Split Complex Lorentzian Dynamics

Part X: A Distinguished Axis, Transverse Channels, Half-Angle State Action, and the First Dynamics Boundary

A formally verified continuation of the dependency audit

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Abstract

Parts I–IX of Experiment 2 remained kinematic and algebraic. They reconstructed longitudinal Lorentz sectors, their $3+1$ gluing, the minimal associative eight-dimensional product closure, the central element S with $S^2 = -1$, and the exact two-branch central-valued multiplicative quadratic continuation of the inherited Lorentz form. No physical state space, particle type, time evolution, Hamiltonian or field equation had been assumed or derived.

Part X reports the first deliberately narrow probe beyond that boundary. Exactly one inherited old spatial generator A is distinguished. Relative to it, the old rest space is proved to split canonically as a longitudinal line and a two-dimensional transverse plane. Ordinary rotations about A are introduced only on the old $3+1$ carrier and then extended, rather than prescribed, to the full Task VIII/IX algebra W. The extension exists uniquely. It fixes $1, A, R, S$, rotates B, C by the full angle, rotates the mixed channels P, Q in parallel, preserves both Task IX quadratic branches separately, and is exactly 2π-periodic on the complete algebra.

The transverse sector acquires additional structure only after the already-derived central plane $Z = \mathrm{span}_{\mathbb{R}}\{1, S\}$ is used. The old transverse plane itself is irreducible under a nontrivial axial rotation, but its Z-linear extension splits into exactly two one-dimensional Z-channels with mutually inverse full-angle factors, while the longitudinal channel remains fixed. Only in the late comparison layer is this recognized as the familiar axial weight pattern $0, \pm 1$ of a vector-like representation. No physical spin-1 particle is inferred.

The more diagnostic step is the internal lift problem. Before any half-angle formula is introduced, Task X asks for a one-parameter family inside $K = \mathrm{span}_{\mathbb{R}}\{1, R\}$ whose conjugation action implements the already-derived algebra automorphisms. A distinguished solution is recovered with half-angle trigonometric dependence and gives $U_{\mathrm{std}}(2\pi) = -1$, $U_{\mathrm{std}}(4\pi) = 1$. However, the proof search exposed a crucial residual freedom: conjugation cannot see a nonzero real scalar multiplier. The exact formal classification is therefore

$$U(\theta) = k(\theta)\, U_{\mathrm{std}}(\theta), \qquad k(0) = 1, \qquad k(\theta + \varphi) = k(\theta)k(\varphi), \qquad k(\theta) > 0.$$

The lift is not unique. This freedom nearly disappeared during proof search when the familiar normalization $a^2 + b^2 = 1$ was considered as a convenient way to force $k \equiv 1$. Doing so would have converted an observed dependency into an unannounced assumption. The final formalization correctly retains the scale freedom and constructs an explicit distinct lift using $k(\theta) = e^{\theta}$. The sharp $2\pi \mapsto -1$ statement therefore belongs to the distinguished representative, or to a lift satisfying an additional explicitly stated compatibility condition with the Task IX quadratic branch, not to the bare conjugation problem itself.

When W is used only as a provisional left module over itself, two distinguished-axis sectors are derived from $R\psi = \pm S\psi$, equivalently $A\psi = \pm\psi$. They have real dimension four, or Z-dimension two, jointly span W, and transform with opposite half-angle factors. Thus the same axial rotation acts with full angle on the algebraic transverse channels and half angle on these candidate state sectors. A specific transverse state orientation is not selected by the present data. This establishes transformation structure, not a fermion.

Finally, Task X classifies all real-linear axial derivations preserving the inherited transverse Euclidean form. Their space is exactly one-dimensional:

$$D = \lambda D_{\mathrm{gen}}, \qquad D_{\mathrm{gen}}(x) = \tfrac{1}{2}(xR - Rx).$$

The normalized generator is the derivative of the algebra action at the identity. Yet every real multiple remains admissible, so the reconstructed kinematic/algebraic structure fixes a generator direction but not an evolution rate. The rotation parameter is never identified with physical time. Part X therefore reaches the intended boundary: vector-like and half-angle transformation sectors emerge, and an infinitesimal flow direction is fixed, but state normalization, state-carrier minimality, transverse state orientation and dynamical rate remain additional questions.

The proposed Task XI should harden these foundations before spin 1 and spin 1/2 are studied separately. It should close the continuous lift classification, remove the artificial restriction to K by classifying all internal implementers up to the centre, test both Task IX quadratic branches symmetrically, and separate the central freedom of a state lift from the unique adjoint generator seen by the algebra. Only after this base layer is stable should later tasks ask which sectors survive as physical spin 1 or spin 1/2 state carriers.

Keywords: longitudinal/transverse decomposition; axial rotation; associative envelope; central plane; full-angle channel; half-angle lift; state action; derivation; generator-rate freedom; formal verification; Lean 4; conservation of difficulty.

