DynamicDays2021 - XL
Online Conference

Coupled behaviour of oscillators under asymmetric forcing

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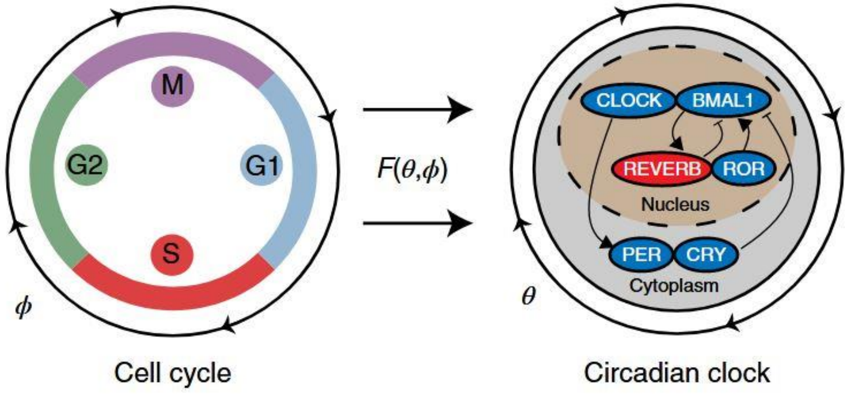
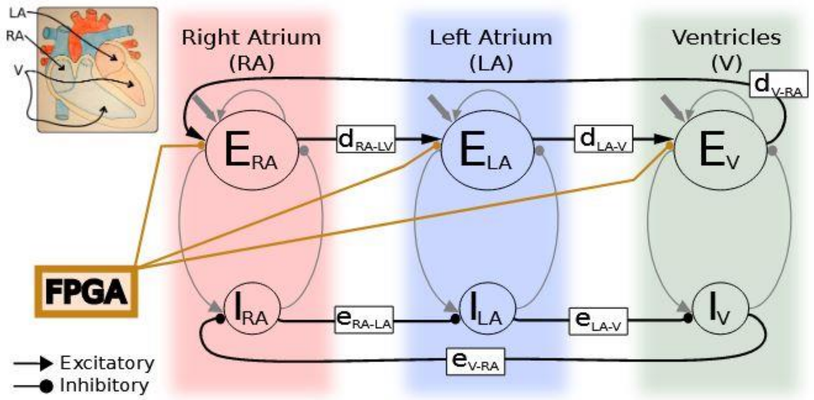
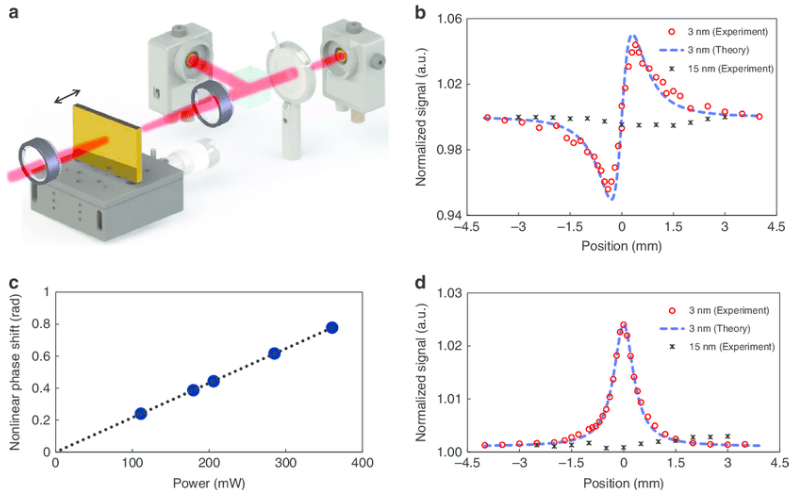


Circadian rhythm cycle



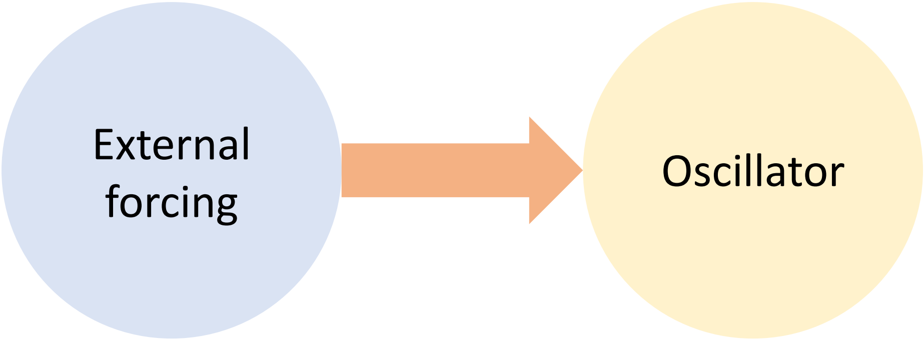
Cardiac pacemaker dynamics



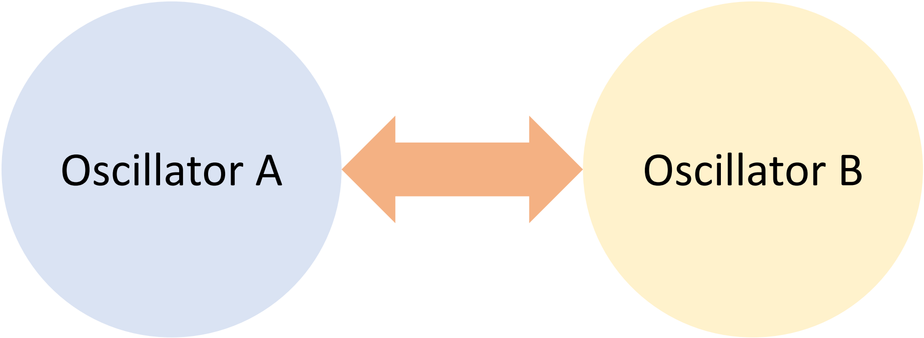
Laser oscillators

Structural oscillations

Droin *et al*. Nature Physics **15**, 2019, Qian *et al*. Nature Communications **7**, 2016
Krause *et al*. arXiv, 2021, BritSync YouTube Channel



