

Modélisation, estimation, simulation des risques climatiques
Session 3

Scenario building of Integrated Assessment Models

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Schedule of the day

Objectives

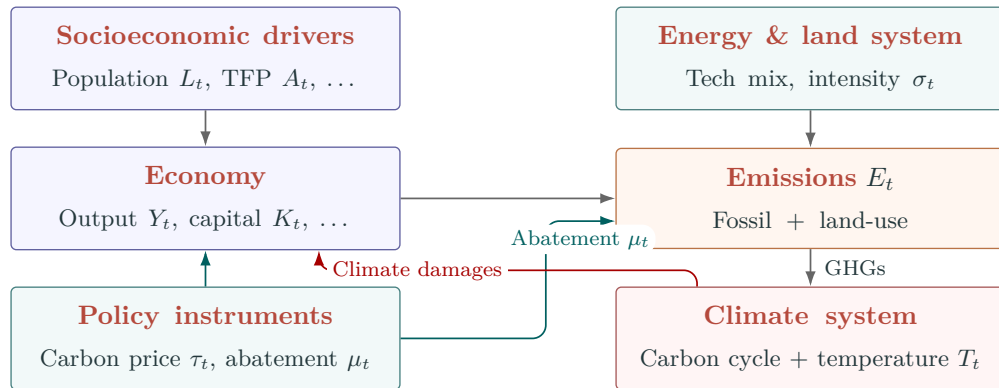
- Understanding the **basics** of **Integrated Assessment Models**.
- Build and **simulating macro-climate scenarios** using IPCC and NGFS methodology.

Materials:

- notebook_session3.ipynb *notebook introduction of model's options.*
- notebook_session3.ipynb *notebook exercises about scenariii simulations.*
- Handout_DICE.pdf a synthetic presentation of the DICE model.

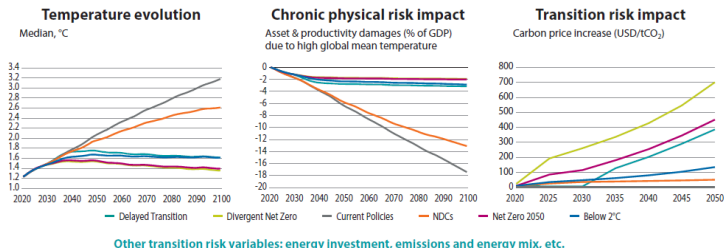
Introduction

Introduction: why IAMs matter for policy and finance



Introduction: IAMs in practice: NGFS Scenarios

- IAMs provide scenario pathways $\Rightarrow$
 - **Temperature evolution** under different policies.
 - **Physical risk impacts**: GDP losses from climate damages.
 - **Transition risk impacts**: carbon price trajectories.
- Used for **stress-testing** and **risk assessment** in finance.



Source: IIASA NGFS Climate Scenarios Database, REMIND model.

NGFS = Network for Greening the Financial System (central banks and supervisors).

Introduction: why IAMs matter for policy and finance

- **For financial institutions:**

- IAM outputs (e.g. from NGFS scenarios) are used by central banks and supervisors for **climate stress-testing exercises**.
- Provide pathways for GDP, energy, emissions, and damages under different policies.

- **For international policy:**

- IAMs feed into **IPCC reports**, underpinning assessments of mitigation costs and impacts.
- Establish a common reference for climate policy design across countries.

- **For carbon pricing:**

- Benchmark for the **social cost of carbon**.
- Example: the US administration (Biden) adopted \$52/ton CO₂ based on IAM results (incl. DICE).

Introduction: Why do we have a variety of scenarios?

