2. $P = P_0 P_{01} P_{12} = 0.3 \times 0.2 \times 0 = 0$

3.

解：(a) $P\{X_0 = 0, X_1 = 0, X_2 = 0\} = p_0 P_{00} P_{00} = 1 \cdot (1-\alpha) \cdot (1-\alpha) = (1-\alpha)^2$

(b) $P = p_0 P_{00} P_{00} + p_0 P_{01} P_{10} = (1-\alpha)^2 + \alpha^2 = 2\alpha^2 - 2\alpha + 1$

(c) 转移概率矩阵 $P = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ 0 & 1-2\alpha \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$

$$\therefore P^{(5)} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ 0 & (1-2\alpha)^5 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1+(1-2\alpha)^5}{2} & \frac{1-(1-2\alpha)^5}{2} \\ \frac{1-(1-2\alpha)^5}{2} & \frac{1+(1-2\alpha)^5}{2} \end{pmatrix}$$

$\therefore P\{X_5 = 0 | X_0 = 0\} = p_0 \cdot \frac{1+(1-2\alpha)^5}{2} = \frac{1+(1-2\alpha)^5}{2}$

4.

解：

$$P_{ij} = \begin{cases} p \cdot \frac{i}{N} + q \cdot \frac{N-i}{N} & ,j = i \\ q \cdot \frac{i}{N} & ,j = i-1 (i = 1, 2, \cdots, N) \\ p \cdot \frac{N-i}{N} & ,j = i+1 (i = 0, 1, \cdots, N-1) \\ 0 & ,\text{其他} \end{cases}$$

其转移概率矩阵为

$$P = \frac{1}{N} \begin{pmatrix} qN & pN & 0 & \cdots & 0 & 0 & 0 \\ q & q(N-1)+p & p(N-1) & \cdots & 0 & 0 & 0 \\ 0 & 2q & q(N-2)+2p & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2q+p(N-2) & 2p & 0 \\ 0 & 0 & 0 & \cdots & q(N-1) & q+p(N-1) & p \\ 0 & 0 & 0 & \cdots & 0 & qN & pN \end{pmatrix}$$

6. 设 u_k 为家鼠从 k 出发在遭到电击前能找到食物的概率

(1) 认为 $u_7=1, u_8=0$

$$\therefore \begin{cases} u_0=\frac{1}{2}(u_1+u_2) \\ u_1=\frac{1}{3}(u_0+u_3+u_7) \\ u_2=\frac{1}{3}(u_0+u_3+u_8) \\ u_3=\frac{1}{4}(u_1+u_2+u_4+u_5) \\ u_4=\frac{1}{3}(u_3+u_6+u_7) \\ u_5=\frac{1}{3}(u_3+u_6+u_8) \\ u_6=\frac{1}{2}(u_4+u_5) \\ u_7=1 \\ u_8=0 \end{cases} \Rightarrow \begin{cases} u_0=\frac{1}{2} \\ u_1=\frac{2}{3} \\ u_2=\frac{1}{3} \\ u_3=\frac{1}{2} \\ u_4=\frac{2}{3} \\ u_5=\frac{1}{3} \\ u_6=\frac{1}{2} \\ u_7=1 \\ u_8=0 \end{cases}$$

(2) 认为出发时刻不考虑电击与食物

由对称性知 $u_0=u_3=u_6=1/2$，以及

$$u_1=\frac{1}{3}(u_0+u_3+1) \quad = \quad u_4=\frac{1}{3}(u_6+u_3+1)$$

$$u_2=\frac{1}{3}(u_0+u_3+0) \quad = \quad u_5=\frac{1}{3}(u_6+u_3+0)$$

$$u_7=\frac{1}{2}(u_1+u_4) \qquad u_8=\frac{1}{2}(u_2+u_5)$$

$\Rightarrow u_0=u_3=u_6=1/2$，$u_1=u_4=u_7=2/3$，$u_2=u_5=u_8=1/3$

7.

$$P=\begin{pmatrix} p_0 & p_1 & p_2 & \cdots \\ p_0 & p_1 & p_2 & \cdots \\ p_0 & p_1 & p_2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

11.

解：$f_{00}^{(1)}=P_{00}=0, f_{00}^{(2)}=\begin{pmatrix}\frac{1}{2} & 0 & \frac{1}{2}\end{pmatrix}\begin{pmatrix}0 & 0 & \frac{1}{2}\end{pmatrix}^T=\frac{1}{4}$

对 $n\geqslant 2$ 有

$$f_{00}^{(n)}=\begin{pmatrix}\frac{1}{2} & 0 & \frac{1}{2}\end{pmatrix}\begin{pmatrix}0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & \frac{1}{2}\end{pmatrix}^{n-2}\begin{pmatrix}0 \\ 0 \\ \frac{1}{2}\end{pmatrix}$$

当 $n=3$ 时, $f_{00}^{(3)}=\frac{1}{8}$

$$f_{00}^{(n)}=\begin{pmatrix}\frac{1}{2} & 0 & \frac{1}{2}\end{pmatrix}\begin{pmatrix}0 & 0 & \frac{1}{2^{n-4}} \\ 0 & 0 & \frac{1}{2^{n-3}} \\ 0 & 0 & \frac{1}{2^{n-2}}\end{pmatrix}\begin{pmatrix}0 \\ 0 \\ \frac{1}{2}\end{pmatrix}=\frac{5}{2^n}$$

