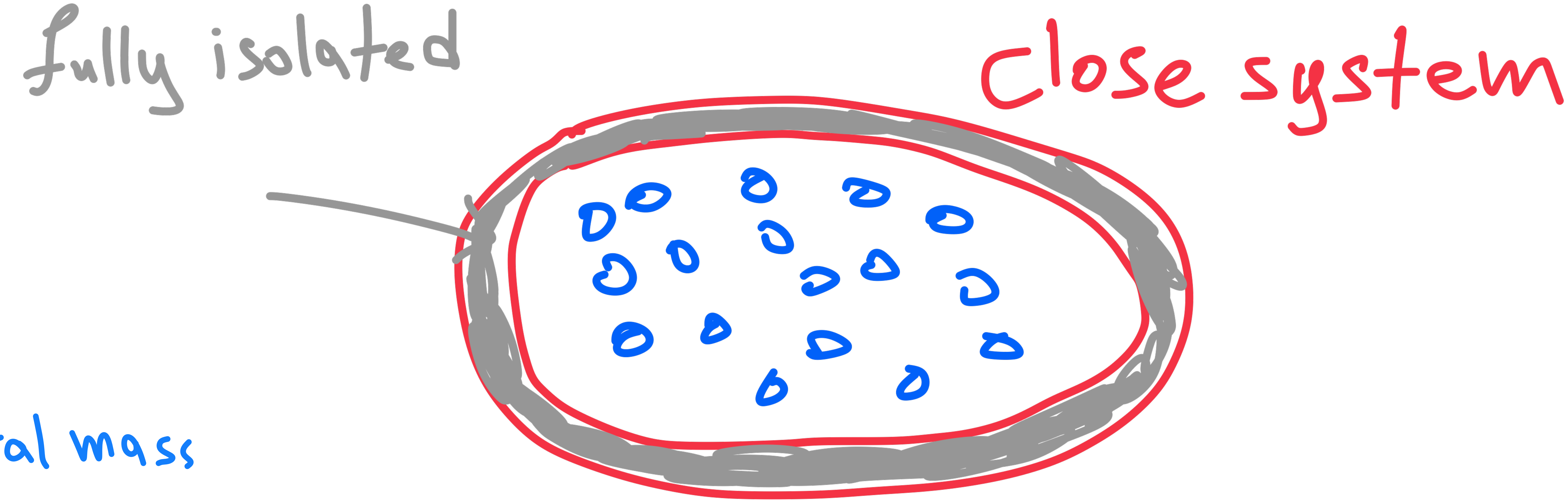
Momentum: The total momentum of an isolated system remains constant. (chapter 3)

Energy: The total energy of an isolated system remains constant. (chapter 4)

Charge: : The total charge of an isolated system remains constant.

What is a close system?

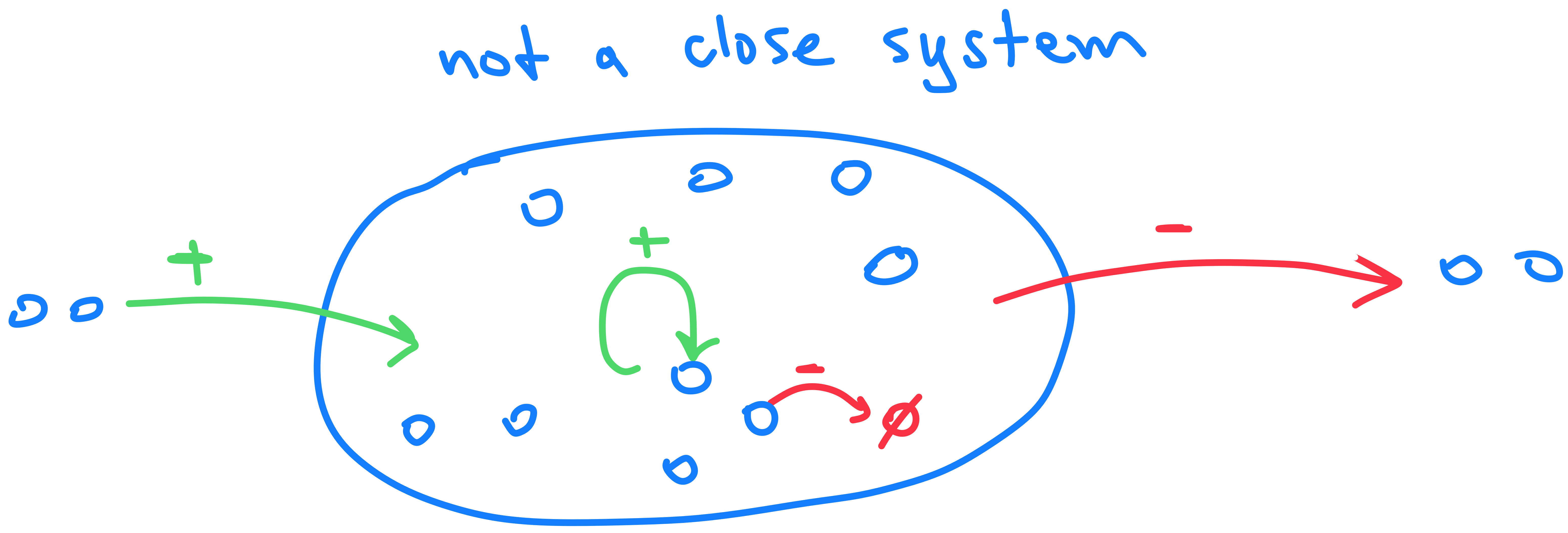
Close system is a system that has no physical interaction or exchange with the outer world. For example no matter is allowed to enter or leave:



total mass

$$M_{\text{closed sytem}} = \text{constant} \Longrightarrow \frac{dM_{\text{closed system}}}{dt} \equiv \dot{M}_{\text{closed sytem}} = 0$$

