Split Complex Lorentzian Dynamics

Part XI: Exact Central Freedom, Full-Carrier Lift Rigidity, and the Separation of Observable and State Generators

A formally verified continuation of the dependency audit

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2 September 2026

Abstract

Part X made the first deliberately narrow move from reconstructed kinematics toward candidate state transformations and infinitesimal evolution. Relative to one distinguished old spatial axis, the already-derived eight-dimensional associative algebra W exhibited a unique full-angle algebra automorphism, a half-angle internal implementation inside the restricted plane $K = \mathrm{span}_{\mathbb{R}}\{1, R\}$, half-angle candidate state sectors, and a one-dimensional family of admissible axial derivations. The same task also exposed two apparently different freedoms: a positive multiplicative scalar factor in the restricted internal lift, and an independent scalar rate multiplying the axial derivation. A premature normalization almost removed the first freedom before its dependency status had been understood.

Part XI is a hardening task. It introduces no new spatial direction, no new particle label, no new field equation and no physical time. Instead it removes the artificial restriction to K and asks for every internal one-parameter implementation of the already fixed algebra automorphism family on the complete carrier W. The first result is an exact center theorem: the central plane found earlier is not merely central but exhausts the center,

$$\mathrm{Center}(W) = Z = \mathrm{span}_{\mathbb{R}}\{1, S\}.$$

Already commutation with the two independent old generators A and B is sufficient to force this form, while commutation with A alone is not. This gives a sharp centralizer negative control.

Before importing any explicit Task-X lift, two completely arbitrary implementations of the same algebra action are compared. Their relative element is proved to commute with all of W, hence to lie in the exact center, and its one-parameter multiplicative law is derived. Consequently every full lift differs from any reference lift by a central multiplicative factor. Only after this upper rigidity theorem is established is the Task-X half-angle representative introduced. The Task-X real scalar freedom is then identified exactly as the intersection of the full central freedom with the restricted plane K; it was only a slice of the true ambiguity.

Continuity closes the principal unresolved obligation of Part X. Every continuous central residual factor is proved intrinsically, without identifying Z with conventional complex numbers, to have the unique two-parameter form

$$z_{\alpha,\beta}(\theta) = e^{\alpha\theta}\big(\cos(\beta\theta)\,1 + \sin(\beta\theta)\,S\big), \qquad \alpha, \beta \in \mathbb{R}.$$

Thus every continuous full implementation is

$$U_{\alpha,\beta}(\theta) = z_{\alpha,\beta}(\theta)\,U_{\mathrm{ref}}(\theta),$$

with exactly two independent real residual parameters. The one-parameter freedom observed in Part X is the subfamily in which the central S-rotation is absent. At the purely algebraic level the residual class is strictly larger; Task XI constructs a discontinuous witness and keeps algebraic and continuous classifications separate.

The periodicity audit further clarifies why algebra-action periodicity and implementer periodicity must not be conflated. Although $\Phi_{2\pi} = \mathrm{id}_W$, the general $U_{\alpha,\beta}(2\pi)$ and $U_{\alpha,\beta}(4\pi)$ retain explicit dependence on (α, β). The familiar values -1 and $+1$ characterize the chosen reference representative only after stronger conditions are imposed; they are not consequences of the bare inner-action problem. This paper also corrects an overstatement in the Task-XI prose audit: the pair of sample values $U(2\pi) = -1$ and $U(4\pi) = 1$ does not by itself imply $\alpha = \beta = 0$; for example $\alpha = 0$ and integer β gives the same two sample values while defining a generally different lift. The formal Lean periodicity theorems themselves are unaffected.

Task XI then tests the two Task-IX quadratic branches symmetrically. Preservation under left multiplication is introduced explicitly as a new conditional compatibility requirement, not as an inherited law. For either N_+ or N_-, and likewise for simultaneous preservation, the condition selects exactly the reference lift. The tempting normalization of Part X therefore reappears only as a *derived consequence of an explicitly added compatibility condition*; it is never inserted into the general lift classification.

Finally every continuous full lift is automatically differentiable, and its infinitesimal left generator is classified exactly as

$$G = \alpha\, 1 + \beta\, S - \frac{1}{2} R.$$

The commutator action of G on the algebra is independent of the central terms and equals the previously reconstructed axial derivation D_{gen}. More generally, two carrier generators induce the same algebra derivation if and only if their difference lies in the exact center. Hence the visible infinitesimal component is $-R/2$, while the entire two-dimensional center is invisible to conjugation. This invisible state-lift freedom is formally distinct from the rate freedom λD_{gen} of Part X: changing λ changes the algebra flow itself, whereas changing (α, β) leaves the fixed algebra flow unchanged.

