CSCA67 TT2 Review Seminar

Held by AMACSS

Oct 22nd (Wed) 17:00 - 19:00

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Translating Logic to English

Let the universe of discourse be all people.

Let:

A(x) mean “x is teaching CSCA67.”

T(x) mean “x is taking CSCA48.”

F(x) mean “x has an Instagram page.”

C(x) mean “x likes to cook.”

From the logical forms, create the corresponding English statements:

A. $\exists x, A(x) \wedge C(x)$
B. $\forall x, T(x) \rightarrow F(x)$
C. $\neg \exists z, T(z) \wedge A(z)$
D. $\neg \forall x, T(x) \rightarrow (F(x) \vee C(x))$

Translating English to Logic

Analyse the logical forms of the following statements.

A. Anyone who thinks 173 is fun is either crazy or an instructor.
B. No prime number except 3 is divisible by 3.
C. Every CSCA67 student must complete an assignment, and at least one CSCA67 student must shovel snow.

Which statements are equivalent?

Select all statements which are equivalent to the following statement:

$\forall n \in Z$, n^2 is even $\Rightarrow$ n is even.

A. All integers have even squares and are even.
B. Given an integer whose square is even, that integer itself is even.
C. There does not exist an integer that is odd and has an even square.
D. n is even is necessary for n^2 to be even.
E. n is even is sufficient for an n^2 is even.
F. All integers that are even have even squares.

Quantifier negation

Let P(x) mean “x is gonna party after the term test”.

Negate $\forall$ **x, P(x)**

(negate “**everyone** is gonna party”)

$\neg(\forall x, P(x))$

$\exists x, \neg P(x)$

“**someone** is **not** gonna party”

Negate $\exists$ **x, P(x)**

(negate “**someone** is gonna party)

$\neg(\exists x, P(x))$

$\forall x, \neg P(x)$

“**everyone** is **not** gonna party”

“nobody is gonna party”

Warm up - intuition-building

1. Warm up
   For each of the following English statements (e.g. everyone is nice), negate them by prefixing them with an "it is not true that" (e.g. it is not true that everyone is nice). Then write an equivalent statement that does not begin with a negation (e.g. some people are not nice).

   (a) All crows are black.

   (b) Somebody once told me the world was gonna roll me.

   (c) For every epsilon greater than zero there is a delta greater than zero such that $f(x)$ is within epsilon of the limit if delta is sufficiently close to x.

Warm-up solutions

(a) Some crows are not black.

(b) Everyone didn't tell me the world was gonna roll me.

(c) There exist some epsilons greater than zero where for all deltas greater than zero it is not the case that $f(x)$ is within epsilon of the limit if delta is sufficiently close to x.

Quantifier Negation - Single Quantifier

2. Single-quantifier negation
   Let $B(x)$ represent "it is x's birthday today" and $C(x)$ represent "x is eating cake". For each of the following statements, negate them formally and translate the original and negated statements into English.

   (a) $\exists x, B(x)$

   (b) $\forall x, B(x) \rightarrow C(x)$

Single Quantifier solutions

(a) $\neg(\exists x, B(x))$

eqv. $\forall x, \neg B(x)$

Original: Someone has a birthday today.

Negated: It is not anyone's birthday today.

(b) $\neg(\forall x, B(x) \rightarrow C(x))$

eqv. $\exists x, \neg(B(x) \rightarrow C(x))$ (q.n.)

eqv. $\exists x, \neg(\neg B(x) \vee C(x))$ (cond.)

eqv. $\exists x, B(x) \wedge \neg C(x)$ (DM)

Original: Everyone whose birthday is today, is eating cake.

Negated: It is someone's birthday today, and they are not eating cake.

Quantifier Negation - Multiple Quantifiers

3. Multi-quantifier negation
   Let $B(x)$ represent the same as before. Let $G(x, y)$ represent "x got y a birthday gift". For each of the following statements, negate them formally and translate the original and negated statements into English.

   (a) $\forall x \exists y, G(x, y)$

   (b) $\exists y \forall x, G(x, y)$

   (c) $\forall x, B(x) \rightarrow \exists y, G(y, x)$

Multiple Quantifiers solutions

(a) $\neg(\forall x \exists y, G(x, y))$

eqv. $\exists x \neg(\exists y, G(x, y))$

eqv. $\exists x \forall y, \neg G(x, y)$

Original: Everyone gave somebody a birthday gift.

Negated: Someone gave no one a birthday gift.

(b) $\neg(\exists y \forall x, G(x, y))$

eqv. $\forall y \neg(\forall x, G(x, y))$

eqv. $\forall y \exists x, \neg G(x, y)$

Original: Someone received birthday gifts from everyone.

Negated: Everyone has at least one person who didn't give them a gift.

