

The Optimism Principle: A Unified Framework for Optimal Robotic Network Deployment in An Unknown Obstructed Environment

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Abstract—We consider the problem of deploying a team of robots in an unknown, obstructed environment to form a multi-hop communication network. As a solution, we present a unified framework, onLinE rObotic Network formAtion (LEONA), that is general enough to permit optimizing the communication network for different utility functions in non-convex environments. LEONA adopts the principle of “optimism in the face of uncertainty” to allow the team of robots to form optimal network configurations efficiently and rapidly without having to map link qualities in the entire area. We demonstrate and evaluate this framework on two specific scenarios concerning the formation of a multi-hop communication path between fixed end-points: one minimizing the total path cost, and another maximizing the bottleneck communication rate. Our simulation-based evaluation shows that the use of the optimism principle can significantly reduce resources spent in exploring and mapping the entire region prior to network optimization. We also present a mathematical modeling of how the searched area scales with various relevant parameters in each case.

I. INTRODUCTION

We consider the problem of deploying a team of robots in an unknown, obstructed environment to form a multi-hop communication network. In obstructed environments, such as inside buildings or outdoors in forested areas, not only is there a concern with moving efficiently through the environment while avoiding obstacles and walls, the communication channels are also cluttered and highly varying due to signal attenuation (shadowing), as well as multi-path scattering (fading).

With the exception of some recent work (e.g., [1]), most research to date on robot network deployments has assumed idealized communication models such as the unit disk model. Further, the problem of network formation has typically been treated assuming convex utility functions that can be optimized through localized potentials [2] and greedy distributed gradient descent algorithms [3]. While radio propagation models such as the simple path loss model can yield such convex optimization problems in unobstructed environments, the presence of walls introduces non-convexities. We argue in this work that a more tractable perspective is to consider a graph theoretic formulation in which vertices correspond to the set of all possible (discretized) locations for the robots in the given environment, and there are labels on the edges between the vertices that indicate the RF path loss

(or a monotonic function thereof) between the corresponding positions. The network formation problem becomes one of finding subgraphs of this graph (in this work we focus on path formation, but the approach could be generalized further to trees or other graph structures) that satisfy the constraint that the number of nodes in the selected subgraph must be equal to or less than the number of available robots, and maximize a desired utility function. Such a problem could then be solved using a suitable centralized or decentralized graph algorithm (e.g., Bellman-Ford algorithm to compute the minimum cost path) to yield the optimal configuration of the robotic network.

While general enough to handle many non-convex network optimization problems such as minimum cost path formation in environments with arbitrary link qualities (which is not possible using a purely distributed potential-based approach in obstructed environments), the implementation of this graph-theoretic approach in practice faces one significant hurdle: it requires prior mapping of the area to determine the link qualities for every pair of locations, which could be prohibitively time-consuming. We address this challenge with an innovative online, iterative, approach that is based on the principle of “Optimism in the Face of Uncertainty” inspired by similar ideas in the domain of online learning and multi-armed bandits [4].

Contributions. We first propose a unified framework, called LEONA (for onLinE rObotic Network formAtion), that is general enough to allow optimizations for different utility functions in non-convex environments. The crux of the approach is the following. At each stage, an optimistic prediction of the graph edge weights (link qualities) is maintained, i.e., it is ensured that the predicted link quality is no worse than the true link quality. And at that iteration, the robots move through the environment to the network configuration computed to be optimal for that predicted graph. As they move through the environment, the robots collaborate to take additional measurements of the link qualities. These measurements, and potentially, additional inferences derived from these measurements¹, are used to update the predicted graph to a new set of values, that are still ensured to be optimistic (though now a bit “closer” to the true graph because of the updates). The iterations continue until the robots are at a configuration whose measured utility

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¹For example, it may be reasonable to assume that if a particular pair of locations has a certain path loss indicative of significant attenuation due to a wall, then links corresponding to all locations that fall on or even near the same line as those locations must experience at least that much loss due to attenuation as well.

is as good as the best possible configuration in the current predicted graph, which can then be shown to be provably optimal because of its optimistic bias.