The comparison with the independent Experiment 1 is now sharper. Experiment 1 reached a Clifford envelope and a central square-root-of-minus-one structure from a different starting direction, beginning with weaker real Euclidean data and reconstructing Lorentz structure before the associative closure. Experiment 2 began with Lorentz data, reconstructed the same associative boundary through longitudinal-sector gluing, and has now proved that its central two-plane is the exact center and exactly the kernel of the inner commutator action. The endpoints are standard mathematics, but the two independent dependency paths again converge on the same central structural boundary without sharing reconstruction results.

The proposed Task XII should remain neutral and harden the candidate state carrier before any dedicated spin-1, spin-1/2, Maxwell or Dirac investigation. It should ask which nonzero proper left-invariant subspaces or one-sided ideals are forced by the already reconstructed algebra and hardened lift action, whether the Part-X half-angle sectors are reducible, and whether any minimal carrier is selected intrinsically or only after new data are supplied. Only after that layer is settled should later tasks split the vector-like and half-angle sectors or test field-equation structure.

Keywords: exact center; internal implementer; continuous one-parameter subgroup; central freedom; inner automorphism; generator kernel; axial derivation; state-action ambiguity; formal verification; Lean 4; conservation of difficulty.

Contents

1 Why Part XI is a hardening task

Part X deliberately stopped at an uncomfortable but informative boundary. A single distinguished spatial axis had already made longitudinal and transverse sectors visible, the old axial rotation extended uniquely to the full associative carrier W, and an internal half-angle implementation was found. Yet the implementation was not unique: inside

$$K = \mathrm{span}_{\mathbb{R}}\{1, R\} \tag{1}$$

all admissible lifts had the form

$$U(\theta) = k(\theta)\, U_{\mathrm{std}}(\theta) \tag{2}$$

with a positive multiplicative real function k. Separately, all structurally admissible axial derivations formed the line λD_{gen} [9, 1].

The immediate temptation was to interpret these results in familiar representation-theoretic language. That would have been premature. Part X had not shown that K contained every possible implementer, had not completed the continuous classification of k, had not established that the known central plane was the *entire* center, and had not proved whether the free lift factor and the derivation-rate parameter were two descriptions of one freedom or logically independent structures.

Part XI therefore does not add a new physical target. It removes restrictions from the previous question and asks what survives.

Methodological principle 1 (Hardening by removing a convenience restriction)**.** If a classification was obtained only after restricting the unknown object to a convenient subspace, the next dependency question is whether the classification survives when that restriction is removed. A standard answer must not be imported to make the unrestricted problem look like the restricted one.

The complete inherited basis remains

$$1,\ A,\ B,\ C,\ P = AB,\ Q = AC,\ R = BC,\ S = ABC, \tag{3}$$

with

$$A^2 = B^2 = C^2 = 1, \qquad AB = -BA, \quad AC = -CA, \quad BC = -CB, \tag{4}$$

and

$$R^2 = S^2 = -1, \qquad Sx = xS \quad (x \in W). \tag{5}$$

Task IX had already derived the central plane

$$Z = \mathrm{span}_{\mathbb{R}}\{1, S\} \tag{6}$$

and the two multiplicative quadratic branches N_+, N_-. Task X had already fixed the one-parameter algebra automorphism family Φ_θ. No further algebraic object is added at the start of Part XI [8, 9, 2].

2 The exact center must be proved, not guessed

It was known before Task XI that every element of Z commutes with W. What had not been proved was the converse. This distinction becomes decisive because the ambiguity between two internal implementers of the same inner action is expected, but not assumed, to lie in the center.

Task XI therefore defines neutrally

$$\mathrm{Center}(W) = \{z \in W :\ zx = xz \text{ for every } x \in W\} \tag{7}$$

and starts with a general eight-coordinate element.

The coefficient calculation can be organized economically. Commutation with A alone eliminates the B, C, P, Q coordinates. Commutation with B then eliminates the remaining A and R coordinates. Exactly the 1 and S coefficients survive. Since 1 and S were already proved central, no additional generator test is required.

Theorem 1 (Exact center)**.** *The full center of the independently reconstructed associative carrier is exactly*

$$\mathrm{Center}(W) = Z = \mathrm{span}_{\mathbb{R}}\{1, S\}. \tag{8}$$

Moreover the centralizer of the pair $\{A, B\}$ already equals the full center.

The negative control is equally useful. The centralizer of A alone is strictly larger than the center. Hence central rigidity does not follow merely from respecting the distinguished axis of Part X; a second independent old spatial generator is required to collapse the commutant to Z.

Remark 1 (A local-axis choice does not determine the full commutant)**.** *Part X made A distinguished for the axial problem. Task XI shows that this does not make the algebra one-dimensional around A: noncentral elements can still commute with A. The exact center becomes visible only after enough independent generator relations are imposed.*

This result is the structural prerequisite for everything that follows. Without it one could show that two lifts differ by an element commuting with the algebra action but could not know whether that ambiguity had been completely classified.

