

# 第一章 几何光学

## 1. 基本规律

光程  $l = ns, \left( V = \frac{c}{n} \right).$

费马原理

$$\delta = 0, l \text{取极大、极小或常数} \Rightarrow i = -i', n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

## 2. 成像

### ① 单球面折射

$$\frac{s^2}{n^2(s+r)^2} - \frac{s'^2}{n'^2(s'-r')^2} = -4r \sin^2\left(\frac{\varphi}{2}\right) \left[ \frac{1}{n^2(s+r)} + \frac{1}{n'^2(s'-r')} \right]$$

保持同心性物像点:

$$\frac{s^2}{n^2(s+r)^2} - \frac{s'^2}{n'^2(s'-r')^2} = 0, \frac{1}{n^2(s+r)} + \frac{1}{n'^2(s'-r')} = 0.$$

傍轴条件:

$$\frac{n'}{s'} + \frac{n}{s} = \frac{n' - n}{r}.$$

$$f = \frac{nr}{n' - n}, f' = \frac{n'r}{n' - n}$$

$$\frac{f'}{s'} + \frac{f}{s} = 1.$$

$$V = \frac{y'}{y} = -\frac{ns'}{n's}.$$

### ② 球面镜成像 $n' = -n.$

$$\frac{1}{s'} + \frac{1}{s} = -\frac{2}{r}, f = f' = -\frac{r}{2}, V = -\frac{s'}{s}$$

### ③ 薄透镜

$$\frac{f'}{s'} + \frac{f}{s} = 1.$$

$$f = \frac{n}{\frac{n_L - n}{r_1} + \frac{n' - n_L}{r_2}}, f' = \frac{n'}{\frac{n_L - n}{r_1} + \frac{n' - n_L}{r_2}}.$$

$$n' = n, f' = f, \frac{1}{s'} + \frac{1}{s} = \frac{1}{f}.$$

$$s = x + f, s' = x' + f',$$

$$xx' = ff', V = -\frac{f}{x} = -\frac{x'}{f'}.$$

#### ④密接透镜组

$$s_2 = -s'_1, \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}.$$

$$P = \frac{1}{f}, P = P_1 + P_2.$$

#### ⑤望远镜 $M \equiv -\frac{f_o}{f_E}$

## 第二章 光的干涉

### 一.光波基本描述

$$1. v = \frac{c}{n} \quad c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.0 \times 10^8 \text{ m/s}, \quad n = \sqrt{\epsilon_r \mu_r}.$$

$$\lambda = \frac{v}{\nu} = \frac{c}{n\nu} = \frac{\lambda_0}{n}, \lambda_0 = \frac{c}{\nu}$$

### 2.单色波

①电场  $\vec{E} = \vec{E}_0(p) \cos(\omega t - \varphi(p))$

磁场  $\vec{B} = \vec{B}_0(p) \cos(\omega t - \varphi(p))$

## ②单色平面波

$$\vec{E} = \vec{E}_0 \cos[\omega t - \vec{k} \cdot \vec{z} + \varphi_0], \quad \tilde{E}(p, t) = E_0 \exp[-i(\omega t - \vec{k} \cdot \vec{r} + \varphi_0)]$$

波数  $k = \frac{2\pi}{\lambda}$ , 波矢  $\vec{k} = \frac{2\pi}{\lambda} = k\vec{k}_0$ ,  $\vec{k}_0$  为传播方向的单位方向矢量。

波的相位  $\omega t - \vec{k} \cdot \vec{r} + \varphi_0 = \omega t - R \frac{2\pi}{\lambda} + \varphi_0$ , 其中  $R$  为  $\vec{r}$  在  $\vec{k}_0$  方向上的投影。

$$\text{复振幅 } \tilde{E}(p) = E_0 \exp(i\varphi(p)).$$

## ③单色球面波

$$E = \frac{A_0}{r} \cos[\omega t - kr + \varphi_0]$$

$$\tilde{E}(p, t) = \frac{A_0}{r} \exp[-i(\omega t - kr + \varphi_0)]$$

$$\text{复振幅为 } \tilde{E}(p) = \frac{A_0}{r} \exp[i(kr - \varphi_0)].$$

## 3.光强度

光强  $I = \langle \vec{s} \rangle = E_0^2 = \vec{E}^*(p)\vec{E}(p)$ .  $\vec{s}$  是电磁波能流密度。

