Geometry

Problem-Solving Session (Junior)
11 Feb 2022

---

1. A point D lies inside triangle ABC. Let A_1, B_1, C_1 be the second intersection points of the lines AD, BD, CD with the circumcircles of BDC, CDA, ADB respectively. Show that

$$\frac{AD}{AA'} + \frac{BD}{BB'} + \frac{CD}{CC'} = 1$$

2. In triangle ABC with circumcircle Γ, the internal angle bisector of $\angle A$ intersects BC at D and Γ again at E. The circle with diameter DE meets Γ again at F. Prove that AF is a symmedian of triangle ABC.

3. Let Γ be the circumcircle of the triangle ABC. The circle ω is tangent to the sides AC and BC, and it is internally tangent to the circle Γ at the point P. A line parallel to AB intersecting the interior of triangle ABC is tangent to ω at Q. Prove that $\angle ACP = \angle QCB$.

4. Let D be the foot of altitude from A to BC in a triangle ABC, and let X and Y be the feet of altitude from D to AB and AC respectively. Denote Z as the orthocentre of ABD. Prove that (XYZ) is tangent to the circle centred at A with radius AD.

5. A convex quadrilateral $ABCD$ satisfies $AB \cdot CD = BC \cdot DA$. A point X is chosen inside the quadrilateral so that $\angle XAB = \angle XCD$ and $\angle XBC = \angle XDA$. Prove that $\angle AXB + \angle CXD = 180$.

6. Let ABC be a triangle with circumcircle ω and ℓ a line without common points with ω. Denote by P the foot of the perpendicular from the center of ω to ℓ. The side-lines BC, CA, AB intersect ℓ at the points X, Y, Z different from P. Prove that the circumcircles of the triangles AXP, BYP, CZP have a common point different from P or are mutually tangent at P.

Sources

1. BAMO 2008
2. Russia 2009
3. EGMO 2013
4. I don't know
5. IMO 2018
6. IMOSL 2012

 Prepared by Tristan Chaang.