BERNSTEIN FILTER: A NEW SOLVER FOR MEAN CURVATURE REGULARIZED MODELS

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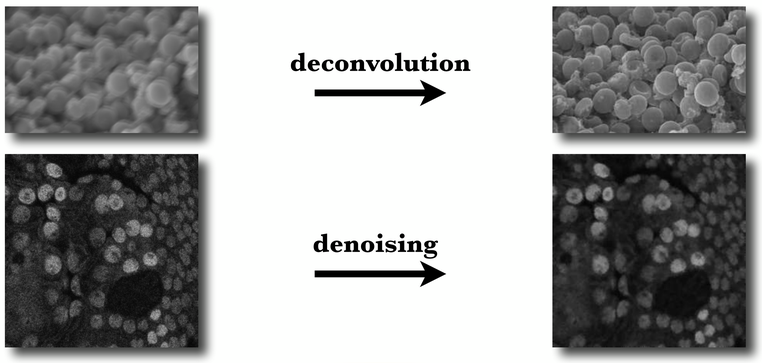
Introduction :

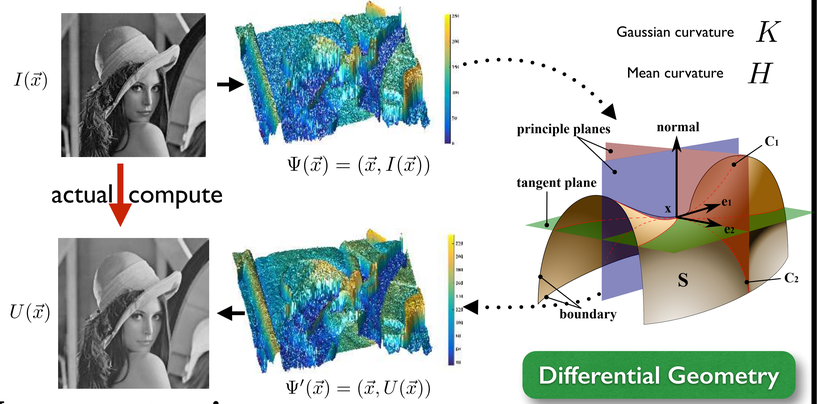
Signal processing problems are usually ill-posed, where a prior or regularization is needed.



Mean Curvature is a good prior

Mean Curvature Regularization:

Images are surfaces:



Mean curvature is:

$$H(U(\vec{x})) = \frac{(1+U_y^2)U_{xx} - 2U_xU_yU_{xy} + (1+U_x^2)U_{yy}}{2(1+U_x^2+U_y^2)^{\frac{3}{2}}}$$

Convexity:

According to Bernstein Theorem, minimizing mean curvature is assuming that the signal is piece-wise linear. Based on this linearity, we can prove that mean curvature regularization term is convex.

Contribution:

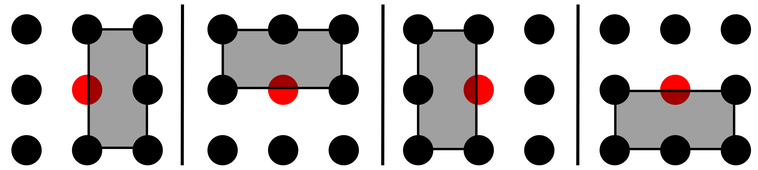
We prove that:

mean curvature is a **convex** term

We show that:

Bernstein Filter is a **fast** solver

Bernstein Filter:



Impulsing the linearity on the four half-windows (above) by Least Square Regression, we get

