Split Complex Lorentzian Dynamics

Part XVIII: Central Bridge Resolution, the Three-Condition Global Obstruction, and a Branching Geometry of Admissibility

When normalization closes one difficulty and exposes several independent global ones

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Abstract

Part XVII ended at an unusually sharp boundary. The local exact-representative problem had closed: on every connected inherited domain the internal family was exhausted by one integer label, arbitrary local overlaps were classified by central integer-valued factors, and all finite overlap composition laws were proved. Yet a covering system with *identical* local representatives and unit overlap factors still failed to reconstruct the unique global sign-relaxed representative by literal equality. The counterexample therefore did not expose an ordinary gluing failure. It exposed a mismatch between two normalization requirements. Task 18 was designed to decide whether an explicit local-to-global central conversion factor removes that mismatch or whether another obstruction survives even after the conversion data are carried explicitly.

The first answer is positive and exact. For every jointly regular family U, Aristotle defines the intrinsic relative factor

$$\mathrm{Bridge}(U; n, \theta) = \mathrm{ovl}(U, U^{\mathrm{ref}}; n, \theta)$$

against the unique global sign-relaxed reference. It is central and unique, and one has the carrier-level factorization

$$U(n, \theta) = \mathrm{Bridge}(U; n, \theta)\, U^{\mathrm{ref}}(n, \theta)$$

at every unit direction. Local exactness forces the bridge rate to be half-odd; on a preconnected exact domain that rate has one label $k + \frac{1}{2}$. The Task 17 integer overlap between two local exact families is then recovered as the difference of their two half-odd bridge rates. Removing the bridge while retaining it as data sends every jointly regular family, not merely every locally exact one, to the same global reference. Hence every covering system of locally exact representatives admits a unique bridge-normalized reconstruction. The Task 17 mismatch was therefore exactly a *literal-normalization mismatch*, not a hidden failure of the overlap laws.

The global exact obstruction nevertheless remains. Task 18 defines a global bridge and proves that global exact existence is equivalent to existence of one such bridge. More importantly, it derives the rate conditions forced by any global bridge: one real function on the unit directions must be simultaneously continuous, half-odd everywhere, and odd under reversal. No such function exists. Task 18 then proves a stronger diagnostic statement: every pair of these three conditions is explicitly realizable. Thus the obstruction belongs to the joint three-way compatibility, not to continuity, half-oddness, or reversal oddness separately, and not to any pair of them.

At the same time the domain problem branches. The Part XVII paper-level theorem is now a named Lean endpoint: on every preconnected set of unit directions, admissibility is equivalent to antipodal-freeness, including the empty case. But maximality depends essentially on the point-set category. Every nonempty inherited $\mathrm{Dom}(v)$ is maximal among *open preconnected admissible* sets, yet it is not maximal among open admissible sets when disconnected enlargements are

allowed, and not maximal among arbitrary admissible sets. Explicit lexicographic domains provide distinct maximal preconnected admissible extensions with identical reconstructed local algebra, proving non-canonicity rather than resolving it by convention.

A third branch appears when ordinary domain overlap is separated from visible coincidence relations. Direction coverage does not imply relation coverage, and no family of antipodal-free domains covers all visible coincidences. The exact Task 16 coincidence theorem shows two cross-domain classes: the antipodal class and a degenerate identity-parameter class in which both parameters are integer multiples of 2π and direction becomes irrelevant. The latter is formally present but harmless for the explicit local models. Disconnected domains likewise admit genuinely richer label patterns: an explicit admissible example carries four different half-odd labels, labels are constant on connected components but not freely assignable, and distinct labels cannot accumulate because they arise from one ambient continuous rate.

Finally, the Task 17 C^1 quantifier gap closes completely: for arbitrary direction sets, continuous admissibility is equivalent to C^1 admissibility, with a constructed C^1 rate obtained by extension, smooth approximation, and a finite staircase that restores the exact half-odd values on the domain.

Part XVIII records all of these findings, including five negative controls, failed positive conjectures, and two important claim-boundary warnings. First, the theorem `globalExact_iff_globalBridge` is correct but partly structural because `IsGlobalBridge` already requires the product with the reference to be jointly regular and globally exact; the substantive new content lies in the derived rate conditions and their minimal three-way incompatibility. Second, the declaration named `no_global_bridge_iff_no_rate` is not syntactically an equivalence: its statement is a conjunction of the two nonexistence results. Task 19 should harden these points with a rate-level bridge notion independent of global exactness, while pursuing the newly separated branches of maximal-domain geometry, relation completeness, disconnected extension data, and higher regularity.

Keywords: central bridge; bridge normalization; global exact obstruction; half-odd rate; reversal oddness; admissible direction domain; maximal domain; relation coverage; disconnected labels; C^1 regularity; formal verification; Lean 4; conservation of difficulty.

Contents

1 The boundary inherited from Part XVII

Part XVII ended after two local questions had closed and one global question had changed category [6, 1]. The jointly regular family remained of the inherited form

$$U(n,\theta) = z_{\alpha(n),\beta(n)}(\theta)\, U_n(\theta), \tag{1}$$

with continuous directional rates. Exactness at a unit direction forced

$$\alpha(n) = 0, \qquad \beta(n) \in \mathbb{Z} + \frac{1}{2}, \tag{2}$$

while the unique globally jointly regular sign-relaxed family was the reference family U^{ref}, with recovered second rate zero.

On every nonempty inherited domain

$$\mathrm{Dom}(v) = \{n : h(n,n) = 1,\ h(n,v) > 0\}, \tag{3}$$

Task 17 proved complete family-level exhaustion:

$$U \text{ exact on } \mathrm{Dom}(v) \iff \exists!\, k \in \mathbb{Z} :\ U(n,\theta) = U_k^{\mathrm{loc}}(n,\theta) \tag{4}$$

for all $n \in \mathrm{Dom}(v)$ and all real θ.

It also classified arbitrary local overlaps. If U_i, U_j are exact on a preconnected common region, the relative factor is central, direction-independent there, and carries one integer rate

$$j_{ij} = k_i - k_j \in \mathbb{Z}. \tag{5}$$

Finite composition is exact.

The surprise came from reconstruction. Task 17 chose every local representative to be the same global function U_0^{loc}. Consequently every ordinary overlap factor was the unit. The local representatives literally glued already. Yet no globally sign-relaxed family could restrict to them by literal equality, because

$$\beta_{U_0^{\mathrm{loc}}} = \frac{1}{2}, \qquad \beta_{U^{\mathrm{ref}}} = 0. \tag{6}$$

Part XVII therefore ended not with a patching problem but with a representation-class mismatch.