- Climate scenarios are not forecasts: they are “**what if**” explorations.
- They differ because of two main ingredients:
 1. **Socioeconomic narratives (SSPs)**
 - Long-run assumptions on population, economic growth, inequality.
 - Energy and land-use technology adoption (renewables, CCS, etc.).
 - Speed of structural change and decarbonization potential.
 2. **Policy implementation (forcing levels)**
 - Presence (or absence) of a carbon price.
 - How fast the carbon tax rises over time.
 - Coverage: local vs. global implementation, uniform vs. fragmented.
- **Together, narratives + policy assumptions build the SSP scenario matrix used by the IPCC.**

Introduction: roadmap for today

- Explain the role of **Integrated Assessment Models (IAMs)** in climate-economy analysis.
- Learn how climate scenarios are **constructed**, not predicted:
 - **Narrative dimension**: socioeconomic pathways (SSPs), growth, technology, demographics.
 - **Policy dimension**: climate policy ambition (carbon pricing, mitigation targets).
- Understand the **IPCC scenario matrix** (SSP - forcing levels) and its role in assessments (e.g. CMIP6).
- Explore the DICE model as a **case study**: its building blocks, trade-offs, and illustrative simulations.

Introduction: where does DICE fit in this landscape?

- **Strength:** standard economic block + climate module (carbon cycle, temperatures) $\Rightarrow$ clear insight into mechanisms and the *social cost of carbon*.
- **Limits:** no sectoral detail, no fine multi-regional structure, limited uncertainty $\Rightarrow$ complement to complex IAMs.
- **Use:** conceptual benchmark, teaching tool, stress-testing policies, part of the five core models used by Biden's administration to set the carbon price (\$52/ton carbon).

1 Introduction

2 Shared Socioeconomic Pathways (SSPs)

3 The DICE Model

- The socioeconomic block
- Economic decisions

Shared Socioeconomic Pathways (SSPs)

SSPs: Key definitions

- **SSPs (Shared Socioeconomic Pathways):** Narratives + quantitative pathways describing alternative futures of population, GDP, technology, land use, energy.
- **CMIP (Coupled Model Intercomparison Project):** International framework where Earth System Models (ESMs) simulate climate under common scenarios.
- **IPCC:** Not a research body itself. It *assesses* the scientific literature (incl. IAMs & CMIP runs) to produce consensus reports.

SSPs: Why Shared Socioeconomic Pathways (SSPs)?

- Provide a **common framework** for socio-economic and climate futures.
- Two building blocks:
 1. **Narratives (qualitative)**: governance, technology, inequality.
 2. **Quantification (IAMs)**: GDP, population, emissions, land-use.
- Combined with **forcing levels** (1.9, 2.6, 4.5, 7.0, 8.5 W/m²) to span policy ambition.

SSPs: The 5 SSP narratives

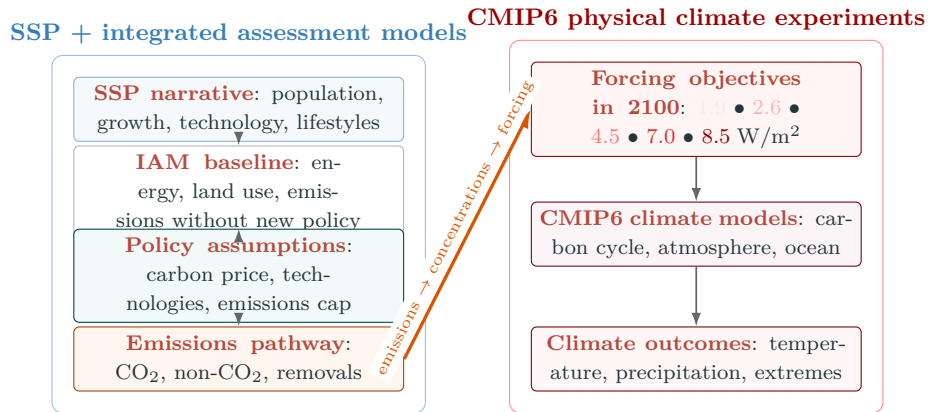
- **SSP1 – Sustainability**: inclusive development, green growth.
- **SSP2 – Middle of the road**: historical trends continue.
- **SSP3 – Regional rivalry**: fragmentation, low cooperation.
- **SSP4 – Inequality**: persistent global disparities.
- **SSP5 – Fossil-fueled growth**: rapid growth, fossil energy use.

