University of Guilan

Computational Intelligence

Subject7: Fuzzy Operators, Calculations and Relations

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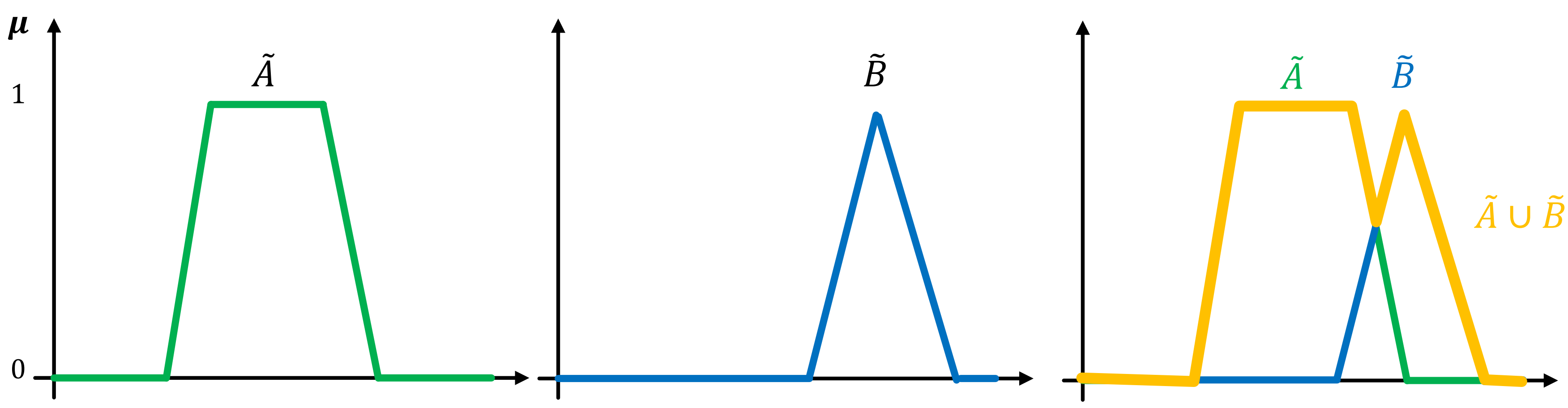
Agenda

- Fuzzy Operators
- Fuzzy Numbers
- Fuzzy Calculations
- Fuzzy Relations

Fuzzy Operators

- For Fuzzy sets $\tilde{A}$ and $\tilde{B}$, the **Union** operator is defined as:
  - Also known as $s - norms$

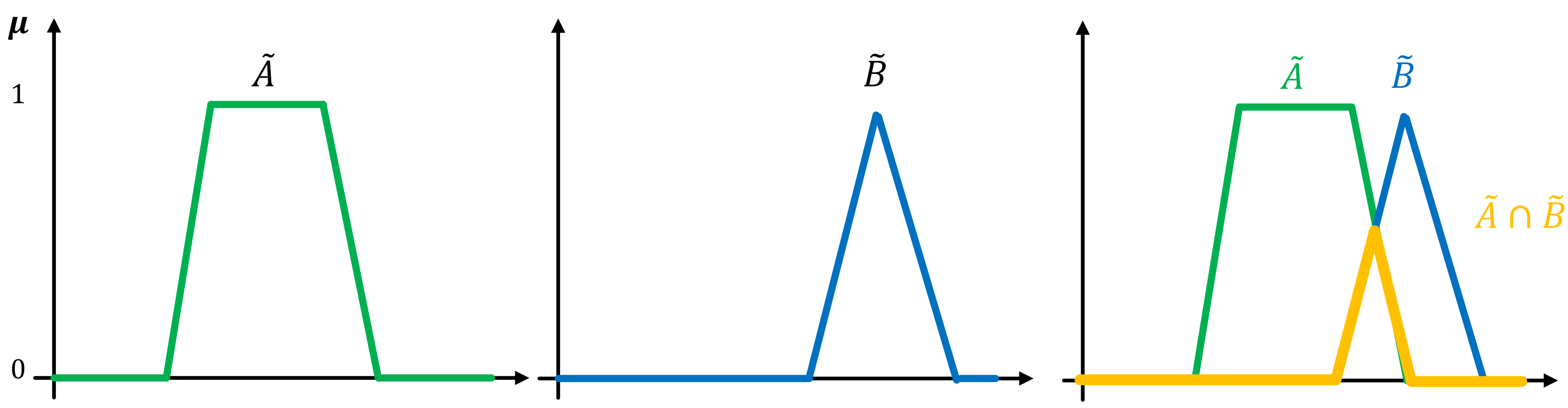
$$\mu_{\tilde{A}\cup\tilde{B}}(y) = \mu_{\tilde{A}} \vee \mu_{\tilde{B}} \quad \forall y \in U$$



Fuzzy Operators

- For Fuzzy sets $\tilde{A}$ and $\tilde{B}$, the **Intersection** operator is defined as:
  - Also known as t$-$*norms*

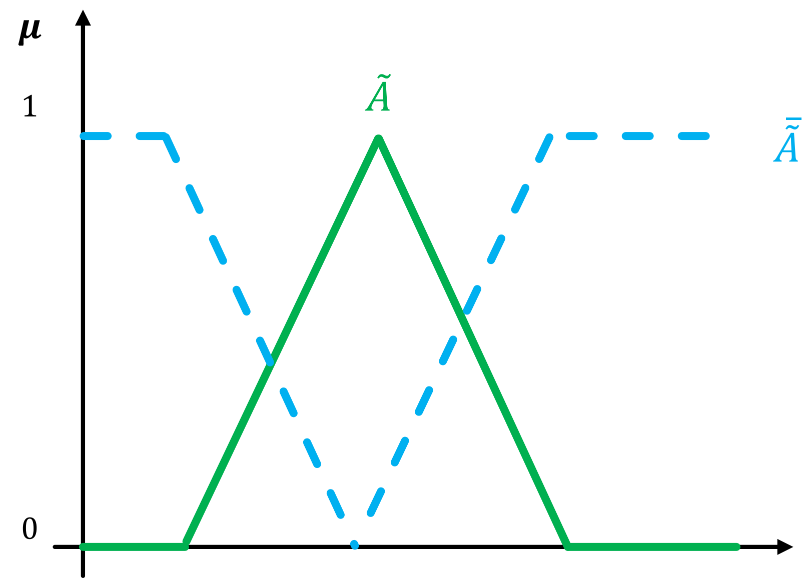
$$\mu_{\tilde{A}\cap\tilde{B}}(y) = \mu_{\tilde{A}} \wedge \mu_{\tilde{B}} \quad \forall y \in U$$



Fuzzy Operators

- For Fuzzy set $\tilde{A}$, the **Complement** operator is defined as:

$$\mu_{\bar{\tilde{A}}} = 1 - \mu_{\tilde{A}}(y) \quad y \in U$$



Fuzzy Operators

Important:

Operation	Modifies Alpha-cut	Modifies Strong Alpha-cut
Fuzzy Union	No	No
Fuzzy Intersection	No	No
Fuzzy Complement	Yes	Yes

Fuzzy Operators

Fuzzy sets algebra

- Commutative property

$$\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A}$$ $$\tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A}$$

- Associative property

$$\tilde{A} \cup \left(\tilde{B} \cup \tilde{C}\right) = \left(\tilde{A} \cup \tilde{B}\right) \cup \tilde{C}$$ $$\tilde{A} \cap \left(\tilde{B} \cap \tilde{C}\right) = \left(\tilde{A} \cup \tilde{B}\right) \cup \tilde{C}$$