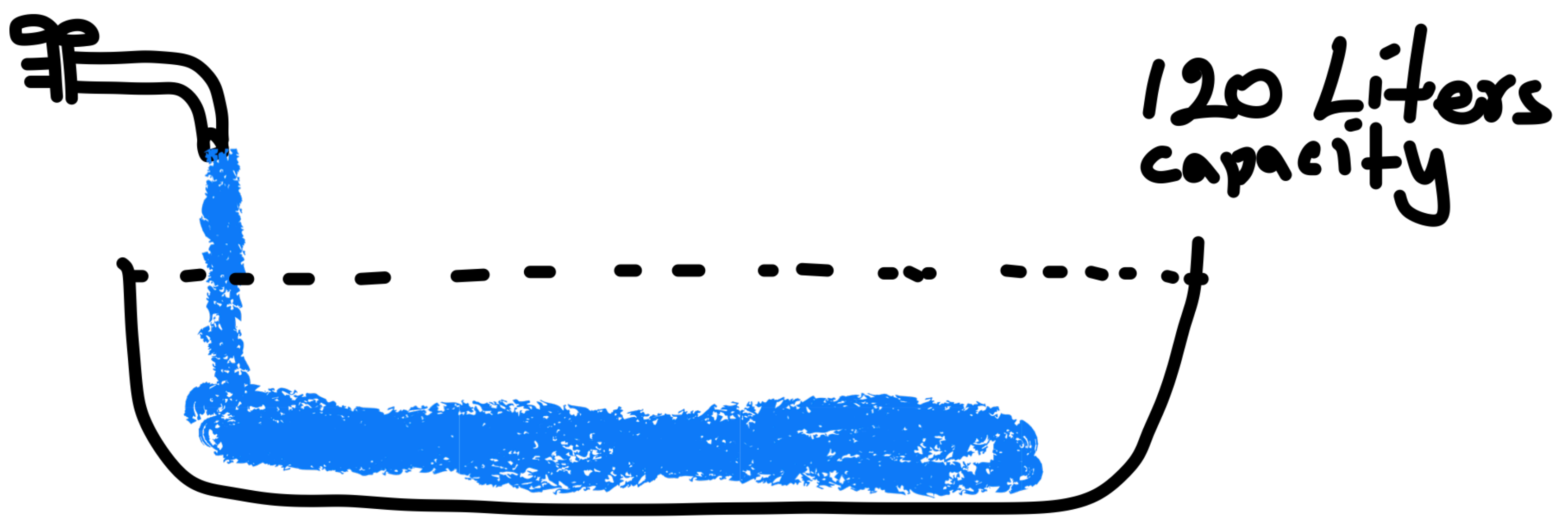
However, the assumption of being fully isolated is too restrictive and not realistic. In reality (for example, in biological systems), there is always some exchange with the surroundings.



Now the total mass M of the blue species in the system is not constant anymore, but we can can use the concept of conservation of mass to write down an mass conservation equation.

But before moving further, we introduce the concept of '**rate**' with the following example:

Example 1: We know that the following bathtub is filled with water in 10 minutes.



Q1: what is the rate of water from the tap? in $\left(\frac{kg}{s}\right)$

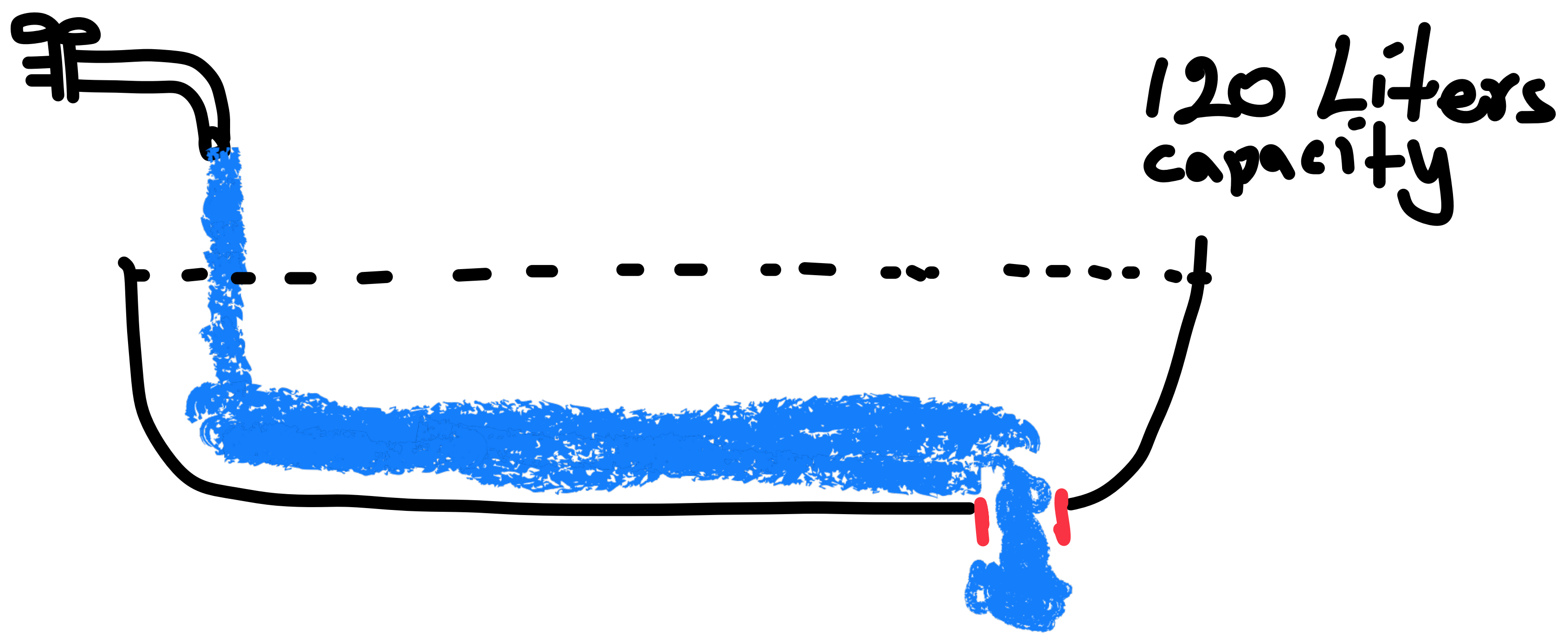
Solution: • we know that $\rho_w = \frac{1000\ kg}{m^3} \Rightarrow \frac{1000\ kg}{1000\ L} = 1\ \frac{kg}{L}$

• therefor total mass needed: $120\ L \times 1\ \frac{kg}{L} = 120\ kg$

• $$\boxed{Rate_{tap} = \frac{\text{mass transferred}}{\text{time}}} \Rightarrow Rate_{tap} = \frac{+120\ kg}{10 \times 60\ s} = +0.2\ \frac{kg}{s}$$

Q2: When the tap is off, and we open the drain, it takes 5 minutes for the tub to get emptied.