Algorithm 1 Bernstein Filter

Require: IterationNum, $I(x_i, y_j)$
$U^0(x_i, y_j) = I(x_i, y_j), t = 0$
while $t <$ IterationNum **do**
for i=2:M-1,j=2:N-1 **do**
$d_1 = \frac{1}{2}\left[U^t(x_{i-1}, y_j) + U^t(x_{i+1}, y_j)\right] - U^t(x_i, y_j)$
$d_2 = \frac{1}{2}\left[U^t(x_i, y_{j-1}) + U^t(x_i, y_{j+1})\right] - U^t(x_i, y_j)$
find d_m such that $|d_m| = \min_{k=1,2}\{|d_k|\}$
$U^{t+1}(x_i, y_j) = U^t(x_i, y_j) + d_m$
end for
$t = t + 1$
end while
Ensure: $U(x_i, y_j)$

Contact and free! software:

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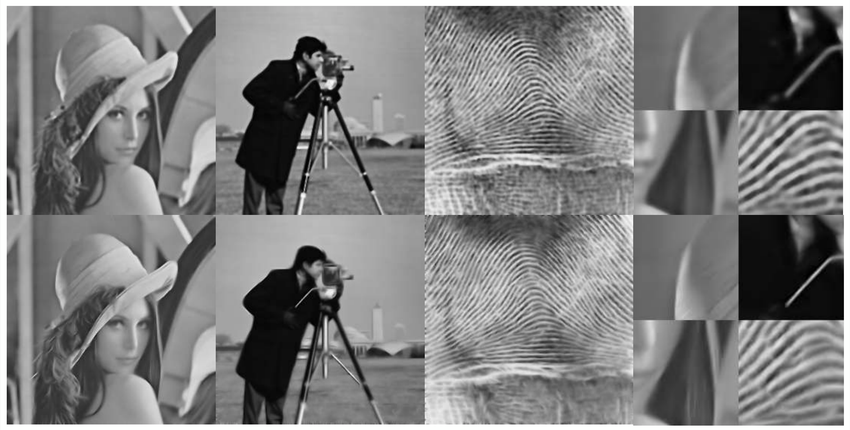
https://github.com/YuanhaoGong/CurvatureFilter

Curvature Filter

Experiments:

Results from Multi Grid Solver(first row) and Bernstein Filter(second row) are similar because both solve the same variational model. However, our filter is much faster.

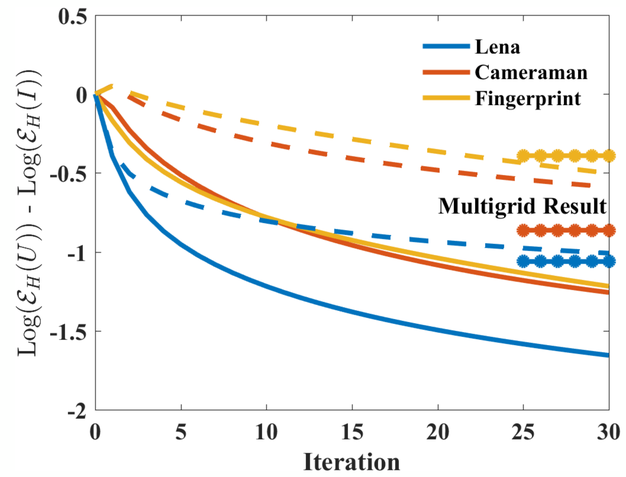


Two or three orders of magnitude FASTER!

solver (language)	Multigrid (Matlab)	Our filter (Matlab)	Our filter (C++)
Lena	183	1.1	0.025
Cameraman	648	1.1	0.025
Fingerprint	587	1.1	0.025

Table 1: time in seconds on 512×512 images. Our filter runs 30 iterations

More effective in the minimization



Solid lines indicate Bernstein Filter.
Dash lines indicate Mean Curvature Filter.
For multi grid solver, only converged states are shown.

Yuanhao Gong et al. *A natural scene gradient distribution prior and its application in light microscopy image processing, IEEE J-STSP, p99-114, Vol.10, Feb., 2016.*
Yuanhao Gong et al., *Local Weighted Gaussian Curvature for Image Processing,* **ICIP2013, oral.**
Yuanhao Gong et al., *Coupled signed distance functions for implicit surface reconstruction,* **ISBI2012, Best Paper Award.**