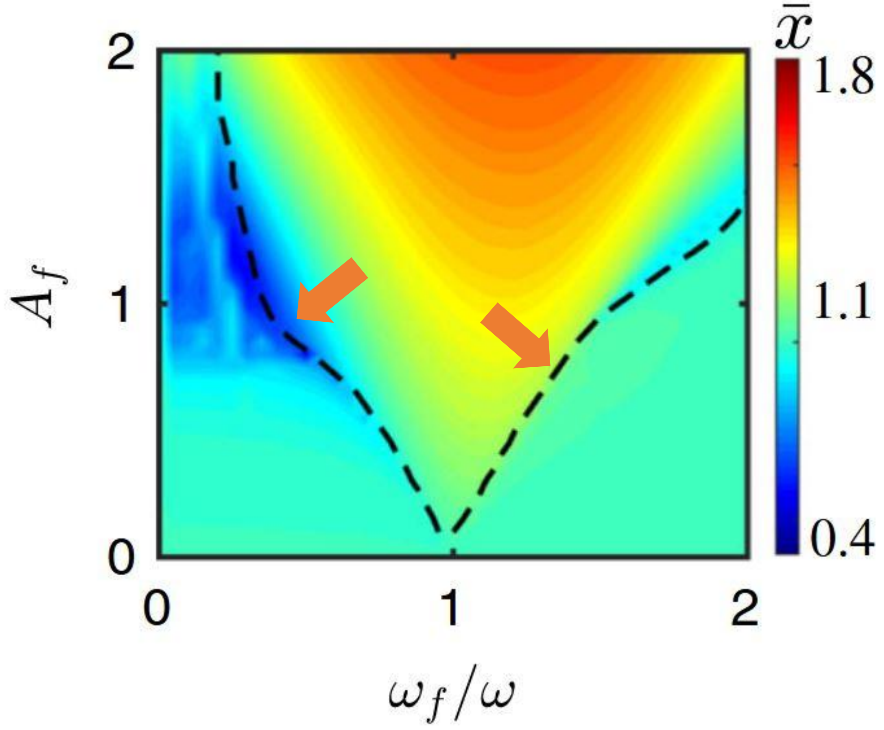
External forcing



Mutual coupling

External forcing

Driven Stuart-Landau oscillator

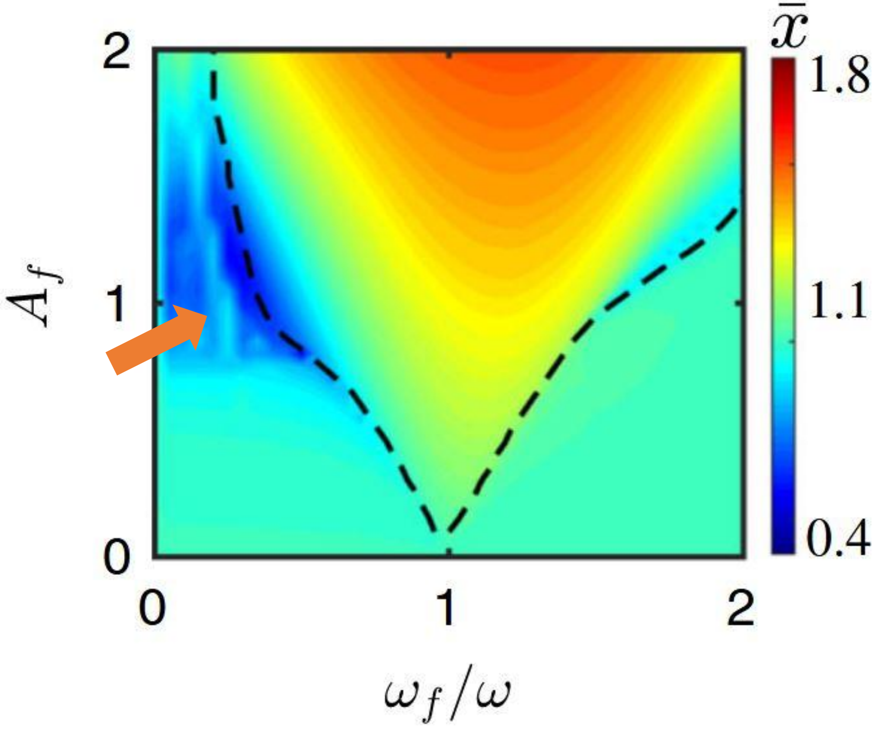


$$\dot{z} = (\lambda_0 + i\omega - a|z|^2 - b|z|^4)z + A\sin(\omega_f t)$$

Premraj *et al*. Nonlinear Dynamics **103**, 2021

External forcing

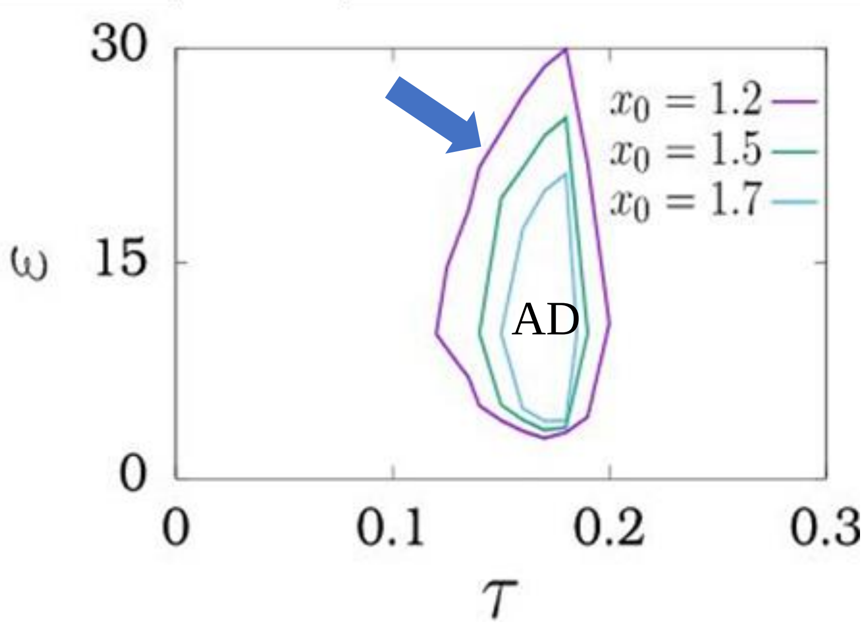
Driven Stuart-Landau oscillator



$$\dot{z} = (\lambda_0 + i\omega - a|z|^2 - b|z|^4)z + A\sin(\omega_f t)$$

Mutual coupling

Coupled Stuart-Landau oscillators



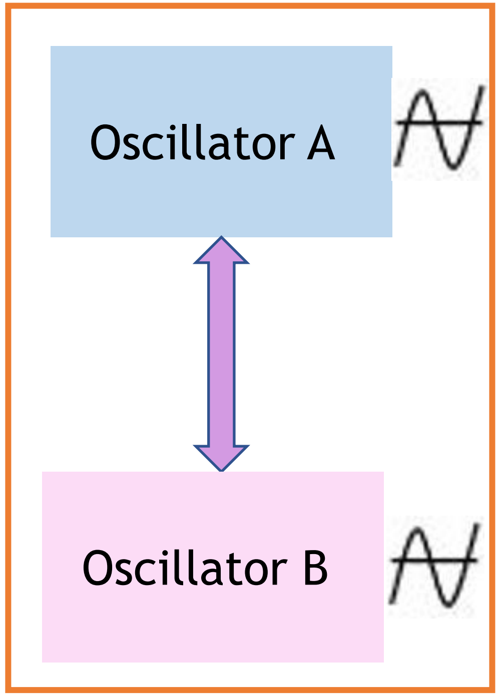
$$\dot{z}_j = (\lambda_0 + i\omega_j - |z_j|^2 - |z_j|^4)z_j + \epsilon[z_k(t-\tau) - z_j]$$

Premraj *et al*. Nonlinear Dynamics **103**, 2021

Can we enhance the characteristics of amplitude suppression in a coupled system through forcing?

$$\dot{z}_j = (\lambda_0 + i\omega_j - |z_j|^2 - |z_j|^4) z_j + \epsilon[z_k(t-\tau) - z_j]$$

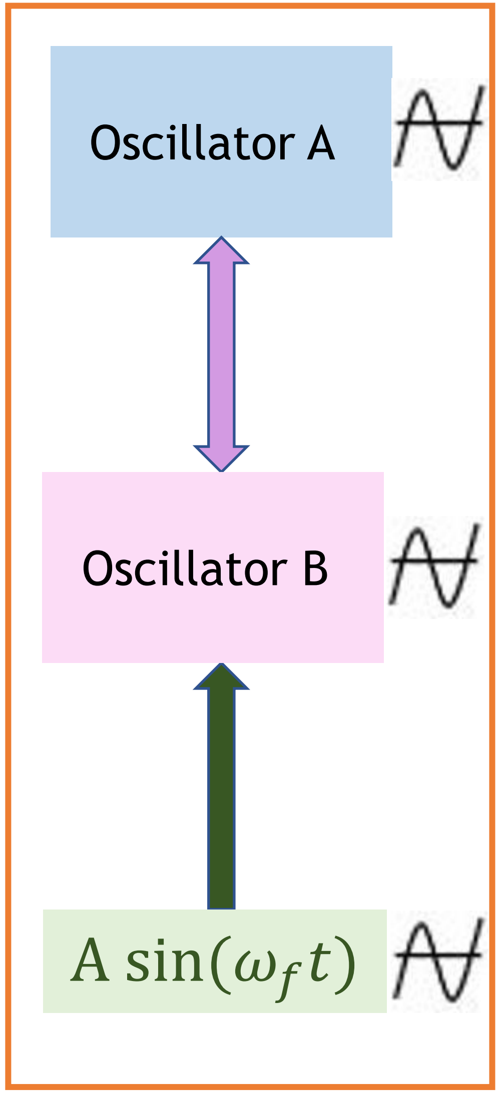
Coupling term



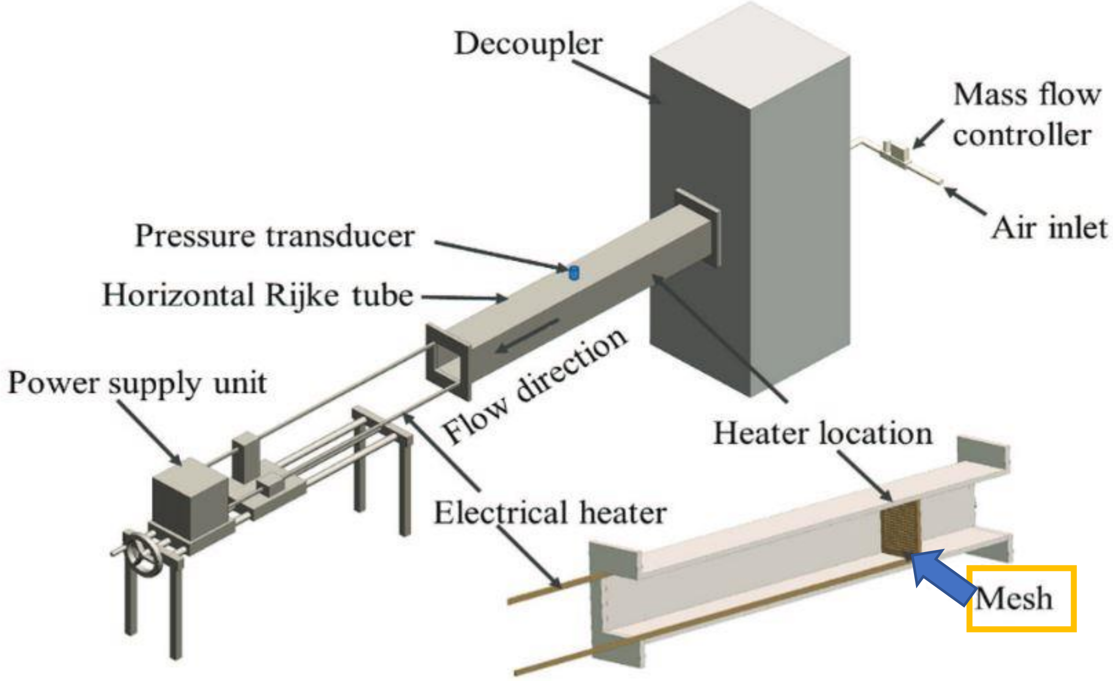
Dynamics of an asymmetrically forced coupled system

$$\dot{z}_j = (\lambda_0 + i\omega_j - |z_j|^2 - |z_j|^4)z_j + \epsilon[z_k(t-\tau) - z_j] + [A\sin(\omega_f t)]^B$$

Coupling term Harmonic forcing



Horizontal Rijke Tube

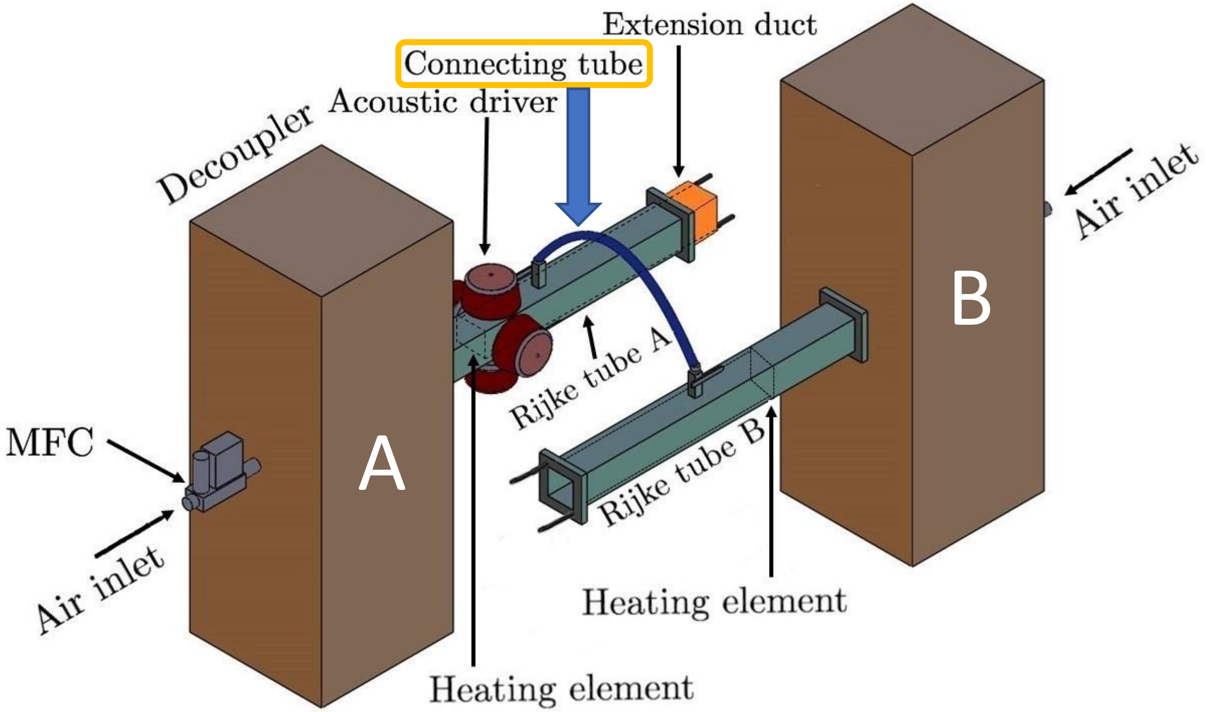


Rijke tube is a prototypical thermoacoustic system

The acoustic duct can exhibit self-excited limit cycle oscillations

Tandon *et al.*, Chaos **30**, 2020

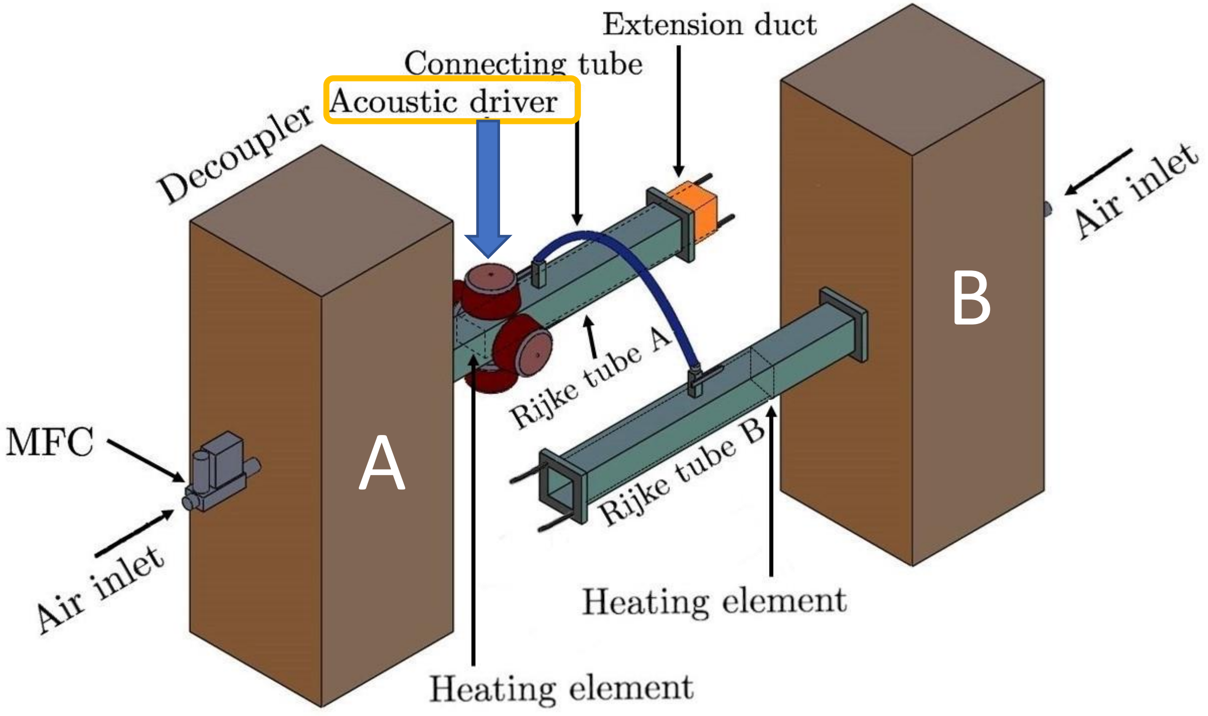
Coupled Rijke tubes with asymmetric forcing