Second, we demonstrate and evaluate how this general framework, i.e., LEONA, works in unknown environments for two specific scenarios concerning the formation of a multi-hop robotic relay path between two fixed end-points in an obstructed environment, that differ in the path utility functions: in one we seek to minimize the total path cost, and in the other, we seek to maximize the bottleneck rate (i.e., the end-to-end data rate). Our simulation-based evaluation shows that the use of the optimism principle can significantly reduce the time spent in exploring and mapping the entire region *a priori* before the optimal network configuration is constructed. We also present a mathematical modeling of how the searched area scales with various relevant parameters in each case.

The rest of the paper is organized as follows: in section II, we present related work; in section III, we define the general utility maximization problem involved in robotic network formation in the environment; in section IV we present the general LEONA framework, while in the succeeding sections V and VI we present two case studies for specific utility functions pertaining to minimization of expected number of transmissions per successfully delivered packet (ETX) and maximization of bottleneck-rate respectively, including both theoretical analysis and simulation-based evaluations; Finally, we present a summary and discuss future work directions in section VII.

II. RELATED WORK

Networked robots have been well investigated in recent years, especially in the field of flocking [5], formation control [6] and swarming [7]. The key idea is to allow a team of robots to cooperate and coordinate in a networked autonomous system to perform a specific task. Therefore, communication among robots plays a significant role in enabling cooperation. While the disk communication model has been used in many of these previous works to maintain connectivity among robots, such a simple model does not fit in a realistic environment, and thus degrades the performance of previous work when applied in reality.

Recently, there has been growing body of work considering more realistic communication performance in networked robots, referred to as Communication-Aware Robotics. Researchers in [8] have studied integrity problems where controllers of robots are designed while considering maintaining a desired transmission rate at the same time. A robotic router formulation problem has been studied in [1], where an optimal configuration of robots is formulated to maintain a maximized successful reception rate in realistic communication environments. ETX has been taken into consideration in [9], where researchers design a hybrid architecture to allow robots optimally configured so that each flow has a minimized ETX in a multiple flow network. But because of the convexity of utility functions each of these papers uses, they can apply potential functions or greedy gradient descent

algorithms, which are implicitly based on the assumption there is only one extreme solution. This assumption no longer holds when the existence of noise sources or obstacles introduces non-convexities.

Controlling a team of robots in a network becomes more challenging when there are obstacles in the environment. Most work designs the controllers of robots under the assumption that the obstructed environment is known *a priori*. This would allow researchers to explicitly add obstacle avoidance by utilizing either linear constraints [10] or artificial potentials [11]. Moreover, when communication-oriented performance is taken into consideration, not only obstacles can block robots' movements, they can also cause signal attenuation. Few works [12], [14] consider wall attenuation when studying robots' coordination. More difficulties arise when the environment is unknown, which further requires robots to take measurements and explore the environment.

In [13], a measurement-based mapping is computed in each spatial direction between a robot's current position and the received signal strength regardless of the robots' environment and this is used to obtain a quadratic optimization yielding the best locations for a set of robotic access points to serve a set of (possibly mobile) clients. However that work does not consider the problem of forming a general utility-optimized multi-hop communication network among the robotic nodes. Another problem similar to the one we address is studied in [14] by proposing an algorithm for maintaining end-to-end network connectivity for a team of robots. They jointly find robot configurations with wireless network routing. However, in order to build the configuration space, the environment has to be known *a priori*.

To our knowledge, this is the first work to present a mechanism for rapid optimal multi-hop network configuration by a team of robots in an unknown realistic RF environment with obstructions, where the problem is non-convex and not amenable to solution using standard potential-based approaches.

III. PROBLEM FORMULATION

We consider a team of $m \geq 3$ robots performing a task in an unknown walled environment. Robot 1 works as a source that transmits information to a destination robot m . Our goal is to find an optimal configuration of the remaining relay robots so that they can form a multi-hop communication path from the source to the destination with optimized performance.

A. Link Quality Metric

The walled environment is represented as a 2-D $L \times L$ grid and each pixel in the grid can either be a possible location for a robot or is occupied by a wall. Among all the m robots, robot 1 and robot m work as a source-destination pair, which are static and their positions are known *a priori*. The rest of the robots can move around in the space to enable and improve the communication between the source and destination. When a robot i communicates with robot j , the strength of signal decreases as it travels through air.

