Split Complex Lorentzian Dynamics

Part VI: Clean-Room Replication, Universal 3+1 Obstructions, and the Localization of a Closure Deficit

A formally verified continuation of the dependency audit

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Abstract

This paper reports the clean-room repetition and strengthening of the higher-dimensional gluing analysis begun in Part IV. There is deliberately no Part V in the present series. The clean-room task originally planned after Part IV was renumbered and executed as Task 06 before a separate Part V artifact was produced. To keep the paper numbering synchronized with the actually executed formal task, the series therefore continues directly from Part IV to Part VI. No mathematical step is omitted by this numbering choice.

Part IV had established an exact local-to-global classification. All longitudinal $1+1$ products inside a fixed real $3+1$ Lorentz carrier force a unique symmetric product but leave an alternating cross-sector defect. Proper spatial isotropy appeared to reduce this defect to one real degree of freedom, whereas full isotropy removed it. The mathematical result was kernel-checked, but its intended blind-reconstruction status was weakened because an explicit nonzero coordinate model of the later alternating operation had been introduced before the proper-isotropy classification was complete.

Task 06 repairs this defect by a strict clean-room source order. Before any explicit nonzero alternating map exists, an abstract defect

$$A : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}^{1,3}$$

is constrained only by bilinearity, alternation and explicit orientation-preserving stabilizer transformations. The temporal output is forced to vanish, the three independent basis-pair values are forced to share one real coefficient, and the solution space injects into $\mathbb{R}$. Only after this rigidity theorem is complete is a nonzero witness constructed. Existence and rigidity together show that the proper-equivariant defect space is exactly one-dimensional. Orientation reversal negates its parameter and full isotropy leaves only the zero defect. The formerly contaminated Task 04 classification is therefore independently replicated under a stronger firewall.

The second phase is more consequential. It abandons isotropy entirely and returns to an arbitrary real bilinear product satisfying only longitudinal sector compatibility. Two independent no-go theorems are proved. No such product is globally associative, and no such product is globally multiplicative for the Lorentz quadratic form Q_4. Both obstructions already occur for two orthonormal rest directions. If $a^2 = b^2 = e_0$ and $ab + ba = 0$, associativity forces $p = ab$ to satisfy $p^2 = -e_0$, while sector compatibility forces the time component of every in-carrier square to be a nonnegative sum of squares. Independently, global Q_4-multiplicativity forces the mixed product to have vanishing time component and positive Lorentz square, which contradicts positivity of the rest-space metric.

These results are not new algebra. Their importance for this experiment is diagnostic. The formal path has reached a boundary that is mirrored exactly by established mathematics: the Jordan/spin-factor product closes on the paravector carrier but is nonassociative, whereas an associative product generated by independent vector directions naturally retains mixed-grade products rather than forcing them back into the paravector space. Clifford algebra provides

the standard realization of this phenomenon, but it is used here only as a late comparison, never as an input to the formal derivation.

The conservation-of-difficulty question is therefore sharpened. The unresolved issue is no longer which alternating gluing coefficient should be chosen inside the original carrier. Task 06 proves that no choice can restore either global associativity or global Q_4 multiplicativity under the inherited sector rules. The next task must consequently isolate one assumption at a time. Task 07 should investigate only the associativity obstruction for two orthogonal rest directions, remove only the requirement that their mixed product remain inside the original paravector carrier, and classify what new product direction and multiplication relations are forced. It must stop before a third spatial generator, norm extension, or conventional algebraic identification is introduced.

Keywords: Lorentzian geometry; split-complex sectors; 3+1 dimensions; clean-room reconstruction; gluing; associativity; quadratic norm; spin factors; paravectors; formal verification; Lean 4; conservation of difficulty; dependency analysis.

Contents

1 A note on numbering: why there is no Part V

Part IV ended with a methodological problem and a proposed repair. The next formal task was designed as a clean-room repetition of the orientation-sensitive gluing classification, followed by a narrow test of whether the associativity and norm obstructions seen in Task 04 were artifacts of the symmetry-restricted family or genuine obstructions for every sector-compatible product.

During the transition from planning to execution, that task was renumbered as TASK 06. No distinct formal Task 05 result intervened, and no separate Part V paper was produced. The present paper therefore follows the identifier of the executed formal development and is titled Part VI. The jump in numbering is administrative rather than mathematical:

$$\text{Part IV / Task 04} \longrightarrow \text{planned clean-room repetition} \equiv \text{executed Task 06 / Part VI.} \tag{1}$$

Nothing in the dependency chain is skipped. On the contrary, the purpose of TASK 06 is precisely to repeat one formerly weakened part of Task 04 under stricter conditions before allowing the programme to move forward.