- Distributive property

$$\tilde{A} \cup \left(\tilde{B} \cap \tilde{C}\right) = \left(\tilde{A} \cup \tilde{B}\right) \cap \left(\tilde{A} \cup \tilde{C}\right)$$ $$\tilde{A} \cap \left(\tilde{B} \cup \tilde{C}\right) = \left(\tilde{A} \cap \tilde{B}\right) \cup \left(\tilde{A} \cap \tilde{C}\right)$$

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Fuzzy sets algebra

- Idempotent laws

$$\tilde{A} \cup \tilde{A} = \tilde{A}$$ $$\tilde{A} \cap \tilde{A} = \tilde{A}$$

- Identity and complement expressions

$$\tilde{A} \cup \varphi = \tilde{A}$$ $$\tilde{A} \cap \varphi = \varphi$$

$$\tilde{A} \cap U = \tilde{A}$$ $$\tilde{A} \cup U = U$$

Fuzzy Operators

Fuzzy sets algebra

- Other rules

$$If\ \tilde{A} \subseteq \tilde{B} \subseteq \tilde{C},\ then\ \tilde{A} \subseteq \tilde{C}$$

$$\overline{\overline{\tilde{A}}} = \tilde{A}$$

$$\overline{\tilde{A} \cap \tilde{B}} = \overline{\tilde{A}} \cup \overline{\tilde{B}}$$

$$\overline{\tilde{A} \cup \tilde{B}} = \overline{\tilde{A}} \cap \overline{\tilde{B}}$$

Fuzzy Operators

Size of a Fuzzy set

- Simply, sum of the membership degrees

$$|A| = \sum_{x \in A} \mu_A$$

- Sample:

$$\tilde{A} = \{(5, 0.2), (10, 0.7), (16, 0.3), (18, 0.4), (19, 0.5)\}$$

$$\bar{\tilde{A}} = \{(5, 0.8), (10, 0.3), (16, 0.7), (18, 0.6), (19, 0.5)\}$$

$$|\tilde{A}| = 0.2 + 0.7 + 0.3 + 0.4 + 0.5 = 2.1$$

$$|\bar{\tilde{A}}| = 0.8 + 0.3 + 0.7 + 0.6 + 0.5 = 2.9$$

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Multiplication operation

- Two Fuzzy sets being multiplied together:

$$\mu_{A.B}(x) = \mu_A . \mu_B$$

- A number multiplied by a Fuzzy set:

$$\mu_{a.A}(x) = a . \mu_A$$

- *Sample:*

$$A = \{(a, 0.2), (b, 0.5), (c, 0.9)\}$$
$$B = \{(a, 0.9), (b, 0.2), (c, 1)(d, 0.1)\}$$

$$A.B = \{(a, 0.18), (b, 0.1), (c, 0.9), (d, 0.1)\}$$
$$0.5 \times A = \{(a, 0.1), (b, 0.25), (c, 0.45)\}$$

Fuzzy Operators

- How to show a Fuzzy set based on Alpha-cut?

$$A(x)_\alpha = \alpha . A(x)^\alpha$$

$$A = \frac{0.2}{x_1} + \frac{0.4}{x_2} + \frac{0.6}{x_3} + \frac{0.8}{x_4} + \frac{1}{x_5}$$

$$A_{0.2} = \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} + \frac{1}{x_5}$$

$$A_{0.4} = \frac{0}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} + \frac{1}{x_5}$$

$$A_{0.6} = \frac{0}{x_1} + \frac{0}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} + \frac{1}{x_5}$$

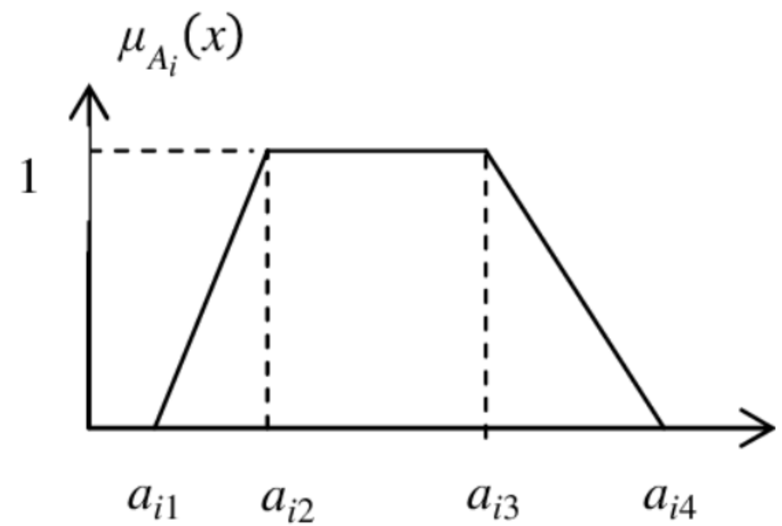
$$A_{0.8} = \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{1}{x_4} + \frac{1}{x_5}$$

$$A_1 = \frac{0}{x_1} + \frac{0}{x_2} + \frac{0}{x_3} + \frac{0}{x_4} + \frac{1}{x_5}$$

Fuzzy Numbers

- A generalization of the real numbers
- They refer to a connected set of possible values

- Applications:
  - Control System
  - Decision Making
  - Optimization
  - Probabilistic Reasoning



Fuzzy Numbers

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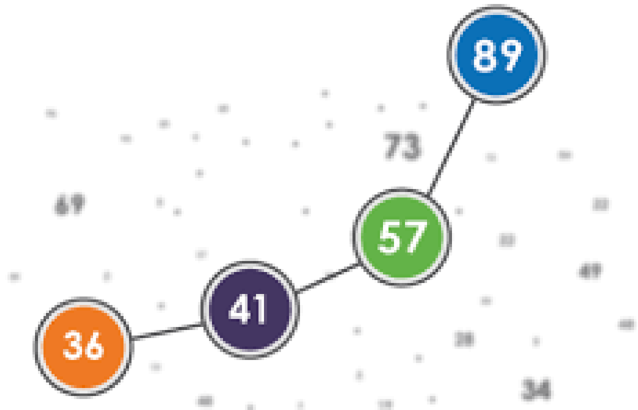
A Fuzzy number:

- Is a connected set of possible values
- Introduces the concept of uncertainty for numbers
- Has a membership degree

For a Fuzzy number *A:*

- Values can be the members of a normal Fuzzy set
- Alpha-cut is defined
- Support set (A^{0+}) is bounded
  - Strong Alpha-cut for α=0