Methodological principle 1 (Task XVIII question)**.** Do not ask whether the already coherent local representatives can be glued once more. Ask whether the exact local class and the unique global sign-relaxed class are related by an explicit central conversion factor, and whether any obstruction remains after that factor is retained rather than suppressed.

2 Verification boundary, archive provenance, and source firewall

The supplied Aristotle archive reports Task 18 as complete [2]. The archive run is

`50b9a9ba-6df6-4894-bf7b-97a39dece12c`

and the supplied archive file has SHA-256

`1164d78406b2672e24bf83f0fa64baa99894d541791e79deb3ddc6ee96d11251`.

It records Lean `leanprover/lean4:v4.28.0` and mathlib `v4.28.0` at revision

`8f9d9cff6bd728b17a24e163c9402775d9e6a365`.

The archive reports:

- focused build `lake build RequestProject.NullSectorTask18.Task18`: successful;
- whole-project `lake build`: successful, 8226 jobs;
- no warning from any Task 18 module;
- no `sorry`, `admit`, custom `axiom`, `@[implemented_by]`, `native_decide`, `bv_decide`, or `unsafe` in the Task 18 source layer;
- all 45 exposed endpoints report exactly `[propext, Classical.choice, Quot.sound]` under `#print axioms`.

The Task 18 layer contains 2838 lines of Lean source in the supplied archive. Its linear source order is

`SafeBase` → `ConnectedAdmissibility` → `MaximalDomains` → `ReconstructionPredicates` → `BridgeFactors` → `BridgeReconstruction` → `GlobalBridgeObstruction` → `RelationCoverage` → `DisconnectedDomains` → `RegularityAudit` → `NegativeControls` → `Identification` → `Task18`.

The reconstruction firewall remains intact. Task 18 enters Task 17 through the reconstruction module `RegularityAudit` only. Task 17's comparison module is not imported. Task 18's own `Identification` module is comparison-only and is imported only by the top module. The archive's vocabulary scan finds no imported spin, covering-space, bundle, cohomological, gauge, or physical-state target in the reconstruction layer. The bare word "cover" occurs only for ordinary indexed coverage of direction sets and relations.

Proof boundary 1 (Build claim versus editorial review)**.** *The full Lean build was reported by the supplied archive and its audit. The present manuscript environment did not independently rerun the pinned Lean/Lake toolchain. The source inventory, source order, theorem statements, archive hash, and static prohibited-construct scan were independently rechecked. As in Part XVII, a green formal build establishes the encoded theorem statements; it does not by itself guarantee that theorem names, prose summaries, or category labels state the strongest correct interpretation.*

3 The bridge is derived, central, and unique

Task 18 begins with the most conservative possible comparison. It does not posit a new object. It reuses the already derived relative-factor operation and compares an arbitrary jointly regular family to the unique reference:

Definition 1 (Local-to-reference bridge)**.** For a jointly regular family U and unit direction n,

$$\mathrm{Bridge}(U;n,\theta) := \mathrm{ovl}(U,U^{\mathrm{ref}};n,\theta). \tag{7}$$

No model formula for U enters the definition.

Theorem 1 (Task 18 bridge factorization)**.** *For every jointly regular U, every unit direction n, and every $\theta \in \mathbb{R}$:*

(i) $\mathrm{Bridge}(U;n,\theta)$ *lies in the exact centre;*

(ii) it is the unique factor satisfying

$$U(n,\theta) = \mathrm{Bridge}(U;n,\theta)\,U^{\mathrm{ref}}(n,\theta); \tag{8}$$

(iii) its central one-parameter form is recovered from the already reconstructed rates of U.

The bridge therefore does not add a hidden degree of freedom. It exposes the degree of freedom that was already present in the jointly regular family.

Local exactness then changes the bridge's rate class.

Theorem 2 (Bridge rate under exactness)**.** *If U is exact at one unit direction n, the bridge rate at n is half-odd:*

$$\beta_U(n) \in \mathbb{Z} + \frac{1}{2}. \tag{9}$$

If U is exact on a preconnected domain D, the bridge is direction-independent on D and has one unique label $k + \frac{1}{2}$.

This cleanly recovers the Task 17 relative integers. For two locally exact representatives,

$$\operatorname{ovl}(U_i, U_j) = \operatorname{Bridge}(U_i)\ \operatorname{Bridge}(U_j)^{-1}, \tag{10}$$

and on a connected overlap the rate of the right-hand side is

$$\left(k_i + \frac{1}{2}\right) - \left(k_j + \frac{1}{2}\right) = k_i - k_j \in \mathbb{Z}. \tag{11}$$

Status 1 (What became of the integer overlap)**.** The integer transition is not an independent layer added after the half-odd local labels. Task 18 derives it as the relative difference of the two local-to-reference bridge labels. Thus the affine lattice $\mathbb{Z} + \frac{1}{2}$ and the difference lattice $\mathbb{Z}$ are one structure viewed absolutely and relatively.

4 Bridge normalization resolves the Task-XVII mismatch

Task 18 now performs the operation that Part XVII had deliberately refused to perform implicitly. The conversion factor is removed explicitly while retained as data. Define

$$\operatorname{BNorm}(U; n, \theta) := \operatorname{ovl}(U^{\mathrm{ref}}, U; n, \theta)\, U(n, \theta). \tag{12}$$

The removed factor is exactly the inverse of the bridge.

Theorem 3 (Normalization identity)**.** *For every jointly regular family U, every unit direction n, and every parameter θ,*

$$\operatorname{BNorm}(U; n, \theta) = U^{\mathrm{ref}}(n, \theta). \tag{13}$$

Local exactness is not required.

This point deserves emphasis. The theorem is stronger than a local exact-family statement but, precisely for that reason, less mysterious than a new gluing theorem. It follows from the inherited one-axis inverse law: the relative factor from the reference to U cancels U back to the reference. Its importance is conceptual rather than algebraically surprising: the operation proves that the Task 17 mismatch was caused by comparing two normalization classes by literal equality.

For an arbitrary indexed covering system of jointly regular locally exact representatives, Task 18 proves existence and uniqueness of the bridge-normalized reconstruction:

$$\operatorname{BNorm}(F_i) = U^{\mathrm{ref}} \qquad \text{on every local domain } D_i. \tag{14}$$

Any jointly regular global family agreeing with all normalized members at all covered unit directions agrees with U^{ref} there. The original local family is recovered from the retained bridge and U^{ref} through equation (8).