谱密度  $i(\lambda) = \frac{dI_\lambda}{d\lambda}$ , ( $dI_\lambda$  是  $\lambda \sim \lambda + d\lambda$  之间光强)。

$$I = \int_0^\infty dI_\lambda = \int_0^\infty i(\lambda) d\lambda$$

$$4. \text{反衬度 } \gamma \equiv \frac{I_M - I_m}{I_M + I_m}$$

二、线性叠加原理(弱光情况下成立):

$$1. \vec{E}(p, t) = \vec{E}_1(p, t) + \vec{E}_2(p, t) + \dots$$

同方向光振动叠加:

$$E(p,t) = E_1(p,t) + E_2(p,t) + \dots$$

2. 同频率、同振向波的叠加

$$E_1(p,t) = E_{10}(p) \cos(\omega t - \varphi_1(p)),$$

$$E_2(p,t) = E_{20}(p) \cos(\omega t - \varphi_2(p))$$

$$E(p,t) = E_0(p) \cos(\omega t - \varphi).$$

$$E_0^2(p) = E_{01}^2(p) + 2E_{10}(p)E_{02}(p) \cos[\varphi_1(p) - \varphi_2(p)] + E_{02}^2(p)$$

$$\tan \varphi(p) = \frac{E_{10}(p) \sin \varphi_1(p) + E_{20}(p) \sin \varphi_2(p)}{E_{10}(p) \cos \varphi_1(p) + E_{20}(p) \cos \varphi_2(p)}$$

$$I(p) = I_1(p) + I_2(p) + 2\sqrt{I_1(p)I_2(p)} \cos \delta.$$

$$I = I_0(1 + \gamma \cos \delta), I_0 = I_1 + I_2.$$

三、光的干涉和相干条件

1. 相干条件

① 位相差判据

当  $\delta = 2\pi m, (m = 0, \pm 1, \pm 2, \dots)$  (同位相),

$I_M = (E_{01} + E_{02})^2$ , 称为干涉极大, 对应亮纹;

当  $\delta = (2m + 1)\pi, (m = 0, \pm 1, \pm 2, \dots)$  (反位相),

$I_m = (E_{01} - E_{02})^2$ , 称为干涉极小, 对应暗纹.

② 光程差判据

$$\text{位相差 } \delta(p) = k(r_2 - r_1) = \frac{2\pi}{\lambda_0} \Delta l(p).$$

其中  $\Delta l(p) = n_1 r_1 - n_2 r_2$ .

干涉极大  $\Delta l(p) = m \lambda_0$ .

干涉极小  $\Delta l(p) = \left(m + \frac{1}{2}\right) \lambda_0$ .

#### 四、杨氏实验

1. 光程差  $\Delta l = r_1 - r_2 \approx d \sin \theta$ .  $\Delta l \approx d \frac{x}{D}$ .

2. 极大位置  $x = \frac{m \lambda_0 D}{d}$  ( $m = 0, \pm 1, \pm 2, \dots$ ).

极小位置  $x = \frac{(m + 1/2) \lambda_0 D}{d}$  ( $m = 0, \pm 1, \pm 2, \dots$ ).

3. 条纹宽度  $\Delta x = \frac{\lambda_0 D}{d}$ .

#### 4. 光强分布

$$\delta(p) = \frac{2\pi}{\lambda_0} \Delta l(p) = \frac{2\pi}{\lambda_0} d \frac{x}{D}.$$

实验中,  $I_1 \approx I_2 = I_0$ ,  $I = 2I_0 \left(1 + \cos \frac{2\pi}{\lambda_0} d \frac{x}{D}\right) = 4I_0 \cos^2 \left(\frac{\pi d}{D \lambda_0} x\right)$ .

5. 最大光程差  $\Delta l_M = m' \lambda_0 = \frac{\lambda_0^2}{\Delta \lambda_0}$ .