12

$$\text{证}: X_{n+1}=\begin{cases} X_{n+1} & ,\text{第 n+1 次试验成功} \\ 0 & ,\text{第 n+1 次试验失败} \end{cases}$$

$\therefore \{X_n\}$ 是 $M.C.$

$$P=\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \vdots \end{array}\begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & \cdots \end{array} \\ \begin{pmatrix} q & p & 0 & 0 & \cdots \\ q & 0 & p & 0 & \cdots \\ q & 0 & 0 & p & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \end{array}$$

$$T=\min\{n: X_n=0, X_s\neq 0(s=1,2,\cdots,n-1)\}$$

$$P(T=k)=p^{k-1}q \qquad k=1,2,\cdots$$

$$\mathrm{E}T=\sum_{k=1}^{+\infty} p^{k-1}qk$$

$$\therefore p\mathrm{E}T=\sum_{k=1}^{+\infty} p^{k}qk$$

$$\therefore (1-p)\mathrm{E}T=q\mathrm{E}T=q+pq+p^2q+\cdots=\frac{q}{1-p}=1$$

$$\therefore \mathrm{E}T=\frac{1}{q}$$

所有状态都是非周期正常返的（不考虑 p 与 q 等于 0 的情况）

14.

(2)不可约马氏链，所有状态遍历，故极限分布等于平稳分布(1/3　1/3　1/3)。三种产品无差异。

(3)可约马氏链！通过求解发现其极限分布与平稳分布仍然相同

由归纳法可知

$$P^n = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2}\left(1-\frac{1}{3^n}\right) & \frac{1}{3^n} & \frac{1}{2}\left(1-\frac{1}{3^n}\right) \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

平稳分布：(1/2　0　1/2)

故应无差别地多生产 A、C 产品，少生产 B 产品

(4)不可约马氏链，三个状态的周期都是 3，非遍历。

平稳分布为(1/3　1/3　1/3)

其极限分布不存在,但从时间平均上可以看出三者无差异

故三者应无差异生产

17.

解；设 π 为该 $M.C.$ 的平稳分布, $\pi = (\pi_0, \pi_1, \pi_2)$

$$\begin{cases} \pi \geqslant 0 \\ \sum_{i=0}^{2} \pi_i = 1 \\ \pi P = \pi \end{cases} \Rightarrow \pi = \left(\frac{5}{14}, \frac{6}{14}, \frac{3}{14}\right)$$

易知该 $M.C.$ 不可约且遍历

$$\therefore \text{极限分布为} \begin{pmatrix} \frac{5}{14} & \frac{6}{14} & \frac{3}{14} \\ \frac{5}{14} & \frac{6}{14} & \frac{3}{14} \\ \frac{5}{14} & \frac{6}{14} & \frac{3}{14} \end{pmatrix}$$

$\frac{3}{7}$

18.

$$\begin{cases} \pi \geqslant 0 \\ \sum_{i=0}^{3} \pi_i = 1 \\ \pi P = \pi \end{cases} \Rightarrow \pi = (\frac{3}{11}, \frac{1}{11}, \frac{1}{11}, \frac{6}{11})$$

π 反映了 $M.C.$ 中各状态在长期中所占的平均比例

$\therefore$ 一年中晴朗的天数 $= \frac{365}{2} \times (\frac{3}{11} \times 2 + \frac{1}{11} + \frac{1}{11}) = 132.7$(天)

26.

$$P_0(t) = e^{-\lambda_0 t} = e^{-N\lambda t}$$

$$P_n(t) = \prod_{i=0}^{n-1} \lambda_i \left[\sum_{i=0}^{n} c_{i,n} e^{-\lambda_i t} \right]$$

$$= \frac{N!}{(N-n)!} \lambda^n \left[\sum_{i=0}^{n} \frac{(-1)^{n-i}}{i!(n-i)!} * \lambda^{-n} * e^{-(N-i)\lambda t} \right]$$

$$= \frac{N!}{(N-n)!} \lambda^n \left[\lambda^{-n} e^{-N\lambda t} * \sum_{i=0}^{n} \frac{(-1)^{n-i}}{i!(n-i)!} * e^{i\lambda t} \right]$$

$$= \frac{N!}{n!(N-n)!} e^{-N\lambda t} \left[\sum_{i=0}^{n} \frac{n!}{i!(n-i)!} * (-1)^{n-i} * e^{i\lambda t} \right]$$

$$= C_N^n e^{-N\lambda t} (e^{\lambda t} - 1)^n = C_N^n e^{(n-N)\lambda t} (1 - e^{-\lambda t})^n \qquad (1 \le n \le N)$$

27.

仅需建立模型即可

$$\begin{cases} P(X(t+h) - X(t) = -1 \mid X(t) = n) = \lambda_n h + o(h) \\ P(X(t+h) - X(t) = 0 \mid X(t) = n) = 1 - \lambda_n h + o(h) \\ P(X(t+h) - X(t) = 1 \mid X(t) = n) = 0 \\ \lambda_n = \lambda n \end{cases}$$