Contents

1 From algebraic closure to a state-and-evolution probe

The first nine parts of Experiment 2 were intentionally kinematic. Even when the algebra grew from the old real $3+1$ carrier to the eight-dimensional associative envelope W, and even when the Lorentz quadratic form acquired its central-valued multiplicative continuation, the objects under study remained products, quadratic forms, embeddings and covariance maps. No proposition required a physical state space or a law of motion [8, 7, 1].

Part X changes the type of question, but only minimally. It does not ask for a particle, a wave equation or a Hamiltonian. It fixes one old spatial direction and asks what the already reconstructed algebra does relative to that axis.

The inherited basis is

$$1,\ A,\ B,\ C,\ P=AB,\ Q=AC,\ R=BC,\ S=ABC, \tag{1}$$

with

$$A^2=B^2=C^2=1, \qquad AB=-BA, \quad AC=-CA, \quad BC=-CB, \tag{2}$$

and

$$R^2=S^2=-1, \qquad Sx=xS \quad (x\in W). \tag{3}$$

The complete old $\mathsf{Vec4}$ is embedded in W, the old Lorentz form Q_4 is recovered exactly there, and Task IX supplies the central plane

$$Z=\operatorname{span}_{\mathbb{R}}\{1,S\} \tag{4}$$

and the two multiplicative quadratic branches N_+, N_- [2].

Task X designates exactly one inherited normalized spatial generator, A, as the *distinguished axis*. The formal core does not call it a photon direction, a fermion momentum or a propagation law. The physical motivation is deferred: a nonzero momentum would distinguish such an axis, but the reconstruction begins only with the geometric selection itself.

Methodological principle 1 (One axis, one subgroup)**.** The task studies only rotations that fix A. General Lorentz representation theory, arbitrary momentum directions, boosts, mass shells, gauge constraints and field equations remain outside the experiment.

This restriction turns out to be sufficient to expose three logically different layers:

1. the full-angle action on algebraic vector channels;
2. a half-angle internal lift and candidate state action;
3. an infinitesimal derivation direction whose overall rate remains free.

Their separation is the main result of Part X.

2 The distinguished-axis decomposition

Let the old rest space be the spatial hyperplane orthogonal to e_0. Task X defines

$$\text{Long} := \operatorname{span}_{\mathbb{R}}\{A\}, \qquad \text{Trans} := \operatorname{span}_{\mathbb{R}}\{B,C\}. \tag{5}$$

No second transverse axis is declared preferred.

The formal development proves

$$\text{Rest} = \text{Long} \oplus \text{Trans}, \tag{6}$$

with trivial intersection and unique decomposition. The inherited positive Euclidean rest-space form makes A orthogonal to every vector in Trans [2].

Theorem 1 (Momentum-adapted geometric split)**.** *Once one normalized old spatial direction is distinguished, the old* 3*-space decomposes canonically into one longitudinal line and one two-dimensional transverse plane relative to that choice.*

The word *canonically* is relative to the chosen axis. Nothing in the data distinguishes B from C inside their plane. This distinction later becomes important: the transverse *plane* is selected, while a transverse *state orientation* is not.

3 Old axial rotations before any internal lift

The next input is deliberately conventional and confined to the already known old geometry. For $\theta \in \mathbb{R}$, define the ordinary rotation about A by

$$\begin{aligned} \mathrm{rot}_\theta(e_0) &= e_0, \\ \mathrm{rot}_\theta(A) &= A, \\ \mathrm{rot}_\theta(B) &= \cos\theta\, B + \sin\theta\, C, \\ \mathrm{rot}_\theta(C) &= -\sin\theta\, B + \cos\theta\, C. \end{aligned} \tag{7}$$

Task X proves directly

$$\mathrm{rot}_0 = \mathrm{id}, \qquad \mathrm{rot}_{\theta+\varphi} = \mathrm{rot}_\theta \circ \mathrm{rot}_\varphi, \tag{8}$$

and

$$Q_4(\mathrm{rot}_\theta X) = Q_4(X). \tag{9}$$

The Euclidean rest-space product is preserved as well. No element of W has yet been chosen to implement this rotation.

This source order matters. A standard Clifford calculation would normally start with an exponential or rotor-like element and derive the vector rotation by conjugation. Such a route would hide the dependency question asked here. Task X instead starts from the old $3+1$ symmetry and asks whether the associative closure is forced to follow it.

4 Unique extension to the full algebra

An unknown real-linear map $\Phi : W \to W$ is called an axial extension at angle θ if it is unital, multiplicative, and agrees with rot_θ on every embedded old vector. Before a candidate is written down, multiplicative generation by $1, A, B, C$ forces the complete action on the derived products.