SSPs: The SSP $\times$ forcing matrix

		Forcing W/m^2				
		1.9	2.6	4.5	7.0	8.5
SSP1	SSP1-1.9	SSP1-2.6	—	—	—	—
SSP2	—	SSP2-2.6	SSP2-4.5	SSP2-7.0	—	—
SSP3	—	—	SSP3-4.5	SSP3-7.0	SSP3-8.5	—
SSP4	—	SSP4-2.6	SSP4-4.5	SSP4-7.0	—	—
SSP5	—	—	SSP5-4.5	—	SSP5-8.5	—

Each cell = a scenario used in IPCC/CMIP6 (e.g., **SSP1-2.6**, **SSP5-8.5**).

From an SSP storyline to a climate outcome



Key point: the forcing label is an *outcome to be reached*, not a climate trajectory mechanically attached to an SSP. IAMs must find a policy and technology pathway consistent with both.

Why are some SSP–forcing combinations missing?

A combination must pass three tests

1. **Narrative consistency:** does the policy path still resemble the SSP?
2. **Model feasibility:** can IAMs reach the target with available technologies and resources?
3. **Scenario selection:** was it retained for CMIP6 experiments?

Example: SSP5 with low forcing

SSP5 starts from rapid, fossil-intensive growth. Reaching 1.9 or 2.6 W/m² would require:

- a very high and rapid carbon constraint;
- accelerated clean-technology substitution;
- often large-scale carbon removal and net-negative emissions.

A blank cell does not automatically mean “physically impossible.” It may mean *not selected, not robustly produced by IAMs, or attainable only under assumptions that strain the SSP narrative.*

SSPs: Two contrasted futures

SSP1-2.6 (Sustainability + strong mitigation)

- Low population, inclusive growth.
- Rapid clean tech diffusion.
- CO₂ emissions peak early, net zero 2070.
- Warming: +1.5–2°C.

SSP5-8.5 (Fossil-fueled growth, no policy)

- High GDP, energy-intensive growth.
- Continued fossil dependence.
- CO₂ emissions keep rising.
- Warming: +4°C or more.

SSPs: Where DICE fits in this exercise

- IAMs provide the **rich global picture** used by IPCC.
- DICE is a **simplified IAM**: Solow–Ramsey economy + climate block.
- **Educational role:**
 - Shows the mechanics of abatement vs damages.
 - Easy to simulate baseline vs policy cases.
 - Helps interpret more complex IAM/CMIP outcomes.

The DICE Model

DICE: Introduction

- DICE (Dynamic Integrated Climate-Economy model) was developed by Nordhaus (1992) and went through several (minor) updates up to the latest 2023 snapshot;
- Simple but powerful model to provide fast estimates of the social cost of carbon and transition pathways for the world economy;
- It has become **an intuitive benchmark**.

- The framework is “old-fashion” (from methodological perspective):
 - Solow growth model with climate externality;
 - Centralized equilibrium based on Ramsey allocations;
 - Purely deterministic setup;
- Physical capital, temperatures and carbon boxes are the main state variables that the planner uses to optimally maximize welfare.
- These slides are based on **2016 snapshot** (a 2023 framework now available).

DICE: setup basics

- Time is discretized in steps $i = 1, \dots, I$.
 - calendar time: $t(i) = t_0 + \Delta \cdot i$
 - t_0 initial period, Δ time step (in years)
 - horizon: $t \in \{t_0, t_0 + \Delta, t_0 + 2 \cdot \Delta, \dots, T\}$
- Variables can be indexed either by *time* or by *step*:

$$Y_t \equiv Y(i) \quad \text{with} \quad t = t(i).$$

- The model is a **discrete-time economy with finite horizon**:
 - horizon length: I steps $\equiv$ final time T .
 - centralized economy: a *single representative agent* chooses all decisions.

DICE: Carbon cycle

- Atmospheric loading of CO₂ is a system of differential equations:

$$M_t = \Phi_M M_{t-\Delta} + \Xi_M E_t, \quad (1)$$

where $M_t = [M_t^{AT}, M_t^{UP}, M_t^{LO}]'$ comprises 3 layers of carbon: atmospheric, upper and lower ocean respectively.