Fuzzy Numbers

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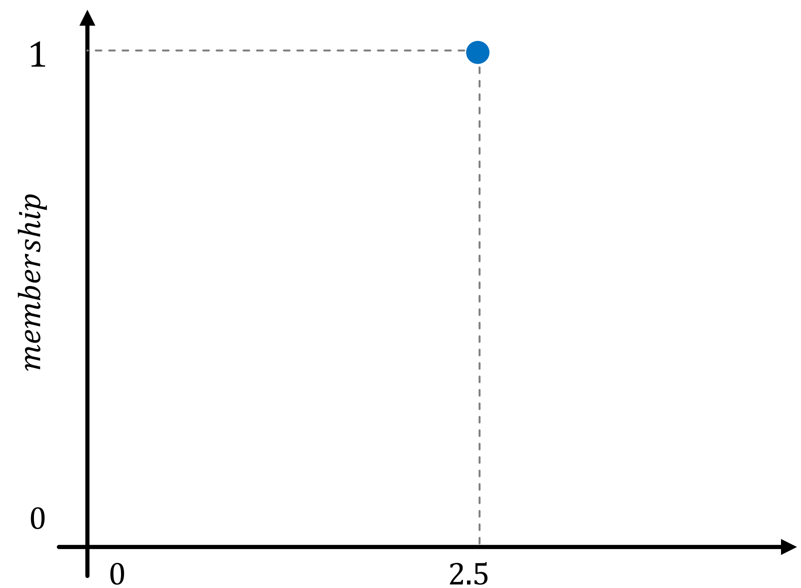
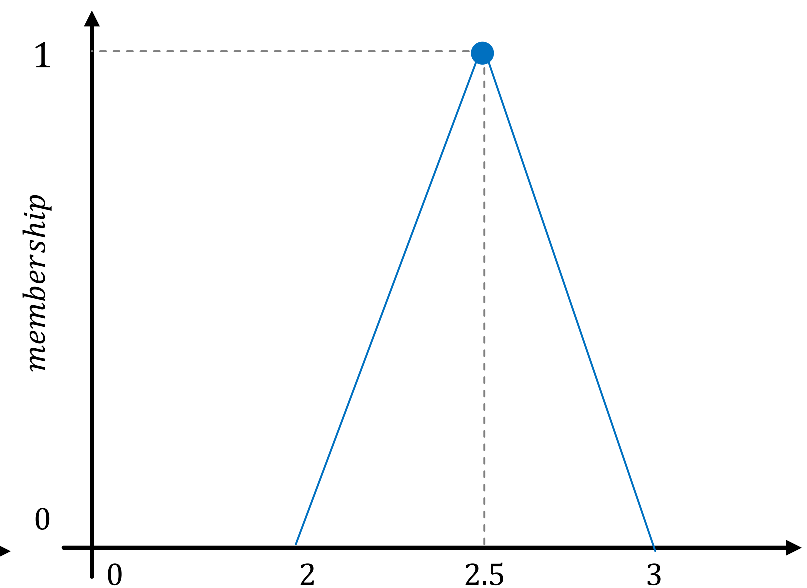
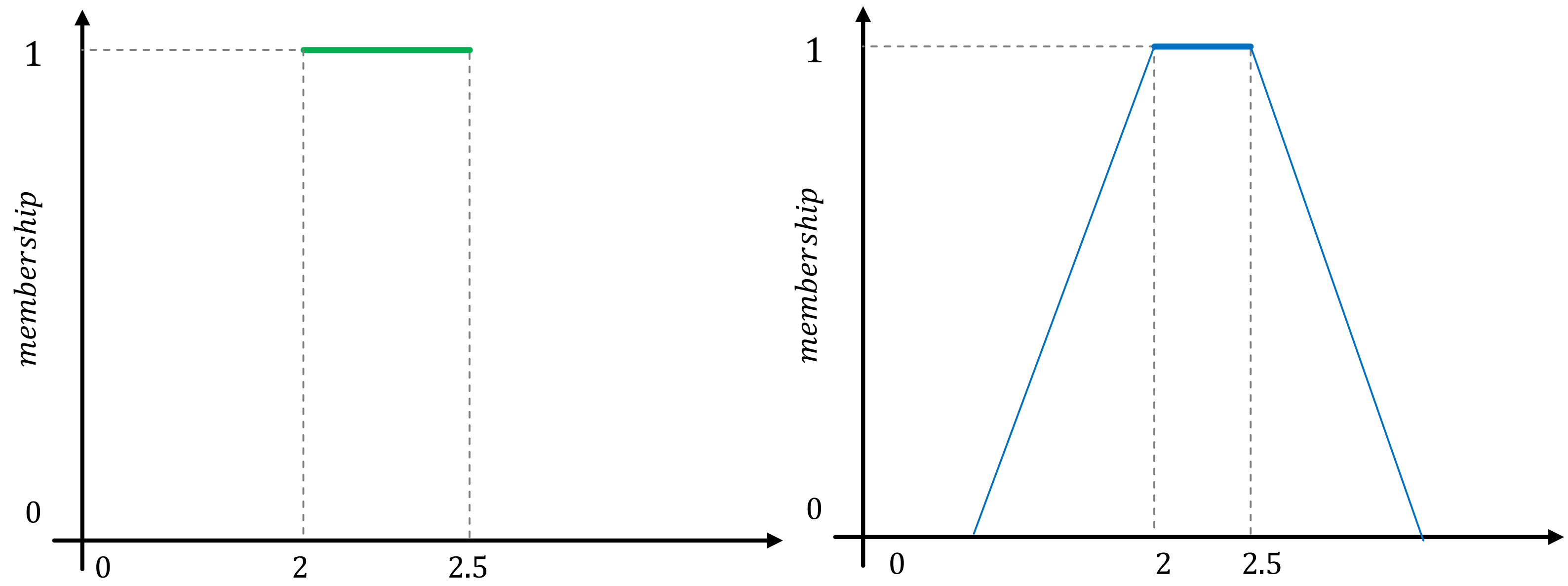
Crisp vs. Fuzzy numbers





