Split Complex Lorentzian Dynamics

Part XIII: Universal Axial Carrier Type, Rigid Relative Sector Action, and the Exact Two-Point Fixed Locus

A formally verified continuation of the dependency audit

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Abstract

Part XII replaced the provisional eight-dimensional regular carrier by an intrinsic classification of minimal left-stable carriers. Every proper nonzero left carrier was shown to be minimal of real dimension four; the family is infinite but consists of one equivalence class. Only after this global carrier layer had been established was a distinguished spatial axis reintroduced, yielding a universal $2 + 2$ intersection

$$L = (L \cap H_+) \oplus (L \cap H_-)$$

for every minimal carrier L. The four-dimensional half-angle sectors themselves were not minimal left carriers. They were right ideals, and the exact minus-sector stabilizer/reducibility theorem remained the one local hardening obligation. The more substantive question was still open: does the half-angle behavior previously found on the full carrier W survive reduction to an arbitrary minimal carrier, and if it does, which part is intrinsic and which part depends on the nonunique internal lift?

Part XIII isolates this one-axis transformation problem. It first closes the inherited minus-sector gap directly: the exact left stabilizer of H_- is a six-dimensional unital subalgebra with coordinate equations $x_2 + x_4 = 0$ and $x_3 + x_5 = 0$, and H_- is reducible under that weaker action. The plus/minus counterpart relation is itself formalized through an inherited axis-reversing inner automorphism rather than inferred from symmetry rhetoric.

The carrier analysis then begins from an arbitrary minimal left carrier L, not from either of the two axial examples. Its slices

$$K_+(L) = L \cap H_+, \qquad K_-(L) = L \cap H_-$$

are each two-dimensional over $\mathbb{R}$, stable under the exact center

$$Z = \mathrm{Center}(W) = \mathrm{span}_{\mathbb{R}}\{1, S\}, \qquad S^2 = -1,$$

and each is exactly the central span of any of its nonzero elements. Thus each slice is intrinsically one-dimensional over the already reconstructed central plane, while no generator of that central line is canonically selected.

The first principal null test is then passed. The reference implementer is restricted to arbitrary L only after the slice structure is fixed, and its action is re-derived from the sector equations rather than transferred from Part X. For every minimal carrier,

$$U_{\mathrm{ref}}(\theta)\psi_+ = \big(\cos(\theta/2)1 - \sin(\theta/2)S\big)\psi_+, \qquad \psi_+ \in K_+(L),$$

and

$$U_{\mathrm{ref}}(\theta)\psi_- = \big(\cos(\theta/2)1 + \sin(\theta/2)S\big)\psi_-, \qquad \psi_- \in K_-(L).$$

The two central factors are mutually inverse. Their relative transformation is the full-angle element

$$\mathrm{Rel}(\theta) = \cos\theta\, 1 - \sin\theta\, S,$$

and this relation is carrier-independent.

The second difficulty is to retain, rather than normalize away, the complete Task-XI internal-lift freedom. Every continuous lift has the form

$$U_{\alpha,\beta}(\theta) = z_{\alpha,\beta}(\theta)U_{\mathrm{ref}}(\theta), \qquad z_{\alpha,\beta}(\theta) = e^{\alpha\theta}\big(\cos(\beta\theta)1 + \sin(\beta\theta)S\big).$$

Part XIII proves that the entire two-parameter residual factor is common to the two slices. The exact slice factors become

$$c_+ = e^{\alpha\theta}\big(\cos((\beta - \tfrac{1}{2})\theta)1 + \sin((\beta - \tfrac{1}{2})\theta)S\big),$$
$$c_- = e^{\alpha\theta}\big(\cos((\beta + \tfrac{1}{2})\theta)1 + \sin((\beta + \tfrac{1}{2})\theta)S\big),$$

yet the relative element remains exactly $\mathrm{Rel}(\theta)$ for every full lift. Hence all residual lift freedom survives on the carrier as common internal freedom but cancels from the relative sector action. Infinitesimally, the restricted generators are

$$G_+ = \alpha 1 + (\beta - \tfrac{1}{2})S, \qquad G_- = \alpha 1 + (\beta + \tfrac{1}{2})S,$$

so $G_+ - G_- = -S$ independently of α and β. The Task-X rate parameter remains logically separate.