- Matrices are determined by:

$$\Phi_M = \begin{bmatrix} 1 - \Delta b_{12} & \Delta b_{12} \frac{M_{AT0}}{M_{UP0}} & 0 \\ \Delta b_{12} & 1 - \Delta b_{12} \frac{M_{AT0}}{M_{UP0}} - \Delta b_{23} & \Delta b_{23} \frac{M_{UP0}}{M_{LO0}} \\ 0 & \Delta b_{23} & 1 - \Delta b_{23} \frac{M_{UP0}}{M_{LO0}} \end{bmatrix} \text{ and } \Xi_M = \begin{bmatrix} \xi \\ 0 \\ 0 \end{bmatrix}$$

- Carbon emissions go into the atmosphere, before being captured by oceans through a long lasting process.

DICE: Temperatures

- Temperature anomalies $T_t = [T_t^{AT}, T_t^{OC}]$ of atmosphere and ocean respectively:

$$T_t = \Phi_T T_{t-\Delta} + \Xi_T F_t, \quad (2)$$

- Matrices are given by:

$$\Phi_T = \begin{bmatrix} (1 - \Delta \cdot c_1 \cdot F_{2 \times CO_2} - \Delta \cdot c_1 \cdot c_3) & (\Delta \cdot c_1 \cdot c_3) \\ (\Delta \cdot c_4) & (1 - \Delta \cdot c_4) \end{bmatrix} \text{ and } \Xi_T = \begin{bmatrix} \Delta \cdot c_1 \\ 0 \end{bmatrix}$$

- Cooling effects of oceans through c_3 , c_1 is the heating sensitivity to radiative forcing F_t , oceans are heating via atmospheric warming up to c_4 degree.

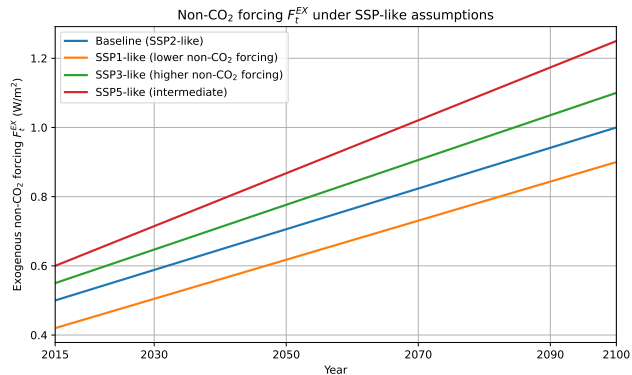
DICE: Radiative forcing

- Greenhouse effect via radiative forcing (W/m^2):

$$F_t = F_{2\times\text{CO}_2} \log \left(M_{t-\Delta}^{AT} / M_{AT0} \right) / \log(2) + F_t^{EX} \quad (3)$$

- Here, $F_{2\times\text{CO}_2}$ denote the forcing (Wm^{-2}) of equilibrium CO_2 doubling ($M_t^{AT} = 2M_{AT0}$).
- F_t^{EX} is an exogenous process tracking the contribution of other sources of GHG (i.e. methane) on F_t

DICE: Example of exogenous forcing



- World population (millions):

$$L_t = L_{t-\Delta} (L_\infty / L_{t-\Delta})^{\ell_g}, \quad (4)$$

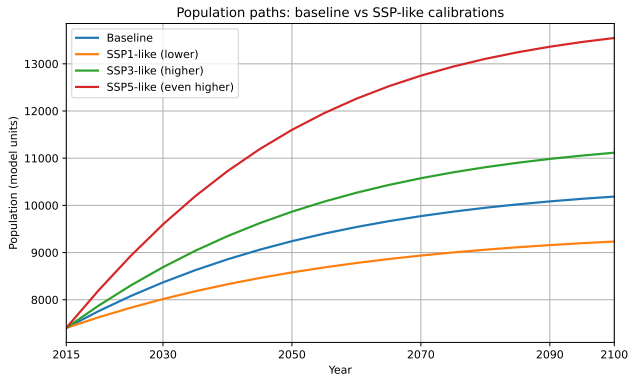
$\ell_g \in [0, 1]$ convergence rate, $L_\infty \in [0, +\infty)$ long term population.