3 Arbitrary full-carrier lifts

Task XI next removes the defining restriction of Part X. An internal lift is now an arbitrary map

$$U : \mathbb{R} \longrightarrow W, \tag{9}$$

not assumed to lie in K or Z. The only requirements are

$$U(0) = 1, \tag{10}$$

$$U(\theta + \varphi) = U(\theta)U(\varphi), \tag{11}$$

$$U(\theta)xU(-\theta) = \Phi_\theta(x) \qquad (x \in W). \tag{12}$$

The inverse relations $U(\theta)U(-\theta) = 1 = U(-\theta)U(\theta)$ are derived from the group law rather than assumed.

Continuity is kept separate. This preserves the distinction between the algebraic one-parameter problem and the regular one-parameter problem that Task X had left open.

Definition 1 (Full lift)**.** A full lift is any map $U : \mathbb{R} \to W$ satisfying equations (10) to (12). A continuous full lift is a full lift whose carrier-valued map is continuous in the inherited finite-dimensional real topology.

The source order is a critical part of the audit: the explicit Task-X half-angle representative is not imported at this stage.

4 Relative rigidity before any reference formula

Let U and V be two completely arbitrary full lifts of the same fixed algebra action Φ. Define only the neutral relative element

$$\rho_{U,V}(\theta) := U(\theta)V(-\theta). \tag{13}$$

Nothing in the definition says that $\rho_{U,V}$ is central.

The two conjugation laws imply that $\rho_{U,V}(\theta)$ commutes with every $x \in W$. The exact center theorem then converts this pointwise commutation statement into a complete location theorem:

$$\rho_{U,V}(\theta) \in Z. \tag{14}$$

The group laws of U and V further imply

$$\rho(0) = 1, \qquad \rho(\theta + \varphi) = \rho(\theta)\rho(\varphi). \tag{15}$$

Theorem 2 (Upper relative-lift classification)**.** *Fix any full lift V. A map $U : \mathbb{R} \to W$ is a full lift of the same algebra action if and only if there exists a multiplicative base-point-normalized map*

$$z : \mathbb{R} \to Z \tag{16}$$

such that

$$U(\theta) = z(\theta)V(\theta) \tag{17}$$

for every θ.

This theorem is proved before any explicit half-angle formula is available to the Task-XI reconstruction layer. The result therefore cannot be an artifact of beginning with the expected representative.

It also supplies the first major Conservation-of-Difficulty verdict of Part XI:

> The non-uniqueness of an internal implementation is neither arbitrary nor merely scalar. It is exactly the freedom to multiply by a one-parameter homomorphism valued in the full center of the reconstructed algebra.

5 Only after rigidity: the reference implementation

Only after equation (17) is proved does Task XI import the explicit Task-X solution. It is renamed neutrally U_{ref} and used only as an existence witness. The full algebraic classification becomes

$$U(\theta) = z(\theta)U_{\mathrm{ref}}(\theta), \qquad z : \mathbb{R} \to Z, \qquad z(\theta + \varphi) = z(\theta)z(\varphi). \tag{18}$$

Writing the exact central element as

$$z(\theta) = f(\theta)\,1 + g(\theta)\,S, \tag{19}$$

the multiplicative law is equivalent to

$$f(\theta + \varphi) = f(\theta)f(\varphi) - g(\theta)g(\varphi), \tag{20}$$
$$g(\theta + \varphi) = f(\theta)g(\varphi) + g(\theta)f(\varphi), \tag{21}$$

with $f(0) = 1$, $g(0) = 0$ and a derived nondegeneracy condition

$$f(\theta)^2 + g(\theta)^2 > 0. \tag{22}$$

No conventional complex number is used to state or prove these relations.

6 What Task X had actually seen

The restricted plane $K = \mathrm{span}\{1, R\}$ from Part X can now be placed exactly inside the full classification.

Task XI proves the equivalence

$$U(\theta) \in K \text{ for every } \theta \quad \Longleftrightarrow \quad g(\theta) = 0 \text{ for every } \theta. \tag{23}$$

Thus the Part-X scalar factor k was not the full implementation ambiguity. It was precisely the $g \equiv 0$ slice of the larger central family.

A continuous full lift with nonzero S-component is constructed explicitly, proving that the inclusion is strict.

Theorem 3 (Strict enlargement of the lift freedom)**.** *The full-carrier internal implementation problem has strictly more solutions than the K-restricted problem of Task X. The latter is exactly the real-central slice of the former.*

This is why the hardening step was necessary. Had Part X been immediately interpreted as a complete state-lift classification, one entire continuous direction of residual freedom would have remained invisible.

7 Continuity converts functional freedom into two parameters

At the purely algebraic level, equations (20) and (21) permit highly irregular solutions. Task XI now imposes exactly the regularity condition that Part X had originally intended but not completed: continuity.