Coupling parameters:

- Length of the coupling tube (L)
- Internal diameter of the coupling tube (d)

Coupled Rijke tubes with asymmetric forcing



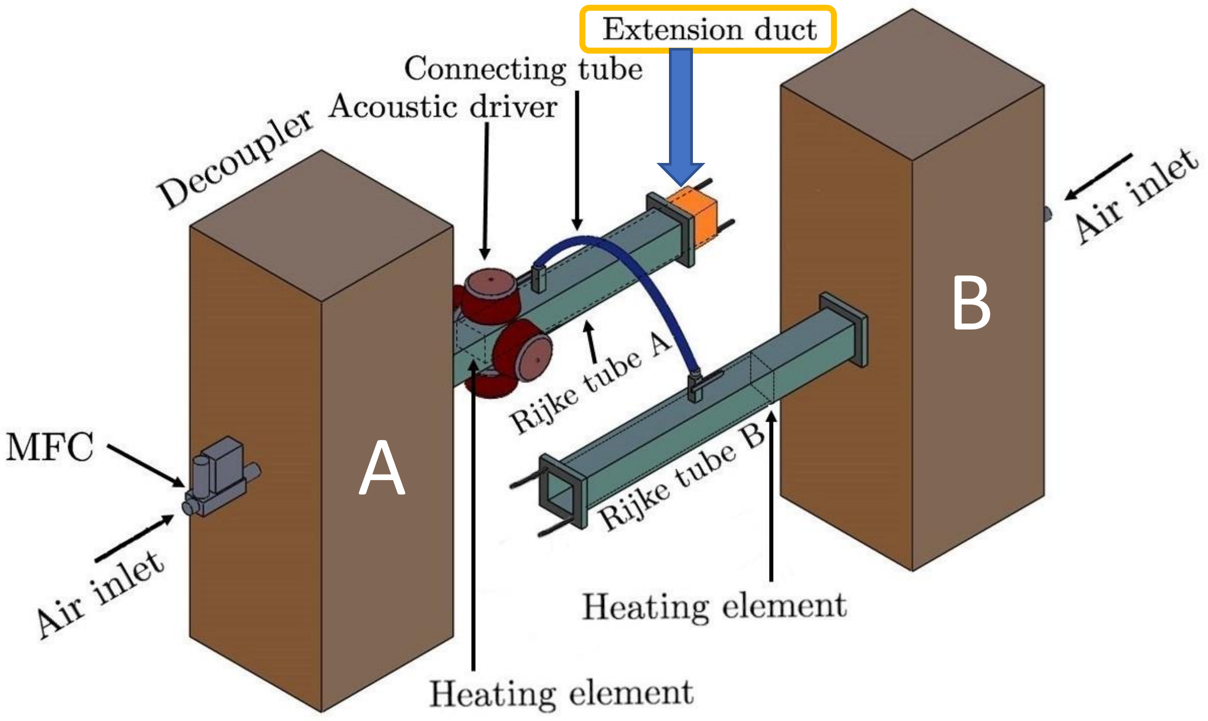
Coupling parameters:

- Length of the coupling tube (L)
- Internal diameter of the coupling tube (d)

Forcing signal parameters:

- Forcing frequency (f_f)
- Forcing amplitude (A_f)

Coupled Rijke tubes with asymmetric forcing



Coupling parameters:

- Length of the coupling tube (L)
- Internal diameter of the coupling tube (d)

Forcing signal parameters:

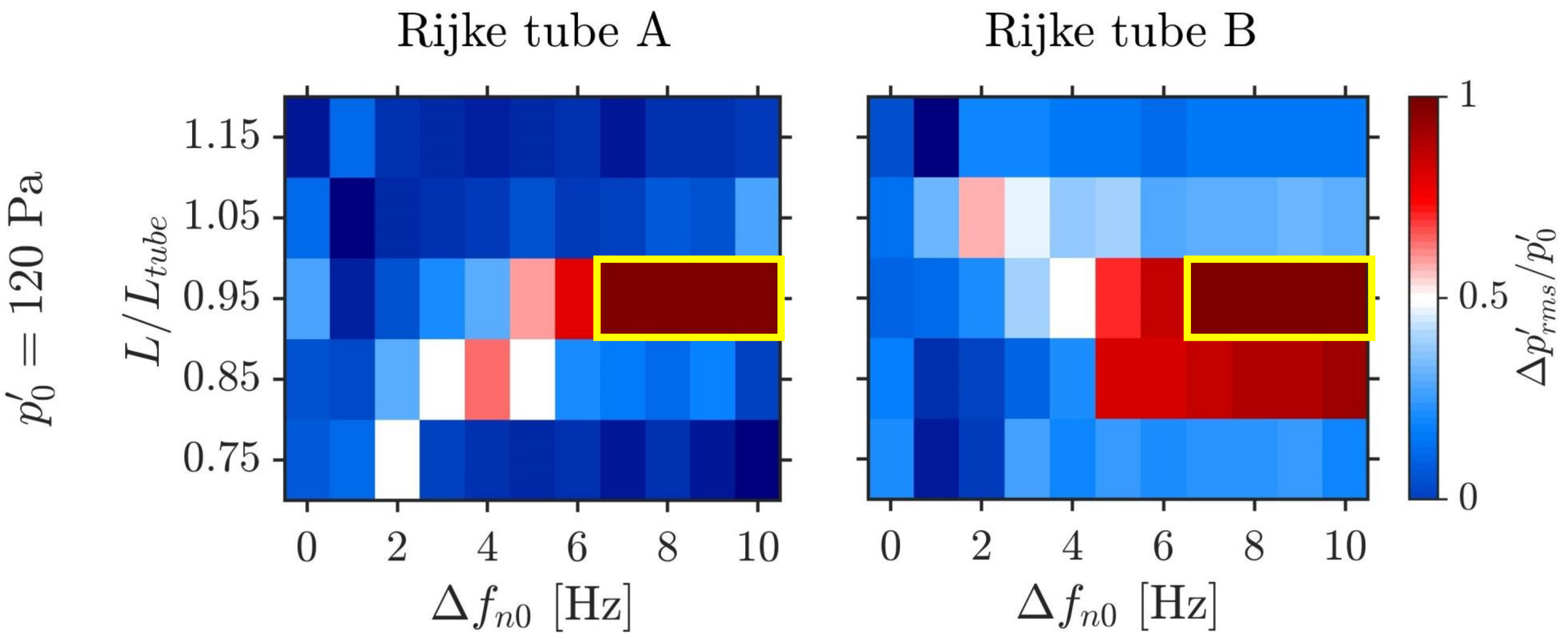
- Forcing frequency (f_f)
- Forcing amplitude (A_f)

Extension duct:

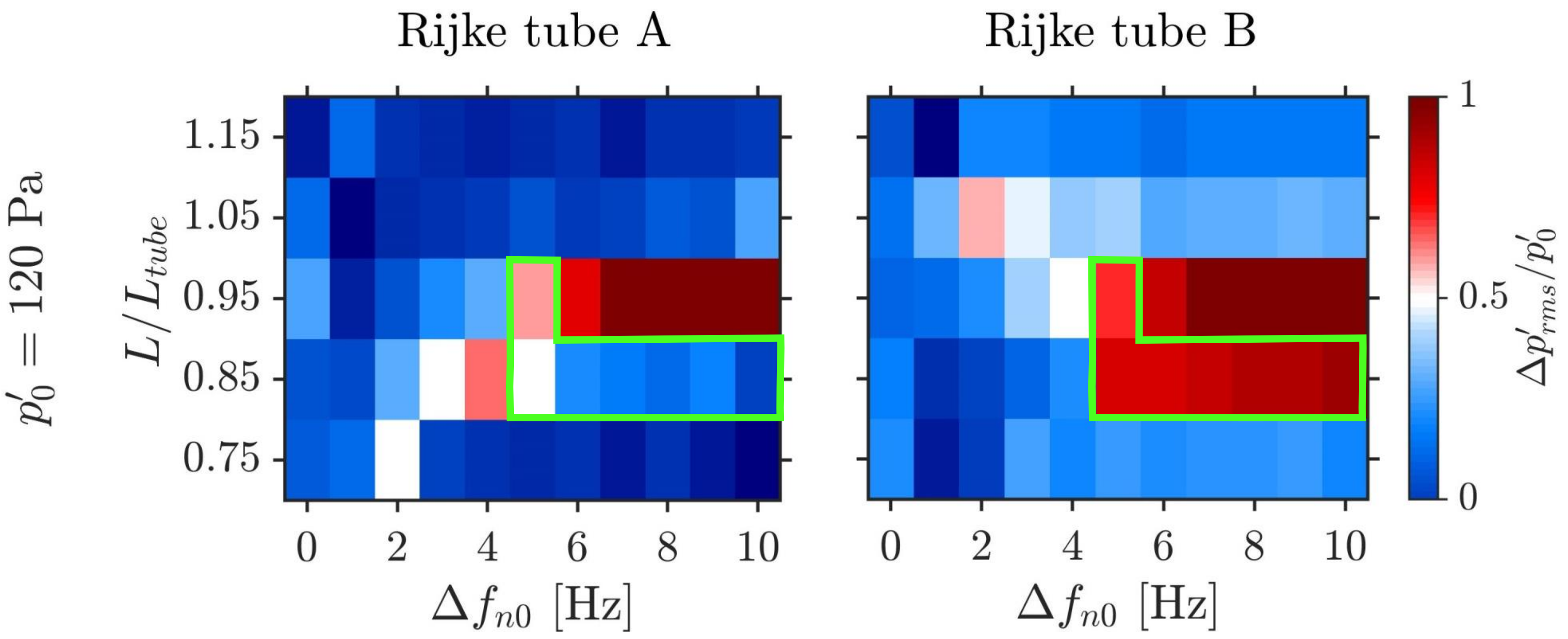
Introduce frequency detuning in the system (Δf)