A third difficulty is representative dependence. Task XII had shown that different minimal carriers are equivalent through bijective restricted right-multiplication intertwiners. Part XIII proves that these intertwiners map plus slices onto plus slices, minus onto minus, and intertwine the reference implementer, every full lift and the infinitesimal implementer. The one-axis result therefore has a precise verdict: *universal axial carrier type.* The infinitely many minimal carriers are different realizations of the same one-axis transformation structure, not different one-axis representation types.

The final difficulty is selection. The algebra automorphism Φ_θ acts not only on elements but on the family of minimal carriers, and this action must be separated from the internal left action of $U(\theta)$. Part XIII closes the global carrier-family gap by deriving an exact parametrization: every minimal carrier has a unique representative

$$e(n) = \frac{1}{2}(1 + n_1A + n_2B + n_3C), \qquad n_1^2 + n_2^2 + n_3^2 = 1,$$

so the family is in bijection with an explicit real unit two-sphere. A coordinate reflection fixing $1, A, B, C$ and reversing P, Q, R, S is used instrumentally to derive this normal form; no intrinsic metric or canonical geometric state-sphere interpretation is attached to that device. Under Φ_θ, n_1 is fixed while (n_2, n_3) rotates. Consequently a carrier is invariant under the whole axial automorphism family exactly when it is one of the two axial carriers generated by $(1 \pm A)/2$. The axis therefore reduces the infinite family to an exact two-element fixed locus but still does not choose between the two.

No carrier property forces $\alpha = \beta = 0$. Minimality, the $2 + 2$ split, central stability, carrier equivalence, slice invariance and implementation of Φ are all compatible with every real (α, β). The familiar reference normalization arises only after an explicitly additional quadratic-compatibility condition, not from carrier structure itself. The final conventional comparison identifies the independently reconstructed one-axis carrier action with the standard axial spinorial action, but no physical spin, fermion, inner product, probability rule, full spatial rotation group, boost representation or field equation is claimed.

The next task should therefore not add interpretation. Part XIV should test whether the one-axis construction extends consistently to two and then arbitrary spatial directions: whether the independently derived axis-relative decompositions and internal lifts glue canonically, whether composition is path-independent, and whether new orientation, sign or central compatibility data appear. This is the next natural Conservation-of-Difficulty boundary between a verified one-axis structure and a full spatial rotation structure.

Keywords: minimal left carrier; axial action; half-angle; central plane; relative sector action; carrier equivalence; fixed locus; unit two-sphere; formal verification; Lean 4; conservation of difficulty.

Contents

1 Why Part XIII is not yet a full rotation theory

Part XIII begins at a point where several structures that are conventionally introduced together have already been separated by proof. Part X derived an axial automorphism and an internal half-angle implementer on the full eight-dimensional algebra. Part XI then removed the artificial restriction on the implementer and exposed a full two-parameter central residual freedom. Part XII finally classified the actual minimal left carriers and showed that the direction selected in Part X does not create them: a whole family of four-real-dimensional minimal carriers already exists before the axis is used [3, 4, 5].

This ordering creates a new hardening requirement. The Part-X half-angle result was obtained on the regular carrier W. It would be easy to assume that the same formula automatically represents the reduced minimal carrier. That assumption would be structurally plausible and conventionally correct, but it would defeat the dependency audit. The reduced carrier is a different object, and its transformation law has to be proved on that object.