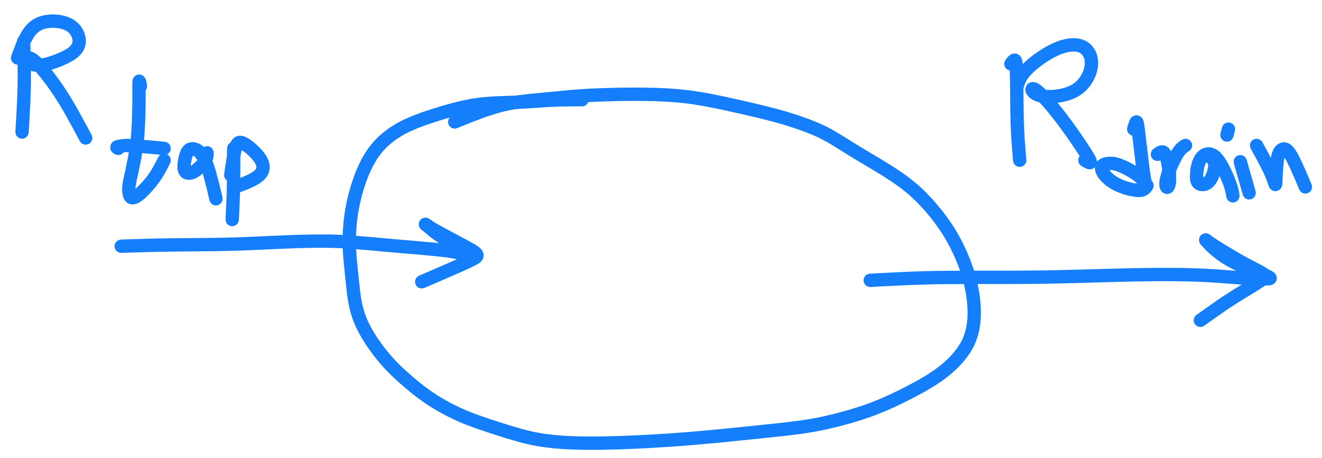
What the rate of accumulation of water in the tub when both the tap and the drain are working?



Solution: first: we can find the rate of draining (assuming is constant)

$$Rate_{drain} = \frac{mass}{time} = \frac{120\,kg}{5 \times 60} = 0.4\ \frac{kg}{s}$$

second: now if the tap and drain are at work at the same time:



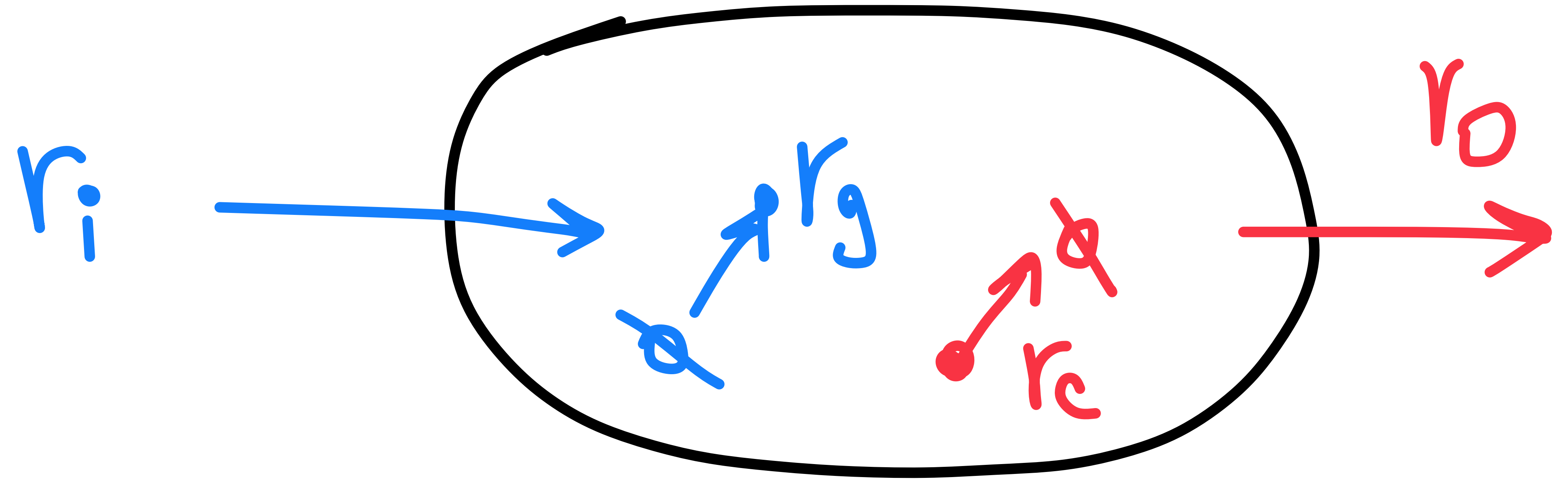
$\dot{m}$ is negative

$$\frac{dM_{water}}{dt} = \overset{0.2}{\cancel{R_{tap}}} - \overset{0.4}{\cancel{R_{drain}}} = -0.2\ \frac{kg}{s}$$

Mass Balance principle:

By applying the conservation of mass of a certain species to an open system, we derive the mass balance principle:

system of interest
that contain x:



r_i : input rate of x
r_o : output rate of x
r_g : generation rate of x
r_c : consumption rate of x

net rate of accumulation of x : $$\frac{dm_x}{dt} = +r_i + r_g - r_o - r_c$$

A few remarks before we proceed:

-This is a fundamental equation. Throughout the rest of this lecture and the next two, we will explore various applications of this principle.