Theorem 4 (Task 17 system reclassified)**.** *The Task 17 system $F_v = U_0^{\mathrm{loc}}$ has three distinct statuses:*

(a) it has an unrestricted literal global extension, namely U_0^{loc} itself;

(b) it has no global sign-relaxed reconstruction by literal equality;

(c) it does have a sign-relaxed reconstruction when the explicit central conversion factors are retained.

The three notions therefore cannot be collapsed into a single word such as “gluing”.

5 Negative control: normalization does not preserve exactness

The previous theorem could otherwise be misread as if the local exactness had simply disappeared as irrelevant normalization. Task 18 explicitly blocks that interpretation.

Theorem 5 (Exactness is carried by the retained bridge)**.** *For nonzero v, bridge-normalization of U_0^{loc} equals U^{ref} throughout* $\mathrm{Dom}(v)$*, but U^{ref} is not exactly transformation-valued on* $\mathrm{Dom}(v)$*.*

Hence normalization changes the representation property. Exactness has not been quotiented away; it has been moved into the conversion factor. Two further controls make this precise:

- the bridge of U_0^{loc} is nontrivial — at parameter π it is not the carrier unit;
- two jointly regular families with the same bridge at a unit direction coincide there, so the retained factor determines the family and no information has been discarded.

A separate control proves that centrality alone is much weaker than exactness: a jointly regular family with bridge rate $1/3$ still has a central bridge, but $1/3$ is not half-odd. Thus the half-odd restriction genuinely comes from exact transformation-valuedness, not from centrality.

6 The global exact no-go survives as a global-bridge no-go

Task 18 next asks whether the now explicit bridge can itself be chosen globally.

The formal predicate used by Aristotle is

$$\mathrm{IsGlobalBridge}(c) \tag{15}$$

meaning that $c(n, \theta)$ is central at every unit direction and that

$$U_c(n, \theta) = c(n, \theta)\, U^{\mathrm{ref}}(n, \theta) \tag{16}$$

is jointly regular and globally exact.

Theorem 6 (Global exactness versus a global bridge)**.** *There exists a globally jointly regular exact family if and only if there exists a global bridge in the sense of equation* (15)*.*

The inherited global exact no-go therefore gives

$$\nexists\, c : \mathrm{IsGlobalBridge}(c). \tag{17}$$

Methodological caveat 1 (Why the bare equivalence is not yet the deepest theorem)**.** *The theorem `globalExact_iff_globalBridge` is correct, but the definition `IsGlobalBridge` already includes that $c\,U^{\mathrm{ref}}$ is jointly regular and globally exact. Consequently the backward direction is largely definitional, and the forward direction is the derived bridge factorization applied to a global exact family. The substantive new diagnostic content comes one step later, when Task 18 derives what a global bridge would force on its rate. Task 19 should introduce a weaker rate-level or factor-level bridge predicate that does not already contain global exactness and prove the strongest genuine equivalence available there.*

7 The obstruction is a minimal three-condition incompatibility

From any global bridge Task 18 recovers a directional rate b satisfying simultaneously:

$$\text{continuity:} \qquad s \mapsto b(s) \text{ is continuous on } \mathbb{S}, \tag{18}$$

$$\text{half-oddness:} \qquad b(n) \in \mathbb{Z} + \frac{1}{2} \quad \text{for every unit } n, \tag{19}$$

$$\text{reversal oddness:} \qquad b(-n) = -b(n) \quad \text{for every unit } n. \tag{20}$$

The global bridge itself then has the central form

$$c(n, \theta) = z_{0,b(n)}(\theta). \tag{21}$$

The inherited Task 17 no-rate theorem excludes the simultaneous conjunction. Task 18 strengthens the diagnosis by exhibiting all three pairwise combinations.

Theorem 7 (Pairwise realizability). *Each pair among equations* (18) *to* (20) *is realizable:*

(i) continuity + reversal oddness: $b \equiv 0$, which is not half-odd;

(ii) continuity + half-oddness: $b \equiv \frac{1}{2}$, which is not reversal-odd;

(iii) half-oddness + reversal oddness: an explicit sign rate taking $\pm\frac{1}{2}$, which is not continuous.

No rate satisfies all three.

This is the cleanest form yet of the global exact obstruction.

Methodological principle 2 (Minimal obstruction). The global failure is not attributable to reversal, discreteness, or continuity separately. It is not even attributable to any pair. The obstruction appears only when all three are demanded of the same global rate.

That distinction is stronger than the earlier shorthand "antipodal comparison causes the obstruction". Reversal supplies one leg of the contradiction, but it is not sufficient by itself.

Methodological caveat 2 (A theorem-name warning). *The declaration named*

`no_global_bridge_iff_no_rate`

does not syntactically state an equivalence. Its conclusion is the conjunction of (i) nonexistence of a rate satisfying all three conditions and (ii) nonexistence of a global bridge. Both conjuncts are proved, but the theorem name suggests a logical $\leftrightarrow$ that is not present in the statement. This is a documentation issue, not a refutation of either nonexistence theorem. Task 19 should either rename the endpoint or prove the actual rate-level equivalence under a bridge predicate independent of global exactness.

8 Connected admissibility is now a formal theorem

Part XVII derived at paper level that connectedness resolves the apparent antipodal paradox. Task 18 packages the result formally.

Theorem 8 (Preconnected admissibility criterion). *Let D contain only unit directions and let its inherited subspace be preconnected. Then*

$$\mathrm{Adm}(D) \iff D \textit{ is antipodal-free.} \tag{22}$$

No nonemptiness hypothesis is required; the empty set satisfies both sides.

Connected and path-connected variants follow only after the preconnected theorem. Task 18 also preserves distinctions among preconnected, connected, path-connected, open, and admissible: they are not interchangeable predicates. In particular, the admissible antipodal pair remains a concrete witness that admissibility does not imply preconnectedness.

This closes one branch opened by Part XVII. But maximality immediately branches again because the answer depends on which point-set properties are retained.

9 The maximality fork: the same domain can be maximal and non-maximal

The inherited $\mathrm{Dom}(v)$ domains acquire a sharply category-dependent status.

Theorem 9 (Maximality with openness and preconnectedness)**.** *For $v \neq 0$, $\mathrm{Dom}(v)$ is inclusion-maximal in the class of open preconnected admissible direction sets.*

Both hypotheses matter. Openness forces any attempted enlargement through a boundary direction to cross transversally into the opposite side. Preconnected admissibility forces antipodal-freeness. Together they prevent a strict enlargement.