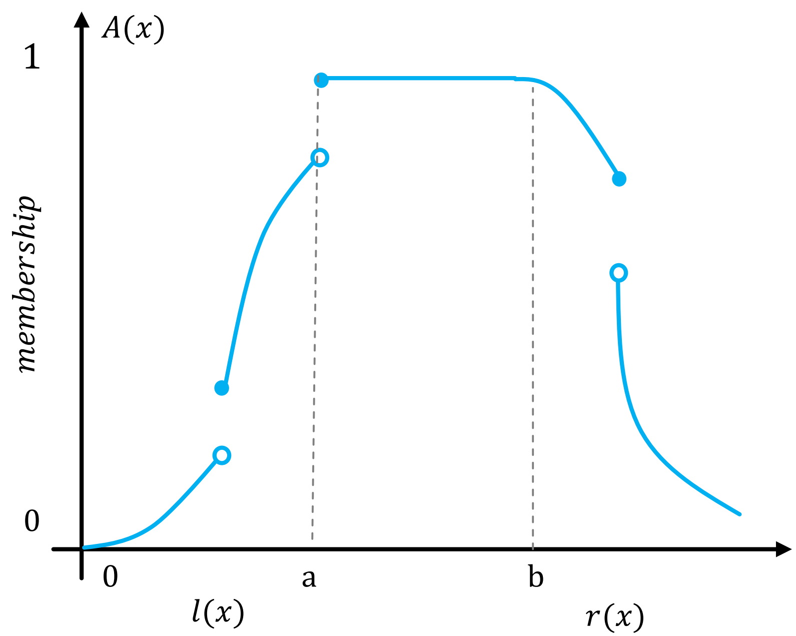
Fuzzy Numbers

Crisp vs. Fuzzy ranges



Fuzzy Numbers

- Fuzzy MFs can be formatted in discrete functions

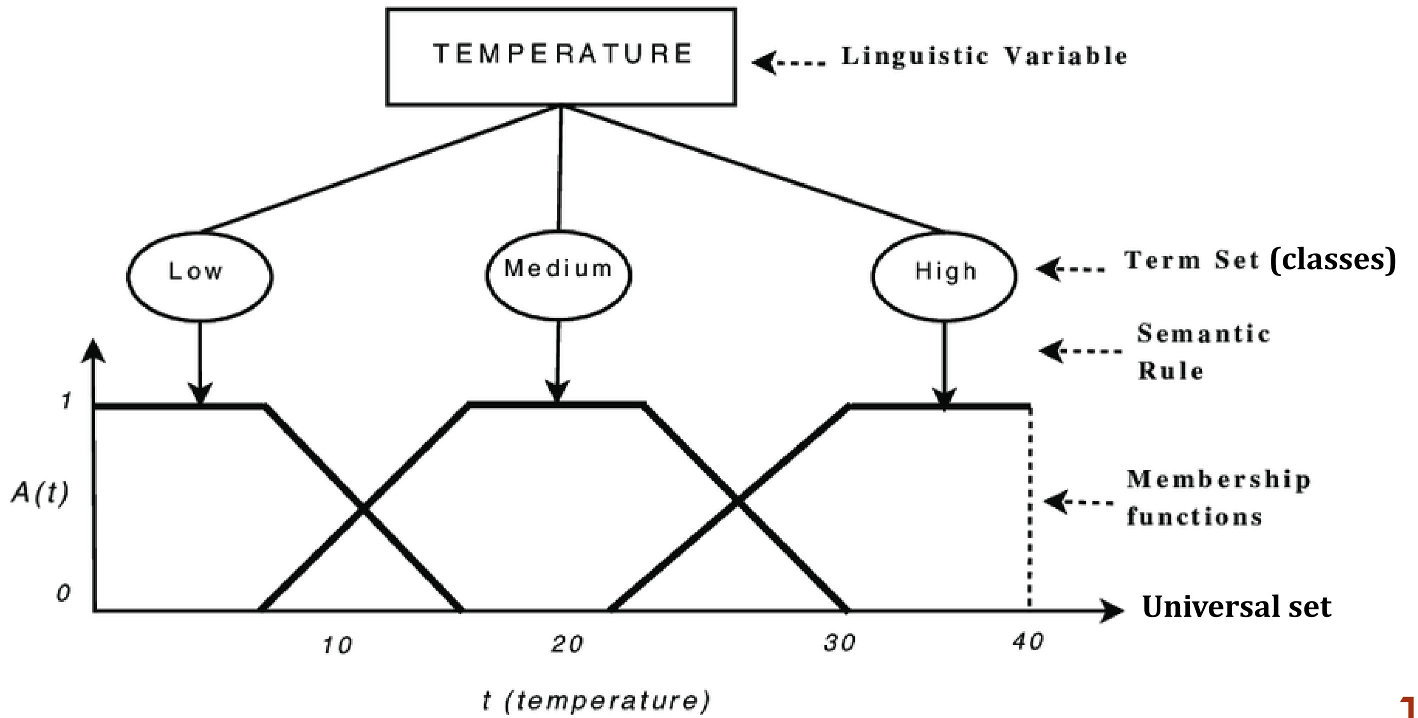


$$A = \begin{cases} 1 & for\ x \in [a, b] \\ l(x) & for\ x \in (-\infty, a] \\ r(x) & for\ x \in [b, \infty) \end{cases}$$

Crisp number: $a = b,\ \ l(x) = r(x) = 0$
Fuzzy number: $a = b,\ \ l(x) = r(x) \neq 0$
Crisp range: $a \neq b,\ \ l(x) = r(x) = 0$
Fuzzy range: $a \neq b,\ \ l(x) = r(x) \neq 0$

Fuzzy Numbers

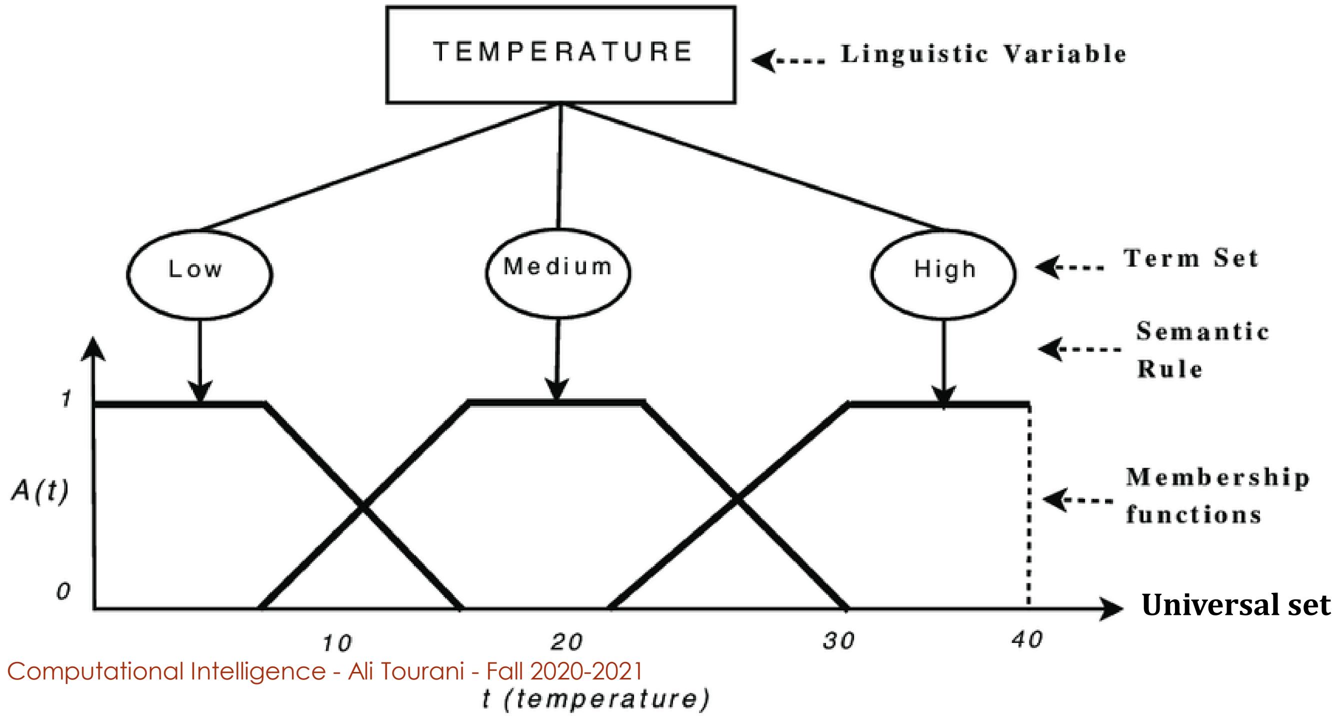
Sample:



Fuzzy Numbers

Sample:

$$Low = \begin{cases} 1 & for\ x \in [0,7] \\ l(x) = 0 & for\ x \in (-\infty, 0] \\ r(x) = \begin{cases} Something & if\ x \in (7,15) \\ 0 & if\ x \in (15, \infty) \end{cases} \end{cases}$$



Fuzzy Calculations

- A Fuzzy set is exclusively defined on its Alpha-cuts
  - Alpha-cuts are closed ranges of real numbers where $\alpha \in (0,1]$
  - Thus, calculations on Alpha-cuts define the calculations on Fuzzy numbers

Calculations in Fuzzy (Interval Arithmetic)

  - Lets consider * as any operation (addition, subtraction, multiplication and division)
  - Note: division is not defined if $0 \in [c, d]$

$$[a, b] * [c, d] = \{f * g \mid a \leq f \leq b, c \leq g \leq d\}$$

Fuzzy Calculations

Accordingly: $[a,b] * [c,d] = \{f * g \mid a \le f \le b, c \le g \le d\}$

$$[a,b] + [c,d] = [a+c, b+d]$$

$$[a,b] - [c,d] = [a-d, b-c]$$

$$[a,b].[c,d] = [\min(ac,ad,bc,bd), \max(ac,ad,bc,bd)]$$

$$[a,b]/[c,d] = \left[\min\left(\frac{a}{c},\frac{a}{d},\frac{b}{c},\frac{b}{d}\right), \max\left(\frac{a}{c},\frac{a}{d},\frac{b}{c},\frac{b}{d}\right)\right]$$