Moreover, if there are walls between the communication pair, additional signal attenuation can occur. Taking signal attenuation caused by travelling distance and walls into consideration, the received signal (in dB) at the receiver j from transmitter i can be expressed as ([15]):

$$Pr_{i,j} = Pr_0 - 10\eta \log\left(\frac{d_{i,j}}{d_0}\right) - n_{i,j}W \quad (1)$$

where $d_{i,j}$ is the distance between robots i and j ; $n_{i,j}$ is the number of walls between them; Pr_0 is the received power strength at a reference point with a small distance d_0 from the transmitter; η is the path loss parameter indicating the rate at which the attenuation increases with distance; W is the attenuation effect of a single wall.

Let the noise power spectral density be N_0 and spectrum bandwidth be B , then the Signal-to-Noise Ratio (SNR) at the receiver j is defined to be:

$$\gamma_{i,j} = \frac{Pr_{i,j}}{N_0B} \quad (2)$$

We define the link quality metric $l_{i,j}$ of the communication link (i,j) as a strictly increasing function of the SNR at receiver j corresponding to transmitter i :

$$l_{i,j} = f(\gamma_{i,j}) \quad (3)$$

B. Mobility, Sensing and Environment Assumptions

Time is divided into discrete time steps of unit duration. At each time step, a mobile robot can move to one of its four neighbor positions (up, down, left, right) or stay at its current position. We assume each robot has the ability to sense and detect walls within one moving step range, which helps a robot avoid colliding with walls. The moving decision is made by each robot itself. We assume the unknown environment is connected, i.e., given any two pixels in the grid that are not occupied by walls, there always exists a path between them. This assumption ensures all the available positions in the environment could be reached by robots.

C. Objective Function

Our goal is to design an optimal configuration of mobile relay robots such that they finally form a communication path² connecting the source and destination with a maximized utility \mathcal{U} , which is a monotonic function of all link qualities:

$$\max_{(x^1, \dots, x^m) \in \mathcal{P}} \mathcal{U}(l_{1,2}, \dots, l_{m-1,m}) \quad (4)$$

where x^k (where $1 \leq k \leq m$) is the position of the k^{th} robot and \mathcal{P} is the set of all possible configurations.

²Configuration, communication path and solution are used interchangeably in the rest of the paper.

IV. ONLINE ROBOTIC NETWORK FORMATION (LEONA)

There are three challenges inherent in the problem we address: 1) The environment is unknown so that robots need to dynamically combine exploration with configuration optimization; 2) The signal attenuation caused by walls results in a non-metric space so that prior metric-based algorithms cannot be applied to our problem; 3) The objective function in general is not convex so that potentials and convex optimization methods do not work. Therefore, one natural question to ask is: Given an unknown environment, is it possible to find an optimal configuration without fully exploring the whole space? The answer is yes, and in the following, we propose a graph-based online approach, which is guaranteed to find the optimal solution with only partial exploration of the environment.

A high-level structure of the proposed unified framework LEONA is presented in **Algorithm 1** with related steps detailed in **Algorithm 2-5**. To begin with, each mobile robot maintains a communication graph, which is represented as a complete directed graph $\mathbb{G}(\mathcal{V}, \mathcal{E})$. The vertices in the graph are all pixels in the grid. And a directed edge $(i,j) \in \mathcal{E}$ represents a communication link with transmitter at pixel i and receiver at pixel j . The graph is complete in the sense that every two vertices are connected by a pair of directed edges. The weight of an edge (i,j) is set as an optimistic prediction of the link quality $l_{i,j}$. We assume robots can share environment information with each other through the communication path between the source and destination, thus all of them maintain the same knowledge of the environment.