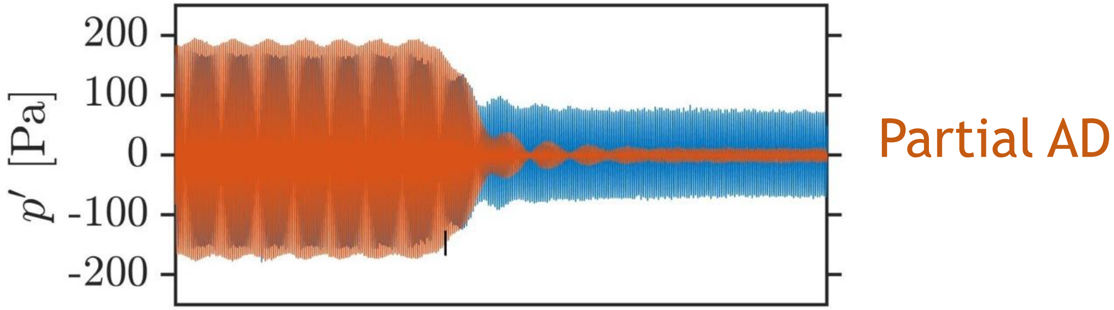
Behaviour of Coupled Two Rijke tubes

During amplitude death, both Rijke tube oscillators reach the same steady state

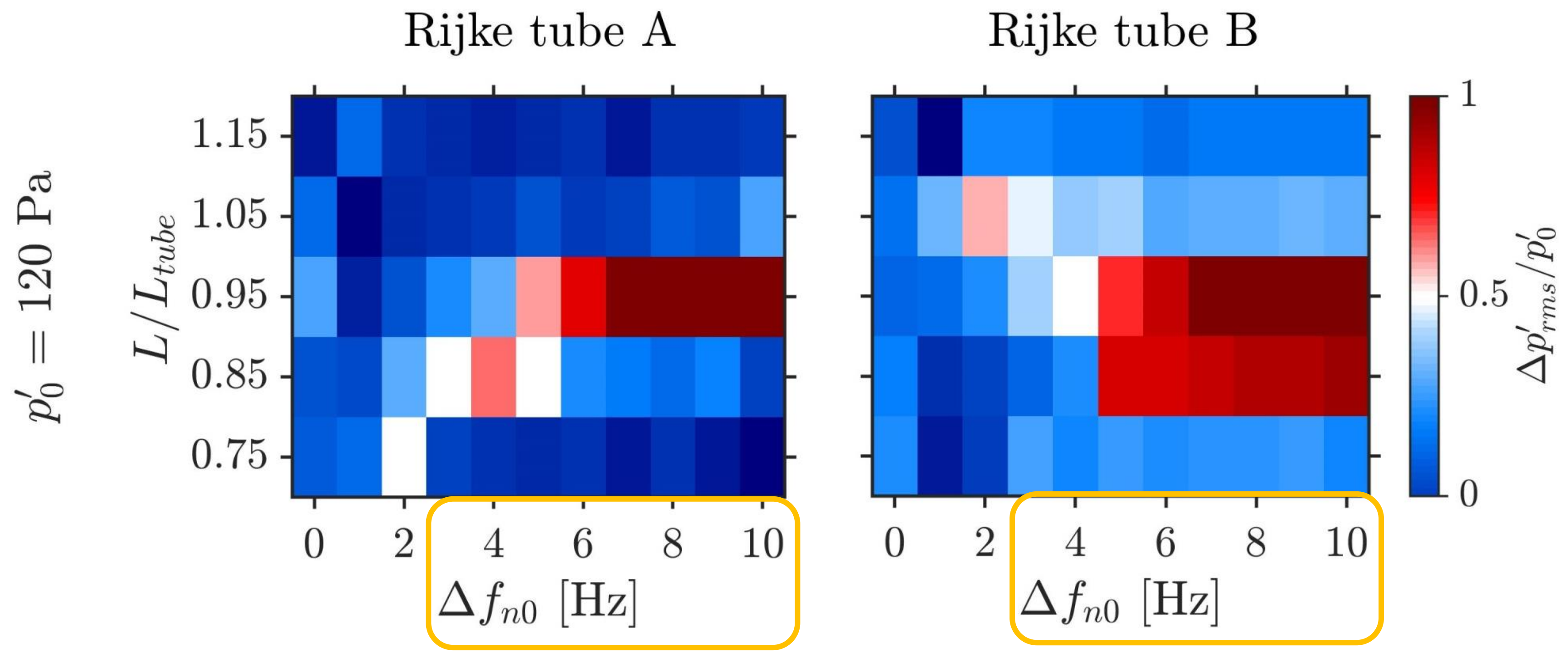


During Partial amplitude death, one Rijke tube oscillator exhibits limit cycle oscillations and another displays nearly quenched state

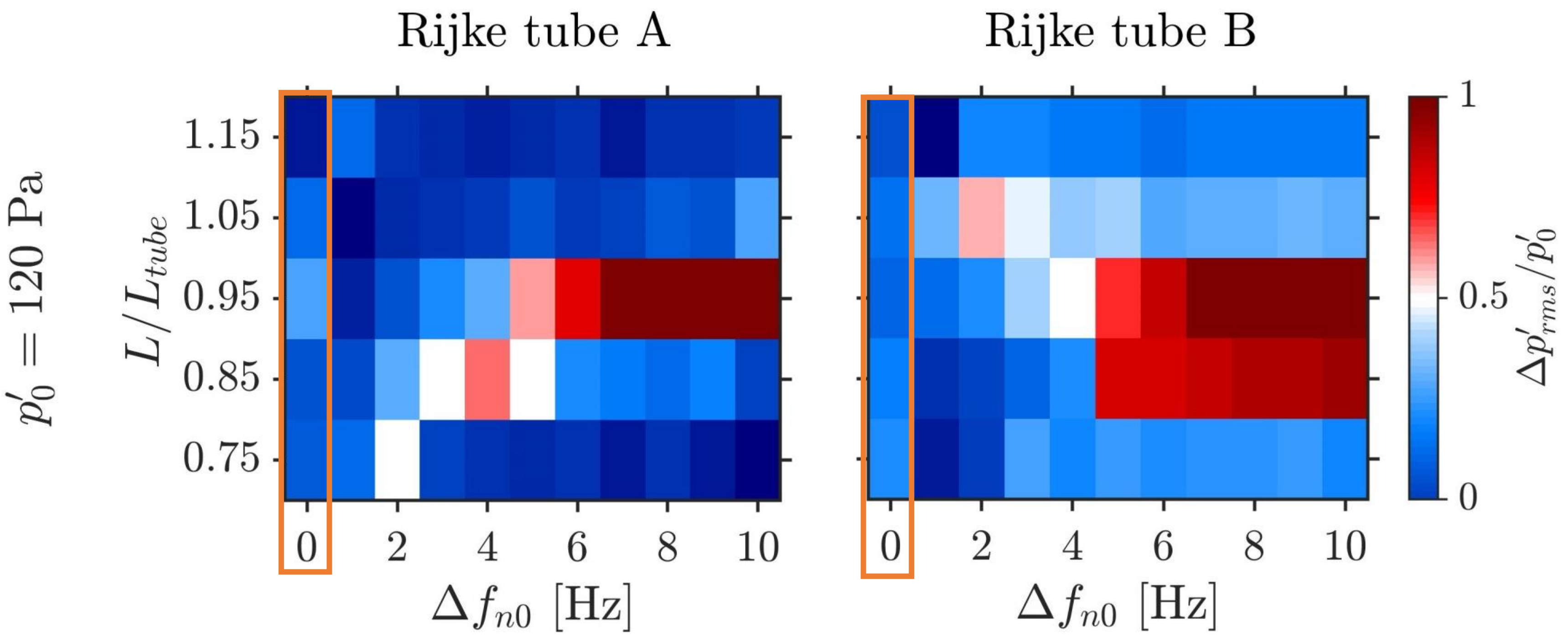




AD and PAD states are observed only for finite frequency detuning in the coupled system

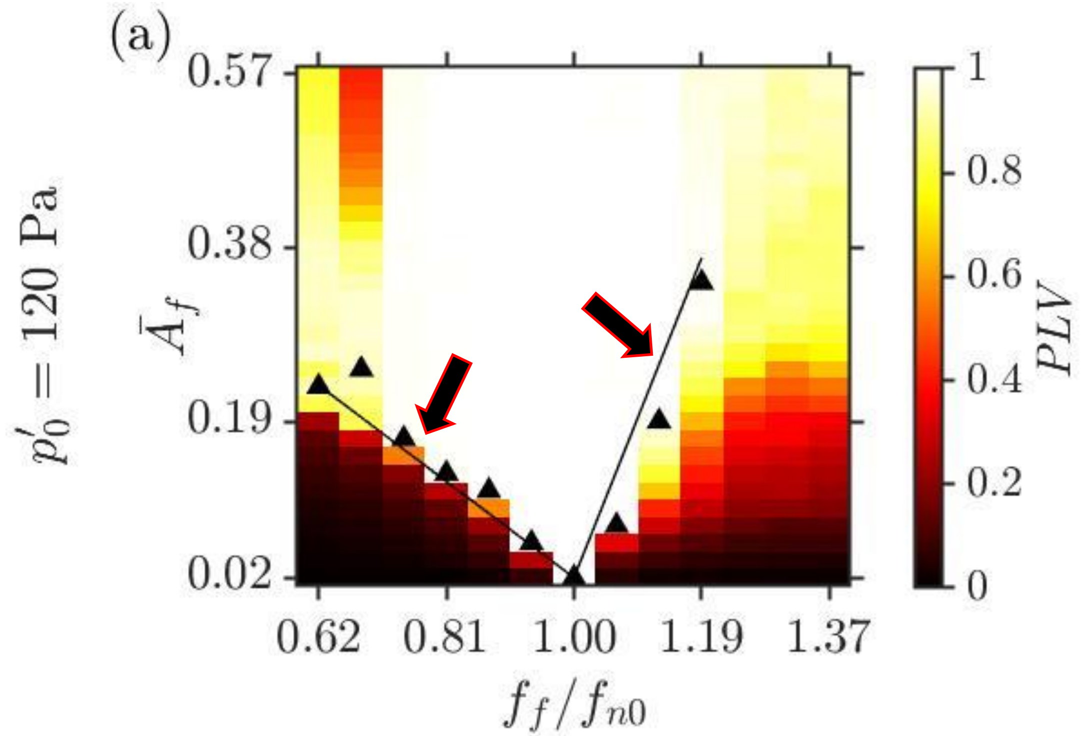


No appreciable amplitude suppression of limit cycle oscillations is observed in coupled system with identical oscillators

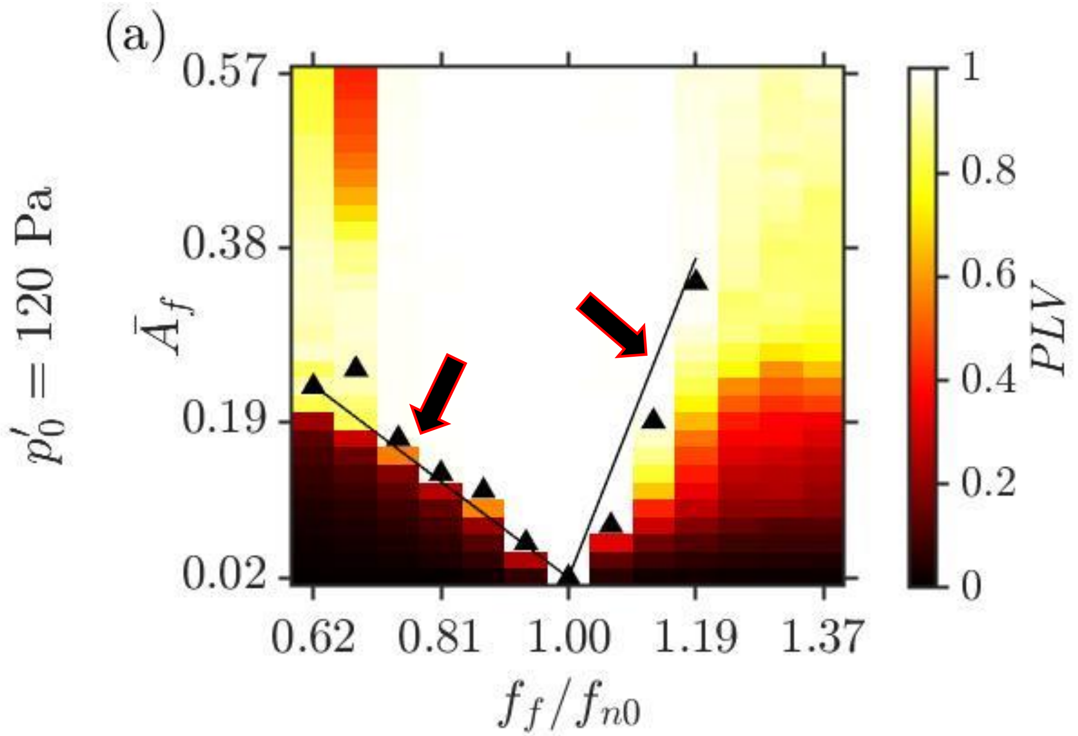


Periodically Forced Rijke Tube Oscillator

Forced limit cycle oscillations display V-shaped Arnold tongue in the synchronization map