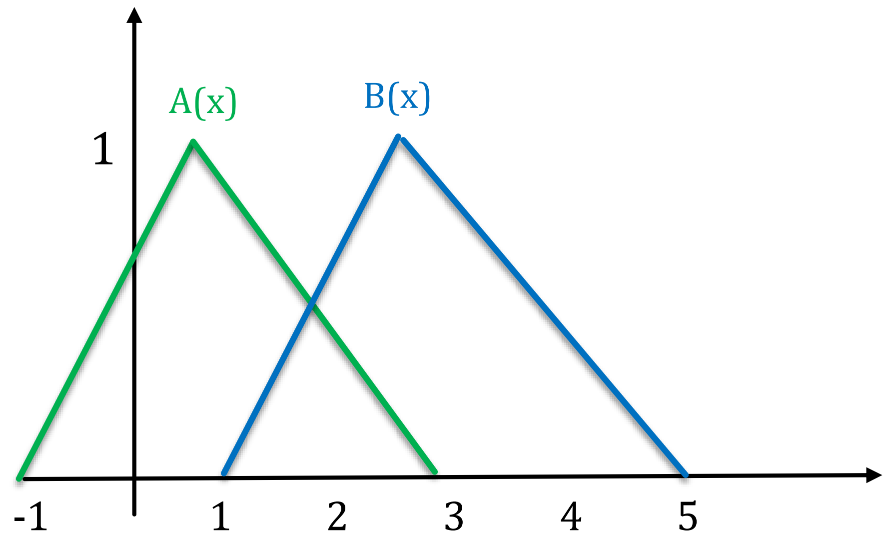
Fuzzy Calculations

Interval Arithmetic

- Basic calculations on below Fuzzy numbers

$$A(x) = \begin{cases} 0 & for\ x < -1\ and\ x > 3 \\ \dfrac{x+1}{2} & for -1 \le x \le 1 \\ \dfrac{3-x}{2} & for\ 1 < x \le 3 \end{cases}$$

$$B(x) = \begin{cases} 0 & for\ x < 1\ and\ x > 5 \\ \dfrac{x-1}{2} & for\ 1 \le x \le 3 \\ \dfrac{5-x}{2} & for\ 3 < x \le 5 \end{cases}$$



Fuzzy Calculations

Interval Arithmetic

- First, let's calculate Alpha

$$A(x) = \begin{cases} 0 & for\ x \leq -1\ and\ x > 3 \\ \dfrac{x+1}{2} & for -1 < x \leq 1 \\ \dfrac{3-x}{2} & for\ 1 < x \leq 3 \end{cases}$$

$$A^{\alpha} = [2\alpha - 1, 3 - 2\alpha]$$

$$B(x) = \begin{cases} 0 & for\ x \leq 1\ and\ x > 5 \\ \dfrac{x-1}{2} & for\ 1 < x \leq 3 \\ \dfrac{5-x}{2} & for\ 3 < x \leq 5 \end{cases}$$

$$B^{\alpha} = [2\alpha + 1, 5 - 2\alpha]$$

Fuzzy Calculations

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Interval Arithmetic

- *Addition*
  - We know that: $$[a, b] + [c, d] = [a + c, b + d]$$
  - Thus, for these Alpha values:

$$A^{\alpha} = [2\alpha - 1, 3 - 2\alpha] \qquad B^{\alpha} = [2\alpha + 1, 5 - 2\alpha]$$

$$(A + B)^{\alpha} = [4\alpha, 8 - 4\alpha]$$

Fuzzy Calculations

Interval Arithmetic

- *Addition*
  - Now, let's see how will the output look like

$$(A+B)^{\alpha} = [4\alpha, 8-4\alpha]$$

$4\alpha = x \rightarrow \alpha = \mathrm{x}/4$

$8 - 4\alpha = x \rightarrow \alpha = (8-\mathrm{x})/4$

$for\ \alpha = \frac{\mathrm{x}}{4}$: $\alpha = 0 \rightarrow x = 0 \quad \alpha = 1 \rightarrow x = 4$

$for\ \alpha = \frac{8-\mathrm{x}}{4}$: $\alpha = 0 \rightarrow x = 8 \quad \alpha = 1 \rightarrow x = 4$

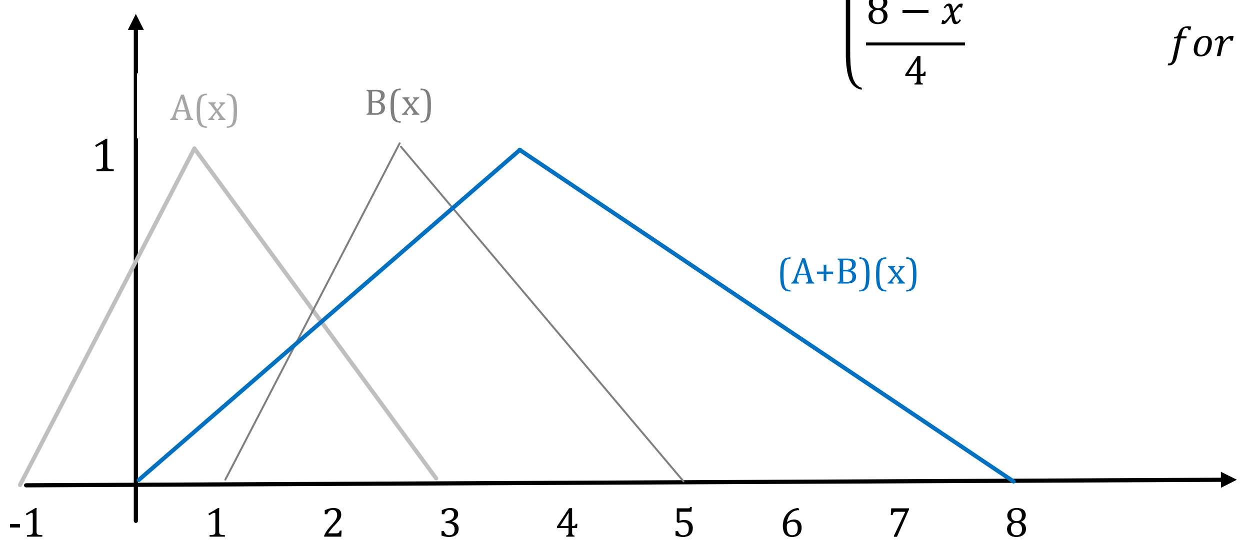
$$(A+B)(x) = \begin{cases} 0 & for\ x < 0\ and\ x > 8 \\ \frac{x}{4} & for\ 0 \le x \le 4 \\ \frac{8-x}{4} & for\ 4 < x \le 8 \end{cases}$$

Fuzzy Calculations

Interval Arithmetic

- *Addition*

$$(A+B)(x) = \begin{cases} 0 & for\ x < 0\ and\ x > 8 \\ \dfrac{x}{4} & for\ 0 \le x \le 4 \\ \dfrac{8-x}{4} & for\ 4 < x \le 8 \end{cases}$$



Fuzzy Calculations

Interval Arithmetic

- *Subtraction*
  - We know that: $$[a, b] - [c, d] = [a - d, b - c]$$
  - Thus, for these Alpha values:

$$A^\alpha = [2\alpha - 1, 3 - 2\alpha] \qquad B^\alpha = [2\alpha + 1, 5 - 2\alpha]$$

$$(A - B)^\alpha = [4\alpha - 6, 2 - 4\alpha]$$

Fuzzy Calculations

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Interval Arithmetic

$$(A-B)^{\alpha}=[4\alpha-6, 2-4\alpha]$$

- *Subtraction*
  - Now, let's see how will the output look like

$$4\alpha-6=x \rightarrow \alpha=(x+6)/4$$
$$2-4\alpha=x \rightarrow \alpha=(2-x)/4$$

$$for\ \alpha=\frac{(x+6)}{4}: \quad \alpha=0 \rightarrow x=-6 \quad \alpha=1 \rightarrow x=-2$$

$$for\ \alpha=\frac{2-x}{4}: \quad \alpha=0 \rightarrow x=2 \quad \alpha=1 \rightarrow x=-2$$