6. 光源 S 沿 x 方向移动  $\delta s$ , 干涉条纹的移动  $\delta x \approx -\frac{D}{l} \delta s$ .

#### 7. 扩展光源

● 临界宽度  $b_c = \frac{l \lambda}{d}$ .

干涉口径角  $\beta \equiv \frac{d}{l}$ , 扩展光源干涉条件为  $b < \frac{\lambda}{\beta}$ .

● 横向相干宽度  $d_c \equiv \frac{l\lambda}{b}$ .

● 光场的空间相干性:  $d < d_c$ , 即  $\beta < \beta_c = \frac{d_c}{l}$  内两点源都是相干点源.

●  $b\beta_c = \lambda$ .

## 五、薄膜干涉

### 1. 光程差

$$\Delta L = 2nt \cos \theta_r + \frac{\lambda}{2} = 2nt \sqrt{n^2 - n_1^2 \sin^2 \theta_i} + \frac{\lambda}{2}.$$

### 2. 等倾干涉

从中心向外数第  $N$  个亮环附近相邻两亮环间的角距离为

$$(\Delta N = 1) \Delta \theta_N = \frac{1}{n'} \sqrt{\frac{n\lambda}{t}} \frac{\Delta N}{2\sqrt{N}}.$$

$$\text{第 } N \text{ 个亮环半径 } r_N \approx \theta_N f = \frac{f}{n'} \sqrt{\frac{nN\lambda}{t}}.$$

$$\text{相邻两亮环间的径向距离为 } \Delta r_N \approx \Delta \theta_N f = \frac{n\lambda f}{2n'^2 t \theta_N}.$$

### 3. 等厚干涉

#### ① 楔形

$$\text{相邻条纹的高度差 } \Delta t = t_{m+1} - t_m = \frac{\lambda}{2n}.$$

$$\text{相邻条纹的间隔 } \Delta l = \frac{\Delta t}{\sin \alpha} = \frac{\lambda}{2n \sin \alpha}.$$

## ②牛顿环

$$\text{光程差 } \theta_i = 0, \Delta L = 2t - \frac{\lambda}{2}.$$

$$m \text{ 级亮纹半径为 } r_m = \sqrt{\left(m + \frac{1}{2}\right)\lambda R}.$$

$$m \text{ 级暗纹半径为: } r'_m = \sqrt{m\lambda R}.$$

$$R = \frac{r'_{m+N}{}^2 - r'_m{}^2}{N\lambda}.$$

## 4.透射光

$$\Delta L = 2nt \cos \theta_r.$$

$$(I_o = I_r + I_t)$$

## 5.薄膜厚度要求

$$\Delta L = 2nt \cos \theta < \Delta L_M = m'\lambda = \frac{\lambda^2}{\Delta\lambda}.$$

$$6. \text{增透膜 } 2nt = \left(m + \frac{1}{2}\right)\lambda, m = 0, 1, 2, \dots$$

$$7. \text{迈克尔逊干涉仪 } \Delta t = \pm N \frac{\lambda}{2}.$$

六、光场的时间相干性:  $t < \tau_0$ .

光波的相干长度  $L_c = \Delta L_{max} = \frac{\bar{\lambda}^2}{\Delta\lambda}$ , 相干时间  $\tau_0 \equiv \frac{L_c}{c}$ .

## 第三章 光的衍射

### 一、惠更斯-菲涅耳原理

$$\tilde{E}(P) = k \iint_{(\Sigma)} \tilde{E}_0(Q) F(\theta_0, \theta) \frac{e^{ikr}}{r} d\Sigma.$$

基尔霍夫公式 
$$\tilde{E}(P) = \frac{-i}{\lambda} \iint_{(\Sigma_0)} \frac{(\cos \theta_0 + \cos \theta)}{2} \tilde{E}_0(Q) \frac{e^{ikr}}{r} d\Sigma.$$

傍轴条件下, 即  $\theta_0 \approx \theta \approx 0, r \approx r_0$

$$\tilde{E}(P) = \frac{-i}{\lambda r_0} \iint_{(\Sigma_0)} \tilde{E}_0(Q) e^{ikr} d\Sigma.$$