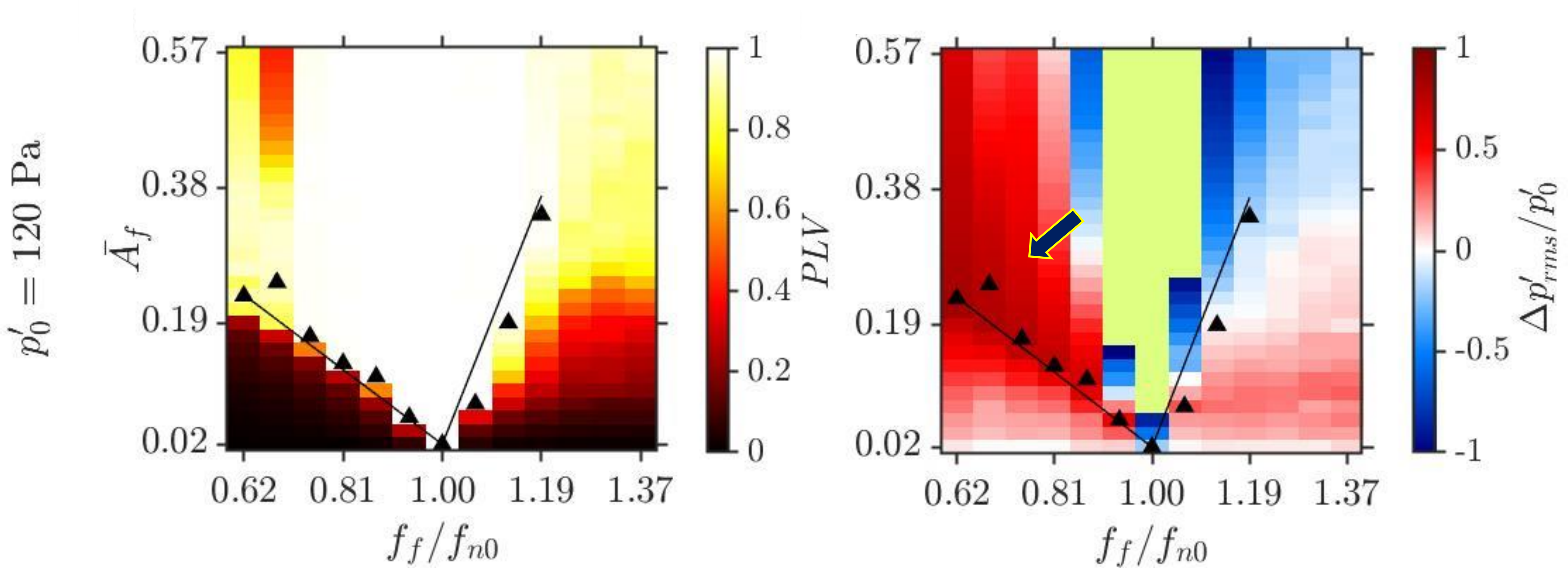


Forced limit cycle oscillations display V-shaped Arnold tongue in the synchronization map



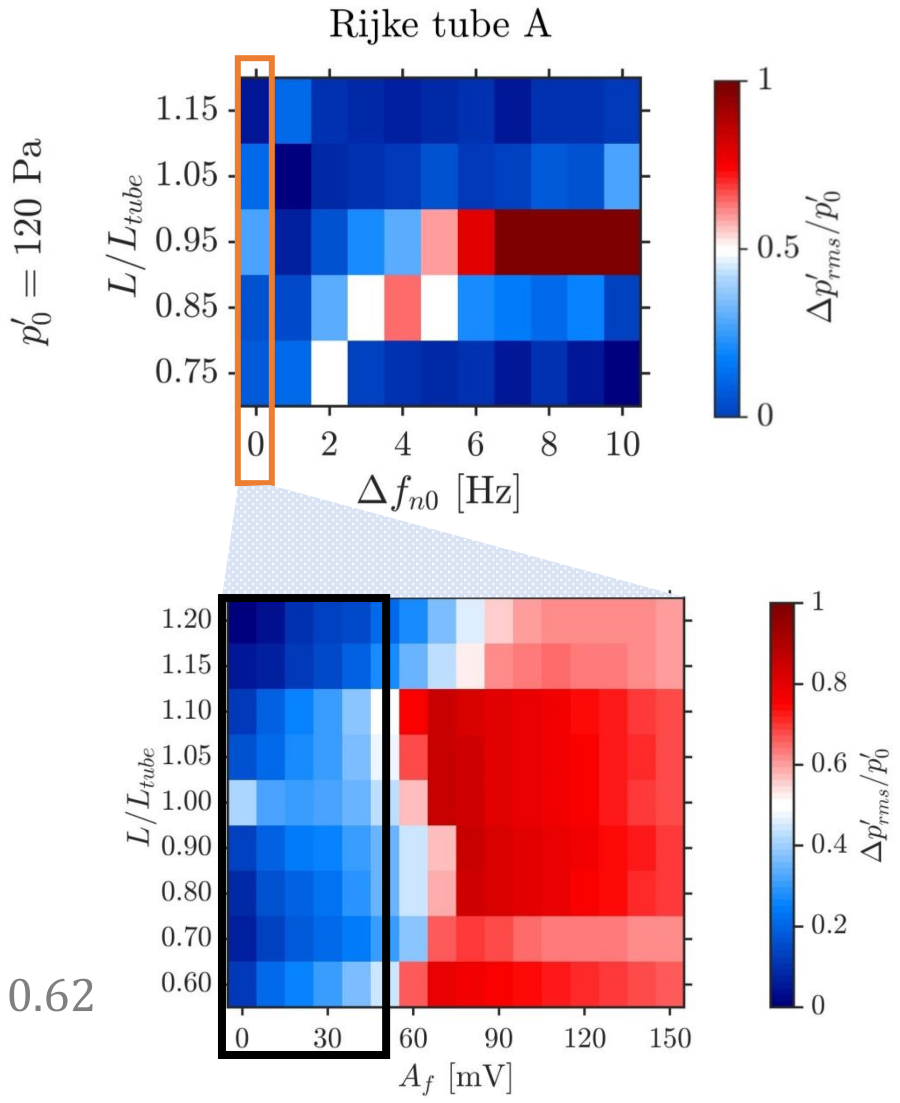
Arnold tongue is asymmetric around the natural frequency

Asynchronous quenching of limit cycle oscillations is observed at non-resonant forcing frequencies



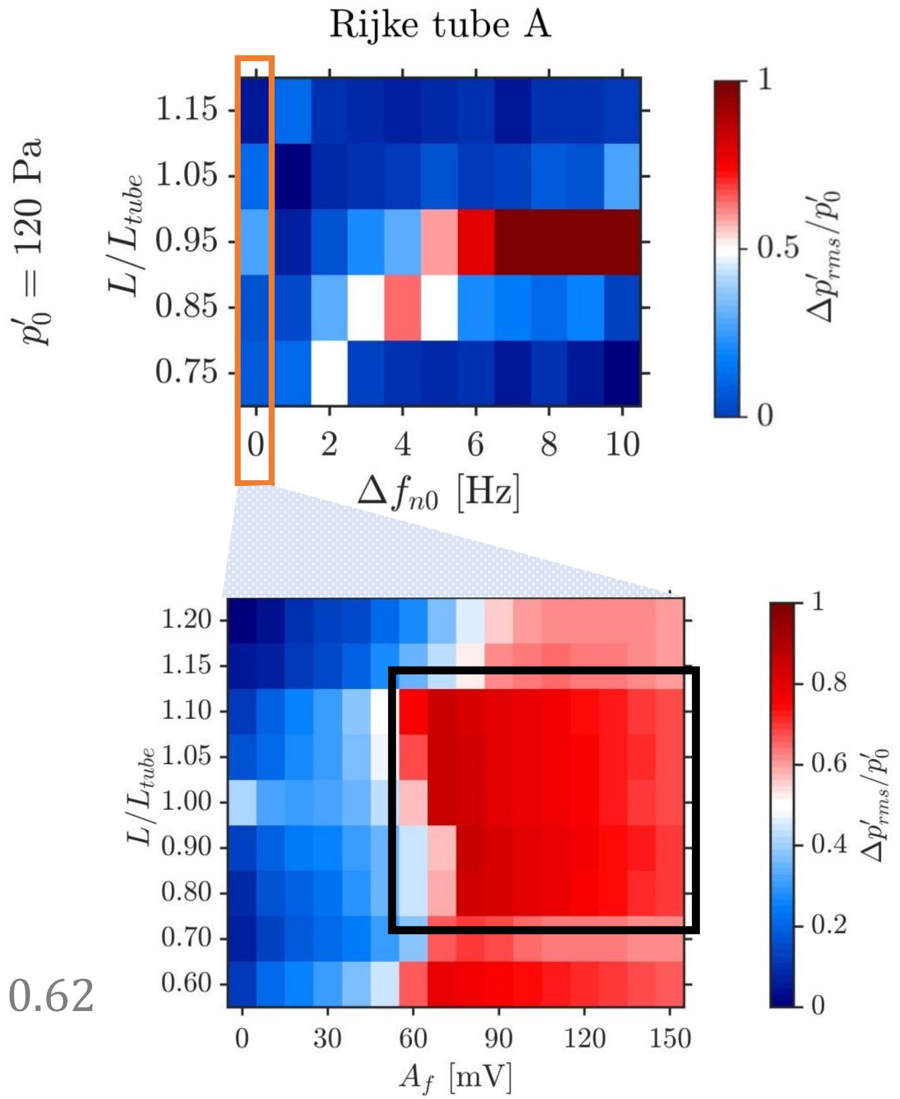
Coupled Rijke Tube Oscillators with Asymmetric Forcing