A second temptation is equally strong. Once Part XII has shown that all minimal carriers are equivalent, one could choose the two particularly simple axial carriers and perform all calculations there. Again, this would likely reproduce the expected answer while hiding whether the answer depends on the representative. Task XIII therefore imposes the stronger quantifier: all principal transformation theorems must hold for an arbitrary minimal carrier L.

A third difficulty is inherited from Part XI. The reference lift is not the unique internal implementation of the fixed algebra automorphism. The complete continuous family carries two real residual parameters (α, β). Any reduced-carrier theorem that begins by setting them to zero would erase the very freedom that Part XI was designed to expose. The correct question is therefore not only what the reference action does, but what survives after all full lifts are retained.

Finally, transformation and selection must be separated. Left multiplication by an implementer acts inside a chosen carrier; the algebra automorphism Φ_θ acts on the family of carrier subspaces. A universal transformation law need not imply a unique carrier selection. This distinction becomes one of the main outputs of Part XIII.

Methodological principle 1 (Re-derive after reduction)**.** A transformation law proved on the regular carrier is not inherited automatically by a reduced minimal carrier merely because the conventional model suggests the same formula. The restriction must be proved from the reconstructed algebra and the defining relations of the reduced sectors.

2 Inherited structure and the one-axis firewall

The associative algebra W and its ordered basis are inherited unchanged:

$$1, A, B, C, P = AB, Q = AC, R = BC, S = ABC. \tag{1}$$

The exact center is

$$Z = \mathrm{Center}(W) = \mathrm{span}_{\mathbb{R}}\{1, S\}, \qquad S^2 = -1. \tag{2}$$

A single old spatial axis A remains distinguished, with the two Part-X sectors

$$H_+ = \{\psi \in W : A\psi = \psi\}, \qquad H_- = \{\psi \in W : A\psi = -\psi\}. \tag{3}$$

Part XII supplies the minimal-carrier class and, for every minimal left carrier L,

$$L = (L \cap H_+) \oplus (L \cap H_-), \qquad \dim_{\mathbb{R}}(L \cap H_+) = \dim_{\mathbb{R}}(L \cap H_-) = 2. \tag{4}$$

Task XIII deliberately does not add a second axis, full spatial rotation group, Hilbert structure or physical state interpretation. Its scope is the complete classification of the already available one-parameter axial system on the already available minimal carriers [1].

We write

$$K_+(L) := L \cap H_+, \qquad K_-(L) := L \cap H_-. \tag{5}$$

These names indicate axis-relative subspaces only; no physical component interpretation is attached.

3 Closing the inherited minus-sector asymmetry

Part XII had fully classified the weaker left action preserving H_+ but had not formalized the corresponding result for H_-. A later theorem comparing the two slices symmetrically would therefore have rested on an unproved symmetry claim. Part XIII closes this gap before the arbitrary-carrier action is studied.

Define

$$\mathrm{Stab}_L(H_-) := \{x \in W : xH_- \subseteq H_-\}. \tag{6}$$

The exact result is:

Theorem 1 (Minus-sector stabilizer)**.** *The left stabilizer of H_- is a six-dimensional unital real subalgebra. In the inherited basis its coordinate constraints are*

$$x_2 + x_4 = 0, \qquad x_3 + x_5 = 0. \tag{7}$$

It contains the center and the selected axis, but not the relevant transverse generator.

The plus sector obeys the opposite pair of equations

$$x_2 - x_4 = 0, \qquad x_3 - x_5 = 0. \tag{8}$$

Crucially, the relation between the two stabilizers is also proved. The inherited inner automorphism generated by B reverses the axis and maps the minus stabilizer onto the plus stabilizer. Thus the symmetry is an output rather than a prose shortcut.

Theorem 2 (Minus-sector reducibility)**.** *The right ideal H_- is reducible under its exact six-dimensional left stabilizer. In particular the two-dimensional central plane generated inside the axial minus sector is a proper nonzero invariant subspace.*

This closes the only local proof obligation exported by Part XII.