The proof remains entirely in the real basis $1, S$. Continuity plus the bilinear recursion is first used to establish differentiability. The group law then produces a constant-coefficient differential system, and two derived conserved combinations fix the solution. The result is the complete classification

$$z_{\alpha,\beta}(\theta) = e^{\alpha\theta} \left(\cos(\beta\theta)\, 1 + \sin(\beta\theta)\, S\right), \qquad \alpha, \beta \in \mathbb{R}. \tag{24}$$

Consequently

$$U_{\alpha,\beta}(\theta) = z_{\alpha,\beta}(\theta) U_{\mathrm{ref}}(\theta) \tag{25}$$

is the general continuous full lift.

Theorem 4 (Exact continuous parameter count)**.** *Every continuous full lift is uniquely represented by exactly one pair $(\alpha, \beta) \in \mathbb{R}^2$ in equation (25). The two real parameters are independent.*

The role of the two parameters is structurally different already at the real-algebra level. The factor $e^{\alpha\theta}$ changes the radial size of the central coefficient pair, while β rotates that pair in the internally derived 1–S plane. These statements need no physical interpretation.

Methodological caveat 1 (Continuous does not mean normalized)**.** *Continuity removes pathological functional freedom. It does not select $\alpha = 0$ or $\beta = 0$. The continuous classification still contains a two-real-parameter family.*

8 Why algebraic and continuous lifts must remain distinct

Task XI also proves that the algebraic solution class is genuinely larger. Using a choice-dependent additive function built from a rational-linear complement, it constructs a central multiplicative residual map that does not belong to the continuous two-parameter family.

This result has two roles in the dependency audit. First, it verifies that continuity is a real additional condition rather than a redundant annotation. Second, it prevents an illicit inference from the familiar continuous answer to the unrestricted algebraic problem.

The project does not attempt to classify every discontinuous residual homomorphism. Such a classification is both unnecessary for the present physics-motivated path and intrinsically dependent on choice-sensitive additive-function structure. The exact continuous class is the relevant hardened layer; the existence of larger algebraic freedom is retained as a formal boundary.

9 Periodicity: what the algebra sees and what an implementer retains

The algebra action from Task X obeys

$$\Phi_{2\pi} = \mathrm{id}_W \,. \tag{26}$$

For the general continuous lift equation (25), however, Task XI derives

$$U_{\alpha,\beta}(2\pi) = -e^{2\pi\alpha}\,(\cos(2\pi\beta)\,1 + \sin(2\pi\beta)\,S)\,, \tag{27}$$

$$U_{\alpha,\beta}(4\pi) = e^{4\pi\alpha}\,(\cos(4\pi\beta)\,1 + \sin(4\pi\beta)\,S)\,. \tag{28}$$

The values $U_{\mathrm{ref}}(2\pi) = -1$ and $U_{\mathrm{ref}}(4\pi) = 1$ are recovered at $(\alpha,\beta) = (0,0)$, but the algebraic return equation (26) alone does not determine either implementer value.

Theorem 5 (Algebraic periodicity does not fix implementer periodicity)**.** *The identity return of Φ after 2π is compatible with continuously many distinct internal values of $U(2\pi)$.*

9.1 Correction to an audit overstatement

The formal equations equations (27) and (28) are correct in the Task-XI development. The prose audit, however, contains the stronger sentence that the pair of values

$$U(2\pi) = -1, \qquad U(4\pi) = 1 \tag{29}$$

occurs *exactly* for $\alpha = \beta = 0$. That statement is false.

For example, $\alpha = 0$ and any integer β gives the same two sample values while the resulting lift generally differs from U_{ref} at intermediate angles. Moreover $U(4\pi) = 1$ alone allows further half-integer values of β when $\alpha = 0$.

Proof boundary 1 (Status of the correction)**.** *No Lean theorem used in the Task-XI classification asserts the false uniqueness claim. The error is confined to an interpretive sentence in the audit. The present paper therefore retains the machine-checked periodicity formulas and corrects only the prose inference drawn from two sampled angles.*

This correction is itself methodologically instructive. A small set of periodicity values cannot replace the full one-parameter classification.

10 The normalization trap revisited correctly

Part X nearly lost its residual factor when the familiar condition $a^2 + b^2 = 1$ was considered as a shortcut to the standard half-angle representative. Part XI now shows how a normalization may enter without corrupting the dependency graph.

The two Task-IX quadratic extensions N_+ and N_- are both retained. A new predicate is introduced only at this late stage:

$$N(U(\theta)x) = N(x) \qquad (\theta \in \mathbb{R},\ x \in W). \tag{30}$$

This is explicitly labelled a *conditional compatibility test.* It is not inherited from Tasks VIII–X and supports none of the center or lift-classification theorems.

For the continuous full-lift family, Task XI proves independently

$$N_+\text{-compatibility} \iff U = U_{\mathrm{ref}}, \tag{31}$$
$$N_-\text{-compatibility} \iff U = U_{\mathrm{ref}}. \tag{32}$$

In fact the two compatibility predicates are equivalent even for arbitrary maps U; this particular condition does not distinguish the two Task-IX sign branches.