Multiple Quantifiers solutions 2

(c) $\neg(\forall x, B(x) \to \exists y, G(y, x))$

eqv. $\exists x, \neg(B(x) \to \exists y, G(y, x))$ (q.n.)

eqv. $\exists x, \neg(\neg B(x) \vee \exists y, G(y, x))$ (cond.)

eqv. $\exists x, B(x) \wedge \neg(\exists y, G(y, x))$ (DM)

eqv. $\exists x, B(x) \wedge \forall y, \neg G(y, x)$ (q.n.)

Original: Everyone gets a gift from someone on their birthday.

Negated: Someone didn't get a gift from anyone on their birthday :(

Identify Valid Applications of EI

Which of the following are valid applications of Existential Instantiation?

(A)

(1) ∃x, P(x)	
(2) P(c)	1, EI

(B)

(1) ∃x, P(x)	
(2) P(c)	1, EI
(3) P(d)	1, EI

(C)

(1) ∃x, y, P(x, y)	
(2) ∃y, P(c, y)	1, EI
(3) P(c, d)	2, EI

(D)

(1) ∃x, y, P(x, y)	
(2) ∃y, P(c, y)	1, EI
(3) P(c, c)	2, EI

Invalid Applications

The following is an **incorrect** “proof” of a clearly invalid argument. Identify the exact step where a rule of inference is misapplied and explain why it is invalid.

Argument

1. $\exists x, A(x) \land \neg B(x)$
2. $\exists x, A(x) \land \neg C(x)$
$\therefore \exists x, \neg B(x) \land \neg C(x)$

“Proof”

(1) $\exists x, A(x) \land \neg B(x)$	Premise 1
(2) $\exists x, A(x) \land \neg C(x)$	Premise 2
(3) $A(c) \land \neg B(c)$	1, EI
(4) $A(c) \land \neg C(c)$	2, EI
(5) $\neg B(c)$	3, Simplification
(6) $\neg C(c)$	4, Simplification
(7) $\neg B(c) \land \neg C(c)$	5, 6, Conjunction
$\exists x, \neg B(x) \land \neg C(x)$	7, EG

Identify Valid Applications of UG

Which of the following are valid applications of Universal Generalization?

(A)

Take arbitrary c
…
P(c)
∀x, P(x) [UG]

(B)

Take arbitrary c
…
P(c) ∧ P(d)
∀x, P(x) ∧ P(d) [UG]

(C)

Take arbitrary c
…
∃x, P(x, c)
∃x∀y, P(x, y) [UG]

(D)

Take arbitrary c
…
∃x, P(x, c)
∀y∃x, P(x, y) [UG]

Practice Applying Instantiation & Generalization

Consider the following premises:

1. $\forall x, C(x) \rightarrow F(x)$
2. $\exists x, C(x) \wedge A(x)$

Where:

C(x): x is a cat	F(x): x likes fish	A(x): x is adorable
Universe of discourse is **all animals**.		

Prove that the following conclusion follows:

$\exists x, F(x) \wedge A(x)$

Practice Applying Instantiation & Generalization 2

“All unicorns are either white or rainbow colored. All white unicorns are magical. All rainbow unicorns can fly. Only magical unicorns fly. Therefore, all unicorns are magical.”

Where:

W(x): x is white	R(x): x is rainbow colored	M(x): x is magical
F(x): x can fly	Universe of discourse is **unicorns**.	

Prove or disprove validity of the argument.

Universal Modus Ponens & Tollens

At Bob's bakery, the following rules are always true:

1. Every item in the display was baked this morning.
2. Every muffin in the display case contains blueberries.
3. Any item that contains blueberries was made by Chef Bob.

Today, you observe that the cookie is in the display case.
You also know that Bob did not make the cookie.

Prove that the cookie is not a muffin.

D(x): Item x is in the display case.	M(x): Item x is a muffin.
B(x): Item x contains blueberries.	C(x): Item x was made by Chef Bob.
T(x): Item x was baked this morning.	Universe of discourse is **bakery items**.

Valid or invalid argument? (with sets!)

(hint: try to visualize first with a Venn Diagram!)

Are the following arguments valid or invalid? Provide a proof!

Q1. If $A \subseteq B$ or $B \subseteq C$, then $A \subseteq C$

Q2. If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

Proving validity of an argument

Prove the following argument is valid:

Let the universe of discourse be unicorns.

For every magical unicorn, there is a blue unicorn. Every unicorn that is blue is happy. There is at least one unicorn that is magical.

Therefore, there must be a unicorn that is happy.

You may use the following predicates:

M(x): “x is magical”	B(x): “x is blue”
H(x): “x is happy”	

Proving an argument is not valid

Prove the following argument is not valid:

Some UTSC students are badminton players.

Some badminton players play pickleball.

All pickleball players play badminton.

∴ Some UTSC students play pickleball.

You may use the following predicates:

U(x): “x is a UTSC student”	B(x): “x plays badminton”
P(x): “x plays pickleball”	