The result is

x	$\Phi_\theta(x)$
1	1
A	A
B	$\cos\theta\, B + \sin\theta\, C$
C	$-\sin\theta\, B + \cos\theta\, C$
P	$\cos\theta\, P + \sin\theta\, Q$
Q	$-\sin\theta\, P + \cos\theta\, Q$
R	R
S	S

(10)

[2].

Only after these values are forced is the explicit Φ_θ defined and verified globally multiplicative. The formal theorem is an existence-and-uniqueness statement:

Theorem 2 (Unique axial algebra extension)**.** *For every real angle θ, there exists exactly one real-linear unital algebra endomorphism of W whose restriction to the embedded old carrier is the axial rotation* rot_θ*.*

The family satisfies

$$\Phi_0 = \mathrm{id}_W, \qquad \Phi_{\theta+\varphi} = \Phi_\theta \circ \Phi_\varphi, \qquad \Phi_{2\pi} = \mathrm{id}_W. \tag{11}$$

Both Task IX quadratic branches are preserved separately:

$$N_+(\Phi_\theta x) = N_+(x), \qquad N_-(\Phi_\theta x) = N_-(x). \tag{12}$$

Thus the axial rotation neither chooses nor exchanges the residual Task IX sign branch.

Remark 1 (The transverse relations propagate into the mixed grades)**.** *The pair (P, Q) rotates in parallel with (B, C) because $P = AB$ and $Q = AC$ while A remains fixed. By contrast, the oriented transverse product $R = BC$ and the central triple product $S = ABC$ are invariant.*

5 Longitudinal and transverse transformation channels

At the purely real level, Φ_θ fixes the longitudinal line pointwise and preserves the transverse plane. For a nontrivial angle with $\sin\theta \neq 0$, the fixed subspace of the old rest carrier is exactly the longitudinal line. The transverse plane contains no further nonzero proper real invariant subspace under that rotation [2].

The situation changes when the already-derived central plane Z is allowed to act. Task X considers the Z-linear transverse span and proves that exactly two one-dimensional Z-channels are invariant. With the neutral representatives

$$\chi_+ = B - P, \qquad \chi_- = B + P, \tag{13}$$

one obtains

$$\Phi_\theta(\psi_+) = (\cos\theta + \sin\theta\, S)\,\psi_+, \qquad \psi_+ \in Z\chi_+, \tag{14}$$

$$\Phi_\theta(\psi_-) = (\cos\theta - \sin\theta\, S)\,\psi_-, \qquad \psi_- \in Z\chi_-. \tag{15}$$

The two factors are mutually inverse. The longitudinal channel carries the constant factor 1.

Theorem 3 (Exact axial channel classification)**.** *Within the Z-linear transverse span, the only nonzero one-dimensional Z-channels on which every Φ_θ acts by multiplication with a single element of Z are the two lines $Z\chi_+$ and $Z\chi_-$. Together with the fixed longitudinal line, they form the complete distinguished-axis channel pattern found in Task X.*

Only after this theorem does the comparison layer use the familiar weights

$$0, \quad +1, \quad -1. \tag{16}$$

This is the standard axial pattern of a vector-like spin-1 representation [12, 4]. The reconstruction core contains no such label.

Methodological caveat 1 (Transformation channel is not physical polarization)**.** *The fixed longitudinal vector channel is a property of the algebraic carrier. It is not a derivation of a physical longitudinal photon mode. The removal of longitudinal modes for a massless spin-1 field requires additional physical constraints not present in Task X [11].*

6 The internal lift problem

The next question is strictly downstream of the unique algebra action. Define

$$K := \mathrm{span}_{\mathbb{R}}\{1, R\} \subset W, \qquad R^2 = -1. \tag{17}$$

Task X asks for an unknown family $U : \mathbb{R} \to W$ satisfying

1. $U(\theta) \in K$;
2. $U(0) = 1$;
3. $U(\theta + \varphi) = U(\theta)U(\varphi)$;
4. conjugation reproduces the already-derived algebra action,

$$U(\theta)xU(-\theta) = \Phi_\theta(x). \tag{18}$$

No half-angle formula is part of this interface.

For a general normalized element $a + bR$ with $a^2 + b^2 = 1$, direct multiplication shows that conjugation rotates the B–C plane with coefficients

$$a^2 - b^2, \qquad -2ab. \tag{19}$$

The doubled angular dependence is therefore derived before the half-angle representative is named. Solving the conjugation equations together with the group law yields the distinguished family

$$U_{\mathrm{std}}(\theta) = \cos\left(\frac{\theta}{2}\right) 1 - \sin\left(\frac{\theta}{2}\right) R. \tag{20}$$

It satisfies

$$U_{\mathrm{std}}(2\pi) = -1, \qquad U_{\mathrm{std}}(4\pi) = 1. \tag{21}$$

At this point a conventional exposition could stop and announce the familiar double-valued lift. Task X does not.