Simultaneous preservation adds nothing further. Either branch alone already eliminates both continuous residual parameters.

Methodological principle 2 (Normalization may be derived only after its cost is named)**.** The reference lift may be selected by an explicit additional compatibility requirement with previously reconstructed structure. It may not be selected by inserting a norm condition into the unrestricted lift problem and then forgetting that the condition was added.

Under equation (30), the unit-like relation for the central coefficient pair follows as a theorem and ultimately forces $\alpha = \beta = 0$. Thus the normalization that almost hid the Task-X ambiguity reappears in Part XI with its dependency status made explicit.

11 Differentiability costs nothing beyond continuity

Because the complete continuous classification is explicit, every continuous full lift is automatically differentiable. No differentiability assumption is needed to obtain the main classification.

For

$$U_{\alpha,\beta}(\theta) = z_{\alpha,\beta}(\theta)U_{\mathrm{ref}}(\theta), \tag{33}$$

Task XI derives the infinitesimal left generator at the identity:

$$G_{\alpha,\beta} = \frac{d}{d\theta}U_{\alpha,\beta}(\theta)\bigg|_{\theta=0} = \alpha\,1 + \beta\,S - \frac{1}{2}R. \tag{34}$$

Equivalently, a carrier element G occurs as the infinitesimal generator of some differentiable full lift if and only if

$$G + \frac{1}{2}R \in Z. \tag{35}$$

The two central parameters therefore survive unchanged at the infinitesimal left-action level.

12 Visible and invisible generator components

A carrier element G acts on a candidate state by left multiplication. The corresponding infinitesimal action on the algebra under conjugation is instead the commutator map

$$\mathrm{ad}_G(x) = Gx - xG. \tag{36}$$

Task XI derives, rather than prescribes, that every generator equation (34) satisfies

$$\mathrm{ad}_{G_{\alpha,\beta}} = D_{\mathrm{gen}}. \tag{37}$$

The central terms vanish because they commute with all of W.

More importantly, the converse is exact:

$$\mathrm{ad}_G = \mathrm{ad}_{G'} \quad\iff\quad G - G' \in \mathrm{Center}(W). \tag{38}$$

Using the center theorem, the entire kernel is therefore

$$\ker(G \mapsto \mathrm{ad}_G) = \mathrm{span}_{\mathbb{R}}\{1, S\}. \tag{39}$$

Theorem 6 (Visible/invisible generator split). *For the fixed axial algebra flow, every infinitesimal internal implementation decomposes uniquely as*

$$G = \underbrace{-\frac{1}{2}R}_{\text{visible under commutator}} + \underbrace{\alpha\,1 + \beta\,S}_{\text{invisible under commutator}}. \tag{40}$$

The invisible component is exactly the full center and no larger subspace.

Task XI also proves that left multiplication by G is not itself a derivation. This avoids another common conflation: the generator acting on a carrier and the derivation acting on algebraic observables are structurally different objects even though the latter is induced by the former.

13 Rate freedom and lift freedom are not the same thing

Part X classified structurally admissible axial derivations as

$$D = \lambda D_{\text{gen}}, \qquad \lambda \in \mathbb{R}. \tag{41}$$

At that stage it remained possible that the free λ and the free lift factor were merely two views of the same missing scale.

Task XI rules this out formally.

Once the automorphism family Φ itself is fixed, every differentiable full lift induces exactly D_{gen}, not λD_{gen} for arbitrary λ:

$$\mathrm{ad}_{G_U} = \lambda D_{\text{gen}} \quad \Longleftrightarrow \quad \lambda = 1. \tag{42}$$

Changing λ changes the algebra flow. Changing α or β leaves that flow unchanged.

Freedom	Location	Effect on the fixed algebra action
λ	derivation line $\mathbb{R}D_{\text{gen}}$	visible; changes the flow itself
(α, β)	exact center $\mathrm{span}\{1, S\}$	invisible; changes the internal implementer only

This is the first fully formal separation in Experiment 2 between a freedom of the algebraic evolution and a freedom of its internal implementation.

14 Comparison with Experiment 1

The comparison with the independently developed Experiment 1 is now stronger than it was after Part VIII, but it must be stated carefully.

Experiment 1 started from weaker real geometric data. In broad form its dependency direction was

$$\begin{aligned} \mathbb{R}^3 \text{ Euclidean datum} &\longrightarrow \text{spin-factor/Jordan structure} \\ &\longrightarrow \text{Lorentz structure} \longrightarrow \text{associative Clifford envelope} \\ &\longrightarrow \text{central square-root-of-minus-one structure.} \end{aligned} \tag{43}$$

The corresponding papers emphasize that the resulting Lorentz, Clifford and central complex structures are standard mathematics; the experiment audits which prior data make them reconstructible [6, 7].