2 Where Part IV left the problem

Parts I–III progressively exhausted the structure of a single longitudinal $1+1$ Lorentz sector. Part IV then asked the first genuinely higher-dimensional question: if every spatial direction through one normalized future unit carries the already reconstructed longitudinal product, do all of those restrictions determine one bilinear product on the full real $3+1$ carrier? [7, 8, 9, 10]

The answer was exact. Let

$$\mathsf{Vec4} = \mathbb{R} \oplus \mathbb{R}^3, \qquad Q_4(t,v) = t^2 - \|v\|^2, \qquad e_0 = (1,0). \tag{2}$$

Every sector-compatible bilinear product M decomposes uniquely as

$$M(X,Y) = \mu_{\mathrm{sym}}(X,Y) + A(v,w), \tag{3}$$

where $X = (a,v)$, $Y = (b,w)$ and A is alternating in the rest-space variables. The forced symmetric branch is

$$\mu_{\mathrm{sym}}(X,Y) = L_4(X,e_0)Y + L_4(Y,e_0)X - L_4(X,Y)e_0, \tag{4}$$

or in split coordinates,

$$\mu_{\mathrm{sym}}((a,v),(b,w)) = (ab + v \cdot w,\ aw + bv). \tag{5}$$

Thus the longitudinal sectors determine the entire symmetric product but not the cross-sector alternating part.

Part IV further reported that full rest-space isotropy forces $A = 0$, whereas orientation-preserving isotropy allows a one-parameter alternating family. The symmetric branch satisfies the Jordan identity but is not globally associative and is not globally Q_4-multiplicative. A late conventional comparison identifies it with the familiar real spin-factor/Jordan product [4, 2].

That mathematical classification was formally correct. The methodological weakness was subtler: before the proper-isotropy rigidity theorem had independently constrained an unknown defect, the implementation already contained an explicit coordinate model with the shape of the later standard three-dimensional alternating operation. The result was proved, but the proof search was no longer fully blind to its eventual representative.

Methodological caveat 1 (The Task-04 defect)**.** *A successful kernel proof answers whether a theorem follows from the formal assumptions. A blind reconstruction experiment asks an additional question: whether the target structure was made available before the dependency analysis had forced it. Task 04 passed the first test but weakened the second.*

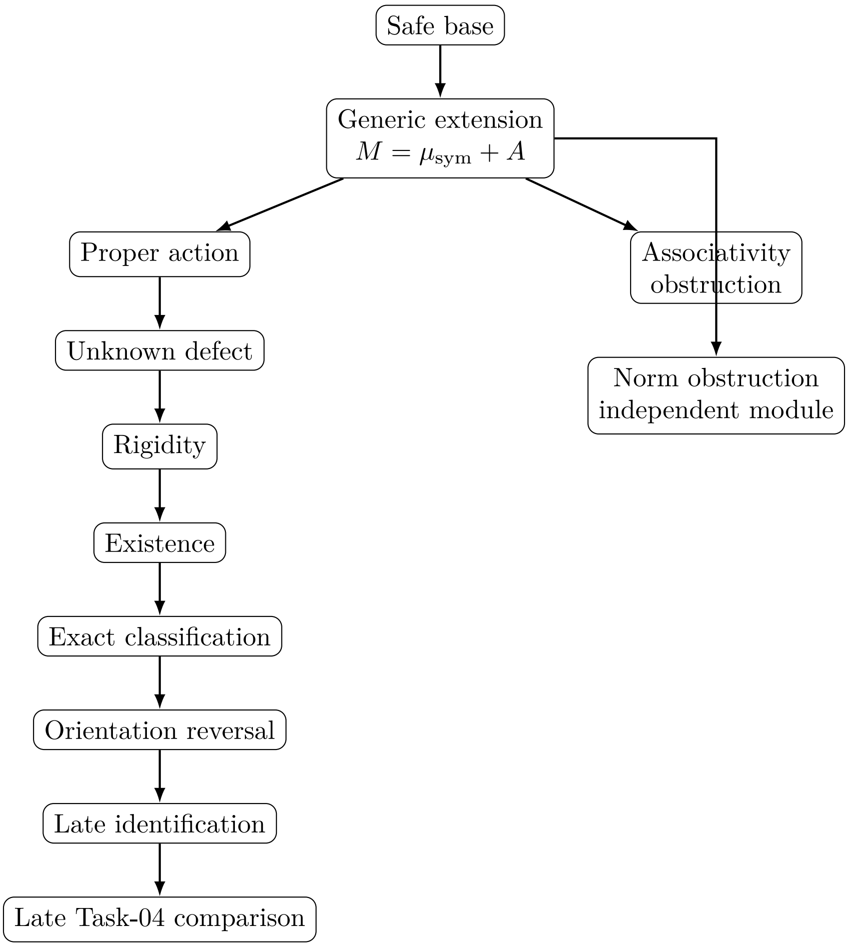
This distinction is central to the present series. The aim is not merely to verify known identities. It is to determine which information does the work, and therefore the order in which candidate structures become available matters.