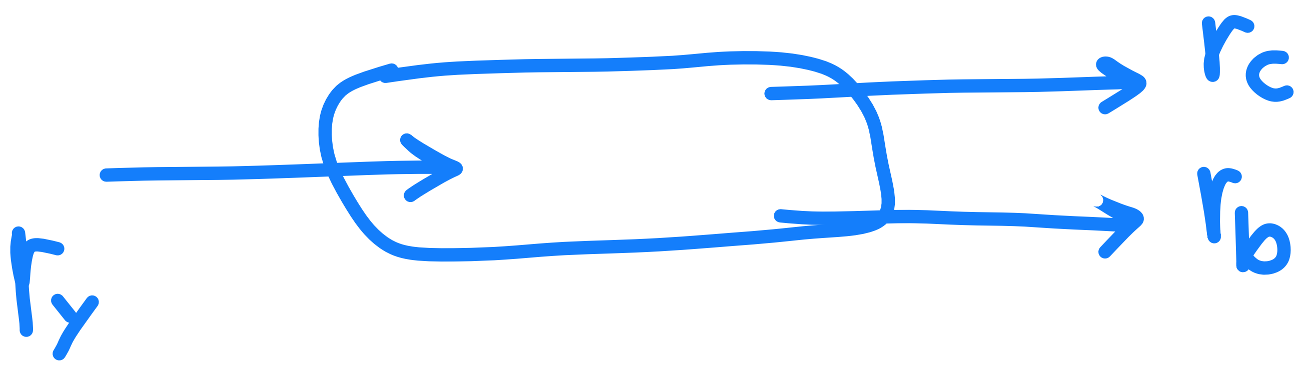
-When applying the mass balance principle, it is helpful to draw the system, including its inputs, outputs, consumption, and generation rates, to have a visual reference before formulating the equations.

-We’ve written the mass balance principle for a single species (x), but it can be applied to two or more species, as we will demonstrate in upcoming examples.

Example 2: In a milk processing plant, curds (yoghurt) is also manufactured. A continuous centrifuge needs to be used to produce buttermilk, that contains less fat, and cream, that contains high fat. If the average need is to process 48,000 Kg of curd containing 4% fat in an 8 hour shift, continuously, into a buttermilk stream that contains 0.5% fat, and a cream stream with 50% fat, what flow rates should the further processing units for buttermilk and cream be equipped to handle, on the average?

hint: assume everything is at steady state

Solution: ① drawing the system:



② putting together what we know: $r_y = \frac{\text{mass}}{\text{time}} = \frac{48000\,\text{kg}}{8\ \text{h}} = 6000\,\frac{\text{kg}}{\text{h}}$

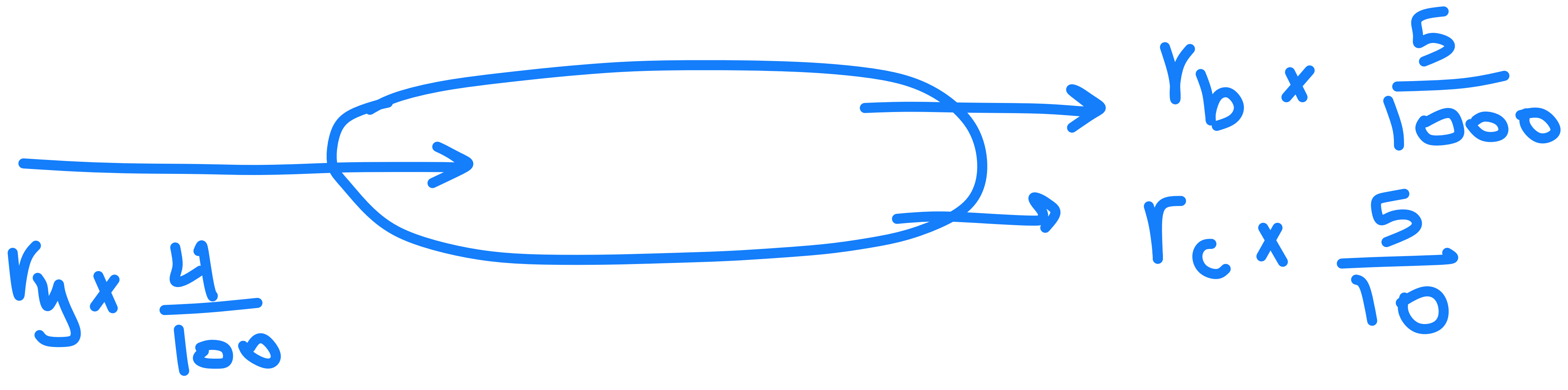
③ writing down the mass balance for total mass:

because of steady state

$$\cancel{\frac{dm_{total}}{dt}}^{\,0} = r_y - r_c - r_b \Rightarrow r_c + r_b = 6000\,\frac{\text{kg}}{\text{h}} \quad (i)$$

④ also we can do above for the fat content:

for fat:



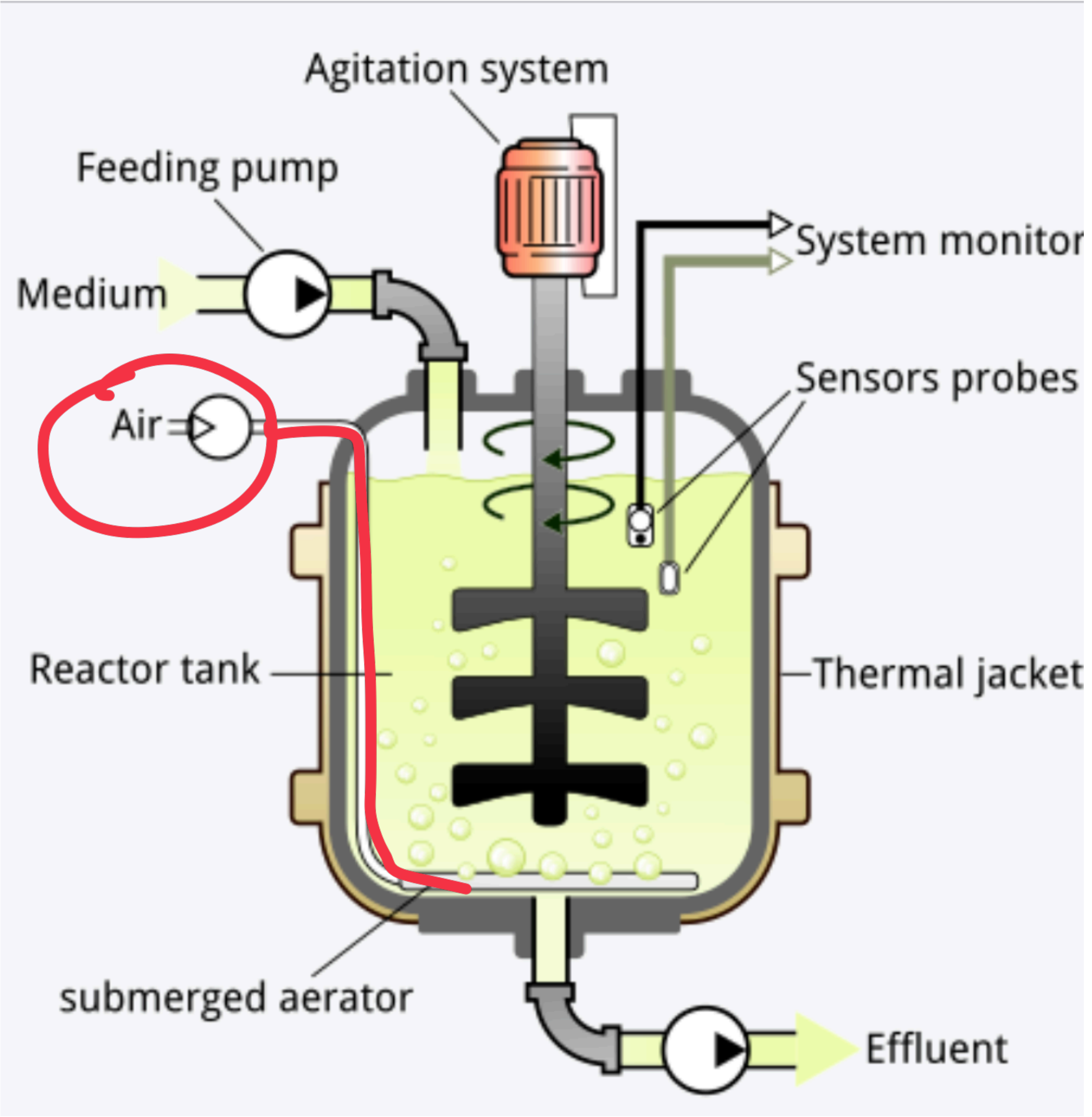
$$\frac{dm_{fat}}{dt} = \left(\frac{4}{100} r_y\right) - \left(\frac{5}{1000} r_b\right) - \left(\frac{5}{10} r_c\right) = 0 \quad s.s$$

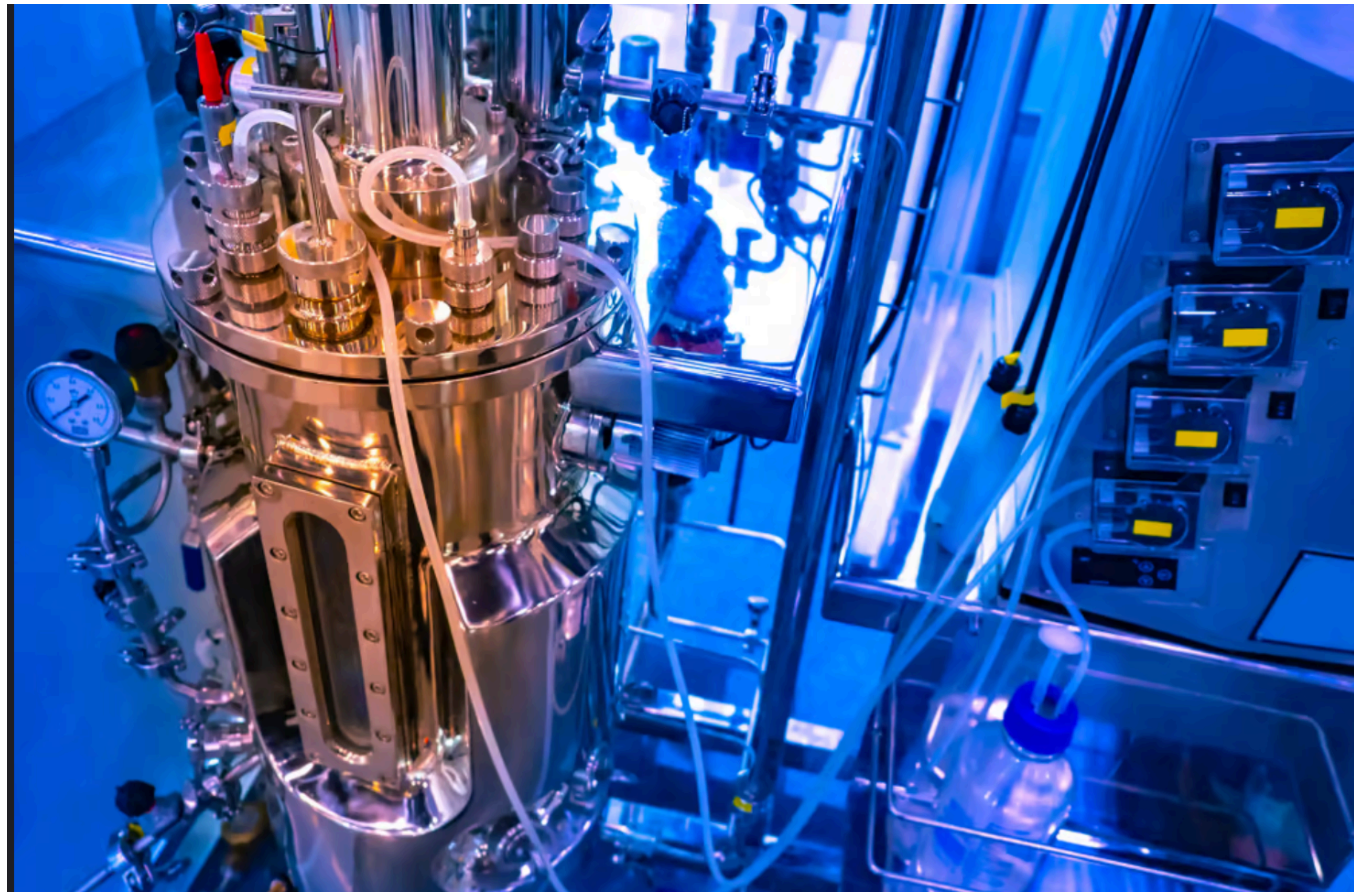
$$\Rightarrow \quad \frac{5}{10} r_c + \frac{5}{1000} r_b = \frac{4}{100} \times 6000 \, \frac{kg}{h} \quad (ii)$$