4 Each carrier slice is a central line

The $2 + 2$ theorem of Part XII determines only real dimensions. Part XIII asks whether the already reconstructed center supplies the exact internal scalar structure of the slices.

Theorem 3 (Central rank one)**.** *Let L be any minimal left carrier. Then both $K_+(L)$ and $K_-(L)$ are stable under Z. If $0 \neq \psi_+ \in K_+(L)$, then*

$$K_+(L) = \{(a + bS)\psi_+ : a, b \in \mathbb{R}\}. \tag{9}$$

Likewise, if $0 \neq \psi_- \in K_-(L)$,

$$K_-(L) = \{(a + bS)\psi_- : a, b \in \mathbb{R}\}. \tag{10}$$

The coefficients are unique in each case.

The proof does not merely rename a two-real-dimensional space as a one-dimensional complex space. Central stability is first established, then the finite-dimensional rank statement is proved inside the reconstructed central algebra. This source order matters because Z itself was derived before any minimal carrier existed.

There is also no canonical generator of either central line. Every nonzero vector in a slice generates that same slice over Z, and nonzero central multipliers act freely and transitively on its generator set. The intrinsic object is the line, not a preferred vector on it.

Methodological principle 2 (Do not confuse a sector with a generator)**.** The axis and algebra determine the two central-line subspaces inside a chosen minimal carrier. They do not, by themselves, select a normalized or otherwise preferred nonzero element of either line.

5 Re-deriving the reference action on an arbitrary minimal carrier

Only after the carrier slices and their central structure are fixed does the reference implementer re-enter. The inherited reference family satisfies the group law and implements the axial automorphism. Its explicit formula in the internal two-plane is known from Part X, but Task XIII does not use the old regular-carrier sector formula as a theorem about L.

For an arbitrary minimal carrier define

$$\rho_{L,\mathrm{ref}}(\theta)\psi := U_{\mathrm{ref}}(\theta)\psi. \tag{11}$$

Since L is left-stable, this is a well-defined real-linear automorphism of L. The inverse is obtained at $-\theta$, and the group law follows from that of U_{ref}.

The exact sector actions are derived from the inherited relations $R\psi = \pm S\psi$ that characterize the two slices.

Theorem 4 (Exact reference-sector action)**.** *For every minimal carrier L and every $\theta \in \mathbb{R}$,*

$$U_{\mathrm{ref}}(\theta)\psi_+ = c_+(\theta)\psi_+, \qquad c_+(\theta) = \cos(\theta/2)1 - \sin(\theta/2)S, \qquad \psi_+ \in K_+(L), \tag{12}$$

$$U_{\mathrm{ref}}(\theta)\psi_- = c_-(\theta)\psi_-, \qquad c_-(\theta) = \cos(\theta/2)1 + \sin(\theta/2)S, \qquad \psi_- \in K_-(L). \tag{13}$$

Both slices are preserved exactly.

The result is stronger than the Part-X observation because the quantifier now ranges over the complete minimal-carrier family.

Corollary 1 (Carrier independence)**.** *The two central elements $c_+(\theta)$ and $c_-(\theta)$ depend only on the sign of the axis-relative sector and the transformation parameter. They do not depend on the literal minimal carrier.*

Thus half-angle behavior survives carrier reduction as a theorem rather than an analogy.

6 The relative sector action is full-angle and rigid

The two factors in equations (12) and (13) are mutually inverse central units:

$$c_+(\theta)c_-(\theta) = c_-(\theta)c_+(\theta) = 1. \tag{14}$$

Their relative difference can therefore be represented without choosing a division convention by the identity

$$c_+(\theta) = \mathrm{Rel}(\theta)c_-(\theta). \tag{15}$$

The unique central solution is

$$\boxed{\mathrm{Rel}(\theta) = \cos\theta\, 1 - \sin\theta\, S.} \tag{16}$$

Equivalently, $\mathrm{Rel}(\theta) = c_+(\theta)^2$.