3 Task 06 as a clean-room experiment

3.1 Two independent phases

Task 06 is deliberately split into two branches [1]. Phase A repairs the contaminated proper-isotropy result. Phase B starts again from the generic extension theorem and asks universal existence questions without using Phase A.

The source graph is schematically



The actual Phase-B associativity and norm modules are siblings: neither imports the other, and neither imports Rigidity, Existence, ExactClassification, Identification, or the Task-04 comparison. This independence is what permits the later no-go statements to be read as theorems about the entire sector-compatible class rather than about one symmetry-selected branch.

3.2 The contamination firewall

The Task 06 clean layer refuses every Task 04 module containing the earlier explicit alternating witness. The generic extension theorem is therefore re-derived rather than imported from the contaminated classification module. The stabilizer machinery is likewise rebuilt without any alternating bilinear map.

Before the rigidity theorem, the development contains only:

- an abstract type of rest-space defects;
- predicates expressing alternation and equivariance;
- extraction and reconstruction operations for an abstract defect;

- unary rest-space transformations and a scalar determinant criterion;
- the zero defect as a negative control.

There is no explicit nonzero alternating map, no basis multiplication table and no conventional three-dimensional product imported under another name.

The first explicit nonzero witness appears only after the rigidity file has compiled. This source-order condition is not cosmetic: it is the central experimental repair.

4 Phase A: rigidity before existence

Let

$$A : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}^{1,3} \tag{6}$$

be an arbitrary real bilinear alternating map. Proper equivariance means that for every Lorentz transformation R fixing e_0, preserving Q_4, and preserving the spatial orientation of the rest space,

$$R(A(v, w)) = A(Rv, Rw). \tag{7}$$

No formula for A is supplied.

Choose an orthonormal rest basis r_1, r_2, r_3. Task 06 then uses explicit determinant-+1 half-turns and quarter-turns only as probes of the unknown map. The first half-turn calculation forces

$$A(r_2, r_3) = c\, r_1 \tag{8}$$

for some $c \in \mathbb{R}$ and simultaneously removes the temporal component and both transverse spatial components of this value. Quarter-turns transport the same coefficient to the other cyclic pairs:

$$A(r_3, r_1) = c\, r_2, \tag{9}$$
$$A(r_1, r_2) = c\, r_3. \tag{10}$$

Alternation fixes the transposed values with the opposite signs.

Bilinearity then extends the basis-pair result to arbitrary rest vectors. In particular,

$$(A(v, w))^0 = 0 \tag{11}$$

for all v, w. Thus proper equivariance does not merely constrain the defect; it proves that its image lies in the rest space.

Theorem 1 (Clean-room upper rigidity)**.** *Before any nonzero alternating witness is defined, an alternating proper-equivariant defect is uniquely determined by the single scalar*

$$c = (A(r_2, r_3))_{r_1}.$$

Two such defects with the same c *are equal, and* $c = 0$ *forces the zero defect.*

This is the key methodological theorem of Phase A. It gives an upper bound on the solution space before existence is known. In particular, the development does not begin from the knowledge that the solution space is one-dimensional; it derives injectivity into $\mathbb{R}$ from the symmetry constraints.

5 Existence after rigidity and exact classification

Only after the preceding rigidity theorem is complete does Task 06 ask whether c can be nonzero. A concrete witness is constructed for $c = 1$, and its alternation, proper equivariance and nontriviality are proved from scratch. Combining this witness with the previous uniqueness theorem gives the exact classification

$$\{A : \; A \text{ alternating and proper-equivariant}\} = \mathbb{R}\, A_0. \tag{12}$$

Equivalently, the solution space is linearly equivalent to $\mathbb{R}$.