Experiment 2 entered from the opposite side. Lorentz structure was accepted at the beginning, but multiplicative structure was withheld. Its path has been

$$\begin{aligned} \text{Lorentz carrier} &\longrightarrow \text{longitudinal } 1+1 \text{ sectors} \longrightarrow \text{global } 3+1 \text{ spin-factor core} \\ &\longrightarrow \text{associative closure obstruction} \longrightarrow W \\ &\longrightarrow S^2 = -1 \text{ central} \longrightarrow \text{exact center and lift kernel.} \end{aligned} \tag{44}$$

After Task VIII the two experiments already met at the familiar three-generator Clifford boundary and the central pseudoscalar-like element whose square is -1. Task XI sharpens the Experiment-2 side of this mirror in two ways:

1. the two-dimensional real plane generated by 1 and the central square root of minus one is proved to be the *entire* center, not merely a central subalgebra;
2. the same center is proved to be exactly the kernel of the commutator map from internal generators to algebra derivations.

This does not mean that Experiment 1 already proved the Task-XI lift classification. It did not address this particular axial internal-implementation problem. The legitimate comparison is structural: both independent reconstructions place a central square-root-of-minus-one structure at the associative boundary, and Experiment 2 now shows that this central structure is precisely where information can vary without changing the inner action on the algebra.

Methodological principle 3 (Independent convergence is calibration, not novelty)**.** The two experiments reaching compatible standard algebraic structures from different dependency directions strengthens confidence in the reconstruction methodology. It does not convert classical Clifford or central-simple-algebra facts into new mathematics and does not establish a fundamental physical ontology.

There is nevertheless a useful asymmetry. Experiment 1 asks how Lorentz and complex structure can emerge from weaker real geometry. Experiment 2 asks what happens after Lorentz structure is granted but products, state transformations and evolution data are withheld. The fact that the second route now localizes its state-implementation ambiguity exactly in the center produced by the first algebraic closure is therefore a nontrivial dependency observation even though the endpoint is conventionally familiar.

15 Late comparison with established algebra

Only in the final comparison module is the previously established isomorphism

$$W \cong M_2(\mathbb{C}) \tag{45}$$

used. Under it, the exact center theorem becomes the standard statement that the center of $M_2(\mathbb{C})$ consists of complex scalar matrices. The intrinsic central factor equation (24) becomes

$$e^{(\alpha+i\beta)\theta} I. \tag{46}$$

The two continuous real parameters are therefore conventionally read as a logarithmic modulus rate and an argument rate. The relative element of two implementers becomes a nonzero complex scalar, and the invisible infinitesimal component becomes the scalar-matrix kernel of the commutator action [4, 3, 2].

These are standard results in matrix and Clifford algebra. The formal interest of Task XI is that none of them is used to prove the reconstruction-core statements. The sequence is instead

$$\begin{aligned} \text{real eight-dimensional multiplication table} &\longrightarrow \text{exact center} \\ &\longrightarrow \text{relative lift rigidity} \\ &\longrightarrow \text{continuous two-parameter family} \\ &\longrightarrow \text{late matrix interpretation.} \end{aligned} \tag{47}$$

No physical meaning is assigned to the central factor. In particular the terms *phase*, *probability amplitude*, *unitarity* and *quantum state* are not reconstruction inputs and are not consequences of Task XI.

16 Heuristic back-translation

Part X showed that simply selecting one spatial direction inside the fully reconstructed $3+1$ structure was enough to make a longitudinal line, a transverse plane, full-angle algebra channels and half-angle candidate state sectors distinguishable. No momentum magnitude or dynamical law was needed for that mathematical selection.

Part XI now adds a different lesson. Once the geometrical/algebraic action Φ is fixed, the complete algebra can still fail to distinguish among different internal implementations. The difference between any two such implementations lies entirely in the center. From the perspective of conjugation on algebraic elements, that central component is invisible.

In the original heuristic language this should not be interpreted as a new hidden band or a new physical motion. It is better read as a distinction between two levels of description:

- the relative geometry of the already reconstructed channels, which fixes the axial algebra action;
- additional central information carried by an internal implementation, which does not alter that relative geometry.

The first level can therefore be completely fixed while the second retains two continuous real degrees of freedom.

The comparison with the selected-axis picture is especially useful. Selecting A made formerly symmetry-equivalent longitudinal and transverse structures distinguishable. By contrast, adding a central component to the internal generator does *not* change what the algebra can distinguish under commutator action. One operation reveals structure by reducing symmetry; the other changes internal implementation while remaining invisible to the already fixed algebra action.

This distinction will matter when a genuine state carrier is eventually selected. At present W itself is still only a provisional left carrier. Task XI says exactly which implementation data the algebra cannot see; it does not yet say which of those data survive or become meaningful on a minimal state carrier.