$$i, \; ii \Rightarrow \quad \boxed{\begin{aligned} r_c &\simeq 5574 \, \frac{kg}{h} \\ r_b &\simeq 424 \, \frac{kg}{h} \end{aligned}}$$

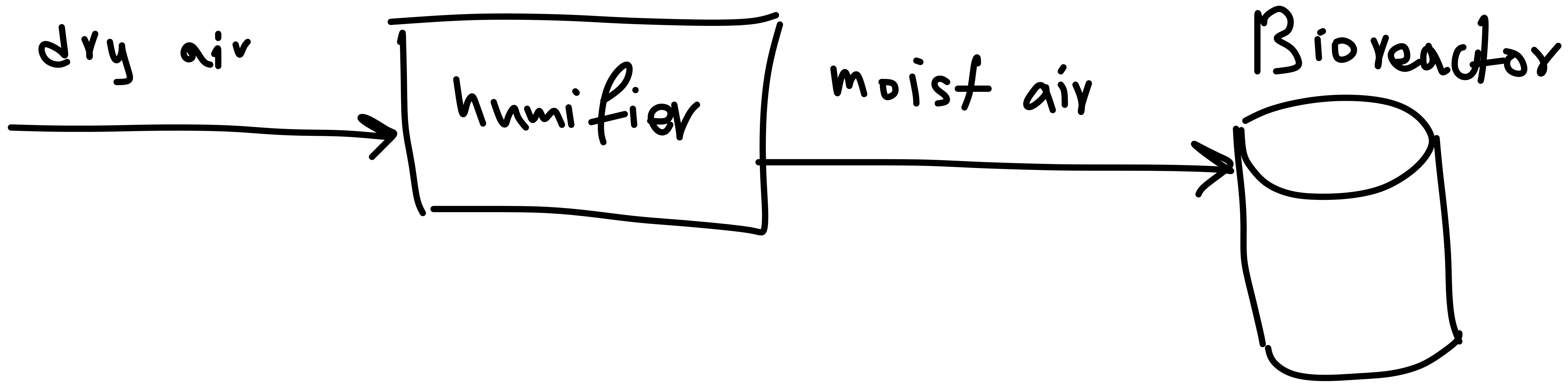
Application of Mass Balance principle in designing the air supply of bioreactors:

A bioreactor is a specialized vessel or container designed to cultivate microorganisms, cells, or tissues under controlled conditions. It not only supports various applications such as fermentation, cell culture, and biochemical production but also provides essential nutrients, oxygen, or carbon dioxide needed for optimal cell growth and productivity.