Drop preconnectedness, however, and maximality fails even while openness is retained. Task 18 constructs

$$\mathrm{bigCap} = \mathrm{Dom}(e_1) \cup \{n : n \text{ unit and } n_1 < -1/2\}, \tag{23}$$

with an explicit continuous clamped rate taking $+\frac{1}{2}$ on $\mathrm{Dom}(e_1)$ and $-\frac{1}{2}$ on the separated cap. This set is open, admissible, and strictly larger than $\mathrm{Dom}(e_1)$.

Drop openness instead, and Task 17's enlargement result remains: a suitable boundary direction can be adjoined while keeping the enlarged set path-connected and admissible. Thus

Class	Status of nonempty $\mathrm{Dom}(v)$
open + preconnected + admissible	maximal
open + admissible	not maximal
all admissible sets	not maximal
path-connected admissible, openness dropped	admits strict extensions

Methodological principle 3 (Category-sensitive maximality)**.** The sentence "$\mathrm{Dom}(v)$ is maximal" is mathematically incomplete. The same set is maximal in one natural class and non-maximal in two broader classes. Any later use of "maximal local domain" must carry its topology and connectedness hypotheses explicitly.

10 Maximal preconnected domains exist, but the algebra does not choose one

Task 18 introduces antipodal completeness:

$$\forall n \text{ unit}, \quad n \in D \ \vee \ -n \in D. \tag{24}$$

Inside the antipodal-free class, inclusion-maximality is equivalent to this completeness property. Aristotle then constructs explicit lexicographic domains lexA and lexB that choose exactly one member of each antipodal pair.

Theorem 10 (Explicit maximal extensions and non-canonicity)**.** *The domains* lexA *and* lexB *are distinct maximal preconnected admissible sets, both contain* $\mathrm{Dom}(e_1)$*, and every explicit model* U_k^{loc} *is exact on both.*

Task 18 also proves that every nonempty $\mathrm{Dom}(v)$ has at least two incomparable connected admissible extensions. The already reconstructed algebraic data therefore do not select one maximal extension over another.

Status 2 (Non-canonicity is a result)**.** The absence of a preferred maximal extension is not merely an unresolved editorial choice. The formal development exhibits different maximal preconnected admissible extensions supporting the same reconstructed local model data. Choosing one by coordinate convention would add structure not forced by the reconstruction.

The plain admissible class remains subtler. Task 18 does not prove that inclusion-maximal members exist there. A naive chain-union argument is unavailable because admissibility carries existence of a continuous rate, and rates associated with members of a chain need not be mutually compatible.

11 Direction coverage is not relation coverage

Task 18 next separates two notions that ordinary patch language tends to conflate.

Definition 2 (Direction versus relation coverage)**.** An indexed family of direction sets covers directions if every unit direction belongs to at least one member. It covers visible relations if every coincidence

$$\Phi(n,\theta) = \Phi(m,\phi) \tag{25}$$

is witnessed with both relevant directions inside a common member, or equivalently by the Task 18 intrinsic relation-coverage predicate.

The inherited family $\{\mathrm{Dom}(v)\}$ covers every direction. It does not cover all relations. In fact:

Theorem 11 (No antipodal-free relation cover)**.** *No family consisting only of antipodal-free domains can cover every visible coincidence relation.*

This follows because antipodal visible coincidences exist while no antipodal-free member may contain both directions.

But Task 18 then refutes the tempting stronger claim that antipodal coincidences are the *only* cross-domain class. Using Task 16's exact coincidence classification, it proves:

$$\Phi(n,\theta) = \Phi(m,\phi) \Longrightarrow \left[m = n \ \vee \ m = -n \ \vee \ \begin{array}{c} \theta \in 2\pi\mathbb{Z}, \\ \phi \in 2\pi\mathbb{Z} \end{array} \right]. \tag{26}$$

The third branch is a degenerate identity class: when both parameters are integer multiples of 2π, direction is not determined by the visible transformation at all. For arbitrary unit n, m one has, for example,

$$\Phi(n, 2\pi) = \Phi(m, 0) = \mathrm{id}. \tag{27}$$

Yet this second cross-domain class is harmless for the explicit local exact models. Aristotle proves that their full-turn values satisfy the required relation, so this degeneracy contributes no additional obstruction.

Status 3 (Relation branch after Task XVIII)**.** Ordinary overlaps do not encode the full visible coincidence relation. The cross-domain relation set contains both an antipodal class and an identity-degenerate class. The latter is present but already satisfied; the former remains the nontrivial relation that cannot live inside a connected admissible domain.

12 Disconnected admissible domains carry componentwise but constrained labels

Task 17 had shown only that the disconnected pair $\{n, -n\}$ can carry opposite half-odd labels. Task 18 proves a broader component structure.

If U is exact on an arbitrary admissible D, its bridge/rate label is constant on every connected component of D. However, different component labels are not freely assignable: they must arise from one globally continuous ambient rate and must satisfy the reversal relation wherever both n and $-n$ lie in the domain.

This is summarized by the formal component-label criterion. Task 18 then constructs an explicit admissible disconnected domain carrying four values

$$+\frac{5}{2}, \quad -\frac{5}{2}, \quad +\frac{3}{2}, \quad -\frac{3}{2}. \tag{28}$$

Hence "one integer per domain" is decisively false outside the connected regime.

At the same time, continuous extension imposes a local rigidity that prevents arbitrary interleaving.

Theorem 12 (No accumulation of distinct labels)**.** *For every point of the ambient unit-direction space there exists a neighborhood in which all points of D carrying half-odd values have the same label. Distinct component labels therefore cannot accumulate arbitrarily close to one another.*

The proof uses only continuity plus the unit spacing of the half-odd lattice: choose an output tolerance strictly below 1/2 and two half-odd values in the resulting neighborhood cannot differ.

Methodological principle 4 (Disconnected freedom is extension-constrained)**.** Disconnectedness permits multiple labels, but it does not turn the label assignment into arbitrary combinatorics. The true datum is still one ambient continuous rate whose restriction happens to be half-odd on the chosen domain.

A complete geometric structure theorem for which disconnected component configurations admit prescribed labels remains open.

13 The arbitrary-domain C^1 gap closes

Part XVII had corrected an overbroad Task 17 prose claim: C^1 neutrality was proved for inherited domains, all antipodal-free domains, an explicit antipodal pair, and local overlaps, but not for every arbitrary continuously admissible set. Task 18 closes exactly that quantifier gap.