7 The difficulty that almost disappeared: scalar lift freedom

During proof search, the uniqueness argument failed for a structural reason. If U implements Φ_θ by conjugation, then multiplying $U(\theta)$ by a nonzero real scalar does not change that conjugation action, because the factor cancels against the inverse. Aristotle's proof-search transcript explicitly records the discovery that a factor $e^{\alpha\theta}$ survives the conjugation equations, followed by the temptation to impose $a^2 + b^2 = 1$ to eliminate it [3].

That normalization would have produced the expected U_{std} immediately. It would also have erased the most important new dependency found by Task X.

The final formal development instead proves the exact classification

$$U(\theta) = k(\theta)\, U_{\mathrm{std}}(\theta), \tag{22}$$

where

$$k(0) = 1, \qquad k(\theta + \varphi) = k(\theta)k(\varphi), \qquad k(\theta) \neq 0. \tag{23}$$

The group law itself implies

$$k(\theta) = k(\theta/2)^2 > 0. \tag{24}$$

Conversely, every such scalar factor gives a valid lift inside K.

Theorem 4 (Internal lift classification)**.** *The conjugation problem fixes the half-angle direction but does not fix the real scalar amplitude of the lift. The complete family of K-valued lifts is $k\, U_{\mathrm{std}}$ with k a positive normalized multiplicative scalar function.*

Non-uniqueness is not merely logical. Task X constructs the explicit second lift

$$U_{\exp}(\theta) = e^{\theta} U_{\mathrm{std}}(\theta). \tag{25}$$

Thus the bare kinematic action on the algebra cannot distinguish $U_{\exp}$ from U_{std} by conjugation.

Methodological principle 2 (Do not normalize away a discovered dependency)**.** A familiar normalization is admissible only if it is already inherited, independently derived, or stated as an additional hypothesis whose logical cost remains visible. It must not be inserted merely because it restores the expected representative.

This near miss is methodologically important. Had $a^2+b^2=1$ been inserted into the definition of a lift, the formal result would have looked stronger but the dependency audit would have become weaker. The experiment would have reconstructed the known half-angle formula while hiding the fact that conjugation alone does not select its normalization.

8 What the 2π sign actually means

The algebra action has exact 2π periodicity:

$$\Phi_{2\pi} = \mathrm{id}_W. \tag{26}$$

For the distinguished lift,

$$U_{\mathrm{std}}(2\pi) = -1, \qquad U_{\mathrm{std}}(4\pi) = 1. \tag{27}$$

For a general lift $U = kU_{\mathrm{std}}$, however,

$$U(2\pi) = -c\,1, \qquad U(4\pi) = c^2\,1, \qquad c = k(2\pi) > 0. \tag{28}$$

Therefore the sharp sign change after one full turn is not a theorem of the unnormalized lift problem.

Task X also proves a separate conditional result. Suppose a lift is additionally required to preserve one of the already reconstructed Task IX quadratic branches under *left multiplication*. For the formal theorem implemented with N_+, this extra compatibility forces $k^2=1$, and positivity then yields $k \equiv 1$. The distinguished lift follows.

The logical wording matters:

> Task IX supplies the quadratic branch as inherited structure. Task X does *not* derive that left multiplication by a state lift must preserve it. Branch preservation is an additional compatibility hypothesis formulated using already-derived data.

It is therefore inappropriate to describe this condition as an automatic normalization law. It is a clean conditional test. The corresponding symmetric question for N_- remains part of the hardening agenda.

9 Using W as a provisional state carrier

Only after the lift problem is understood does Task X let W act on itself from the left:

$$\mathcal{U}_\theta(\psi) := U_{\mathrm{std}}(\theta)\psi. \tag{29}$$

This is explicitly a probe. No claim is made that the regular left module W is the physical state space.

For U_{std}, the action satisfies

$$\mathcal{U}_{2\pi}(\psi) = -\psi, \qquad \mathcal{U}_{4\pi}(\psi) = \psi. \tag{30}$$

Thus the same axial rotation that returns every algebra element after 2π can act with a sign on a candidate state.

For a general lift $U = kU_{\mathrm{std}}$, the common scalar survives:

$$\mathcal{U}^U_{2\pi}(\psi) = -c\,\psi, \qquad \mathcal{U}^U_{4\pi}(\psi) = c^2\psi. \tag{31}$$

The algebra/observable action is therefore insensitive to precisely the factor that a left-state action can detect.

Theorem 5 (Algebra action versus state action)**.** *For the distinguished representative, $\Phi_{2\pi}$ is the identity on W while left multiplication by $U_{\mathrm{std}}(2\pi)$ is minus the identity. For the general lift, the state action retains the scalar factor invisible to conjugation.*

This is the first formal separation in Experiment 2 between a transformation of the algebra and a transformation of a candidate state.