## 二、巴俾涅原理

几何像点之外,

$$\because \tilde{E}_a(P) + \tilde{E}_b(P) = \tilde{E}_0(P) = 0,$$

$$\therefore |\tilde{E}_a(P)| = |\tilde{E}_b(P)|, \Rightarrow I_a(P) = I_b(P).$$

## 三、菲涅耳圆孔衍射和圆屏衍射

$$1. E_0(P) = \frac{1}{2} \Delta E_{10} + (-1)^{n+1} \frac{1}{2} \Delta E_{n0}.$$

$$2. k = \frac{\rho^2}{\lambda} \left( \frac{1}{R} + \frac{1}{r} \right).$$

平行光入射圆孔, 则  $R \rightarrow \infty$ ,  $k = \frac{\rho^2}{\lambda R}.$

$$3. \text{自由传播 } E_0(P) = \frac{1}{2} \Delta E_{10}.$$

$$4. \text{圆屏衍射 } E_0(P) = \frac{1}{2} \Delta E_{k+10}(P) \neq 0$$

## 5. 波带片

• 遮住偶数带, 轴上 P 点的振幅为

$$E_0(P) = \Delta E_{10}(P) + \Delta E_{30}(P) + \Delta E_{50}(P) \cdots + \Delta E_{2n+10}(P).$$

• 遮住奇数带, 轴上 P 点的振幅为

$$E_0(P) = -(\Delta E_{20}(P) + \Delta E_{40}(P) + \Delta E_{60}(P) \cdots + \Delta E_{2n0}(P)).$$



•半波带半径

$$\rho = \sqrt{k} \rho_1, \rho_1 = \sqrt{\frac{Rb\lambda}{R+b}}$$

•透镜作用:  $\left(\frac{1}{R} + \frac{1}{b}\right) = \frac{k\lambda}{\rho_k^2}$ .

#### 四、夫琅禾费衍射

##### 1. 单缝

###### ①光强

$$\tilde{E}_0(P_\theta) = \tilde{E}_0(P_0) \frac{\sin(\alpha)}{\alpha}, I_\theta = I_0 \left(\frac{\sin(\alpha)}{\alpha}\right)^2,$$

其中  $I_0$  为衍射场中心光强度,

$\left(\frac{\sin(\alpha)}{\alpha}\right)^2$  为单缝衍射因子.

②次极强  $\sin\theta = \pm 1.43 \frac{\lambda}{a}, \pm 2.46 \frac{\lambda}{a}, \pm 3.67 \frac{\lambda}{a}, \dots$

③暗纹位置  $\sin\theta = m \frac{\lambda}{a}, (m = \pm 1, \pm 2, \pm 3, \dots)$

④零级亮斑的半角宽度  $\Delta\theta \approx \frac{\lambda}{a}$ .

##### 2. 圆孔

中心角半径:  $\theta = 0.610 \frac{\lambda}{a} \approx 1.22 \frac{\lambda}{D}, D = 2a.$

最小分辨角  $\delta\theta_m = \Delta\theta = 1.22 \frac{\lambda}{D}.$

##### 3. 光栅

###### ①光强

- $I(P_\theta) = A_0^2(P_0) \left( \frac{\sin(\alpha)}{\alpha} \right)^2 \left( \frac{\sin(N\delta/2)}{\sin(\delta/2)} \right)^2$ .

- 主极大:  $d \sin \theta = k\lambda, k = 0, \pm 1, \pm 2, \pm 3, \dots$

$$I_{MAX} = N^2 A_0^2 \left( \frac{\sin(\alpha)}{\alpha} \right)^2 \cdot k_{MAX} = \frac{d}{\lambda}$$

- 极小:  $\sin \theta = \left( k + \frac{m}{N} \right) \frac{\lambda}{d}, m = 1, 2, \dots, N-1 (m \neq 0, N)$

- 主极大的半角宽度  $\Delta \theta = \frac{\lambda}{Nd \cos \theta_k}$ .