Theorem 13 (Continuous admissibility equals C^1 admissibility)**.** *For every set D of directions,*

$$\mathrm{Adm}(D) \quad \Longleftrightarrow \quad \mathrm{Adm}_{C^1}(D). \tag{29}$$

The forward construction is nontrivial. Starting from the continuous admissible rate, Task 18:

1. extends the rate from the unit-direction carrier using the standard Tietze extension machinery;
2. introduces compact support and obtains a uniform smooth approximation;
3. bounds the finitely relevant half-odd range on the compact unit sphere;
4. composes the approximation with an explicit finite C^1 staircase, flat on intervals around precisely the required half-odd levels;
5. uses an approximation error at most 1/4 so that the staircase returns the exact original half-odd value on every point of D;
6. preserves reversal compatibility on D because the constructed rate agrees with the original there.

This replaces a failed first route through infinitely many level-set separations and a smooth Urysohn-style construction. The successful staircase route avoids both an a priori finite-range argument for arbitrary level sets and an infinitely indexed smooth separation theorem.

Proof boundary 2 (Higher regularity)**.** *Task 18 proves C^1, not C^k for $k \geq 2$ and not C^∞. The audit observes that the successful construction appears compatible with smooth choices, but that observation is not a theorem. Task 19 may test the strongest regularity actually supported by the construction instead of promoting the appearance of smoothness to a result.*

14 Five negative controls and what they protect

Task 18 includes a dedicated negative-control module. These controls matter because the positive bridge result could otherwise be overinterpreted.

1. **Centrality does not imply half-oddness.** A jointly regular example with bridge rate 1/3 has a central bridge but is not locally exact.

2. **Direction-independence needs preconnectedness.** An exact family on the disconnected four-label domain has bridge rates 5/2 and 3/2 at different directions.

3. **Normalization does not preserve exactness.** The normalized family is U^{ref}, which is not exact on any nonempty inherited domain.

4. **The bridge is not the identity.** The bridge of U_0^{loc} at e_1 and parameter π is a nontrivial central element.

5. **Nothing is quotiented.** Equality of bridges for two jointly regular families at a unit direction forces equality of the families there.

Together these controls establish the correct reading of the bridge construction: the bridge is explicit information, exactness constrains that information, and bridge normalization reorganizes rather than deletes it.

15 Failed positive conjectures are part of the mathematics

Five failed or abandoned routes materially changed Task 18.

1. The proposed theorem "Dom(v) is maximal among open admissible sets" is false. The counterexample bigCap shows that preconnectedness is essential.

2. The proposed theorem "lexA is maximal among all admissible sets" is false. Maximality is retained only inside the explicitly named preconnected admissible class.

3. The expected answer "the antipodal relation is the only cross-domain relation" is false. Task 16's exact coincidence theorem forces the additional degenerate identity class.

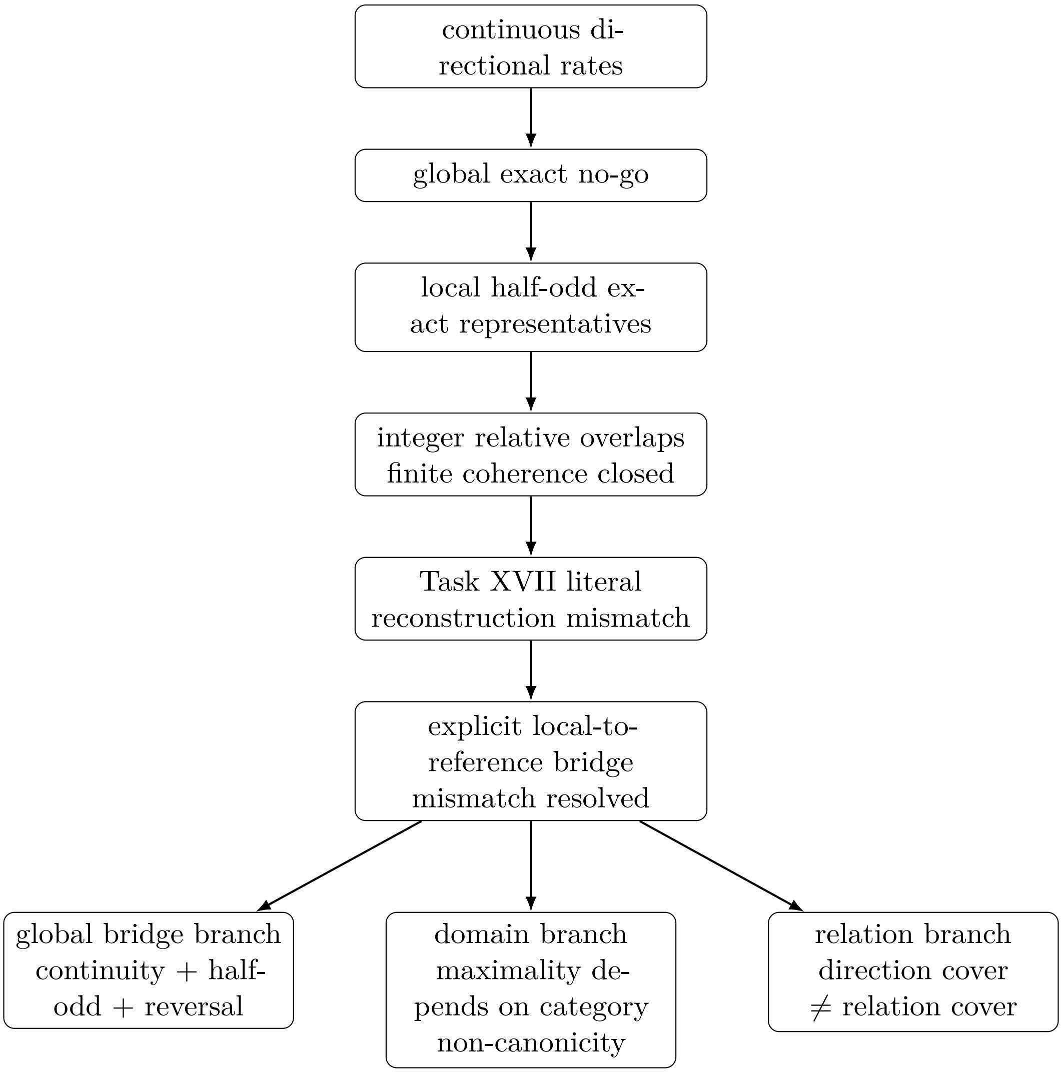
4. Component labels cannot be assigned freely. They must extend to one ambient continuous reversal-compatible rate, and distinct labels cannot accumulate.