10 Distinguished-axis half-angle sectors

Task X defines two neutral state sectors by

$$\mathcal{S}_+ := \{\psi \in W \mid R\psi = S\psi\}, \tag{32}$$
$$\mathcal{S}_- := \{\psi \in W \mid R\psi = -S\psi\}. \tag{33}$$

Using $SR = -A$, these are proved equivalent to the axis-eigenspace conditions

$$A\psi = +\psi \quad \text{and} \quad A\psi = -\psi, \tag{34}$$

respectively.

Both sectors are nonzero and satisfy

$$\dim_{\mathbb{R}} \mathcal{S}_+ = \dim_{\mathbb{R}} \mathcal{S}_- = 4, \tag{35}$$

which the already-derived central scalar plane organizes as Z-dimension two. They have trivial intersection and together span the whole regular left module:

$$W = \mathcal{S}_+ \oplus \mathcal{S}_-. \tag{36}$$

One explicit Z-decomposition is

$$\mathcal{S}_+ = Z(1 + A) \oplus Z(B + P), \tag{37}$$
$$\mathcal{S}_- = Z(1 - A) \oplus Z(B - P). \tag{38}$$

On these sectors, the distinguished state action reduces to the transverse full-angle factors evaluated at half the angle:

$$\psi \in \mathcal{S}_+ \Longrightarrow \mathcal{U}_\theta(\psi) = \left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2} S\right)\psi, \tag{39}$$
$$\psi \in \mathcal{S}_- \Longrightarrow \mathcal{U}_\theta(\psi) = \left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2} S\right)\psi. \tag{40}$$

This is an exact half-angle relation, not merely a 2π sign test.

Only in the final comparison layer are these channels described by the familiar axial labels $\pm\frac{1}{2}$ [4, 5]. No physical fermion is derived. In particular, each sector still has Z-dimension two inside the full regular left module, so Task X has not selected a minimal irreducible state carrier.

11 Why transverse state orientation is not yet selected

The distinguished axis produces a longitudinal state decomposition, but it does not choose a specific direction inside the transverse plane. Task X formalizes this with a negative control.

The axis eigenspaces $\mathcal{S}_\pm$ are preserved by the state action. By contrast, the analogous eigenspace condition for a chosen transverse generator is not preserved under the same axial action. An explicit witness at $\theta = \pi$ is provided in the audit [2].

The precise conclusion is intentionally narrow:

The tested transverse-axis eigenspace criterion is not invariant under the distinguished axial state action. The current data therefore do not provide a canonical transverse state orientation analogous to the distinguished longitudinal axis.

This should not be inflated into a theorem that no conceivable canonical transverse structure can ever exist. What has been shown is enough for the dependency audit: a specific transverse spin orientation is not already supplied by the same data that select the momentum-adapted longitudinal axis.

12 The first infinitesimal evolution probe

The final stage asks for infinitesimal structure preservation without identifying the group parameter with time. Let $D : W \to W$ be real-linear and require:

1. Leibniz:
$$D(xy) = D(x)y + xD(y); \tag{41}$$
2. $D(1) = 0$ and $D(A) = 0$;
3. the old transverse plane is preserved;
4. the induced transverse action is skew with respect to the inherited Euclidean rest-space form.

No commutator formula is assumed.

Solving these conditions forces

$$D(B) = \lambda C, \qquad D(C) = -\lambda B. \tag{42}$$

Leibniz then propagates the same scalar to the derived channels:

$$D(P) = \lambda Q, \qquad D(Q) = -\lambda P, \qquad D(R) = D(S) = 0. \tag{43}$$

Hence the complete solution space is one-dimensional:

$$D = \lambda D_{\text{gen}}. \tag{44}$$

After this classification is established, Task X proves the intrinsic formula

$$D_{\text{gen}}(x) = \frac{1}{2}(xR - Rx). \tag{45}$$

It also verifies directly that D_{gen} is the derivative of Φ_θ at $\theta = 0$.

Theorem 6 (Axial derivation rigidity)**.** *The existing kinematic/algebraic data determine a unique axial derivation direction up to a real scalar multiple.*

13 The second free scale: direction is fixed, rate is not

Every real multiple λD_{gen} obeys the same structural requirements. Different values of λ give different derivations. Therefore no value of λ is selected by the Task X axioms.

This is the first explicit dynamics negative control of the project:

$$\boxed{\text{generator direction fixed}} \qquad \text{but} \qquad \boxed{\text{generator rate free}}. \tag{46}$$

The scalar λ is not called energy, mass, frequency or Hamiltonian. Likewise θ is not identified with physical time.

The result should be compared carefully with the lift-scale freedom $k(\theta)$. They arise in different classification problems:

- $k(\theta)$ multiplies a state lift by a scalar that conjugation cannot see;
- λ rescales the infinitesimal algebra generator itself.