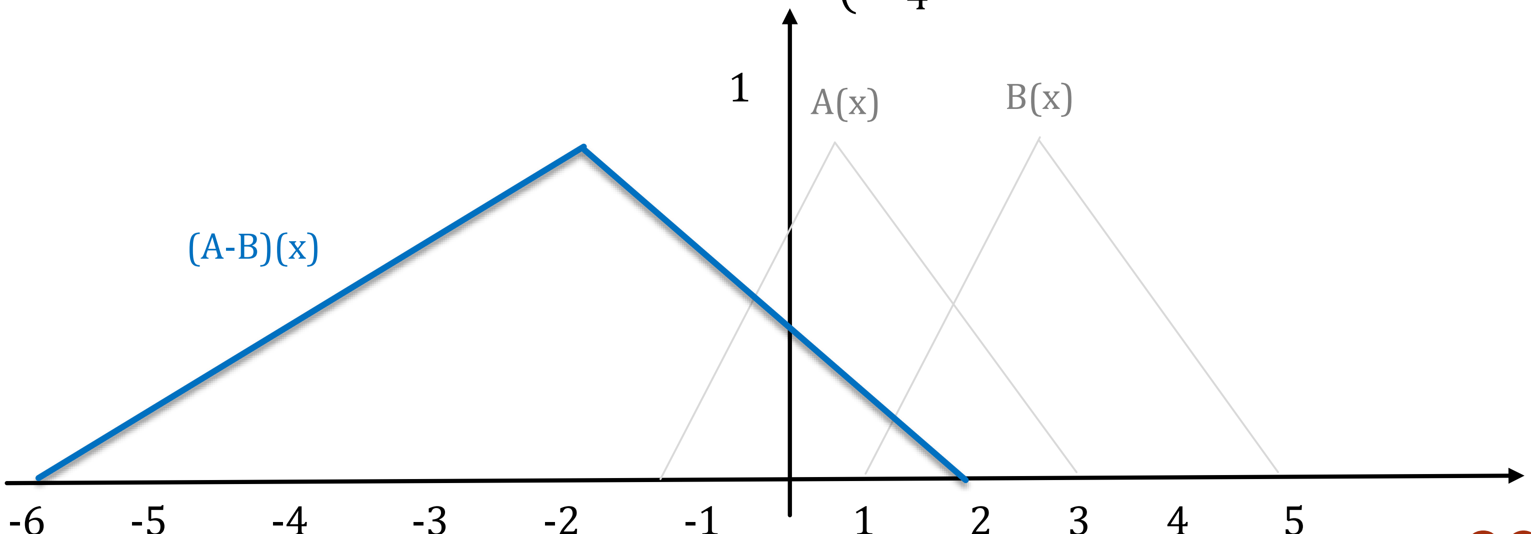
$$(A-B)(x)=\begin{cases} 0 & for\ x<-6\ and\ x>2 \\ \dfrac{x+6}{4} & for\ -6\leq x\leq -2 \\ \dfrac{2-x}{4} & for\ -2<x\leq 2 \end{cases}$$

Fuzzy Calculations

Interval Arithmetic

- *Subtraction*

$$(A-B)(x) = \begin{cases} 0 & for\ x < -6\ and\ x > 2 \\ \dfrac{x+6}{4} & for\ -6 \le x \le -2 \\ \dfrac{2-x}{4} & for\ -2 < x \le 2 \end{cases}$$



Fuzzy Calculations

Check it yourself!

Interval Arithmetic

- *Multiplication*
  - We know that: $[a,b].[c,d] = [\min(ac,ad,bc,bd)\,,\max(ac,ad,bc,bd)]$
  - Thus, for these Alpha values:

$$A^{\alpha} = [2\alpha - 1, 3 - 2\alpha] \qquad B^{\alpha} = [2\alpha + 1, 5 - 2\alpha]$$

$$(A.B)^{\alpha} = \begin{cases} [-4\alpha^2 + 12\alpha - 5, 4\alpha^2 - 16\alpha + 15] & for\ \alpha \in (0, 0.5] \\ [4\alpha^2 - 1, 4\alpha^2 - 16\alpha + 15] & for\ \alpha \in (0.5, 1] \end{cases}$$

Fuzzy Calculations

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Check it yourself!

Interval Arithmetic

- *Division*
  - We know that: $$[a,b]/[c,d] = \left[\min\left(\frac{a}{c},\frac{a}{d},\frac{b}{c},\frac{b}{d}\right), \max\left(\frac{a}{c},\frac{a}{d},\frac{b}{c},\frac{b}{d}\right)\right]$$
  - Thus, for these Alpha values:

$$A^{\alpha} = [2\alpha - 1, 3 - 2\alpha] \qquad B^{\alpha} = [2\alpha + 1, 5 - 2\alpha]$$

$$(A/B)^{\alpha} = \begin{cases} [(2\alpha - 1)/(2\alpha + 1), (3 - 2\alpha)/(2\alpha + 1)] & for\ \alpha \in (0, 0.5] \\ [(2\alpha - 1)/(5 - 2\alpha), (3 - 2\alpha)/(2\alpha + 1)] & for\ \alpha \in (0.5, 1] \end{cases}$$

Fuzzy Calculations

Check it yourself!

Interval Arithmetic

- *Multiplication*

$$(A.B)(x) = \begin{cases} 0, & x < -5 \text{ and } x \geq 15 \\ \dfrac{[3-(4-x)^{1/2}]}{2}, & -5 \leq x < 0 \\ \dfrac{(1+x)^{\frac{1}{2}}}{2}, & 0 \leq x < 3 \\ \dfrac{\left[4-(1+x)^{\frac{1}{2}}\right]}{2}, & 3 \leq x < 15 \end{cases}$$

- *Division*

$$(A/B)(x) = \begin{cases} 0, & x < -1 \text{ and } x \geq 3 \\ \dfrac{x+1}{2-2x}, & -1 \leq x < 0 \\ \dfrac{5x+1}{2+2x}, & 0 \leq x < 1/3 \\ \dfrac{3-x}{2+2x}, & 1/3 \leq x < 3 \end{cases}$$

Fuzzy Calculations

- For the Fuzzy numbers A, B, and C:

$$A + B = B + A \quad \text{and} \quad A.B = B.A$$
$$(A + B) + C = A + (B + C) \quad and \quad (A.B).C = A.(B.C)$$
$$A = A + 0 = 0 + A \quad and \quad A = A.1 = 1.A$$
$$A.(B + C) \subseteq A.B + A.C$$
$$A.(B + C) = A.B + A.C \quad if \;\; b.c \geq 0 \; for \; all \; b \in B \; and \; c \in C$$
$$a.(B + C) = a.B + a.C \quad if \;\; A = [a, a]$$
$$0 \in A - A \quad and \quad 1 \in \frac{A}{A}$$
$$if \;\; \mathrm{A} \subseteq \mathrm{E} \text{ and } \mathrm{B} \subseteq \mathrm{F}, \text{then } A * B \subseteq E * F \; where \;\; * \; can \; be +, -, . \; and \; /$$

Fuzzy Relations

We know from classical relations:

- A classic relation R is a subset of $A \times B$

$$R(x_i \mid i \in N_n) \subseteq X_1 \times X_2 \times \cdots \times X_3$$

Sample:

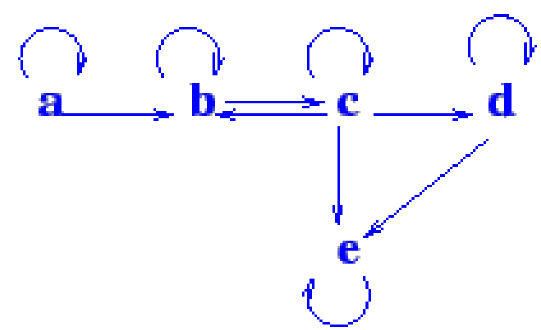
$$X = \{English, French\} \qquad Y = \{Dollar, Pound, Euro, Mark\}$$

$$Z = \{US, Britain, Canada, France\}$$

$$R(X, Y, Z) = \{(English, Dollar, US), (French, Euro, France), (English, Dollar, Canada)\}$$