5. A level-set/Urysohn route to C^1 admissibility was abandoned. The finite staircase construction supplied the successful proof instead.

These are not engineering footnotes. They are precisely where the attempted simplifications failed and the mathematical picture branched.

16 Conservation of Difficulty: one obstruction closes, three branches remain

Parts XVI–XVIII now admit a sharper dependency story than a single linear chain.



The key event of Part XVIII is that the Task 17 mismatch does *not* become a new hidden gluing obstruction. It closes positively once the conversion factor is made explicit. Conservation of Difficulty therefore cannot be maintained by artificially preserving that problem. Instead the difficulty branches into mathematically distinct questions.

Methodological principle 5 (Branched Conservation of Difficulty)**.** A resolved obstruction must be recorded as resolved. Its difficulty is conserved only if the proof itself exposes new independent constraints. After Task 18 these constraints no longer form one linear sequence: global bridge compatibility, maximal-domain geometry, and full relation coverage are separate branches that share inherited data but require different next theorems.

The global branch has already been reduced to the minimal triple incompatibility. The domain branch contains both existence and non-canonicity of maximal preconnected domains, while leaving the plain admissible maximality problem open. The relation branch shows that ordinary domain overlaps are not the full comparison structure, even though the extra identity-degenerate class is harmless. A fourth analytic sub-branch concerns the exact regularity class beyond C^1.

17 Two formal warnings that should not be hidden by the narrative

The dramatic structure is strongest when theorem boundaries are not blurred.

17.1 The global-bridge equivalence is partly definition-shaped

As noted in section 6, `IsGlobalBridge` contains the global exactness of the reconstructed family. Therefore `globalExact_iff_globalBridge` should not be presented as if a completely independent bridge criterion had unexpectedly turned out to be equivalent to exactness. Its useful role is to move the representation problem onto the bridge factor. The new mathematics is the subsequent theorem forcing the bridge rate into the simultaneous continuity/half-odd/reversal class, together with the pairwise realizability theorem.

17.2 The advertised "iff no rate" theorem is a conjunction

The theorem `no_global_bridge_iff_no_rate` proves both nonexistence statements but not an explicit logical equivalence between a rate predicate and a bridge predicate. A manuscript should not silently repair a theorem statement by reading its name as its content. Task 19 should either produce the genuine equivalence or retain the two nonexistence results as separate consequences.

17.3 Unit-direction uniqueness is not unrestricted uniqueness

Bridge-normalized reconstruction is unique on unit directions, which is exactly the carrier region constrained by every notion used here. Values of a global family at non-unit directions are not fixed by these theorems. The uniqueness claim should therefore remain unit-direction uniqueness unless a later task introduces a reason to constrain the ambient non-unit carrier.

18 Late conventional comparison

Only after the intrinsic reconstruction has been fixed may one compare the cumulative pattern with familiar mathematics. By Part XVIII the reconstructed structure contains:

1. a visible 2π family with an inherited internal $2\pi/4\pi$ sign structure from earlier tasks;
2. connected exact local representatives indexed by the affine lattice $\mathbb{Z} + \frac{1}{2}$;
3. relative local factors indexed by the difference lattice $\mathbb{Z}$;
4. explicit local-to-reference central conversion factors;
5. an obstruction to one global continuous half-odd reversal-odd conversion;
6. maximal antipodal selections that are not canonically chosen by the reconstructed algebra;
7. a distinction between coverage of points and coverage of the complete visible coincidence relation.

Central signs, two-fold refinements, local choices, and compatibility data are familiar themes in conventional spin geometry [3, 4]. That comparison is structurally suggestive. It is not a substitute for the intrinsic Task 18 theorems and should not be read backwards into them. No spin group, spin structure, covering-space classification, bundle cocycle, cohomology class, fermion, Hilbert state, or physical spin observable has been derived by Task 18.

The new non-canonicity result also matters for comparison discipline. A conventional formalism may offer a preferred language for maximal local choices, but the present reconstruction has proved that its currently inherited algebra does not itself select between at least two explicit maximal preconnected extensions. Any preferred selection must therefore be either newly derived or openly imported as additional structure.

19 Formal source ledger

The Task 18 audit separates standard imported mathematics from inherited project endpoints and new derived theorems.

The non-elementary standard imports are ordinary point-set topology, connectedness and path-connectedness, metric-space facts, compactness, Tietze extension, compact-support uniform continuity, smooth approximation, smooth transition functions, elementary trigonometry, and basic scalar-injectivity/square identities [7, 8, 5]. No algebraic-topology or conventional spin/bundle classification theorem enters the reconstruction.

The inherited project layer contains the already reconstructed carrier and centre, unit-direction geometry, Φ, the reference implementers, Task 16 exact coincidence classification, jointly regular family classification, global exact no-go, global sign-relaxed uniqueness, the Task 17 domain geometry, arbitrary-domain admissibility criterion, local exact exhaustion, integer overlap classification, and the continuous C^1 notion.

Everything described in sections 3, 4 and 6 to 13 is derived in Task 18, subject to the proof boundaries stated there.

20 Unresolved obligations after Task XVIII

Task 18 closes more than Task 17, but it does not close the project. The remaining obligations are now unusually well separated.

1. **Plain admissible maximality.** It is unknown whether the class of all admissible sets, with no openness or connectedness assumption, has inclusion-maximal elements.
2. **Complete maximal preconnected classification.** Antipodal completeness is sufficient for maximality and is equivalent to maximality inside the antipodal-free class, but the audit does not prove the full converse stated purely inside the preconnected admissible class.
3. **Disconnected geometry.** The component-label criterion is exact but existential: it does not geometrically classify which configurations of components realize a prescribed family of labels through one continuous extension.
4. **Bridge criterion independent of exactness.** The present global-bridge predicate already contains global exactness of the product family. A more primitive bridge/rate predicate is needed if one wants a genuinely independent equivalence theorem.
5. **Full relation completion.** The coincidence classes are classified, but a minimal and sufficient cross-domain comparison data structure for arbitrary local systems has not been formulated and proved complete.
6. **Non-unit carrier.** Bridge-normalized reconstruction uniqueness is only a unit-direction theorem. No current physical or mathematical requirement constrains non-unit directions.
7. **Higher regularity.** C^1 equivalence is closed; C^k and C^∞ remain unproved.
8. **Arbitrary spatial word problem.** It remains untouched, intentionally.