Only now is the coordinate formula derived:

$$A(v, w) = c \begin{pmatrix} & v_2w_3 - v_3w_2 \\ 0, & v_3w_1 - v_1w_3 \\ & v_1w_2 - v_2w_1 \end{pmatrix}. \tag{13}$$

The logical order is therefore

$$\text{unknown } A \to \text{symmetry constraints} \to \dim \le 1 \to \text{existence} \to \dim = 1 \to \text{coordinate formula}. \tag{14}$$

This is exactly the order that Part IV had intended but had not fully enforced.

5.1 Orientation reversal

An explicit orientation-reversing rest-space reflection acts on the classified family by

$$c \longmapsto -c. \tag{15}$$

Consequently, the only defect invariant under the full rest-space isotropy group is $c = 0$. At product level,

$$\mathsf{SectorCompatible}(M) + \text{ full isotropy} \quad \Longrightarrow \quad M = \mu_{\text{sym}}. \tag{16}$$

The distinction between proper and full isotropy is therefore exact:

symmetry condition	surviving defect freedom
proper spatial isotropy	one real parameter c
full spatial isotropy	zero defect only

6 The Task-04 replication verdict

Only after the clean classification and the orientation-reversal theorem are complete does Task 06 import the formerly contaminated Task 04 proper-isotropy module. The two developments are then compared theorem by theorem. Their determinant criteria agree, their stabilizer predicates agree, their symmetric branches agree, their independently constructed nonzero defects agree, their one-parameter families agree, and orientation reversal acts in the same way.

The formal verdict is therefore

$$\boxed{\text{TASK04 PROPER-ISOTROPY RESULT REPLICATED.}} \tag{17}$$

This does not erase the Task-04 methodological defect. It resolves its epistemic status. The earlier theorem was correct, and the result can now be obtained without exposing the prover to the later target shape before rigidity.

Methodological principle 1 (A repaired reconstruction is stronger than a defended one)**.** When a blind-test defect is discovered, the appropriate response is not to argue that the target hint was harmless. The stronger response is to remove the hint and reproduce the result. Task 06 does this.

The replication matters because the next phase relies on distinguishing a genuine mathematical obstruction from an artifact of the chosen symmetric or isotropic branch. That distinction cannot be made confidently if the preceding classification itself remains methodologically ambiguous.

7 Phase B: remove isotropy again

After the clean-room result has been secured, Task 06 deliberately discards it for the next question. Let

$$M : \mathsf{Vec4} \times \mathsf{Vec4} \to \mathsf{Vec4} \tag{18}$$

be any real bilinear product satisfying only $\mathsf{SectorCompatible}$. No commutativity, isotropy, orientation condition, chosen defect, or one-parameter formula is assumed.

The longitudinal sector rules alone imply three identities that will be sufficient. For any rest vector v,

$$M(v, v) = \|v\|^2 e_0. \tag{19}$$

For orthogonal unit rest vectors a, b,

$$a^2 = e_0, \qquad b^2 = e_0, \qquad ab + ba = 0, \tag{20}$$

where multiplication denotes M. The global unit e_0 is itself derived from sector compatibility.

The crucial point is that ab remains completely unknown. It is precisely one of the cross-sector values not visible inside either longitudinal plane. Phase B asks whether some clever choice of that unknown value could restore global properties that had failed in the isotropic family.

The answer is negative twice, for two logically independent reasons.

8 Universal associativity obstruction

Assume for contradiction that M is globally associative. Put

$$p = ab. \tag{21}$$

From equation (20), $ba = -ab = -p$. Associativity and the unit relations give

$$pb = (ab)b = a(b^2) = a, \tag{22}$$
$$bp = b(ab) = (ba)b = (-ab)b = -a. \tag{23}$$

Therefore

$$p^2 = (ab)(ab) = a\big(b(ab)\big) = a(-a) = -e_0. \tag{24}$$

This part is purely algebraic. It uses no coordinates, no positivity, no isotropy and no classification theorem.

Now write the unknown in-carrier value as

$$p = (\tau, u) \in \mathbb{R} \oplus \mathbb{R}^3. \tag{25}$$

Sector compatibility gives a general formula for the temporal component of every square:

$$(M(Z, Z))^0 = (Z^0)^2 + \|Z_{\mathrm{sp}}\|^2 \geq 0. \tag{26}$$

Applying this to $Z = p$ contradicts equation (24), whose time component is -1:

$$\tau^2 + \|u\|^2 = -1. \tag{27}$$

Hence:

Theorem 2 (Universal associativity no-go)**.** *For every real bilinear product M on the fixed $3+1$ carrier,*

$$\mathsf{SectorCompatible}(M) \quad \Longrightarrow \quad \neg\mathsf{Associative4}(M).$$

No isotropy, commutativity or orientation hypothesis is required.