The environment information includes SNR measurements and detected wall positions. Each robot constructs its communication graph $\mathbb{G}(\mathcal{V}, \mathcal{E})$ based on its current information. For each $(i,j) \in \mathcal{E}$, if its link quality has been measured, the edge weight is set as measured. Otherwise, the edge weight is predicted according to eqns. (1)-(3) based on explored walls' information. Thus, if all walls along link (i,j) are fully explored, the predicted edge weight is the same as the actual link quality; if there is still some wall information missing, the predicted edge weight is optimistic or overestimated. Based on the current communication graph \mathbb{G} with optimistic prediction, robots apply $FindPath(\mathbb{G})$ to find the best possible communication path. After moving to form the best possible communication path, robots take measurements to find actual link qualities along the path, and update their communication graph \mathbb{G} based on new information. They run $FindPath(\mathbb{G})$ again to find the best possible communication path. If the current communication path's measured utility is as good as the best possible path, the algorithm terminates. Otherwise, robots move to form the new best-possible communication path, and repeat above procedures until the termination condition is met.

Theorem 1: The robotic network configuration obtained from LEONA is optimal.

Proof: In LEONA, by construction, when updating the communication graph \mathbb{G} at each step based on measurements,

the predicted weight of each edge is always no worse³ than its actual weight. Thus the communication graph is always optimistic. When LEONA terminates, the actual utility of final configuration is at least as good as the best possible estimated configuration, which indicates the actual utility of the final configuration is as least as good as the actual utilities of all other possible configurations. Therefore, the final configuration is optimal. ■

Algorithm 1 Online Robotic Network Formation (LEONA)

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1: ▷ Initialization
2: Robots start from current initial positions  $p^* = (x^1, \dots, x^m)$  and initialize utility of the current path  $p^*$  as  $U^* = 0$ . Initialize the measured SNR set and detected walls' positions set as  $\mathcal{S} = \phi$  and  $\mathcal{W} = \phi$  respectively
3: ▷ Update communication graph
4:  $\mathbb{G} \leftarrow \text{UpdateGraph}(\mathbb{G}(\mathcal{V}, \mathcal{E}), \mathcal{S}, \mathcal{W})$ 
5: ▷ Find best possible path and its utility
6:  $(p, U) \leftarrow \text{FindPath}(\mathbb{G})$ 
7: while  $U^* < U$  do
8:   ▷ Robots move to form the best possible path
9:    $(p^*, \mathcal{W}) \leftarrow \text{Move}(p^*, p)^4$ 
10:  ▷ Measure SNR of each link on current path  $p^*$ 
11:   $\mathcal{S} \leftarrow \text{MeasureSNR}(p^*)$ 
12:  ▷ Set the utility of current path  $p^*$ 
13:   $U^* \leftarrow \text{SetUtility}(p^*, \mathcal{S})$ 
14:  ▷ Probe walls on the current path  $p^*$ 
15:   $\mathcal{W} \leftarrow \text{ProbeWall}(p^*)$ 
16:  ▷ Update communication graph
17:   $\mathbb{G} \leftarrow \text{UpdateGraph}(\mathbb{G}(\mathcal{V}, \mathcal{E}), \mathcal{S}, \mathcal{W})$ 
18:  ▷ Find best possible path and its utility
19:   $(p, U) \leftarrow \text{FindPath}(\mathbb{G})$ 
20: end while

```

Remark 1: LEONA provides a unified framework to finding an optimal communication path that combines both environment exploration and configuration optimization. And it is general enough to permit optimizing for different utility functions in non-convex environments. In the following, we provide two specific case studies in which we apply LEONA with $\text{FindPath}(\mathbb{G})$ instantiated to find optimal configurations with respect to two different metrics.

³The predicted edge weight is the same as the actual weight if the link quality has been measured or all related wall information has been detected; overestimated if there is still some related walls' information missing.

⁴This $\text{Move}(p^*, p)$ algorithm follows the same idea of *Robotic Routing Protocol* [16], except for allowing a robot to choose using either *Righthand Traversal Rule* or *Lefthand Traversal Rule* in the *Recovery Mode*: A robot first greedily moves to its goal position in *Forwarding Mode*. When meeting a wall, it switches to move in *Recovery Mode* by following either *Righthand Traversal Rule* if its goal position is on the right side of its current position or *Move in Recovery Mode* by following *Lefthand Traversal Rule* if its goal position is on the left side of its current position to avoid walls. When arriving in a position that is closer to its goal position than the start of *Recovery Mode*, the robot switches back to *Forwarding Mode*. A robot can also record positions of walls Ω_r sensed during moving and update \mathcal{W} as $\mathcal{W} \cup \Omega_r$.

⁵An element in \mathcal{S} is denoted as $(i, j, \gamma_{i,j})$, where i is the transmitter's position, j is the receiver's position and $\gamma_{i,j}$ is the corresponding SNR of link (i, j) . An element in \mathcal{W} is a detected wall position.

Algorithm 2 UpdateGraph($\mathbb{G}(\mathcal{V}, \mathcal{E}), \mathcal{S}, \mathcal{W}$)

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1: for each directed edge  $(i, j) \in \mathcal{E}$  do
2:   if  $(i, j, \gamma_{i,j}) \in \mathcal{S}^5$  then
3:     Update the edge weight as  $l_{i,j} = f(\gamma_{i,j})$ 
4:   else
5:     Predict  $n_{i,j}$  according to  $\mathcal{W}$ 
6:     Calculate  $\gamma_{i,j}$  according to (1) and (2)
7:     Update the edge weight as  $l_{i,j} = f(\gamma_{i,j})$ 
8:   end if
9: end for
10: return  $\mathbb{G}$ 

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Algorithm 3 MeasureSNR(p^*)