21 Task XIX: harden the branches before opening dynamics

Task 19 should not jump from the new structural resemblance to physical spin or dynamics. Part XVIII has produced several sharper internal questions that can be tested without importing a conventional target.

A suitable neutral title is

Task XIX – Intrinsic Global-Obstruction Decomposition: Primitive Bridge Criteria, Maximal Admissible Geometry, Relation Completion, and Higher-Regularity Closure.

The next task should be organized around the following work packages.

21.1 WP1: replace the definition-shaped global bridge by a primitive criterion

Define a central bridge candidate using properties of the factor itself and its rate, without putting "cU^{ref} is globally exact" directly into the definition. Possible ingredients to test include centrality, one-parameter multiplicativity, normalization at $\theta = 0$, continuous dependence on direction, and an explicitly recovered rate.

Then determine, rather than assume, the strongest equivalence among:

(a) existence of a global exact jointly regular representative;

(b) existence of a primitive bridge satisfying the necessary factor laws;

(c) existence of one continuous half-odd reversal-odd rate;

(d) existence of a central factor with the exact visible-coincidence compatibility law.

If a full equivalence fails, isolate the missing implication and provide a counterexample.

This work package should also replace or rename `no_global_bridge_iff_no_rate` so theorem names and logical forms coincide.

21.2 WP2: prove minimality of the three-condition obstruction at theorem level

Task 18 proves that all three conditions cannot coexist and that each pair is realizable. Task 19 should package this as an exact minimal-unsatisfiable-set theorem for the primitive rate/bridge predicate: removing any one of continuity, half-oddness, or reversal oddness restores existence, while all three forbid it. The result should distinguish a purely rate-level statement from consequences for internal families.

21.3 WP3: complete maximal preconnected admissible-domain classification

Determine whether every inclusion-maximal preconnected admissible domain is antipodally complete, with exact handling of the empty case and of any closure/boundary subtleties. If true, prove a necessity-and-sufficiency classification. If false, construct a maximal preconnected admissible counterexample that is not antipodally complete and identify the missing hypothesis.

Then characterize the relation between maximal preconnected domains and maximal *open* preconnected domains. In particular, determine whether the inherited $\mathrm{Dom}(v)$ are precisely the maximal members of some intrinsically defined open subclass or merely one family of them.

21.4 WP4: decide maximality in the plain admissible class

Investigate whether arbitrary admissible sets have inclusion-maximal extensions. Do not invoke a maximality principle until closure under the relevant chain operation is proved. If chain unions fail admissibility, exhibit the failure mechanism in terms of incompatible ambient rates. If a maximal extension theorem can be proved with additional compatibility data, state those data explicitly.

21.5 WP5: classify disconnected component-label extension data

Replace the existential component-label criterion by the strongest explicit extension theorem available. Determine which families of connected components with assigned half-odd labels can be realized as restrictions of one continuous reversal-compatible ambient rate. Address accumulation, closures of differently labelled components, antipodal pairing between components, and whether local constancy plus a separation condition is sufficient.

21.6 WP6: complete relation coverage intrinsically

Use the exact Task 16 coincidence classification as fixed input. Define the minimal comparison data required when two coincident visible parameterizations do not lie in a common connected admissible domain. Separate:

(i) same-direction relations;

(ii) antipodal relations;

(iii) degenerate identity-parameter relations.

Prove which data are automatic from the local bridge/overlap laws and which require an independent cross-domain comparison. Determine whether the antipodal class is the only *nonautomatic* class after the degenerate class is discharged.

21.7 WP7: connect relation completion to the primitive global bridge

Test whether a primitive global bridge exists exactly when all local bridge data satisfy the complete visible coincidence relation, not merely ordinary overlaps. This should be a theorem to prove or refute, not a conventional interpretation. A counterexample with all ordinary overlaps coherent but one cross-domain relation failing would be particularly informative.

21.8 WP8: harden regularity beyond C^1 only as far as the construction supports

Determine whether the Task 18 extension-and-staircase construction yields C^k for all finite k or C^∞ admissibility for arbitrary domains. State separate theorems for existence of a smooth rate and preservation of the exact half-odd values. Do not infer analyticity; if analyticity is tested, treat it as a different problem because exact plateaus and analytic continuation may change the situation drastically.

21.9 WP9: preserve negative controls and the reconstruction firewall

Task 19 should continue to forbid conventional spin-group, bundle, covering-space, cohomological, gauge, and physical-state input in reconstruction modules. It should include counterexamples showing where connectedness, openness, relation completeness, or higher regularity are genuinely used. The arbitrary spatial word problem should remain outside scope unless a specific relation-completion theorem formally requires it.

21.10 Task-XIX decision table

The final Task 19 audit should distinguish at least:

Question	Required verdict
Primitive global bridge predicate independent of exactness?	PROVED / REFUTED / PARTIAL
Primitive bridge $\leftrightarrow$ global exact representative?	PROVED / REFUTED / CONDITIONAL
Primitive bridge $\leftrightarrow$ continuous half-odd reversal-odd rate?	PROVED / REFUTED / CONDITIONAL
Three-condition obstruction minimal in the exact logical sense?	PROVED / REFUTED
Maximal preconnected admissible $\leftrightarrow$ antipodally complete?	PROVED / REFUTED / CONDITIONAL
Classification of maximal open preconnected domains?	PROVED / PARTIAL / OPEN
Maximal elements of the plain admissible class?	PROVED / REFUTED / OPEN
Explicit disconnected component-label extension theorem?	PROVED / PARTIAL / OPEN
Minimal complete cross-domain relation data?	PROVED / PARTIAL / OPEN
Only antipodal cross-domain class nonautomatic?	PROVED / REFUTED
Relation completion $\leftrightarrow$ primitive global bridge?	PROVED / REFUTED / CONDITIONAL
Continuous admissibility $\leftrightarrow$ C^∞ admissibility?	PROVED / REFUTED / OPEN
Arbitrary spatial word problem	remain OPEN unless forced

A positive Task 19 result should not be phrased as "the conventional structure has appeared". The desired endpoint is instead an intrinsic decomposition of exactly which global compatibility requirement fails, exactly which domain choices are forced or noncanonical, and exactly which cross-domain relations must be supplied beyond ordinary overlaps.