The obstruction uses only two orthonormal rest directions. Task 06 does not claim formal minimality in the sense that no smaller set of assumptions could ever yield a contradiction; it proves the stronger practical fact needed here: two directions already suffice.

8.1 Why the result is stronger than Task 04

Task 04 had established nonassociativity for the symmetric branch and for the proper-isotropic one-parameter family. That still left a logical possibility: perhaps some highly anisotropic alternating defect, excluded by the symmetry assumptions, could restore associativity.

The theorem above closes that loophole. There is no hidden parameter and no exotic in-carrier gluing rule to discover. The entire generic sector-compatible class is excluded.

9 Universal Lorentz-norm obstruction

The second no-go theorem is proved independently, in a module that does not import the associativity proof. Assume only global multiplicativity of the Lorentz quadratic form:

$$Q_4(M(X,Y)) = Q_4(X)Q_4(Y). \tag{28}$$

Again let a, b be orthonormal rest directions and put $p = ab = (\tau, u)$.

Because

$$Q_4(a) = Q_4(b) = -1, \tag{29}$$

equation (28) gives

$$Q_4(p) = 1, \qquad \text{hence} \qquad \tau^2 - \|u\|^2 = 1. \tag{30}$$

Bilinearity and $a^2 = e_0$ imply

$$a(a+b) = e_0 + p. \tag{31}$$

Since $Q_4(a+b) = -2$, multiplicativity also gives

$$Q_4(e_0 + p) = 2, \qquad \text{hence} \qquad (1+\tau)^2 - \|u\|^2 = 2. \tag{32}$$

Subtracting equation (30) from equation (32) yields

$$\tau = 0. \tag{33}$$

Equation equation (30) then becomes

$$-\|u\|^2 = 1, \tag{34}$$

contradicting positivity of the derived rest-space metric.

Theorem 3 (Universal Q_4-multiplicativity no-go)**.** *For every real bilinear product M on the fixed $3+1$ carrier,*

$$\mathsf{SectorCompatible}(M) \implies \neg\mathsf{Q4Multiplicative}(M).$$

The proof uses neither associativity nor isotropy.

The two no-go results are therefore genuinely separate. Neither is a corollary of the other, and neither depends on the orientation-sensitive classification of Phase A.

10 What has actually been localized

The significance of Task 06 is not merely that two desirable global properties fail. Part IV already contained failure results. Task 06 localizes where the failure cannot be repaired.

Before Phase B, the remaining alternating defect A was a plausible location for the unresolved difficulty. One might have imagined that the symmetric product failed only because the wrong cross-sector coupling had been selected, or that the proper-isotropic one-parameter family was too restrictive. The universal theorems rule out that interpretation.

The dependency picture is now:

$$\begin{gathered} 3+1 \text{ Lorentz form } Q_4 \text{ and chosen } e_0 \\ \Downarrow \\ \text{all longitudinal } 1+1 \text{ sector products} \\ \Downarrow \\ \mu_{\text{sym}} + \text{arbitrary alternating defect} \\ \Downarrow \\ \boxed{\text{no globally associative in-carrier product}} \\ \boxed{\text{no globally } Q_4\text{-multiplicative in-carrier product}}. \end{gathered} \tag{35}$$

The boxes do not depend on how the arbitrary defect is chosen.

Methodological principle 2 (Conservation of difficulty after Task 06)**.** Once every admissible choice of the remaining free datum has been excluded, the unresolved difficulty is no longer a parameter-selection problem. At least one structural assumption defining the search space must be opened.

This is a sharper form of the diagnostic principle used throughout the experiment. Earlier tasks repeatedly moved apparent primitive structure into weaker data. Here the process reaches a different kind of boundary: the present carrier-plus-closure problem has no solution satisfying either of the tested global requirements.

It is important not to overread the theorem. Task 06 does not prove that Nature requires an enlarged algebra, that associativity is physically fundamental, or that Q_4 must be a composition norm. It proves only a conditional statement: if one insists on the inherited longitudinal rules, real bilinearity, closure inside the fixed paravector carrier, and either global property, the requirements are incompatible.

11 A short mirror against established mathematics

The point of the present experiment is dependency localization, not mathematical novelty. The Task 06 boundary is reassuring precisely because it mirrors standard algebra once the formal result has been obtained independently.