Task X does not identify these freedoms. Their coexistence instead sharpens the boundary between kinematic symmetry and a fully specified dynamical evolution.

14 The normalization trap as a Conservation-of-Difficulty test

The most informative event in Task X occurred before the final proof script stabilized. The half-angle representative was already visible, but the attempt to prove uniqueness exposed an extra multiplicative factor. The proof-search transcript then considered the familiar normalization $a^2 + b^2 = 1$ as a way to force that factor to unity [3].

This was mathematically legitimate as a *new assumption*. It was methodologically dangerous as a silent repair.

Had the normalization been inserted early, the reconstruction would have reported

$$U(\theta) = U_{\mathrm{std}}(\theta) \tag{47}$$

as if conjugation and the group law alone forced it. In fact they force only

$$U(\theta) = k(\theta)U_{\mathrm{std}}(\theta). \tag{48}$$

The familiar normalized representative is one point in a larger family.

Methodological caveat 2 (A successful formula can hide a failed dependency claim)**.** *Producing the standard half-angle expression is not enough. The experiment asks which assumptions make it necessary. An early normalization would have preserved the formula while destroying the answer to that dependency question.*

This episode is an unusually clear example of the project's Conservation-of-Difficulty method. The difficulty did not disappear when the right formula was found. It reappeared as a normalization ambiguity. Retaining the ambiguity makes visible the precise point at which additional state structure may be required.

15 Late comparison with established representation theory

Only after the neutral reconstruction is complete does Task X attach conventional terminology.

The fixed longitudinal channel and the two opposite full-angle transverse Z-channels reproduce the standard axial weight pattern $0, \pm 1$ of a vector representation. The distinguished internal representative has the familiar 4π return and, on the two axis sectors, acts with half-angle factors associated with axial projections $\pm\frac{1}{2}$ [12, 5, 4].

These identifications are classical. In the conventional Clifford description, rotations are implemented by elements of the even algebra and act on vectors by conjugation; half-angle parameters in the implementing element yield full-angle rotations on vectors [5]. Task X does not claim new representation theory.

What is specific to the present experiment is the source order and the negative controls:

1. the old axial rotation is defined first;
2. its full-algebra extension is derived uniquely;
3. only then is an internal implementer sought;
4. the half angle is recovered from the doubled conjugation action;

5. the implementer is found to be non-unique because of an invisible scalar factor;
6. only after that is W used as a provisional state carrier;
7. the infinitesimal algebra generator is classified separately and retains its own scale freedom.

The standard names are therefore comparison labels, not construction inputs.

16 Heuristic back-translation

The original counter-running-band picture was introduced only as a heuristic visualization of longitudinal causal sectors. It remains useful if interpreted structurally rather than literally.

One selected propagation or momentum direction corresponds to one distinguished longitudinal sector. In $3+1$, choosing that direction automatically leaves a two-dimensional transverse plane. A single trajectory can remain longitudinal while the surrounding $3+1$ geometry still carries transverse transformation freedom. Task X formalizes precisely this relative decomposition: the selected axis is fixed, while the orthogonal plane rotates.

The new information is that the full algebra distinguishes two types of angular response. The transverse vector-like channels respond with the full angle. The candidate state sectors produced by left action of the internal lift respond with half the angle. In the heuristic language, this is not a claim that a particle literally moves transversely while another moves longitudinally. It is a statement about how degrees of freedom *relative to a selected propagation axis* transform.

Likewise, the absence of a canonical direction inside the transverse plane has a natural heuristic reading. The momentum axis selects what counts as longitudinal and what counts as transverse, but it does not by itself mark one transverse orientation as special. Additional state data would be needed to choose such an orientation.

The free lift factor k has no established physical interpretation. Heuristically it says only that the same geometric rotation of the reconstructed algebra can correspond to more than one left-state transformation unless an additional normalization/compatibility rule is supplied. The free derivation scale λ says something parallel but distinct: even once the infinitesimal rotation direction is known, the current structure does not tell us how fast any evolution should run.

Thus the heuristic survives Task X in a sharpened form:

> A selected propagation direction organizes the $3+1$ geometry into longitudinal and transverse sectors. The algebra fixes their relative rotation structure and even supports half-angle candidate state sectors, but it does not yet fix a physical state normalization, a unique transverse state orientation, or a dynamical rate.

17 Conservation of Difficulty after Task X

The new dependency chain can be summarized as follows.

Tasks VIII–IX: fixed associative carrier W, central plane Z, and two multiplicative quadratic branches $N_{\pm}$.

Task X: distinguish one old axis A. The old rest space splits as Long $\oplus$ Trans.

Ordinary axial rotation extends uniquely to the full algebra: one fixed longitudinal channel and two opposite full-angle transverse Z-channels.

Internal implementer in K: half-angle dependence is forced, but $U = k\,U_{\mathrm{std}}$ with a positive multiplicative scalar factor k.