Fuzzy Relations

We know from classical relations:



- Reflexive, $if\ R(x,x)$
- Irreflexive, $if\ !\ R(x,x)$
- Symmetric, $if\ (x,y) \in R,\ then\ (y,x) \in R$
- Antisymmetric, $if\ (x,y) \in R,\ then\ (y,x) \notin R$
- Transitive, $if\ (x,y) \in R\ and\ (y,z) \in R, then\ (x,z) \in R$
- Anti-transitive, $if\ (x,y) \in R\ and\ (y,z) \in R, then\ (x,z) \notin R$

Fuzzy Relations

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- Membership degrees in relations

$$America = \{New\ York, Washington, Atlanta\}$$

$$Europe = \{Paris, Milan, Barcelona, Rome\}$$

$$Asia = \{Tehran, Beijing, Tokyo, Dubi\}$$

$$R(Europe, Asia) = 0.7\ (Paris, Tehran) + 0.8\ (Milan, Dubai) + 0.2\ (Rome, Dubai)$$

Fuzzy Relations

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Projection

- We can project a fuzzy relation R ⊆ A × B with respect to A or B as in the following manner

For all $x \in A, y \in B$

Projection to A: $$\mu_{R_A}(x) = Max\ \mu_R\ (x, y)$$

Projection to B: $$\mu_{R_B}(x) = Max\ \mu_R\ (x, y)$$

Fuzzy Relations

Projection

- For instance:

M_R

	b_1	b_2	b_3
a_1	0.1	0.2	1.0
a_2	0.6	0.8	0.0
a_3	0.0	1.0	0.3

M_{R_A}

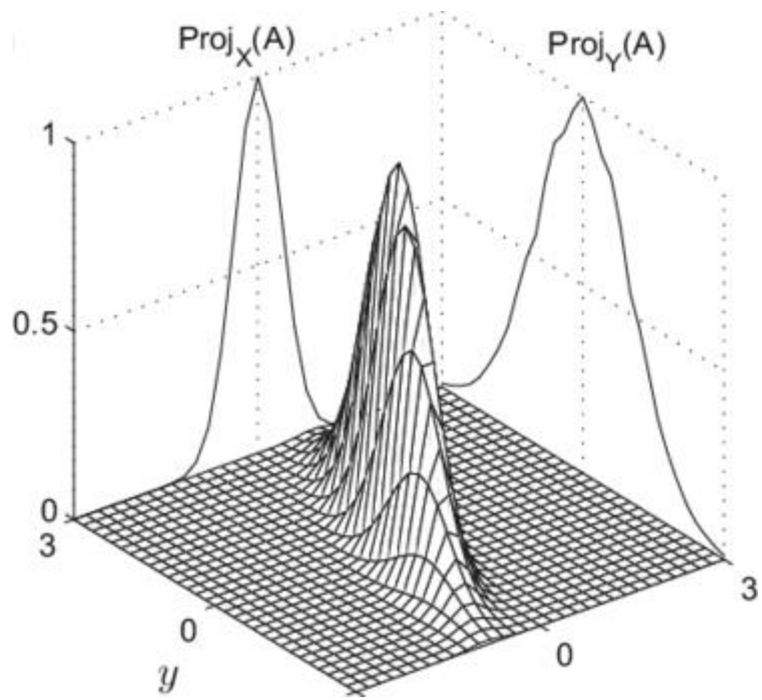
a_1	1.0
a_2	0.8
a_3	1.0

M_{R_B}

b_1	b_2	b_3
0.6	1.0	1.0

Fuzzy Relations

Projection



Fuzzy Relations

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Projection

$X_1 = \{0,1\}$ $X_2 = \{0,1\}$ $X_3 = \{0,1,2\}$

X1	X2	X3	R(x1, x2, x3)
0	0	0	0.4
0	0	1	0.9
0	0	2	0.2
0	1	0	1.0
0	1	1	0.0
0	1	2	0.8
1	0	0	0.5
1	0	1	0.3
1	0	2	0.1
1	1	0	0.0
1	1	1	0.5
1	1	2	1.0

Max

Max

Max

Max

R(x1, x2)
0.9
0.9
0.9
1.0
1.0
1.0
0.5
0.5
0.5
1.0
1.0
1.0

Fuzzy Relations

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Projection

$X_1 = \{0,1\}$ $X_2 = \{0,1\}$ $X_3 = \{0,1,2\}$

X1	X2	X3	R(x1, x2, x3)
0	0	0	0.4
0	0	1	0.9
0	0	2	0.2
0	1	0	1.0
0	1	1	0.0
0	1	2	0.8
1	0	0	0.5
1	0	1	0.3
1	0	2	0.1
1	1	0	0.0
1	1	1	0.5
1	1	2	1.0

Max (0.4 , 1.0)

R(x1, x3)
1.0
1.0

Fuzzy Relations

Projection

$X_1 = \{0,1\}$ $X_2 = \{0,1\}$ $X_3 = \{0,1,2\}$

X1	X2	X3	R(x1, x2, x3)
0	0	0	0.4
0	0	1	0.9
0	0	2	0.2
0	1	0	1.0
0	1	1	0.0
0	1	2	0.8
1	0	0	0.5
1	0	1	0.3
1	0	2	0.1
1	1	0	0.0
1	1	1	0.5
1	1	2	1.0

Max (0.9 , 0.0)

R(x1, x3)
1.0
0.9
1.0
0.9

Fuzzy Relations

Projection

$X_1 = \{0,1\}$ $X_2 = \{0,1\}$ $X_3 = \{0,1,2\}$

X1	X2	X3	R(x1, x2, x3)
0	0	0	0.4
0	0	1	0.9
0	0	2	0.2
0	1	0	1.0
0	1	1	0.0
0	1	2	0.8
1	0	0	0.5
1	0	1	0.3
1	0	2	0.1
1	1	0	0.0
1	1	1	0.5
1	1	2	1.0

Max (0.8 , 0.2)

R(x1, x3)
1.0
0.9
0.8
1.0
0.9
0.8

Fuzzy Relations

Projection

$X_1 = \{0,1\}$ $X_2 = \{0,1\}$ $X_3 = \{0,1,2\}$

X1	X2	X3	R(x1, x2, x3)
0	0	0	0.4
0	0	1	0.9
0	0	2	0.2
0	1	0	1.0
0	1	1	0.0
0	1	2	0.8
1	0	0	0.5
1	0	1	0.3
1	0	2	0.1
1	1	0	0.0
1	1	1	0.5
1	1	2	1.0

Likewise ...

R(x1, x3)
1.0
0.9
0.8
1.0
0.9
0.8
0.5
0.5
1.0
0.5
0.5
1.0

Fuzzy Relations

Projection

$X_1 = \{0,1\}$ $X_2 = \{0,1\}$ $X_3 = \{0,1,2\}$

X1	X2	X3	R(x1, x2, x3)
0	0	0	0.4
0	0	1	0.9
0	0	2	0.2
0	1	0	1.0
0	1	1	0.0
0	1	2	0.8
1	0	0	0.5
1	0	1	0.3
1	0	2	0.1
1	1	0	0.0
1	1	1	0.5
1	1	2	1.0

Max

Max

R(x1)
1.0
1.0
1.0
1.0
1.0
1.0
1.0
1.0
1.0
1.0
1.0
1.0

Fuzzy Relations

University of Guilan

Projection

$X_1 = \{0,1\}$ $X_2 = \{0,1\}$ $X_3 = \{0,1,2\}$

X1	X2	X3	R(x1, x2, x3)
0	0	0	0.4
0	0	1	0.9
0	0	2	0.2
0	1	0	1.0
0	1	1	0.0
0	1	2	0.8
1	0	0	0.5
1	0	1	0.3
1	0	2	0.1
1	1	0	0.0
1	1	1	0.5
1	1	2	1.0

Max

R(x2)
0.9
0.9
0.9
0.9
0.9
0.9

Fuzzy Relations

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Projection

$X_1 = \{0,1\}$ $X_2 = \{0,1\}$ $X_3 = \{0,1,2\}$

X1	X2	X3	R(x1, x2, x3)
0	0	0	0.4
0	0	1	0.9
0	0	2	0.2
0	1	0	1.0
0	1	1	0.0
0	1	2	0.8
1	0	0	0.5
1	0	1	0.3
1	0	2	0.1
1	1	0	0.0
1	1	1	0.5
1	1	2	1.0

Likewise ...