- Law of motion of capital:

$$K_t = (1 - \delta_K)^\Delta K_{t-\Delta} + I_t, \quad (5)$$

$\delta_K \in [0, 1]$ is the depreciation rate, while Δ is the time step (here $\Delta = 5$ years)

DICE: Example of population dynamics



DICE: Production of goods

- Production is Cobb-Douglas in thousands USD:

$$Y_t = A_t K_{t-\Delta}^\gamma \left(\frac{L_t}{1,000} \right)^{1-\gamma} \quad (6)$$

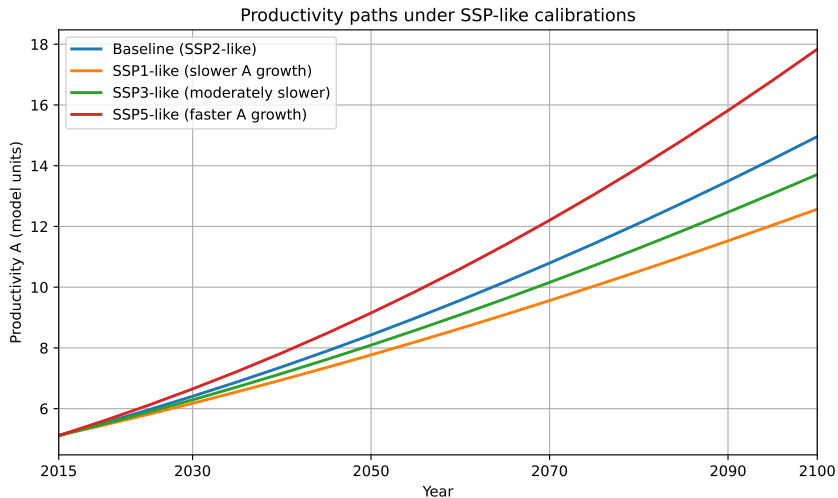
where $\gamma \in [0, 1]$ is capital intensity, physical capital $K_{t-\Delta}$.

- Exogenous total productivity A_t (with vanishing growth)

$$A_t = A_{t-\Delta} / (1 - g_A \exp(-\delta_A (t - t_0))) \quad (7)$$

$g_A \geq 0$ the initial productivity growth and decay rate $\delta_A \in (0, 1]$ to match declining productivity growth

DICE: Example of productivity dynamics



- Emissions (GtCO₂):

$$E_t = \Delta \cdot \left(\sigma_t (1 - \mu_t) Y_t + E_t^{\text{land}} \right) \quad (8)$$

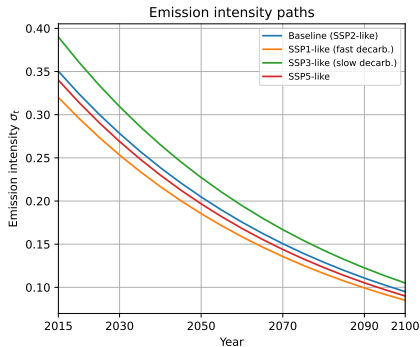
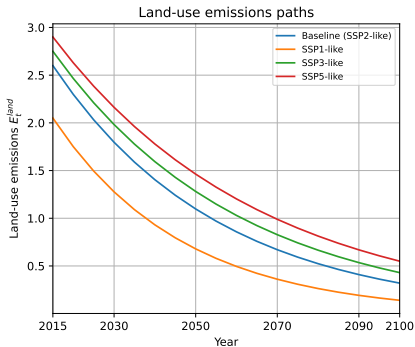
where σ_t decoupling rate, μ_t is abatement share, E_t^{land} is exogenous process of carbon emission from change in land use (i.e. deforestation, ~10% of emissions).

- Exogenous decoupling of emissions to GDP

$$\sigma_t = \sigma_{t-\Delta} \exp\left(-g_\sigma (1 - \delta_\sigma)^{(t-t_0)} \Delta \right) \quad (9)$$

$g_\sigma \geq 0$ the initial decoupling rate and decay rate $\delta_\sigma \in (0, 1]$ to match emissions to GDP data.

DICE: Example of technology dynamics



DICE: Equilibrium in goods and climate

- Resource constraint:

$$Y_t \left(1 - \theta_{1,t} \mu_t^{\theta_2}\right) \Omega \left(T_{t-\Delta}^{AT}\right) = C_t + I_t \quad (10)$$

LHS is output loss from abatement and damage, RHS is demand side.