- 主极大缺级:

主极大  $d \sin \theta = k\lambda, k = 0, \pm 1, \pm 2, \pm 3, \dots$

单缝极小  $a \sin \theta = n\lambda, n = \pm 1, \pm 2, \pm 3, \dots$

当  $\sin \theta = \frac{k\lambda}{d} = \frac{n\lambda}{a}$  时, 即  $k = \frac{dn}{a}$  缺级.

## ② 光谱

- 色散本领定义为  $D_\theta = \frac{\delta \theta_k}{\delta \lambda} = \frac{k}{d \cos \theta_k}$ .

- 瑞利判据: 最小分辨角  $\delta \theta'$  等于光谱线的半角宽度, 即  $\delta \theta' = \Delta \theta$ .

- 色分辨本领  $R = \frac{\lambda}{\delta \lambda} = kN$ .

③ 闪耀光栅  $d \sin 2\theta_B = k\lambda_B^k$ ,  $k$  级最亮.

同时,  $a \approx d$ ,  $a \sin(2\theta_B) = k\lambda_B^k$  也成立, 即其它干涉级均成为缺级.

④布拉格条件  $2d \sin\theta = k\lambda$ .

## 第五章 光的电磁性

### 一、偏振

1. 偏振度:  $P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}, 0 \leq P \leq 1$ .

2. 马吕斯定律  $I_2 = I_1 \cos^2 \alpha$ .

3. 布儒斯特角  $\theta_B$   $\text{tg} \theta_B = \frac{n_2}{n_1}$ .

4. e 光主折射率  $n_e = c / V_e = \frac{\sin \theta_i}{\sin \theta_r^e}$ .

### 5. 波晶片

若  $(n_o - n_e)d = \pm \frac{\lambda}{4} + m\lambda, m$  为整数,  $\varphi_{o-e} = \pm \frac{\pi}{2}$ , 则称波晶片为  $\frac{\lambda}{4}$  片.

若  $(n_o - n_e)d = \pm \frac{\lambda}{2} + m\lambda, \varphi_{o-e} = \pm \pi, 2\pi$ , 则称波晶片为  $\frac{\lambda}{2}$  片.

6. 椭圆偏振光  $\vec{E} = \vec{E}_x + \vec{E}_y$ .

其中  $E_x = E_{x0} \cos\left(\omega t - \frac{2\pi}{\lambda} z\right), E_y = E_{y0} \cos\left(\omega t - \frac{2\pi}{\lambda} z - \delta\right)$ .

### 二、光的吸收、色散和散射

1. 光的吸收  $I = I_0 e^{-\alpha d}$ .

2. 色散

•柯西公式:  $n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$ .

•色散率  $\frac{dn}{d\lambda} < 0$ , 正常色散;  $\frac{dn}{d\lambda} > 0$ , 反常色散

3.瑞利散射:  $I_s(\lambda) \propto \frac{f(\lambda)}{\lambda^4}, a < \lambda$ .

## 第六章 光的量子性

1.普朗克能量子  $\epsilon_0 = h\nu$

### 2.光电效应

•  $\frac{1}{2}mv_m^2 = eV_0 = eK(\nu - \nu_0)$ .

•  $E = h\nu$ .

•  $\frac{1}{2}mv_m^2 = h\nu - A$ ,

•  $\nu_0 = \frac{A}{h}$ .

### 3.康普顿散射

$\Delta\lambda = 2\lambda_c \sin^2 \frac{\theta}{2}, \lambda_c = \frac{h}{m_0c} = 0.0243 \text{ \AA}$

4. 光的波粒二象性  $E = h\nu, p = \frac{h\nu}{c} = \frac{h}{\lambda}$ .

## 第七章 激光

自发辐射:  $\frac{dN_{21}}{dt} = A_{21}N_2$ ,

受激辐射:  $\frac{dN'_{21}}{dt} = B_{21}\rho(\nu)N_2$ ,

受激吸收:  $\frac{dN_{12}}{dt} = B_{12}\rho(\nu)N_1,$

谐振腔纵模  $\nu_j = j \frac{c}{2nL}, j = 1, 2, \dots$  间隔  $\Delta\nu = \frac{c}{2nL}.$

说明：打黑框的公式必须理解，并且记住！不打黑框的公式要求理解其物理意义。