17 Conservation of Difficulty after Task XI

The hardening sequence can now be summarized without invoking conventional particle language.

Task X: fixed axial algebra action Φ; internal implementation classified only inside $K = \mathrm{span}\{1, R\}$.

Task XI removes the K restriction and first proves the exact center $\mathrm{Center}(W) = \mathrm{span}\{1, S\}$.

Two arbitrary full lifts of the same Φ differ exactly by a central multiplicative map. This is proved before any reference formula is imported.

Task-X real scale is only the K-slice of the full central freedom. The full continuous family has exactly two real parameters (α, β).

Algebraic and continuous classes separate: discontinuous central homomorphisms exist, while continuity rigidifies the family to $z_{\alpha,\beta}$.

Conditional compatibility with either Task-IX quadratic branch eliminates both residual parameters and selects the reference lift; normalization is derived only conditionally.

Infinitesimal implementer $G = \alpha 1 + \beta S - R/2$: center invisible, $-R/2$ visible. This freedom is distinct from the algebra-flow rate λ.

Next difficulty: the state carrier itself is still provisional. Before separate physical sectors are studied, its intrinsic reducibility and minimal stable pieces must be classified.

Three distinct freedoms are now cleanly separated:

1. **algebraic pathologies:** unrestricted discontinuous central homomorphisms;
2. **continuous implementation freedom:** the two central parameters (α, β) for a fixed algebra action;
3. **flow-rate freedom:** the separate real parameter λ that changes the algebra derivation itself.

The difficulty has therefore not accumulated as one opaque missing "dynamics axiom". It has split into logically different locations that can be tested one after another.

18 Formal audit and proof boundary

The supplied Task-XI archive has SHA-256

`4c208650cdaa95493425a8785fba96557b74f74fac9cf0d6b470bcb4fff8b198`.

The pinned environment is Lean 4.28.0 with mathlib revision

`8f9d9cff6bd728b17a24e163c9402775d9e6a365`.

Aristotle reports a successful full build of 8133 jobs [2, 5, 10, 11].

The reconstruction chain is

`SafeBase` $\to$ `CenterRigidity` $\to$ `FullLift` $\to$ `RelativeLiftRigidity`
$\to$ `ReferenceExistence` $\to$ `ExactLiftClassification` $\to$ `ContinuousClassification`
$\to$ `PeriodicityAudit` $\to$ `QuadraticCompatibility` $\to$ `InfinitesimalLift`
$\to$ `GeneratorSeparation` $\to$ `Identification`.

The explicit Task-X lift is first imported in `ReferenceExistence`; the center and relative-lift rigidity theorems are already complete before that point. Conventional matrix/complex identification appears only in `Identification`.

The Task-XI audit reports no `sorry`, `admit`, custom axiom or `implemented_by`. For the principal theorems, `#print axioms` reports only

$$\texttt{propext}, \quad \texttt{Classical.choice}, \quad \texttt{Quot.sound}. \tag{48}$$

The axiom of choice is additionally visible in the deliberately discontinuous residual witness through the construction of a rational-linear complement.

Proof boundary 2 (What was independently checked for this paper)**.** *The full Lean build is Aristotle's reported build result. For this paper, the archive, audit, import order and theorem statements were inspected independently. The exact center and centralizer claims, the commutator kernel, the generator formula and the periodicity formulas were also checked against the explicit eight-dimensional multiplication data. The complete pinned Mathlib build was not rerun in the paper-generation environment.*

The only substantive correction introduced by this paper is the periodicity-uniqueness sentence discussed in section 9; no formal theorem is altered.

19 Open obligations after Task XI

Task XI closes the full-carrier continuous lift problem, but several questions remain deliberately open.

1. **Discontinuous algebraic residuals.** Existence beyond the continuous family is proved, but no complete choice-sensitive parameterization of all such maps is attempted.
2. **Intermediate regularity.** Measurable, Baire, locally bounded or related classes are not classified. They are not presently needed for the main reconstruction path.
3. **Right and two-sided quadratic compatibility.** The Task-XI conditional test uses left multiplication. Right-sided and two-sided variants remain open.
4. **State carrier.** The whole algebra W is still only a provisional left carrier. No minimal one-sided ideal, irreducible module or projective state carrier has been selected.
5. **Half-angle sector reducibility.** The Part-X distinguished-axis sectors have central dimension two. It is not yet known within the reconstruction whether they contain smaller nonzero stable carriers selected by the existing algebra alone.
6. **Local subalgebra structure.** The possibility that locally distinguished directions or candidate states induce natural observable subalgebras remains a later marker, not a Task-XI result.

7. **Physical dynamics.** No physical time, Hamiltonian, field equation, mass relation or interaction law has been introduced.

These obligations should not be collapsed into one broad search. Part XI has made the opposite lesson clear: apparently small restrictions can hide whole dimensions of residual structure.