```

1: for each receiver robot  $k$  on  $p^*$ , where  $k \in \{2, \dots, m\}$  do
2:   Measure its SNR with transmitter robot  $k-1$ :  $\gamma_{k-1,k}$ 
3:    $\mathcal{S} \leftarrow \mathcal{S} \cup (x^{k-1}, x^k, \gamma_{k-1,k})$ 
4: end for
5: return  $\mathcal{S}$ 

```

Algorithm 4 SetUtility(p^*, \mathcal{S})

```

1: for each communication link  $(x^{k-1}, x^k)$  along path  $p^*$ , where  $k \in \{2, \dots, m\}$  do
2:   Find its corresponding  $\gamma_{k-1,k}$  from  $\mathcal{S}$ 
3:   Update edge weight as  $l_{k-1,k} = f(\gamma_{k-1,k})$ 
4: end for
5:  $U^* \leftarrow \mathcal{U}_{(x^1, \dots, x^m)=p^*}(l_{1,2}, \dots, l_{m-1,m})$ 
6: return  $U^*$ 

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Algorithm 5 ProbeWall(p^*)

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1: for each robot  $k$  on  $p^*$ , where  $k \in \{1, \dots, m\}$  do
2:   Send a probing signal to neighbor robots respectively
3:   Detect the position of closest wall  $w_d$  that reflects probing signal back to robot  $k$ :  $\mathcal{W} \leftarrow \mathcal{W} \cup \{w_d\}$ 
4: end for
5: return  $\mathcal{W}$ 

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V. CASE STUDY I: FINDING MINIMIZED ETX PATH

One commonly used metric to measure the link quality is the expected number of transmissions per successfully delivered packet (ETX), which can be modeled as the inverse of the successful packet transmission rate $\lambda_{i,j}$ over a link (i, j) . Once specifics of a communication system (modulation and coding scheme, etc.) are fixed, $\lambda_{i,j}$ can typically be expressed in terms of either a *Q function* or an exponential function of $\gamma_{i,j}$ ([17]). We use *Q function* as an example here, and the corresponding successful packet transmission rate of link (i, j) is

$$\lambda_{i,j} = (1 - Q(\sqrt{c\gamma_{i,j}}))^h \quad (5)$$

where h is the length of a packet and c is some positive constant.

And the corresponding link quality, defined as $-ETX$, is

$$l_{i,j} = \frac{-1}{(1 - Q(\sqrt{c\gamma_{i,j}}))^h} \quad (6)$$

The ETX of a path is the summation of all link ETXs, and our goal is to let robots form a communication path between the source and destination which has the minimized ETX or, equivalently, maximized utility:

$$\max_{(x^1, \dots, x^m) \in \mathcal{P}} \sum_{k=2}^m l_{k-1,k} \quad (7)$$

The optimal configuration of robots can be found by applying LEONA with the $FindPath(\mathbb{G})$ implemented as running a *Shortest Path Algorithm* (e.g., Bellman-Ford algorithm) with a constraint that the total number of hops along a path is at most $m - 1$ if the weight of each edge is set to the associated ETX on the current graph \mathbb{G} .