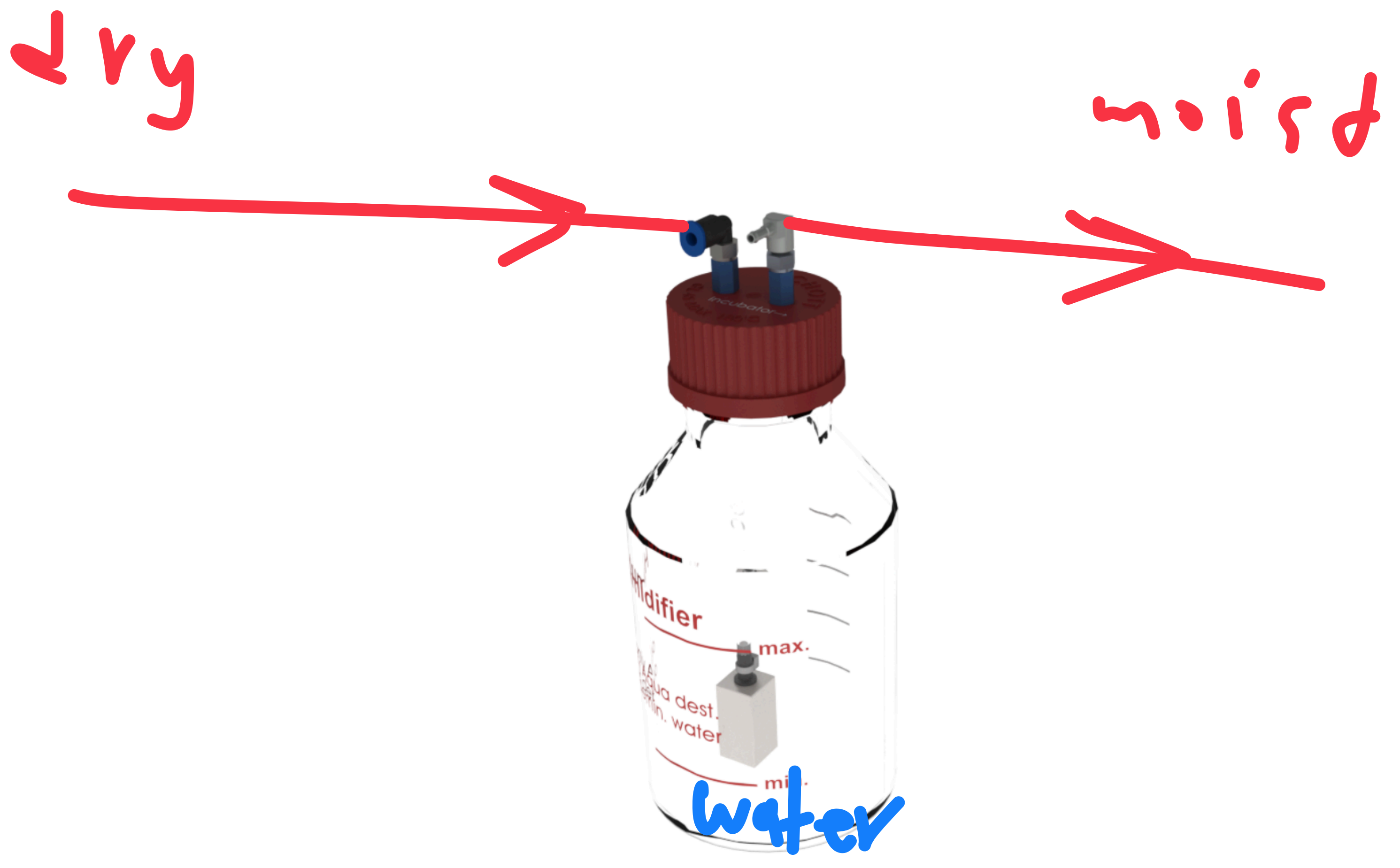


To supply the oxygen or carbon dioxide needed for optimal cell growth and productivity, bioreactors typically have an air supply system. However, this poses a challenge: if dry air is continuously supplied to the bioreactor, the liquid medium can evaporate, leading to changes in the concentration of nutrients or cytokines. To address this issue, engineers install a humidifier in the air supply line to the bioreactor. The humidifier adds moisture to the air (ideally up to saturation), which helps prevent the evaporation of the liquid medium. In the following example, we will explore how the principle of mass balance can be used to determine the appropriate airflow to a bioreactor through a humidifier.

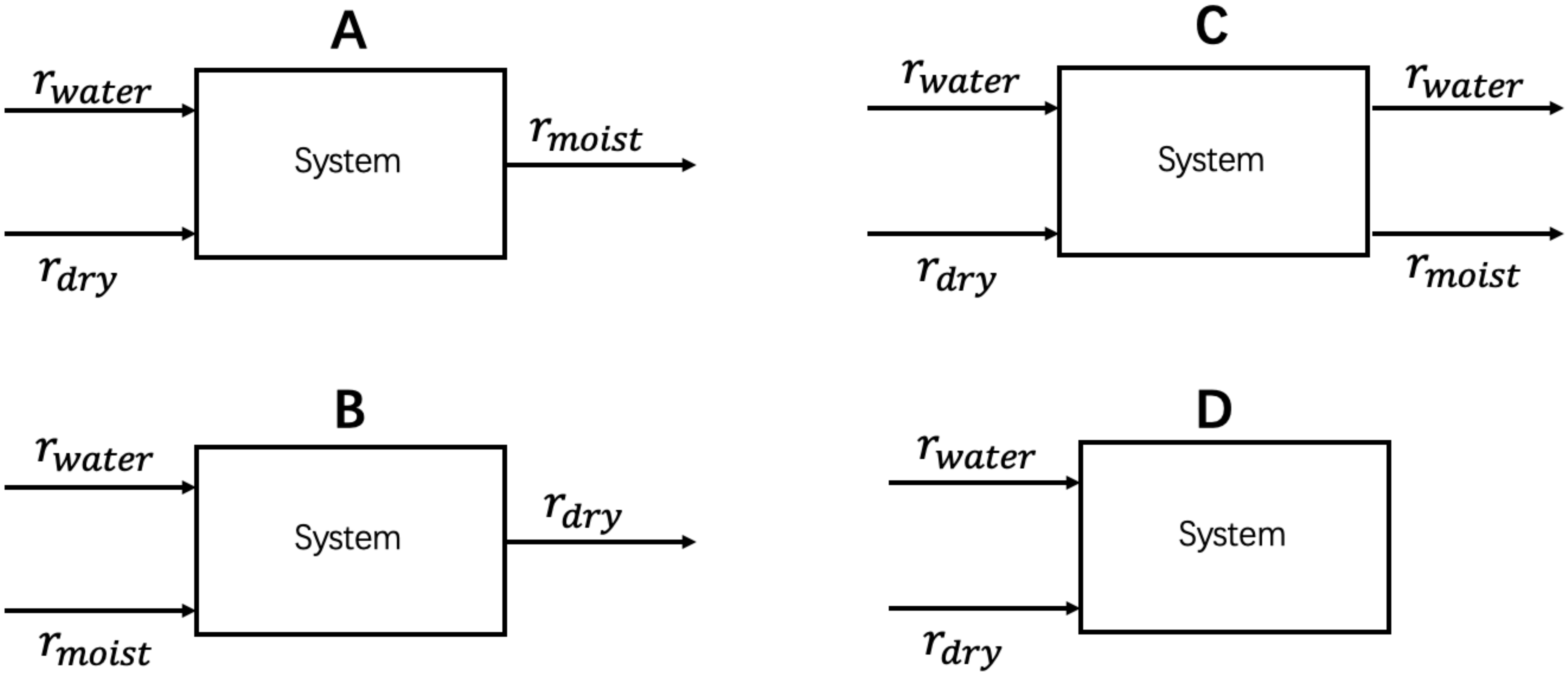


Example 3: In a humidifier, dry air is mixed with liquid water to create humidified air. The liquid water flow rate into the humidifier is 1 mole per minute. The air leaving the humidifier needs to contain 5 mole% oxygen for use in a bioreactor.