11.1 The symmetric closed product: the spin-factor side

The product forced by full isotropy,

$$(a, v) \circ (b, w) = (ab + v \cdot w,\ aw + bv), \tag{36}$$

is the standard spin-factor Jordan product after conventional identification [4, 2]. It closes on $\mathbb{R} \oplus \mathbb{R}^3$, is commutative, and satisfies the Jordan identity, but it is not associative. That is not a defect of the formal reconstruction; it is exactly the established algebraic character of this closed symmetric product.

11.2 The mixed-direction product: the Clifford side

Standard Clifford algebra starts from a quadratic space and imposes relations in which products of independent vector generators are retained as new algebra elements rather than projected back into the vector space. For orthogonal generators with positive square convention,

$$a^2 = b^2 = 1, \qquad ab = -ba, \tag{37}$$

associativity immediately gives

$$(ab)^2 = -1. \tag{38}$$

This is exactly the algebraic equation independently isolated in section 8. In a Clifford algebra the element ab is not required to be another vector; it occupies a different product grade. With three independent positive-square spatial generators, the full real Clifford algebra has dimension $2^3 = 8$, and the paravector subspace $\mathbb{R} \oplus \mathbb{R}^3$ is not closed under the associative Clifford product [5].

This gives a precise retrospective reading of Task 06. The formal no-go is not mysteriously fighting known algebra. It is detecting the cost of demanding that a mixed-direction product which standard associative algebra keeps outside the paravector space be forced back into that four-dimensional carrier while preserving all inherited longitudinal square relations.

11.3 Composition norms

The norm no-go is likewise consistent with the classical rigidity of composition algebras. Multiplicative nondegenerate quadratic forms place strong restrictions on admissible real bilinear algebras [3, 11]. Task 06 does not import or invoke that classification; it proves a much narrower statement directly for the Lorentzian paravector carrier and its inherited sector rules. The established theory is used only afterwards as a check that the obstruction lies in familiar territory rather than as evidence of novelty.

Remark 1 (Why this comparison matters)**.** *A calibration programme should become suspicious if a highly studied object suddenly produces exotic new algebra from elementary assumptions. Here the opposite happens. Once the blind derivation reaches its obstruction, standard Jordan, Clifford and composition-algebra theory provide an exact conceptual mirror. That agreement strengthens confidence in the dependency diagnosis while adding no claim of new mathematics.*

12 Relation to Experiment 1

The comparison with Experiment 1 becomes sharper at this point. Experiment 1 approached Lorentzian structure from weaker real/euclidean data and independently encountered a spin-factor/Jordan layer followed by a Clifford envelope. Experiment 2 began with Lorentz data, reconstructed each split-complex longitudinal sector, glued all sectors, and independently recovered the same symmetric spin-factor product before proving that no sector-compatible associative product can remain closed in the original $\mathbb{R} \oplus \mathbb{R}^3$ carrier [6].

The experiments are not proofs of one another. Their value lies in the opposite direction of travel:

$$\begin{array}{ccc} \textsc{Experiment 1} & & \textsc{Experiment 2} \\ \text{Euclidean } \mathbb{R}^3 & & \text{Lorentz } 3+1 \\ \downarrow & & \downarrow \\ \text{spin factor / Jordan} & \longleftrightarrow & \text{spin factor / Jordan} \\ \downarrow & & \downarrow \\ \text{Clifford envelope} & & \text{in-carrier associative obstruction.} \end{array} \tag{39}$$

The left-hand lower node is not an input to the right-hand path. Consequently, the new Task 06 result should not be summarized as "Experiment 2 has derived Clifford algebra." It has not. It has derived an obstruction whose established mathematical resolution is known to involve retaining products outside the paravector carrier. That difference is precisely where the next blind task must begin.

13 The strengthened conservation-of-difficulty statement

The phrase *conservation of difficulty* is used throughout this project as a research heuristic, not as a theorem of mathematics. Its operational meaning is that an explanatory difficulty should not be declared solved merely because it has been hidden inside a stronger primitive, an unexamined representation choice, or a convenient target candidate.

Task 06 produces three distinct examples of the principle.

First: the Task-04 methodological defect. The proper-isotropy result could have been defended as correct despite the prematurely available target shape. Instead the experiment removed the target and repeated the derivation. The difficulty was not allowed to disappear into proof guidance.