Identical coupled Rijke tubes asymmetrically forced



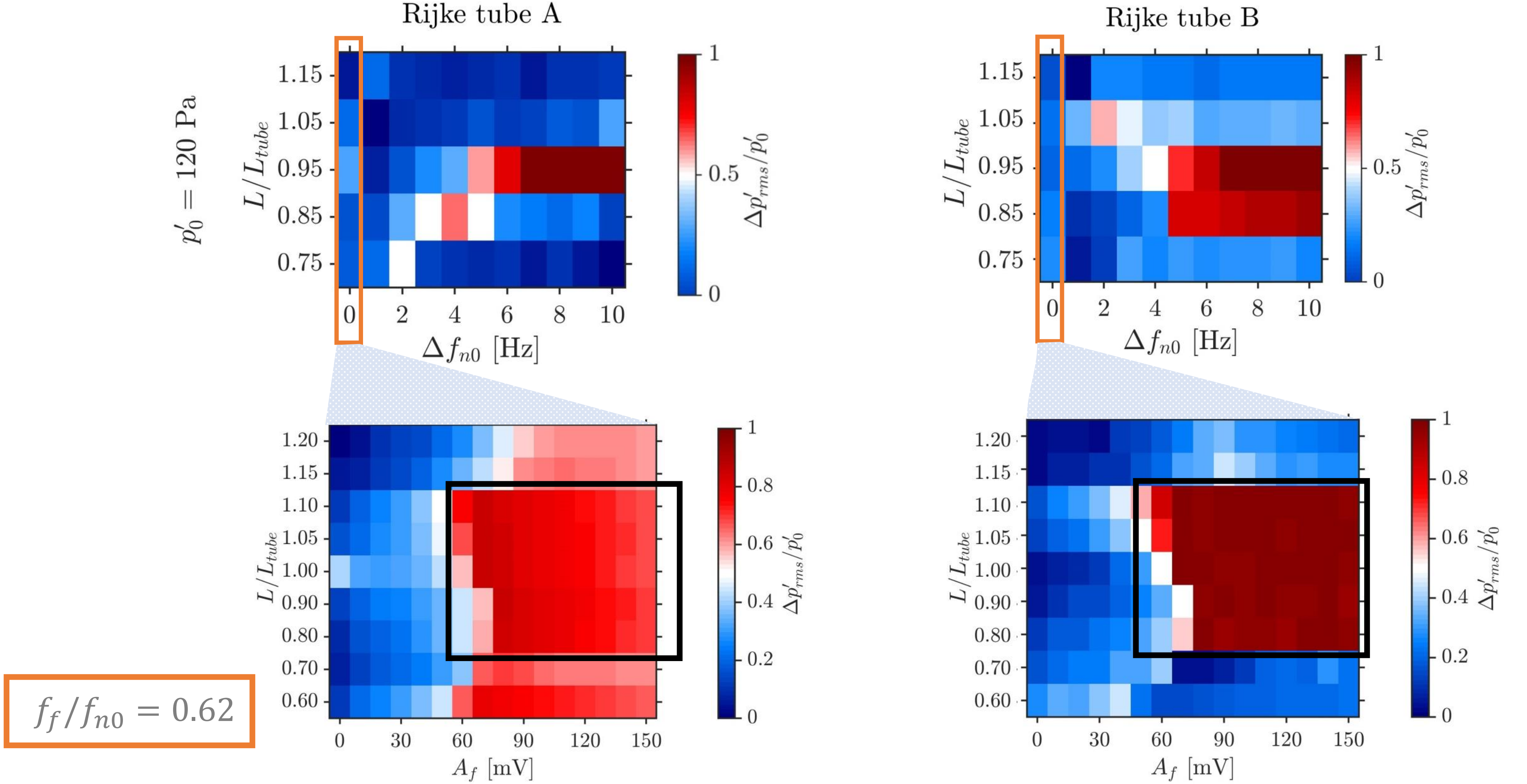
$f_f/f_{n0} = 0.62$

Identical coupled Rijke tubes asymmetrically forced



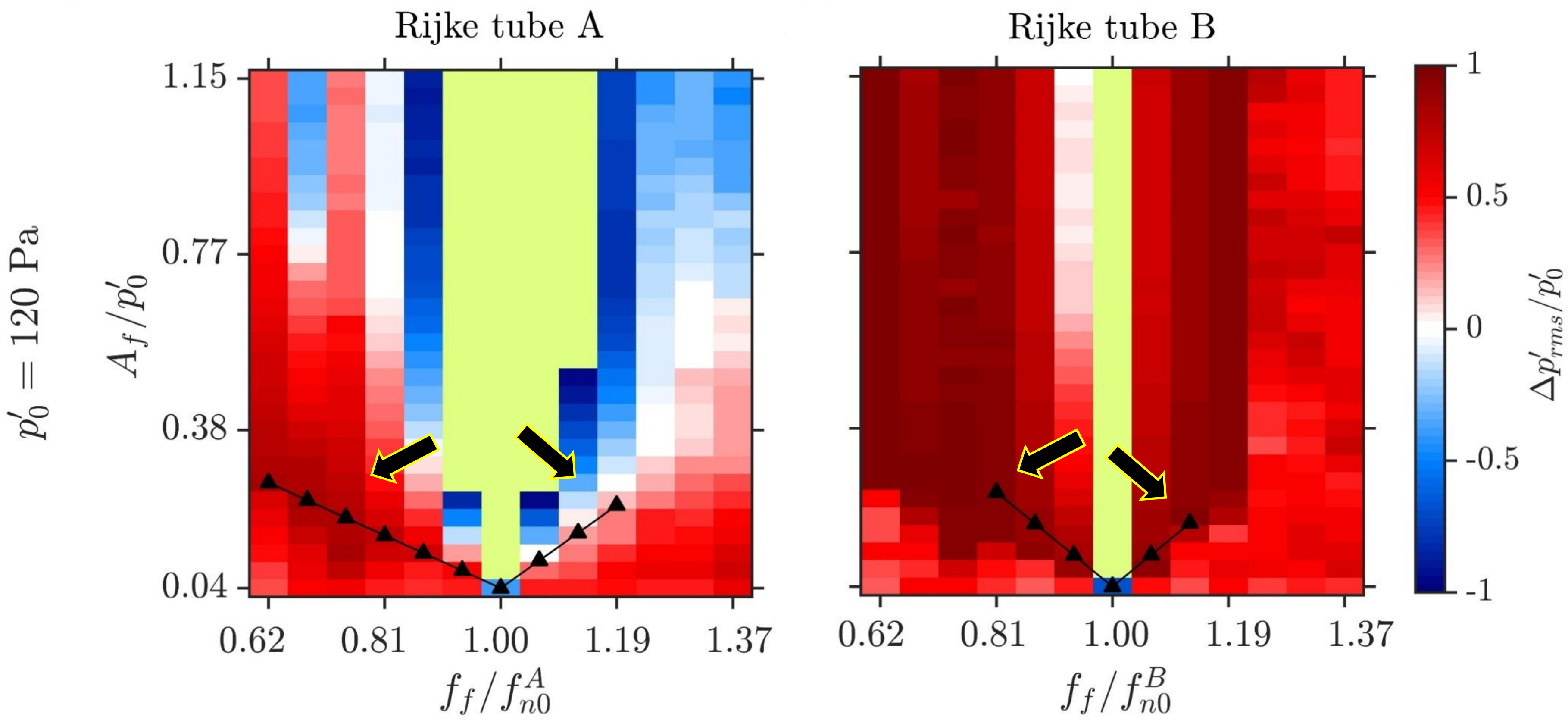
$f_f/f_{n0} = 0.62$

Identical coupled Rijke tubes asymmetrically forced



Complementary forcing and coupling enhances the region oscillations suppression in both the Rijke tube oscillators

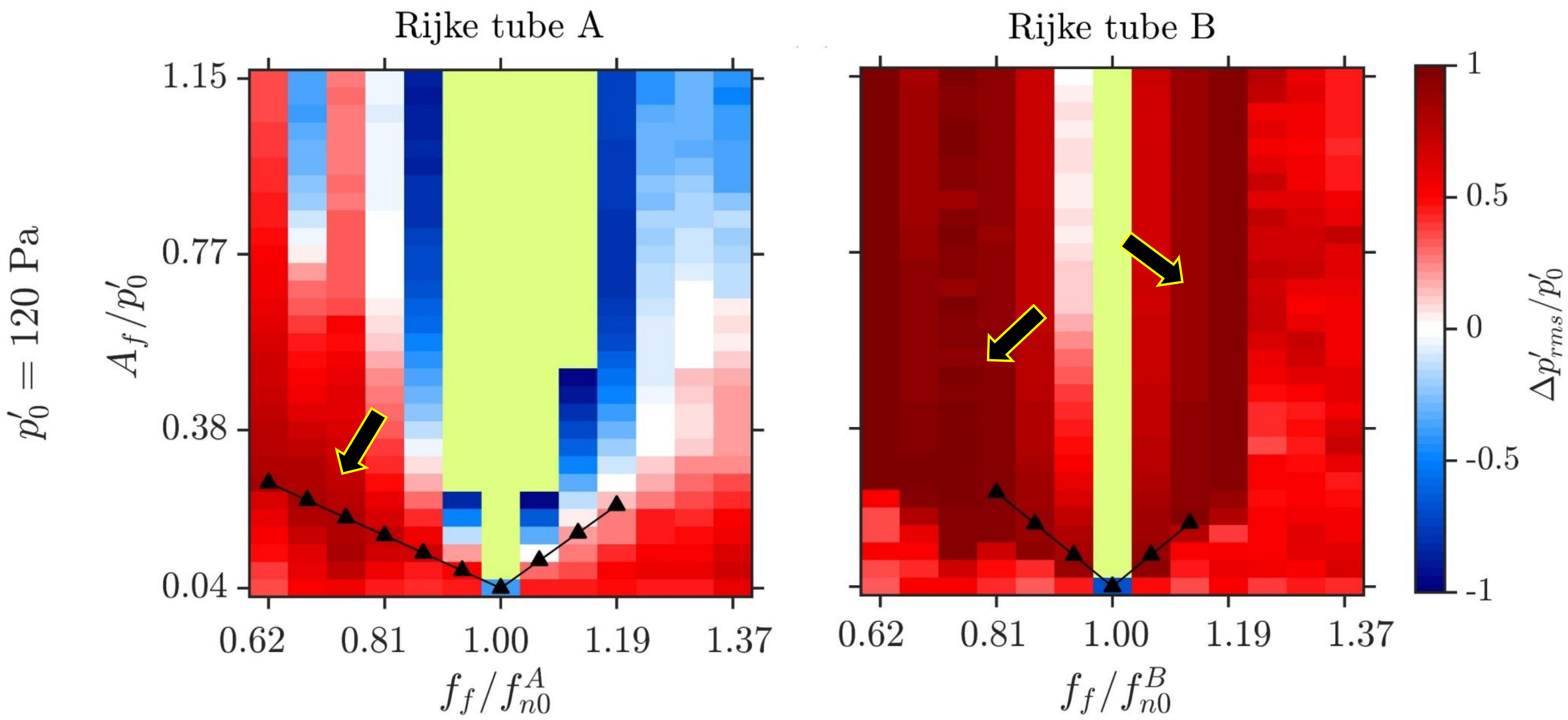
Identical coupled Rijke tubes asymmetrically forced



$$\frac{L}{L_{duct}} = 1.05$$

Forced synchronization region is wider for Rijke tube A than that observed for Rijke tube B

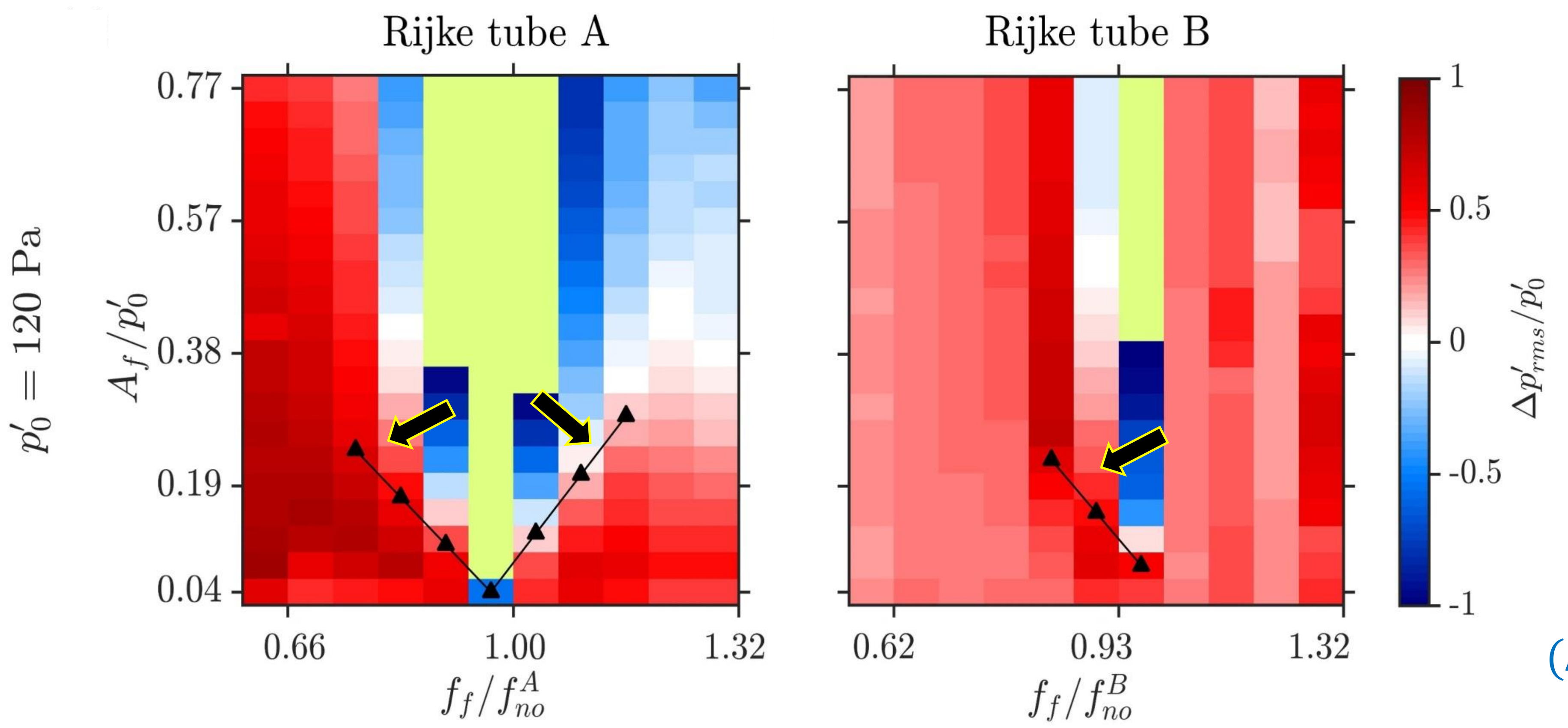
Identical coupled Rijke tubes asymmetrically forced



$$\frac{L}{L_{duct}} = 1.05$$

Larger magnitude of suppression of LCOs is observed in Rijke tube B as compared to that in Rijke tube A.