- Damages are given by:

$$\Omega \left(T_{t-\Delta}^{AT}\right) = 1/(1 + a \left(T_{t-\Delta}^{AT}\right)^2) \quad (11)$$

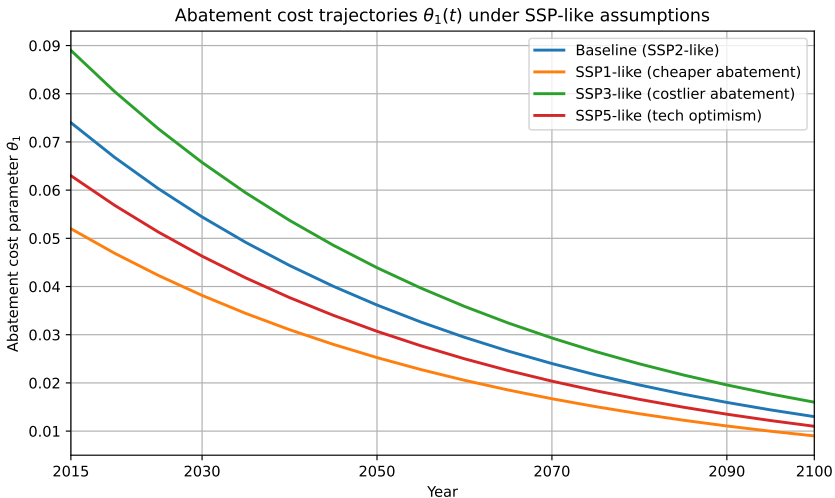
This expression is subject to deep uncertainty, and should be taken as a normative measure of the social loss for the society of rising temperatures. Tipping points could be introduced [Weitzman, 2012].

- Abatement good technology decreases over time:

$$\theta_{1,t} = \frac{p_b}{1000 \theta_2} (1 - \delta_{pb})^{(t-t_0)} \sigma_t. \quad (12)$$

where $p_b > 0$ price of carbon to reach net zero, efficiency factor $\delta_{pb} \in [0, 1]$.

DICE: Example of abatement technology



Total factor productivity:

$$A_t = A_{t-\Delta} / (1 - g_A \exp(-\delta_A (t - t_0)))$$

Decoupling rate of carbon emission to GDP:

$$\sigma_t = \sigma_{t-\Delta} \exp\left(-g_\sigma (1 - \delta_\sigma)^{(t-t_0)} \Delta\right)$$

Cost of abating carbon emissions:

$$\theta_{1,t} = \frac{p_b}{1000 \theta_2} (1 - \delta_{pb})^{(t-t_0)} \sigma_t$$

Non-CO2 forcing law of motion:

$$F_t^{EX} = \min\left(F_{t-\Delta}^{EX} + \Delta \cdot d_F, F_{CAP}\right)$$

Land-use law of motion:

$$E_t^{\text{land}} = (1 - \delta_e)^\Delta E_{t-\Delta}^{\text{land}}$$

World population dynamic:

$$L_t = L_{t-\Delta} (L_T / L_{t-\Delta})^{\ell_g}$$

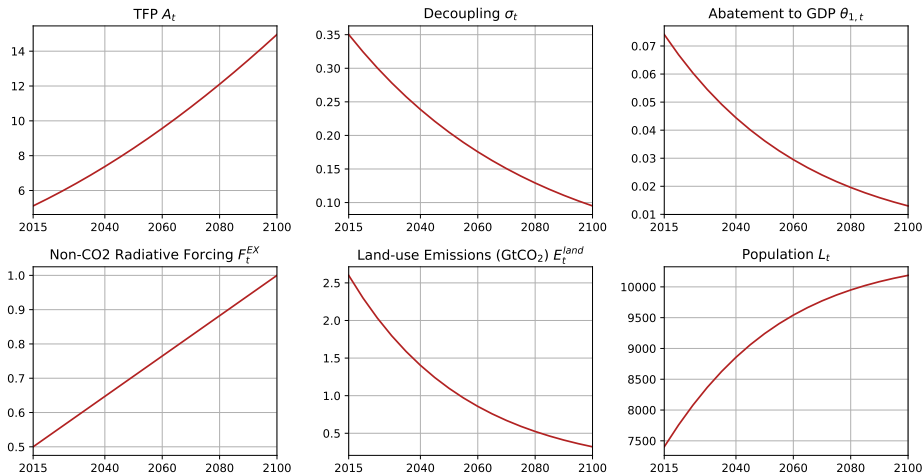


Figure. Summary of exogenous variables

DICE: Introducing a saving rate

- How should we determine optimally C and I ?
- We introduce optimal saving rate s , consumption and investment becomes:

$$C_t = \Delta \cdot (1 - s_t) \times Y_t \left(1 - \theta_{1,t} \mu_t^{\theta_2}\right) \Omega \left(T_{t-\Delta}^{AT}\right) \quad (13)$$

$$I_t = \Delta \cdot s_t \times Y_t \left(1 - \theta_{1,t} \mu_t^{\theta_2}\right) \Omega \left(T_{t-\Delta}^{AT}\right) \quad (14)$$

→implicitly assuming that income spent between C_t and I_t .

- Here, s is “decision” variable, which usually is pinned down by optimal control problem in economics.

Assumption (1)

In what follows, we assume that $\{s_t\}_{t=t_0}^T = 0.25$ as simplifying assumption.

DICE: Introducing the abatement rate

- Environmental policy here, denoted by abatement share μ_t , is the second decision variable.
- Increasing μ_t has several economic and social implications:
 - $\partial C_t / \partial \mu_t < 0$: $\mu_t \nearrow$ crowds out **consumption**. Yellow vest effect which can threaten the implementation of the policy.
 - $\partial I_t / \partial \mu_t < 0$: $\mu_t \nearrow$ crowds out **investment**. Lower growth prospects in the future.
 - $\partial E_t / \partial \mu_t < 0$: $\mu_t \nearrow$ reduces carbon emissions.

Assumption (2)

In what follows, we will give as an input (exogenously) the sequence $\{\mu_t\}_{t=t_0}^T$ of abatement.

DICE: model summary

- **Exogenous variables** each driven by an exogenous process (growth/decline path).

$$x_t = \{A_t, \sigma_t, \theta_{1,t}, F_t^{EX}, E_t^{\text{land}}, L_t\}$$

- **Endogenous state/flow variables**

$$y_t = \{M_t^{AT}, M_t^{UP}, M_t^{LO}, T_t^{AT}, T_t^{LO}, Y_t, K_t, C_t\}$$

- **Decision variables (or controls)**

$$z_t = \{\mu_t, s_t\}$$

- **Horizon and time step:** $t \in \{2015, 2020, 2025, \dots, T\}$, $\Delta = 5$ years (quinqennial).

DICE: A sketch of the numerical solution

Under Assumptions (1) and (2), the numerical solution is straightforward through an open loop:

$$\begin{aligned}x_t &= g(x_{t-1}), & g : \mathbb{R}^{N_x} &\rightarrow \mathbb{R}^{N_x}, \\y_t &= f(y_{t-1}, x_t, z_t), & f : \mathbb{R}^{N_y} \times \mathbb{R}^{N_x} \times \mathbb{R}^2 &\rightarrow \mathbb{R}^{N_y}.\end{aligned}$$

- $x_t \in \mathbb{R}^{N_x}$: **exogenous variables** ($A_t, \sigma_t, \dots, L_t$).
- $y_t \in \mathbb{R}^{N_y}$: **endogenous states/flows** ($M_t^{AT}, M_t^{UP}, \dots, Y_t, K_t$).
- $z_t = [\mu_t, s_t]' \in \mathbb{R}^2$: **decision variables** (abatement, saving).

Thank you!

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References

- Nordhaus, W. (1992). The ‘DICE’ model: Background and structure of a dynamic integrated climate-economy model of the economics of global warming. Technical report, Cowles Foundation for Research in Economics, Yale University.
- Weitzman, M. L. (2012). Ghg targets as insurance against catastrophic climate damages. *Journal of Public Economic Theory*, 14(2):221–244.