20 Outlook: Task XII should harden the carrier before physical sectors

The next task should therefore remain neutral. It should not ask Aristotle to derive a spinor, a fermion, a photon, Maxwell equations or any other conventional target. The next minimal question is structural:

> Given the already reconstructed associative algebra, the fixed axial automorphism and the now-complete internal-lift classification, what are the smallest nonzero left carriers that are forced or permitted by the existing multiplication alone?

A suitable Task XII should begin from the complete W and the Part-X axis decomposition but introduce no conventional module target. It should classify proper left-invariant subspaces or left ideals, determine which are stable under the hardened axial implementation family, and test the reducibility of the two Part-X half-angle sectors.

The source order should again be restrictive:

$$\begin{gathered}\text{arbitrary nonzero left-stable subspace / one-sided carrier}\\ \downarrow\\ \text{closure constraints from the derived multiplication table}\\ \downarrow\\ \text{minimality / reducibility classification}\\ \downarrow\\ \text{compatibility with the fixed axial action and full lift family}\\ \downarrow\\ \text{relation to the Part-X distinguished-axis sectors}\\ \downarrow\\ \text{late conventional comparison only after classification.}\end{gathered} \tag{49}$$

The critical negative control is whether a minimal carrier is *intrinsically selected.* It is not enough that minimal left ideals exist. The experiment must distinguish:

1. existence of smaller stable carriers;
2. classification of all such carriers;
3. uniqueness or non-uniqueness;
4. whether choosing one requires new orientation, sign, idempotent or normalization data.

Only after this carrier layer has been hardened should the program split into dedicated later tasks. One branch may then investigate the vector-like full-angle sector and determine which additional constraints, if any, remove or retain longitudinal degrees of freedom. Another may investigate minimal half-angle carriers and their longitudinal/transverse projection structure. Still later, if independently derived structures justify it, one can ask whether local subalgebras, differential constraints or field-like equations emerge and only then compare them with Maxwell-, Dirac- or algebraic-observable frameworks.

Methodological principle 4 (Do not ask for the name before deriving the carrier)**.** The next task should classify the smallest available state-like carriers without telling the prover which standard representation they are expected to resemble. Conventional spin or field terminology belongs after the structural classification, not in its premises.

This sequencing reflects the present state of the project. The remaining difficulty is distributed across multiple layers: carrier selection, local versus global structure, vector constraints, half-angle sectors and eventually dynamical laws. Each should be reopened only after the layer beneath it has been formally hardened.

21 Conclusion

Part XI removes the main artificial restriction left by Part X and converts an apparent one-dimensional normalization ambiguity into an exact structural theorem. The full center of the independently reconstructed associative algebra is precisely the plane generated by 1 and S. Any two internal one-parameter implementations of the same fixed axial algebra action differ exactly by a multiplicative map into this center. The Task-X scalar factor was therefore only the slice visible after restricting the unknown lift to $K = \mathrm{span}\{1, R\}$.

Continuity rigidifies the full residual map to exactly two real parameters. The general continuous implementation is $U_{\alpha,\beta} = z_{\alpha,\beta} U_{\mathrm{ref}}$, where one central parameter controls radial scaling and the other rotates within the internally derived central plane. Algebraic lifts are more numerous: discontinuous residual homomorphisms exist, so continuity is a genuine extra assumption rather than a redundant regularity label.

The periodicity analysis reinforces the distinction between algebra and implementation. The algebra returns after 2π, but an implementer retains central information not seen by conjugation. A small prose overstatement in the formal audit is corrected here: the reference values at 2π and 4π do not uniquely identify the reference lift from those two samples alone. What uniquely selects the reference lift in the present development is instead a stronger, explicitly named conditional compatibility with either Task-IX quadratic branch.

At the infinitesimal level, every continuous internal implementation has generator

$$G = \alpha 1 + \beta S - \frac{1}{2} R.$$

The entire center is invisible to the induced commutator derivation, while $-R/2$ is the fixed visible component. The kernel statement is exact: two generators induce the same algebra derivation if and only if they differ centrally. This freedom is distinct from the Part-X rate parameter λ, which changes the algebra flow itself.

The comparison with Experiment 1 again shows independent convergence on standard structure from opposite dependency directions. Experiment 1 reconstructed Lorentz and central complex/Clifford structure from weaker real geometry; Experiment 2 accepted Lorentz structure but withheld multiplication and state implementation, independently reached the same associative-central boundary, and now identifies that center as the complete invisible freedom of inner implementation. This agreement calibrates the reconstruction method without making a novelty claim for the underlying algebra.

The correct next move is therefore not yet a dedicated spin-1, spin-1/2 or Maxwell task. The state carrier remains provisional. Task XII should classify its intrinsic reducibility and the smallest stable one-sided carriers before conventional representation names are allowed to guide the search. Only after that hardening will separate investigations of vector-like, half-angle, local-observable and eventually dynamical structures rest on a sufficiently audited dependency base.

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