A. Analysis of the Sufficient Searched Area

As the optimality of the LEONA is guaranteed, one additional question to ask is: How much space needs to be explored in order to find an optimal configuration?

Assume the distance and the number of walls between the source robot 1 and destination robot m are denoted as d and n respectively, then the following theorem provides an upper bound on the sufficient searched area.

Theorem 2: When applying LEONA, the size of the sufficient searched area \mathcal{A} that guarantees robots to find the optimal configuration with minimized ETX is

$$\mathcal{A} = O(10^{n \frac{W}{5\eta}} d^2) \quad (8)$$

Further, if the distribution of n walls along the straight line connecting the source and destination allows each communication pair to have same number of walls when robots are evenly spaced along the straight line, the sufficient searched area can reduce to $O(10^{\lceil \frac{n}{m-1} \rceil \frac{W}{5\eta}} d^2)$.

Proof: See [19]. ■

We omit the detailed proof due to space constraints. But briefly, it takes into account the following considerations. After the first run of $FindPath(\mathbb{G})$ in LEONA with the communication graph \mathbb{G} constructed under the assumption the environment is wall-free, robots move to be evenly spaced along the straight line connecting the source and destination⁶. And if this is not the optimal configuration, robots' exploration starts from here and expands outwards. If the searched area is so large that any communication path (even if without wall attenuation) outside the searched area has a worse ETX than that in the evenly-spaced-along-the-straight-line case, there is no need to search further and the optimal communication path is guaranteed to be found within the searched area. This results in a bound on the searched area.

⁶According to [9], it may be the case that not all relay robots are needed to form the path. However, the benchmark case, i.e., all relay robots are evenly spaced, can give no better ETX and thus, the sufficient searched area is still large enough to find the optimal configuration.

Remark 2: 1) The sufficient searched area scales polynomially with the distance between the source and destination and exponentially with the number of walls along the straight line connecting them. But any other wall that is not along the straight line has no effect on the size of the sufficient searched area; 2) When the distance and the number of walls along the straight line are fixed, the distribution of walls plays a crucial role in the size of searched area. The searched area becomes large when walls gather close to each other; And it becomes small when walls separately locate along the straight line; 3) Given an unknown environment, one possible way to reduce the size of searched area is to send more robots in the space which can provide a better chance to have walls separated among communication pairs, which suffers less signal attenuation and thus have a better ETX.

B. Simulation Results

We present numerical simulation results for a network containing 11 robots in a 50×50 environment. The robot 1 and robot 11, as the source and destination, are statically located at $(3, 3)$ and $(48, 48)$ respectively. The rest are mobile robots moving around with the purpose of formulating a communication path between the source and destination with minimized ETX. The initial locations of mobile robots are set to be as equally spaced as possible along the straight line connecting the source and destination⁷. We use $P_0 = -20$, $d_0 = 1$, $\eta = 3.3$, $W = 20$, $N_0 = 10^{-14}$, and $B = 2 \times 10^6$. When taking wall attenuation into account, the packet reception rate of a link (i, j) is well approximated by a sigmoidal function of distance as $\lambda_{i,j} = 1 - \frac{1}{1 + e^{-\alpha(d_{i,j} 10^{\frac{n_{i,j} W}{10\eta}} - \beta)}}$ ([9]), which considers different kinds of specifics of a communication system. We use this model in our simulations and set $\alpha = 0.2$ and $\beta = 8$.

We fix the shape and size of walls, and use the number of walls as an indicator of the complexity of an unknown environment. We randomly add walls in the space and the number of walls is varied as shown in Figure 1. We compare LEONA with an *Offline Algorithm*, which is guaranteed to find the optimal configuration. In the *Offline Algorithm*, robots first fully explore the environment to find the prior mapping of the area and build the corresponding communication graph \mathbb{G} . Then they apply *Shortest Path Algorithm* to find the optimal configuration. In addition to presenting the ETX of the optimal path, we also present the total number of moving steps robots take before finding the optimal path, which serves as an indicator of searched area size. As can be seen in the Figure 1, LEONA can always find optimal configurations; And as the number of walls (or the complexity of the environment) increases, the number of steps taken during exploration increases under both schemes, but LEONA takes far less.