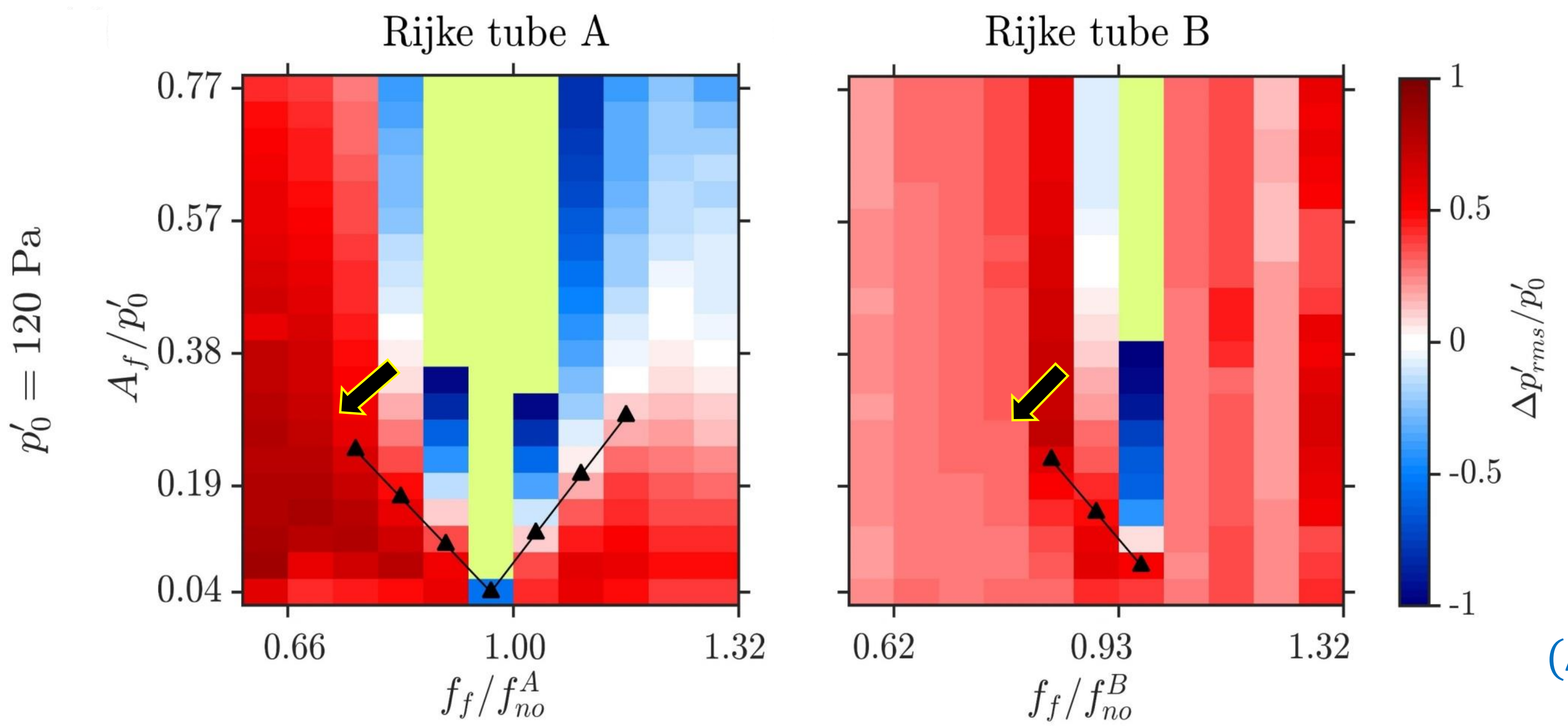
Non-identical coupled Rijke tubes asymmetrically forced



($\Delta f_{n0} = 10$ Hz)

Effect of forcing is less effective in synchronizing and quenching of LCOs in Rijke tube B

Non-identical coupled Rijke tubes asymmetrically forced



$(\Delta f_{n0} = 10$ Hz$)$

Significant suppression of LCOs can still be achieved at nonresonant conditions

Model

$$\frac{d^2\dot{\eta}_j^a}{dt} + 2\zeta_j\left(\frac{\omega_j}{r^a}\right)\frac{d\dot{\eta}^a}{dt} + \left(\frac{k_j}{r^a}\right)^2 \eta_j^a$$

$$= \dot{Q}$$

Source Term

$$\dot{Q} = K * \eta(t-\tau)$$

The source term represents the heater power needed to induce LCO in the Rijke tube

Model

$$\frac{d^2\dot{\eta}_j^a}{dt} + 2\zeta_j\left(\frac{\omega_j}{r^a}\right)\frac{d\dot{\eta}^a}{dt} + \left(\frac{k_j}{r^a}\right)^2\eta_j^a$$

$$= \dot{Q} + K_d\left(\dot{\eta}_j^b - \dot{\eta}_j^a\right)$$

Source Term Dissipative coupling

K_d encapsulates the interaction that arises from the mass transfer between the two ducts

Model

$$\frac{d^2\dot{\eta}_j^a}{dt} + 2\zeta_j\left(\frac{\omega_j}{r^a}\right)\frac{d\dot{\eta}^a}{dt} + \left(\frac{k_j}{r^a}\right)^2 \eta_j^a$$

$$= \dot{Q} + K_d\left(\dot{\eta}_j^b - \dot{\eta}_j^a\right) + K_\tau\left(\dot{\eta}_j^b(t-\tau_{tube}) - \dot{\eta}_j^a(t)\right)$$

Source Term Dissipative coupling Time-delay coupling

τ_{tube} quantifies the time taken by acoustic waves to propagate through the coupling tube connecting the two Rijke tubes

Model

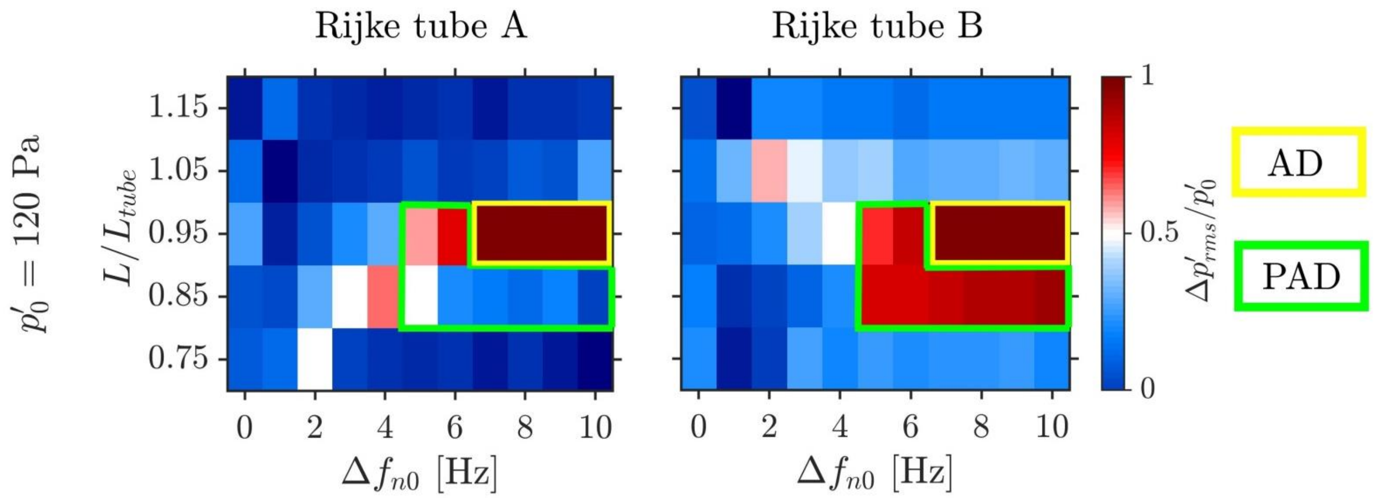
$$\frac{d^2\dot{\eta}_j^a}{dt} + 2\zeta_j\left(\frac{\omega_j}{r^a}\right)\frac{d\dot{\eta}^a}{dt} + \left(\frac{k_j}{r^a}\right)^2 \eta_j^a$$

$$= \dot{Q} + K_d\left(\dot{\eta}_j^b - \dot{\eta}_j^a\right) + K_\tau\left(\dot{\eta}_j^b(t - \tau_{tube}) - \dot{\eta}_j^a(t)\right) + \left[A_f \sin(2\pi f_f t)\right]^a$$

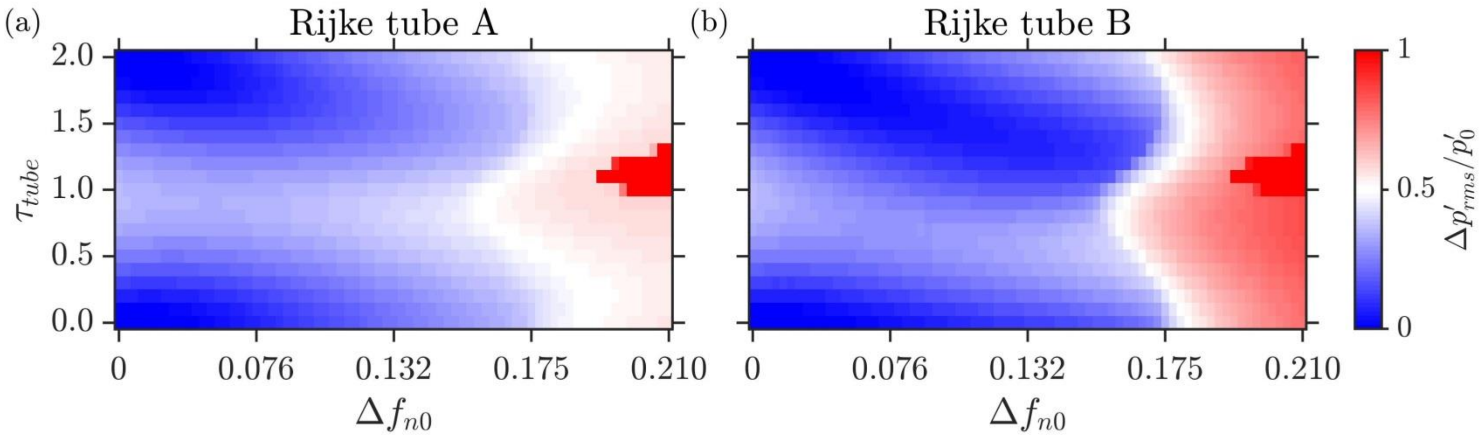
Source Term Dissipative coupling Time-delay coupling Harmonic forcing