Second: the gluing parameter. After Part IV, the coefficient of the orientation-sensitive alternating defect looked like a possible remaining location of the global problem. Phase B shows that changing this parameter, or even abandoning isotropy and allowing an arbitrary alternating defect, cannot restore the tested global properties. The difficulty cannot be solved by tuning the cross-sector wiring inside the same carrier.

Third: closure itself becomes visible. The associativity proof isolates a mixed product $p = ab$ whose algebraically required square is $-e_0$. The contradiction arises only when p is required to be another element of the same paravector carrier, where sector compatibility controls the time component of its square. This does not yet prove that closure is the only assumption worth relaxing, but it identifies closure as the first narrowly testable suspect.

The conservation-of-difficulty chain has therefore changed character:

$$\begin{aligned} \text{explicit split-complex product} &\longrightarrow \text{derived from a chosen unit}, \\ \text{choice of unit} &\longrightarrow \text{one Lorentz orbit}, \\ \text{cross-sector gluing} &\longrightarrow \text{classified alternating freedom}, \\ \text{choice of gluing freedom} &\longrightarrow \text{cannot repair associativity or norm}, \\ \text{remaining diagnostic suspect} &\longrightarrow \boxed{\text{in-carrier closure of mixed products}}. \end{aligned} \tag{40}$$

The final arrow is intentionally labelled a *diagnostic suspect*, not a derived repair.

14 What Task 07 must test

Task 07 should be narrower than Task 06. The temptation at this point is to jump directly to the standard Clifford construction, enlarge the carrier to the familiar dimension, add all three spatial generators, and verify that the known algebra works. That would defeat the purpose of the dependency audit. It would replace a localized obstruction by a known solution package.

The next task should instead open exactly one assumption and follow exactly one obstruction.

14.1 Single target question

The Task 07 core question should be:

> For two orthogonal unit rest directions satisfying the inherited longitudinal relations, what does global associativity force if their mixed product is no longer required to lie in the original paravector carrier?

No norm condition should be imposed in the core. No third spatial direction should be introduced. No conventional associative algebra should be named.

14.2 Allowed starting relations

Task 07 may inherit only the two-direction algebraic consequences already proved in Task 06:

$$e_0 \text{ is a two-sided unit}, \qquad a^2 = b^2 = e_0, \qquad ab + ba = 0, \tag{41}$$

for two abstract orthogonal rest directions a, b.

The mixed product

$$p := ab \tag{42}$$

may be introduced only as a derived abbreviation after multiplication is available. It must not be declared in advance to be a “bivector”, “imaginary unit”, matrix, quaternionic component, or Clifford generator.

14.3 The one assumption to remove

Task 06 required

$$M : \mathsf{Vec4} \times \mathsf{Vec4} \to \mathsf{Vec4}. \tag{43}$$

Task 07 should remove only the codomain closure of the mixed two-direction product. The original paravector sector must embed linearly and unitaly into an otherwise unspecified real associative algebraic carrier. The carrier should not be assigned a familiar dimension in advance.

The task must then determine, rather than assume:

1. whether $p = ab$ can belong to the embedded span of $\{e_0, a, b\}$;
2. whether associativity forces
$$p^2 = -e_0; \tag{44}$$
3. which products among e_0, a, b, p are forced;
4. whether p is linearly independent of e_0, a, b under the inherited relations;
5. whether the span generated by $\{e_0, a, b, p\}$ is closed under multiplication;
6. whether the resulting two-generator completion is unique up to a transparent unit-preserving algebra isomorphism under the stated relations.

The multiplication table should be derived from associativity, not supplied. For example, equations such as

$$ap = b, \qquad pa = -b, \qquad bp = -a, \qquad pb = a \tag{45}$$

should appear only if and when they follow from equation (41).

14.4 What Task 07 must not do

The hard stop should occur as soon as the two-generator associative completion is classified. Task 07 must not:

- add the third orthogonal rest direction;
- construct the full completion of the $3 + 1$ paravector space;
- extend or repair the Lorentz quadratic norm;
- impose global norm multiplicativity;
- introduce exterior powers or graded language as input;
- name Clifford, geometric, quaternionic, Pauli, matrix, or spinorial structures before a late comparison layer;
- infer physical dynamics from the algebraic completion.

A conventional identification is allowed only after the independently reconstructed two-generator multiplication table has been proved. Even then it is comparison only.

14.5 Why this is the right width

Task 07 should answer one diagnostic question: does removing in-carrier closure for the smallest two-direction obstruction force a new independent product direction, and is that one direction sufficient to restore associative closure for the subalgebra generated by those two directions?