In Figure 2, we present the optimal robot configurations obtained in an environment with 12 walls with single wall attenuation effect $W = 20$ in (a) and $W = 3.3$ in (b). In

⁷Mobile robots can initially start from any locations in the environment. And they can find the optimal configurations when LEONA terminates.

Figure 2 (a), wall attenuation is high so that the optimal configuration is a path free from walls, along which robots are roughly evenly spaced. In Figure 2 (b) the optimal path still has walls affecting it, since the wall attenuation is small which does not cause robots to move away from the straight line. However, robots are no longer evenly spaced and those communication robot pairs suffering from wall attenuation compensate by having shorter link distance. As can be seen from both cases, robots only explore part of the environment before finding the optimal configuration. One thing needs to be noted is that due to the fact that the utility in the ETX case is a unimodal (increasing-then-decreasing) function of the number of robots, not all available robots are always required in the optimal path configuration; though, generally, more robots are needed as the number of walls increases.

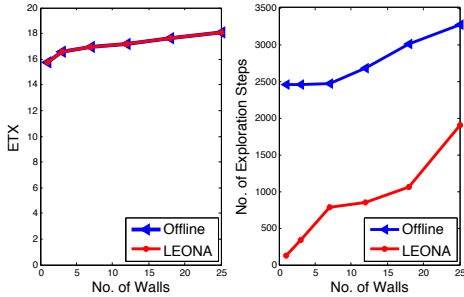
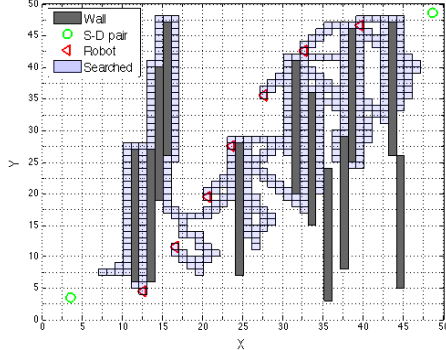
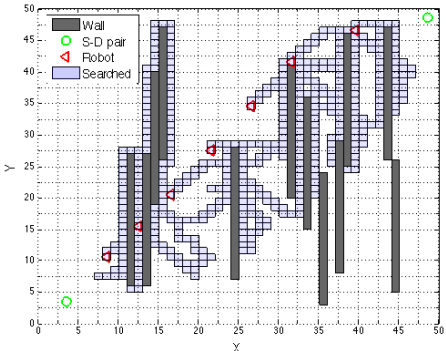


Fig. 1: ETX (left) and moving steps (right)



(a) Configuration with strong wall attenuation ($W=20$)



(b) Configuration with weak wall attenuation ($W=3.3$)

Fig. 2: Illustration of robot configurations

VI. CASE STUDY II: FINDING MAXIMIZED TRANSMISSION-RATE PATH

Another important metric to measure link quality is transmission rate. From the classic Shannon-Hartley Formula, the transmission rate of a link (i, j) is a function of the SNR at receiver j corresponding to transmitter i :

$$l_{i,j} = B \log(1 + \gamma_{i,j}) \quad (9)$$

The transmission rate of a communication path formed by multiple links is determined by the transmission rate of its bottleneck link. Thus, to have a maximized transmission rate between the source and destination, the $m - 2$ mobile robots need to form a transmission path which has a maximized bottleneck rate. Therefore, the objective function in (4) is instantiated as

$$\max_{(x^1, \dots, x^m) \in \mathcal{P}} \min(l_{1,2}, \dots, l_{m-1,m}) \quad (10)$$

The optimal configuration of robots in this case can be found by LEONA with the $FindPath(\mathbb{G})$ implemented as *Widest Path Algorithm* [18] with a constraint that the total number of hops along a path is at most $m - 1$.

A. Analysis of the Sufficient searched Area

In the case of finding maximized transmission-rate path, we have the same sufficient searched area result:

Theorem 3: When applying LEONA, the size of the sufficient searched area \mathcal{A} that guarantees robots to find the optimal configuration with maximized transmission rate is

$$\mathcal{A} = O(10^{n \frac{W}{5n}} d^2) \quad (11)$$

Further, if the distribution of n walls along the straight line connecting the source and destination allows each communication pair to have same number of walls when robots are evenly spaced along the straight line, the sufficient searched area can reduce to $O(10^{\lceil \frac{n}{m-1} \rceil \frac{W}{5n}} d^2)$.