22 Conservation-of-Difficulty ledger

Question	Part-XVIII status
Does every jointly regular family have a central local-to-reference bridge?	Yes; central, unique, and exact factorization proved.
What does local exactness do to the bridge?	Forces a half-odd rate; one label on preconnected exact domains.
Where do Task-17 integer overlaps come from?	Difference of the two half-odd bridge labels.
Does explicit bridge normalization solve the Task-17 mismatch?	Yes; every normalized local member equals the unique global sign-relaxed reference.
Was Task 17 an ordinary gluing failure?	No; its local data already glued. The failure was literal equality across normalization classes.
Does normalization preserve exactness?	No. Exactness is retained in the explicit bridge factor.
Is the bridge trivial or quotiented?	No. It is nontrivial and determines the family.
Does one global exact bridge exist?	No.
What conditions would its rate satisfy?	Continuity + half-oddness everywhere + reversal oddness.
Is any one condition itself the obstruction?	No.
Is any pair already inconsistent?	No; all three pairs are explicitly realized.

Question	Part-XVIII status
Is the global exact obstruction therefore genuinely three-way?	Yes, at the proved rate level.
Is `globalExact_iff_globalBridge` fully independent of the bridge definition?	No; warning: global exactness is built into `IsGlobalBridge`.
Does `no_global_bridge_iff_no_rate` literally state an iff?	No; warning: it states a conjunction of two nonexistence claims.
Preconnected admissibility criterion?	Admissible iff antipodal-free, now a named Lean theorem; empty case included.
Is $\mathrm{Dom}(v)$ maximal?	Yes in open-preconnected-admissible class; no in broader open or arbitrary admissible classes.
Do maximal preconnected admissible domains exist?	Yes, explicit lexicographic constructions.
Is a maximal extension canonical?	No, explicit distinct maximal extensions support the same local algebra.
Do $\mathrm{Dom}(v)$ cover directions?	Yes.
Do antipodal-free domains cover all visible relations?	No.
What cross-domain coincidence classes exist?	Antipodal plus degenerate identity-parameter class.
Does the degenerate class create a new obstruction?	No for the explicit local models; its relation is satisfied.
Can a disconnected admissible domain carry several labels?	Yes; explicit four-label example.
Are component labels arbitrary?	No; they must extend to one continuous reversal-compatible ambient rate.
Can distinct labels accumulate?	No; local constancy proved.
Does C^1 restrict arbitrary-domain admissibility?	No; continuous iff C^1 admissibility proved constructively.
Does the same hold for C^∞?	Open.
Does the plain admissible class have maximal members?	Open.
Is disconnected admissibility geometrically classified?	Not yet; functional criterion remains stronger than available geometric taxonomy.
Has physical spin or quantum dynamics been derived?	No.

23 Conclusion

Task 18 resolves the precise question left by Part XVII. The local exact representatives and the unique global sign-relaxed reference are related by an explicit central bridge. That bridge is not optional bookkeeping: it is central and unique, it carries the local half-odd exactness, it recovers the integer overlap differences, and it determines the local family. Once the bridge is explicitly removed but retained as data, every covering system normalizes to the same global reference. The Task 17 reconstruction failure was therefore a literal-normalization mismatch, exactly as the sharper reading of its counterexample suggested.

The global exact no-go survives untouched in substance but is now localized more precisely. A global conversion would require one rate to be continuous, half-odd everywhere, and reversal-odd. Each pair is compatible. All three are not. The difficulty has therefore contracted to a minimal three-condition global incompatibility rather than an unspecified local-to-global failure.

At the same time, the geometry refuses to remain linear. The inherited $\mathrm{Dom}(v)$ domains become maximal only when openness and preconnectedness are both retained; they cease to be maximal as soon as either condition is relaxed. Explicit maximal preconnected extensions exist, but more than one supports the same reconstructed algebra, so no canonical choice has emerged. Ordinary direction coverage also proves too weak: the complete visible coincidence relation has cross-domain content that no antipodal-free family can internalize, although the additional identity-degenerate class turns out to be harmless. Disconnected admissible sets support multiple half-odd labels, but continuous extension prevents arbitrary accumulation or free componentwise assignment. Finally, the analytic branch strengthens from a partial Task 17 audit to an arbitrary-domain C^1 equivalence.

The Conservation of Difficulty has therefore changed shape. It no longer moves along a single path. One difficulty closed positively, and the proof exposed several independent frontiers: a primitive global bridge criterion, maximal-domain geometry and non-canonicity, complete relation coverage, disconnected extension data, and higher regularity. Task 19 should harden these branches without using a conventional interpretation to merge them prematurely.

The current boundary is therefore:

The local-to-global normalization mismatch is resolved by an explicit, retained central bridge. The remaining global exact obstruction is the simultaneous incompatibility of continuity, half-oddness, and reversal oddness. Independently, maximal-domain choice is noncanonical and ordinary direction covers do not encode the full visible coincidence relation.

No physical interpretation is required to make any of these questions sharp. The mathematics has again generated the next experiment by closing one route and branching the difficulty into several formally distinguishable ones.

References

[1] Aristotle (Harmonic). *Task 17: Hardening the Local Exact-Representative Problem.* Run 1fe8d6fc-ec55-42db-a7f4-350971665d0a; formal predecessor of Task 18. 2026.

[2] Aristotle (Harmonic). *Task 18: Local-to-Global Conversion Audit.* Run 50b9a9ba-6df6-4894-bf7b-97a39dece12c; Lean archive supplied by the editor; archive SHA-256 1164d78406b2672e24bf83f0fa64baa99894d541791e79deb3ddc6ee96d11251. 2026.

[3] H. Blaine Lawson and Marie-Louise Michelsohn. *Spin Geometry.* Princeton University Press, 1989.

[4] Pertti Lounesto. *Clifford Algebras and Spinors.* 2nd ed. Cambridge University Press, 2001.

[5] Leonardo de Moura and Sebastian Ullrich. "The Lean 4 Theorem Prover and Programming Language". In: *Automated Deduction – CADE 28.* Springer, 2021, pp. 625–635.

[6] Jan Seeck, OpenAI ChatGPT, and Aristotle (Harmonic). *Split Complex Lorentzian Dynamics – Part XVII: Local Exhaustion, Non-Hemispherical Admissibility, and the Reconstruction Mismatch.* Internal project manuscript, 4 September 2026. Sept. 2026.

[7] The mathlib Community. "The Lean Mathematical Library". In: *Proceedings of the 9th ACM SIGPLAN International Conference on Certified Programs and Proofs.* ACM, 2020, pp. 367–381.

[8] The mathlib Community. *mathlib4 source tree, pinned revision 8f9d9cff6bd728b17a24e163c9402775d9e6a365.* Revision recorded by the Task-18 lake manifest. 2026.