External harmonic forcing is applied to Rijke tube A only

Coupled Rijke tubes



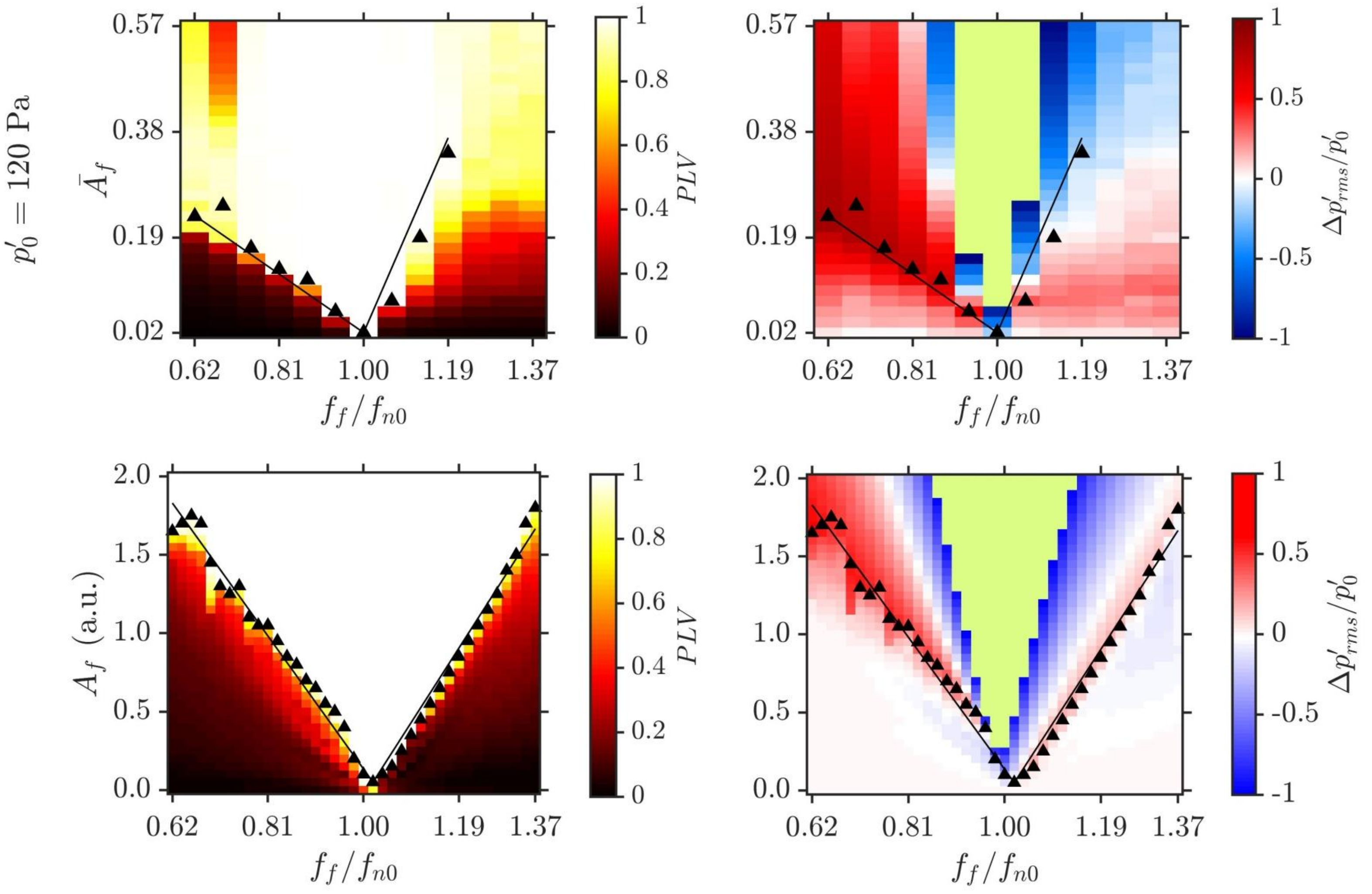
Experimental results



Numerical results

The results from the numerical model show AD, but cannot reproduce partial AD

Externally forced single Rijke tube

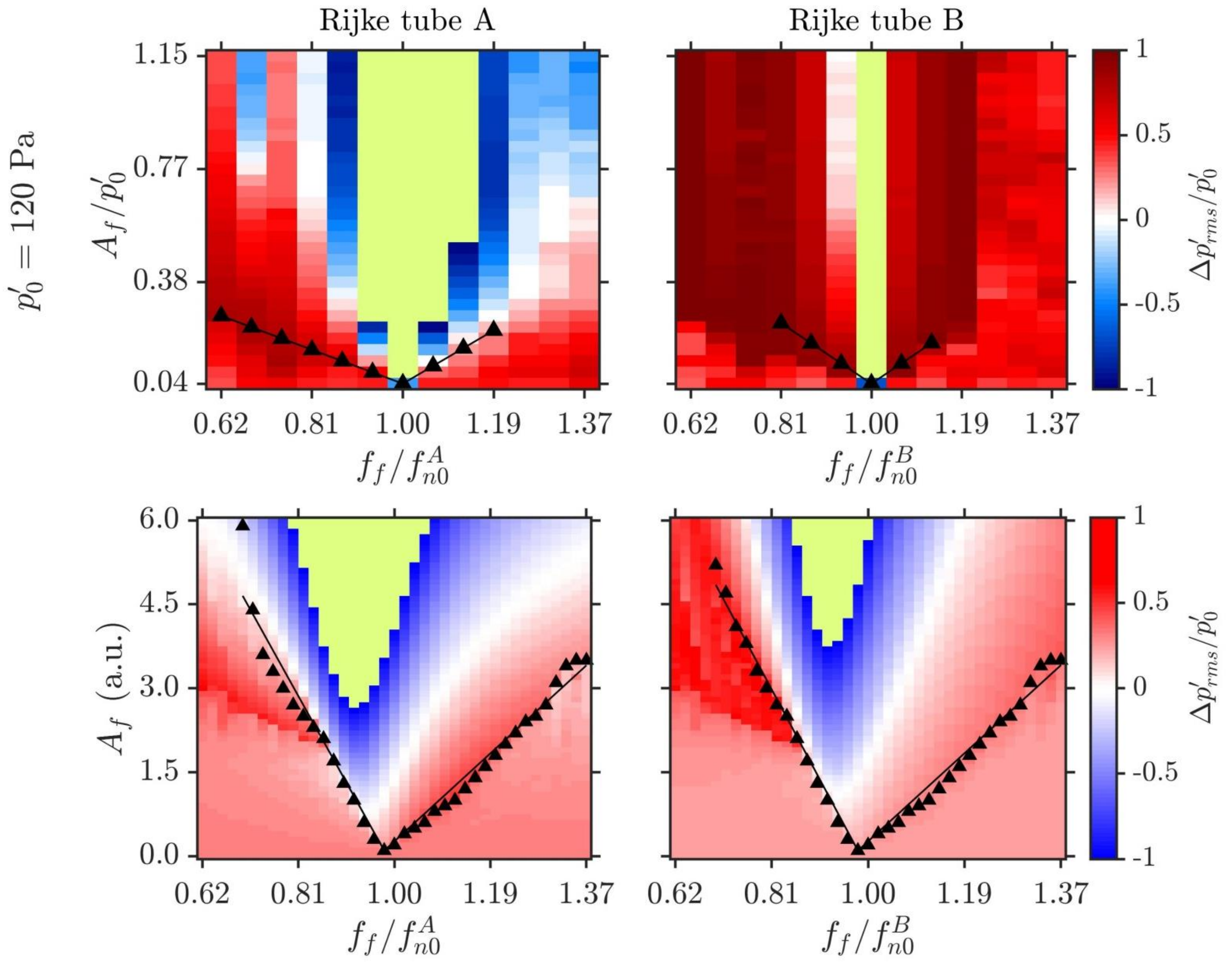


Experimental results

Numerical results

Arnold Tongue, asynchronous quenching, and resonant amplification are replicated through the numerical model

Identical coupled Rijke tubes asymmetrically forced

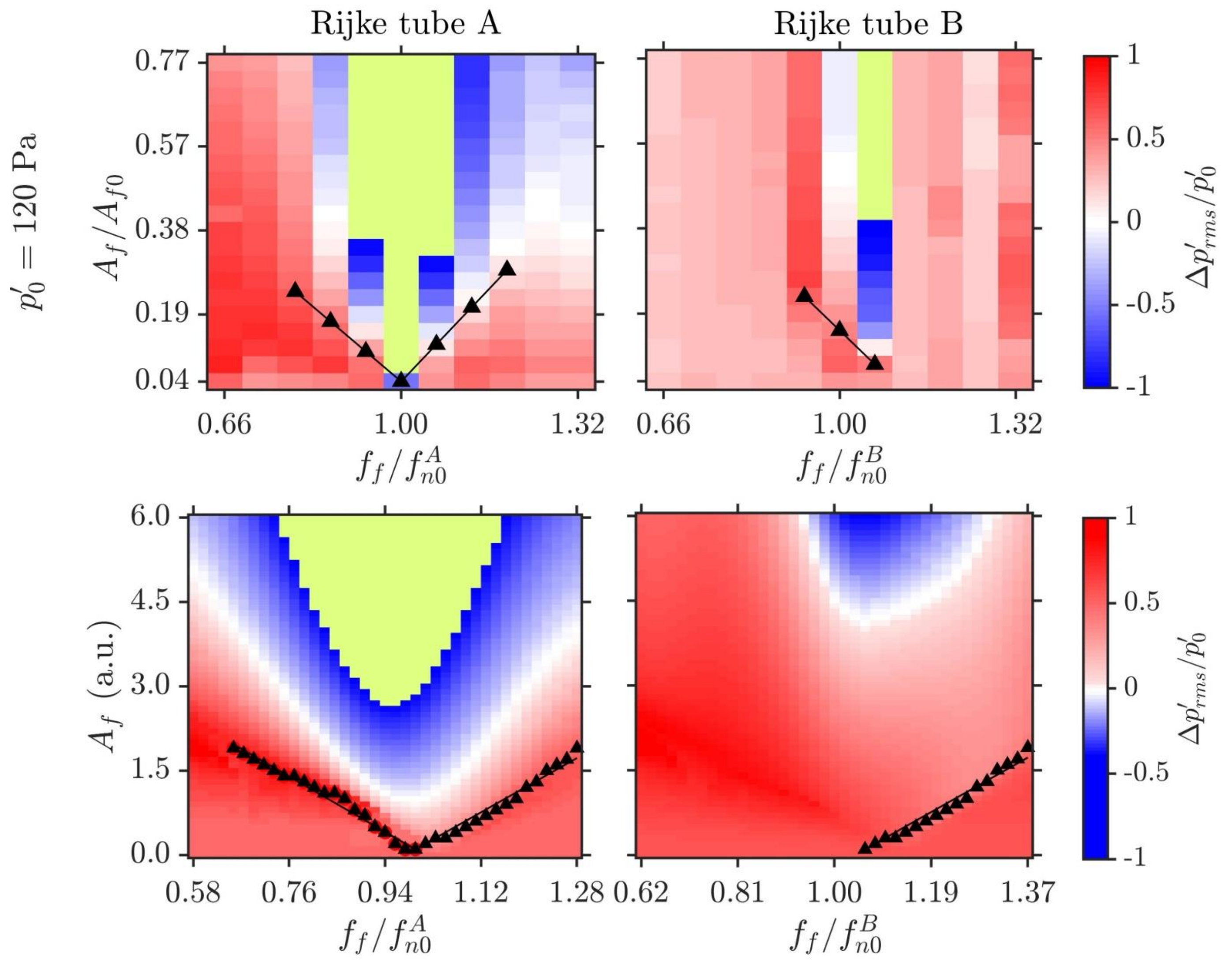


Experimental results

Numerical results

Resonant amplification region is shifted in the results obtained from numerical model

Non-identical coupled Rijke tubes asymmetrically forced



Experimental results

Numerical results

In the numerical results, the phase-locked region in Rijke tube B shifts to the right side of f_{n0}

Key takeaways

On externally forcing a single oscillator, we observe asynchronous quenching and Arnold tongue

Mutual coupling of two non-identical oscillators can lead to AD and partial AD

External forcing widens the region of coupling over which amplitude suppression is observed

Suppression of LCOs is observed predominantly for $f_f < f_{n0}$ in Rijke tube A, while it is observed on both sides of f_{n0} in Rijke tube B.

In a detuned system, Rijke tube A exhibits phase-locked region, while Rijke tube B (not directly forced) remains desynchronized with the forcing signal

Reference: Sahay, A., Roy, A., Pawar, S. A., & Sujith, R. I. (2021). Dynamics of Coupled Thermoacoustic Oscillators Under Asymmetric Forcing. *Physical Review Applied*, *15*(4), 044011.