Proof: See [19]. ■

B. Simulation Results

We conduct simulations in the same network scenario as Case Study I. The *Offline Algorithm* is the same as Case Study I with only one difference that robots apply *Widest Path Algorithm* to find the optimal configuration. As seen in Figure 3, similar results are found: LEONA can always find the optimal configuration and takes less amount of movements and explorations than the offline scheme. We also present robot configurations in an environment with 12 walls with single wall attenuation $W = 20$ in Figure 4 (a) and $W = 3.3$ in Figure 4 (b). In the strong wall attenuation case, robots form a wall-free optimal path on which they are roughly evenly spaced. However, in the weak wall attenuation case, the optimal path still has walls along it where communication pairs suffering from wall attenuation compensate by having shorter link distance. One difference from Case Study I is that since the maximized transmission rate of a path increases as the number of robots increases when environmental configuration is fixed, thus, all 9 relay robots are required in the optimal configuration.

VII. SUMMARY AND FUTURE WORK

We have shown in this work how the adoption of an iterative online search combined with a graph based approach can allow for the formation of optimal robotic network configurations in unknown environments with obstructions. We have illustrated our general LEONA framework with two specific case studies. It is straightforward to incorporate many other utility functions and constraints into this framework. In this work, we assumed a simple path loss model with wall attenuation in order to make and update predictions — the framework is flexible enough to accommodate other models / approaches to prediction. The only hard requirement is that at each step an optimistic estimate be generated.

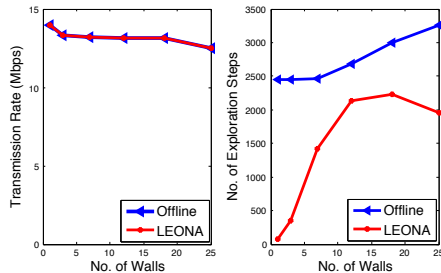
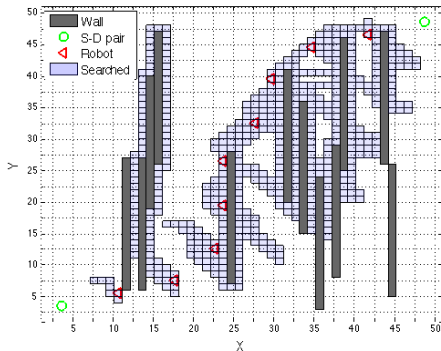
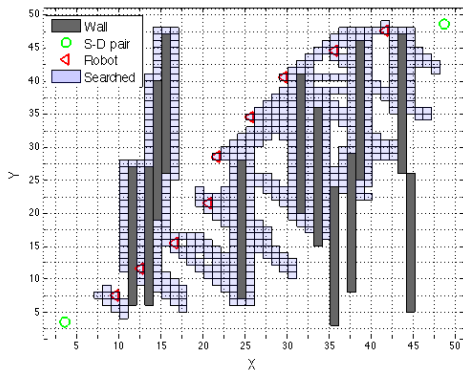


Fig. 3: Transmission rate (left) and moving steps (right)



(a) Configuration with strong wall attenuation ($W = 20$)



(b) Configuration with weak wall attenuation ($W = 3.3$)

Fig. 4: Illustration of robot configurations

as the ones considered in this study, it is possible to obtain a global solution in polynomial time. So long as the network of robots moves in such a way as to ensure connectivity is maintained at all times, they can exchange their measurements in an online fashion, and the predicted graph can be updated in a consistent manner by all robots in the network allowing them to each compute the optimal predicted location for themselves in a parallel fashion. It may be possible to adopt and interleave more sophisticated message passing mechanisms with the iterations of the online algorithm to further improve the robustness and efficiency of the system. Real-world factors such as multi-path fading, heterogeneous wall attenuation, communication failures and time-varying environments also need to be incorporated. We leave the design of such enhancements as open problems to be considered in future work.

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For many path and network optimization problems such