A positive answer would justify a later task asking what changes when a third orthogonal spatial direction is added. A negative answer would instead show that the present diagnosis is incomplete. Either result is useful. What Task 07 must not do is decide the answer by importing the known global solution.

15 Questions deliberately deferred beyond Task 07

The following questions are intentionally left for later work:

1. Does adding a third orthogonal rest generator force further independent product directions beyond the two-generator completion?
2. What is the smallest associative carrier containing the entire $\mathbb{R} \oplus \mathbb{R}^3$ paravector space with all inherited sector relations?
3. Can a quadratic form extending Q_4 be defined on such a carrier, and what multiplicativity or involution properties can it satisfy?
4. Is there a canonical relation between the orientation-sensitive one-parameter defect of Task 06 and any new product directions appearing in an associative completion?
5. How does the independently reconstructed larger structure compare with the Clifford envelope encountered in Experiment 1?

These questions are close enough to the established mathematical answer that they must be kept out of Task 07 itself. Their purpose here is only to state where the dependency programme could proceed after the next narrow test.

16 Formal status and audit summary

The audited Task 06 archive has SHA-256 `4a70d9dc5ade31c9f74765c1499d48581955b3a90cfe 7e19a885d54c4fd55b52`. The artifact uses Lean 4.28.0 and mathlib revision `8f9d9cff6bd728 b17a24e163c9402775d9e6a365`. The reported full build completed successfully with 8068 jobs. Fourteen new Task 06 modules contain 2082 lines of Lean source. Phase B is import-independent of the clean-room classification branch after the generic extension layer.

For nineteen principal Task 06 theorems, the reported `#print axioms` result is

$$[\texttt{propext},\ \texttt{Classical.choice},\ \texttt{Quot.sound}], \tag{46}$$

with no custom axiom, `sorry`, or `admit` introduced in the project sources.

The principal results may be summarized as follows:

Question	Formal result
Can the contaminated generic Task-04 classification be reused safely?	No; the needed generic theorem is re-derived.
Does a target-shaped nonzero defect appear before rigidity?	No.
Proper-equivariant defect space	Exactly one-dimensional over $\mathbb{R}$.
Temporal defect component	Forced to vanish.
Orientation reversal	$c \mapsto -c$.
Full-isotropy defect space	Zero only.
Task-04 proper-isotropy result	Replicated.
Any sector-compatible associative in-carrier product?	No; universal obstruction.
Any sector-compatible globally Q_4-multiplicative in-carrier product?	No; universal obstruction.
Isotropy needed for either no-go?	No.
Sufficient obstruction configuration	Two orthonormal rest directions.

No unresolved obligation affects these theorem statements. Open issues concern only subsequent generalization: the surviving proper-isotropy scale remains a free datum, covariance under changing the chosen timelike unit was not revisited here, and no claim of formal minimality is made for the two-direction witness.

17 Conclusion

Part VI performs two different jobs, and their conjunction is what sharpens the programme.

First, it repairs the experimental procedure. The orientation-sensitive gluing structure reported in Part IV is reconstructed again with no nonzero candidate available before the rigidity theorem. The same one-dimensional solution space reappears, orientation reversal negates its parameter, and full isotropy removes it. The Task 04 result is therefore not merely defended but independently replicated under a stricter blind firewall.

Second, Part VI shows that this gluing freedom was never the solution to the deeper global problem. Once isotropy is discarded and every sector-compatible bilinear extension is allowed, global associativity is still impossible and global Q_4 multiplicativity is independently impossible. Two orthogonal rest directions suffice to expose both contradictions.

The mathematical tension is resolved, rather than amplified, by comparison with established algebra. The closed symmetric product is the known spin-factor/Jordan product. An associative product of independent vector directions is known not to remain inside the paravector subspace; mixed products are retained in a larger algebraic envelope. Task 06 rediscovers the boundary from the opposite direction without being given that envelope.

This is the strongest conservation-of-difficulty localization reached so far in Experiment 2. The difficulty is no longer hidden in the split-complex product, the unit choice, the null-sector labels, or the alternating gluing coefficient. Under the current assumptions, no choice inside the original carrier can do the required work. The next experiment must therefore open one structural wall instead of tuning another parameter.

Task 07 should open only one wall: mixed-product closure for two orthogonal directions under associativity. It should determine whether a new independent product direction is forced, derive the resulting two-generator multiplication rules, and stop. If that narrow clean-room completion reproduces a familiar algebra only at the comparison stage, the experiment will have moved one dependency step deeper without importing the known answer.

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