R(x2)
0.9
0.9
0.9
1.0
1.0
1.0
0.9
0.9
0.9
1.0
1.0
1.0

Fuzzy Relations

Projection (we are reducing the dimensions and losing data)

X1	X2	X3	R(x1, x2, x3)	R(x1, x2)	R(x1, x3)	R(x2, x3)	R(x1)	R(x2)	R(x3)
0	0	0	0.4	0.9	1.0	0.5	1.0	0.9	1.0
0	0	1	0.9	0.9	0.9	0.9	1.0	0.9	0.9
0	0	2	0.2	0.9	0.8	0.2	1.0	0.9	1.0
0	1	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0	1	1	0.0	1.0	0.9	0.5	1.0	1.0	0.9
0	1	2	0.8	1.0	0.8	1.0	1.0	1.0	1.0
1	0	0	0.5	0.5	0.5	0.5	1.0	0.9	1.0
1	0	1	0.3	0.5	0.5	0.9	1.0	0.9	0.9
1	0	2	0.1	0.5	1.0	0.2	1.0	0.9	1.0
1	1	0	0.0	1.0	0.5	1.0	1.0	1.0	1.0
1	1	1	0.5	1.0	0.5	0.5	1.0	1.0	0.9
1	1	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Fuzzy Relations

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Cylinder Extension

- The opposite concept of Projection
- A fuzzy relation R ⊆ A × B can be extended to A × B × C to generate a new Fuzzy set. Thus for the new Fuzzy set C(R):

$$\mu_{C(R)}(a,b,c) = \mu_R\,(a,b) \qquad a \in A, b \in B, c \in C$$

- Sample:

M_{R_A}

a_1	1.0
a_2	0.8
a_3	1.0

	b_1	b_2	b_3
a_1	1.0	1.0	1.0
a_2	0.8	0.8	0.8
a_3	1.0	1.0	1.0

$M_{C(R_A)}$

Fuzzy Relations

Let’s say: R12 = R(x1,x2)

Cylinder Extension

$X_1 = \{0,1\} \quad X_2 = \{0,1\} \quad X_3 = \{0,1,2\}$

X1	X2	X3	R123	R12	R13	R23	R1	R2	R3
0	0	0	0.4	0.9	1.0	0.5	1.0	0.9	1.0
0	0	1	0.9	0.9	0.9	0.9	1.0	0.9	0.9
0	0	2	0.2	0.9	0.8	0.2	1.0	0.9	1.0
0	1	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0	1	1	0.0	1.0	0.9	0.5	1.0	1.0	0.9
0	1	2	0.8	1.0	0.8	1.0	1.0	1.0	1.0
1	0	0	0.5	0.5	0.5	0.5	1.0	0.9	1.0
1	0	1	0.3	0.5	0.5	0.9	1.0	0.9	0.9
1	0	2	0.1	0.5	1.0	0.2	1.0	0.9	1.0
1	1	0	0.0	1.0	0.5	1.0	1.0	1.0	1.0
1	1	1	0.5	1.0	0.5	0.5	1.0	1.0	0.9
1	1	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Min

Min

Cylinder(R12,R13,R23)
0.5
0.5

Fuzzy Relations

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Cylinder Extension

$X_1 = \{0,1\} \quad X_2 = \{0,1\} \quad X_3 = \{0,1,2\}$

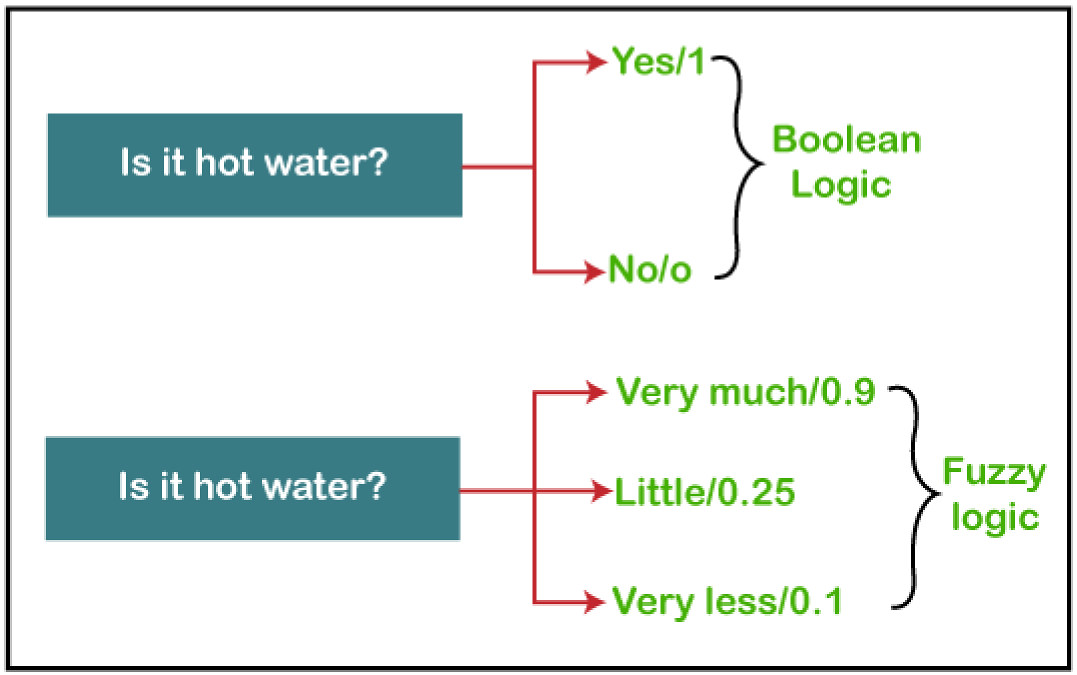
X1	X2	X3	R123	R12	R13	R23	R1	R2	R3
0	0	0	0.4	0.9	1.0	0.5	1.0	0.9	1.0
0	0	1	0.9	0.9	0.9	0.9	1.0	0.9	0.9
0	0	2	0.2	0.9	0.8	0.2	1.0	0.9	1.0
0	1	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0	1	1	0.0	1.0	0.9	0.5	1.0	1.0	0.9
0	1	2	0.8	1.0	0.8	1.0	1.0	1.0	1.0
1	0	0	0.5	0.5	0.5	0.5	1.0	0.9	1.0
1	0	1	0.3	0.5	0.5	0.9	1.0	0.9	0.9
1	0	2	0.1	0.5	1.0	0.2	1.0	0.9	1.0
1	1	0	0.0	1.0	0.5	1.0	1.0	1.0	1.0
1	1	1	0.5	1.0	0.5	0.5	1.0	1.0	0.9
1	1	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Min

Cylinder(R12,R13,R23)
0.5
0.9
0.2
1.0
0.5
0.8
0.5
0.5
0.2
0.5
0.5
1.0

What's Next?

- Fuzzy Logic and Inference



Questions?