hint: 1. If we assume that dry air contains 21% oxygen and the rest is nitrogen, calculate the molar flow rate of dry air (in moles per minute) that should be supplied to the humidifier. 2. Everything is at steady state

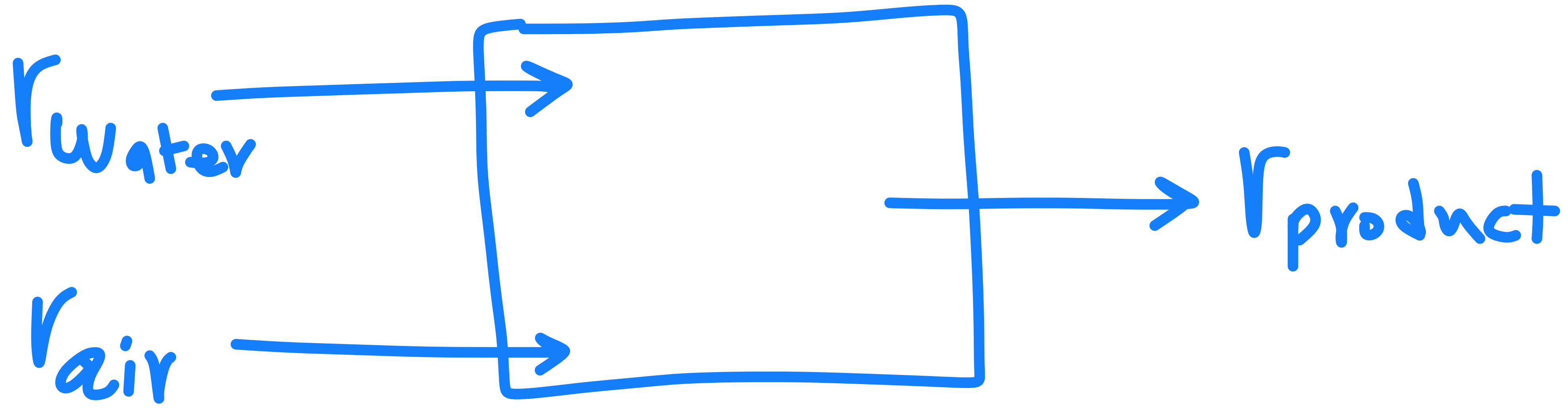


iClicker: Which of the following is the correct drawing of the system?



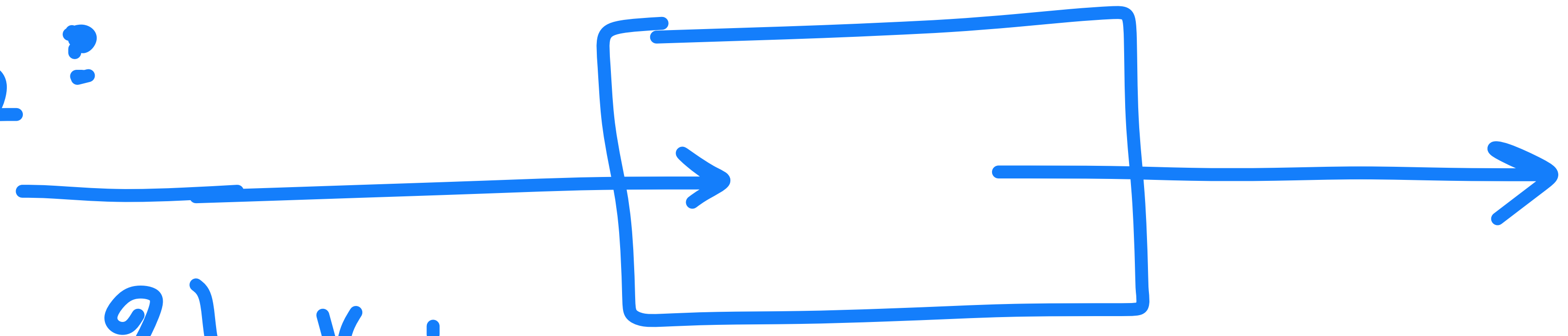
Solution:

① drawing the system: for total mass



② writing mass balance for total mass: $r_a + r_w - r_p = 0$ (s.s) ⓘ

③ drawing the sys. for only O_2:



$r_{O_2}^{in} = \frac{21}{100} r_{air}$

$r_{O_2}^{out} = \frac{5}{100} r_p$

Question asked for it

④ mass balance for only O_2: $r_{O_2}^{in} - r_{O_2}^{out} = 0$ (S.S)

$1 \frac{mol}{min}$

$$\Rightarrow \frac{21}{100} r_a - \frac{5}{100} r_p = 0 \overset{ⓘ}{\Rightarrow} \frac{21}{100} r_a - \frac{5}{100}(r_a + r_w) = 0 \Rightarrow r_a = 0.3 \frac{mol}{min}$$

We can summarize what we learned from these examples:

We can apply the mass balance principle to a variety of problems.

To tackle a problem using this method, start by drawing the system of interest. Identify what is entering and leaving the system and determine if anything is being consumed or generated.

Next, write down the mass balance equation. Determine if the system is at steady state (s.s). If it is, then dm/dt=0.

Apply the mass balance principle to both the total mass and individual species to solve the problem.

In the next lecture:

we will introduce another important application of mass balance: studying how drugs behave in our body (pharmacokinetics). Specifically, we will move beyond the steady-state assumption, making the analysis more interesting as the mass balance equations turn into ordinary differential equations (ODEs) where dmdtdtdm is no longer zero.