Left-state probe: two distinguished-axis sectors show opposite half-angle factors; no canonical transverse state orientation is selected.

Axial derivations form exactly $\mathbb{R}D_{\mathrm{gen}}$ with $D_{\mathrm{gen}} = \frac{1}{2}(\cdot R - R\cdot)$.

Two unresolved scales remain: state-lift normalization k and infinitesimal generator rate λ. Physical time and particle interpretation are still absent.

The main lesson is not that spin labels appeared. It is that the first bridge from kinematics to states immediately generated new dependency questions. The algebra action is rigid; the internal implementer is not. The axial derivation direction is rigid; its rate is not. The longitudinal axis is selected; a transverse state orientation is not.

18 Formal audit and proof boundary

The audited Task X archive supplied with this work has SHA-256

`fb3a725954af00f2488bded1e33965428a67cb51c7ff415ffea36e8081c3e5c7`.

The pinned environment is Lean 4.28.0 with mathlib revision

`8f9d9cff6bd728b17a24e163c9402775d9e6a365`.

Aristotle reports a successful full build of 8120 jobs and a successful targeted build of the Task X entry point [2, 6, 9, 10].

The reconstruction chain is

`SafeBase` → `AxisDecomposition` → `OldAxialRotation` → `AlgebraExtension`
→ `VectorChannels` → `InternalLift` → `StateAction` → `HalfAngleSectors`
→ `AxialDerivation` → `DynamicsDiagnostic`,

followed only then by

`Identification` → `Task10`.

Task X has one external Experiment 2 import, `Task09.Existence`; the Task IX conventional identification layer is not imported by the reconstruction core [2].

The target-structure firewall excludes conventional spin, helicity, spinor, Pauli, $SU(2)$, Spin(3), Dirac, Maxwell, Hamiltonian and gauge constructions from the reconstruction modules. The final identification module alone uses conventional language.

The audit reports no `sorry`, `admit`, custom axiom or `implemented_by` occurrence in Task X. For twenty-one principal theorems, `#print axioms` reports only

$$\texttt{propext},\quad \texttt{Classical.choice},\quad \texttt{Quot.sound}. \tag{49}$$

Proof boundary 1 (What was and was not independently rechecked)**.** *The full Lean build reported above is Aristotle's build result. For this paper, the archive, source ordering, theorem statements and audit were inspected independently; the central multiplication identities, half-angle sector relations and commutator generator formula were also cross-checked algebraically. No separate local rebuild of the complete pinned Mathlib environment was performed in the paper-generation environment.*

19 Open obligations left by Task X

Task X is successful, but it deliberately stops with several foundational questions open.

1. **Continuous scalar classification.** The lift theorem is algebraic. No continuity or measurability is assumed for k. Under continuity one expects the standard positive homomorphism form $k(\theta) = e^{\alpha\theta}$, but this analytic classification is not formalized in Task X.
2. **Implementers outside K.** The full algebra may contain other one-parameter implementers of the same inner automorphism. Task X classifies only those lying in $K = \operatorname{span}\{1, R\}$.
3. **Central ambiguity.** Because S is central, full-algebra implementers may differ by more than a real scalar once the restriction to K is removed. This must be classified before a unique state lift is claimed.
4. **Branch symmetry.** The conditional normalization theorem is formulated with N_+. The corresponding statement for N_-, and its behaviour under $S \mapsto -S$, should be audited symmetrically.
5. **State carrier.** The regular left module W is only a probe. The half-angle sectors have Z-dimension two and are not yet established as minimal state carriers.
6. **Transverse state data.** No canonical state orientation inside the transverse plane is selected.
7. **Generator rate.** The one-dimensional derivation direction is fixed, but its scale is not.
8. **Physical time.** No relation between the rotation parameter and a physical time variable has been introduced.

These are not defects to be patched by importing standard physics. They are the current dependency boundary.

20 Outlook: Task XI should harden the base before splitting spin sectors

The natural next move is *not* to launch separate spin-1 and spin-1/2 programs immediately. Task X has uncovered enough new structure that the foundations should first be hardened. A premature specialization could otherwise inherit an unnoticed normalization or state-carrier assumption.

A narrow Task XI should therefore remain representation-neutral and focus on the lift/state interface itself.

20.1 Close the continuous lift classification

First add only the regularity that Task X originally intended for the internal lift: continuity in the group parameter. Within K, classify continuous scalar factors rather than merely arbitrary multiplicative ones. The expected endpoint is not to set the factor to one, but to prove the exact residual family, conventionally

$$k(\theta) = e^{\alpha\theta}, \tag{50}$$

with a real parameter α, if that follows from the chosen regularity assumptions.

The key acceptance criterion is that continuity may reduce pathological freedom but must not be used as a hidden normalization.

20.2 Remove the artificial K restriction

Second, classify internal one-parameter implementers of the already fixed algebra automorphism Φ_θ in the full algebra W, not only inside K. The reconstruction should ask whether two implementers of the same conjugation action necessarily differ by an invertible element of the already-derived centre Z.

This is a foundational algebra question, not yet a spin question. It should be proved without invoking the conventional matrix identification until a late comparison layer.

If the centre is exactly the residual freedom, Task XI can classify the continuous central one-parameter factors intrinsically. This will reveal whether the real k of Task X was only the K-visible part of a larger central ambiguity.

20.3 Separate adjoint and state generators

For a general continuous implementer $U(\theta)$, Task XI should compare two infinitesimal objects:

1. the derivative of the conjugation action on the algebra, already fixed by D_{gen};
2. the derivative of left multiplication on the candidate state carrier.

Any central term in the state generator will disappear from the adjoint action. Proving this separation intrinsically would make precise why the algebra can determine a rotation generator while a state evolution still carries extra data.

20.4 Audit both quadratic branches symmetrically

The conditional Task X normalization test should be repeated for both N_+ and N_-, with the $S \mapsto -S$ relation kept explicit. Task XI should determine whether branch-preserving left action eliminates the full central freedom, only its real modulus, or something else. No branch should be declared physically preferred.

20.5 Harden the provisional state carrier

Finally, before the words spin 1 and spin 1/2 are allowed to drive separate tasks, Task XI should state exactly what has and has not been established about W as a left module. In particular it should test whether the Task X half-angle sectors admit an intrinsic decomposition into smaller nonzero invariant left-state carriers under the hardened lift, or whether selecting such a carrier requires new data.

This question should be asked neutrally. No minimal ideal, Weyl spinor or Dirac module should be imported as a target.

Methodological principle 3 (Hardening before interpretation)**.** Task XI should eliminate ambiguity in the mathematical interface between algebra automorphism, internal implementer and candidate state action. Only after that interface is stable should later tasks split into dedicated spin-1 and spin-1/2 investigations.

A concise Task XI dependency target is therefore

$$\begin{array}{c}\text{fixed axial algebra action}\\ \downarrow\\ \text{all continuous internal implementers}\\ \downarrow\\ \text{exact central freedom}\\ \downarrow\\ \text{adjoint generator versus state generator}\\ \downarrow\\ \text{symmetric } N_{\pm} \text{ compatibility audit}\\ \downarrow\\ \text{minimality or non-minimality of the provisional state sectors.}\end{array} \tag{51}$$

The hard stop should come there. Spin 1 and spin 1/2 can then be investigated separately on a cleaner foundation.

21 Conclusion

Part X makes the first controlled move beyond the purely kinematic/algebraic program of Parts I–IX. A single distinguished old spatial axis is enough to split the old rest space into a longitudinal line and a transverse plane. The ordinary axial rotation extends uniquely to the complete associative algebra, fixes the longitudinal direction, rotates the transverse and mixed channels with the full angle, leaves R and S invariant, preserves both Task IX quadratic branches, and returns exactly after 2π.

Using the central plane reveals two exact transverse Z-channels with opposite full-angle factors. Using the internal plane generated by 1 and R reveals a half-angle implementer. When the algebra is used as a provisional left module, two distinguished-axis sectors transform with opposite half-angle factors and show the familiar 4π return pattern for the distinguished representative. None of this requires a spin label in the reconstruction core.

The decisive methodological result is the failure of lift uniqueness. Conjugation fixes the half-angle structure but cannot see a scalar multiplier. During proof search, a standard unit normalization almost removed this freedom before it had been classified. The final formalization instead keeps it explicit and proves that every K-valued lift has the form kU_{std}, with a genuine nontrivial example. This is precisely the kind of dependency that the experiment is intended to expose: a standard formula can be correct while a standard normalization hides the assumption that makes it unique.

The infinitesimal analysis produces a second boundary. Axial derivations form exactly a one-dimensional line generated by $D_{\text{gen}} = \frac{1}{2}(\cdot R - R\cdot)$, and D_{gen} is the derivative of the algebra action at the identity. Yet the overall scalar rate remains free. The reconstructed structure therefore fixes how the axial symmetry acts but not a physical clock rate for that action.

Part X should consequently not be read as a derivation of spin-1 particles, spin-1/2 fermions or dynamics. It establishes something more basic and, for the dependency audit, more useful: full-angle vector-like channels, half-angle candidate state sectors, a unique adjoint generator direction, and two explicit places where further state/dynamical information is still required.

The next task should harden exactly those places. By classifying continuous implementers in the full algebra, isolating their central ambiguity, comparing state and adjoint infinitesimal generators, auditing both Task IX branches symmetrically and testing the minimality of the provisional state carrier, Task XI can stabilize the bridge between kinematics and states. Only then will separate spin 1 and spin 1/2 investigations have a clean dependency base.

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