This exposes a structural distinction that was less visible on the regular carrier:

individual slice action	half-angle central factor
relative plus/minus action	full-angle central factor

The relative transformation is already independent of the carrier. The next hardening step is to ask whether it is also independent of the nonunique lift.

7 Keeping the complete Task-XI lift freedom

Part XI classified every continuous full lift of the fixed algebra automorphism as

$$U_{\alpha,\beta}(\theta) = z_{\alpha,\beta}(\theta)U_{\mathrm{ref}}(\theta), \tag{17}$$

where

$$z_{\alpha,\beta}(\theta) = e^{\alpha\theta}(\cos(\beta\theta)1 + \sin(\beta\theta)S). \tag{18}$$

The factor $z_{\alpha,\beta}$ is central. Task XIII explicitly forbids setting $\alpha = \beta = 0$ before the reduced-carrier action is classified.

For every minimal L, every full lift preserves L and both $K_+(L)$ and $K_-(L)$. The exact continuous sector factors are

$$c_+(\alpha, \beta, \theta) = e^{\alpha\theta}(\cos((\beta - \tfrac{1}{2})\theta)1 + \sin((\beta - \tfrac{1}{2})\theta)S), \tag{19}$$
$$c_-(\alpha, \beta, \theta) = e^{\alpha\theta}(\cos((\beta + \tfrac{1}{2})\theta)1 + \sin((\beta + \tfrac{1}{2})\theta)S). \tag{20}$$

The apparent complexity of equations (19) and (20) is diagnostically useful. The two-parameter freedom remains fully visible on each slice. It has not disappeared through carrier reduction.

Theorem 5 (Common-versus-relative classification)**.** *For an arbitrary full lift implementing the inherited axial automorphism, the residual factor multiplying the reference lift is common to the plus and minus slices. The unique relative factor between the two slice actions is always the same element equation* (16)*. In particular, for the complete continuous family it is independent of α and β.*

The formal relative theorem is stated in a division-free manner and applies even to general full lifts beyond the continuously parameterized subclass. Only the closed form equations (19) and (20) uses the continuous (α, β) classification.

Methodological principle 3 (Residual freedom need not disappear to become irrelevant relatively)**.** The two-parameter lift freedom is not removed. It remains visible on the internal action of a carrier. What is rigid is the relation between the two axis-relative slices: all residual freedom is common and cancels from that relation.

This is the central Conservation-of-Difficulty result of Part XIII.

8 Infinitesimal restriction and the fixed relative generator

The general infinitesimal implementer inherited from Part XI is

$$G(\alpha, \beta) = \alpha 1 + \beta S - \frac{1}{2}R. \tag{21}$$

Restricting left multiplication by G to the two slices yields a complete diagonalization over the central plane:

$$G\psi_+ = (\alpha 1 + (\beta - \tfrac{1}{2})S)\psi_+, \qquad \psi_+ \in K_+(L), \tag{22}$$
$$G\psi_- = (\alpha 1 + (\beta + \tfrac{1}{2})S)\psi_-, \qquad \psi_- \in K_-(L). \tag{23}$$

Hence

$$\boxed{G_+ - G_- = -S.} \tag{24}$$

The common infinitesimal part is $\alpha 1 + \beta S$; the sector-distinguishing part is the fixed pair $\mp\frac{1}{2}S$.

The Task-X algebra-derivation rate remains a different parameter. For every (α, β), the induced algebra derivation is the same normalized D_{gen}; changing the Task-X rate λ changes the algebra automorphism itself, whereas changing (α, β) changes only an internal implementation of the same automorphism. Part XIII therefore retains the separation established in Part XI.

9 Carrier equivalence respects the entire one-axis structure

Part XII showed that for any two minimal carriers L and L' there exists an element $r \in W$ such that restricted right multiplication

$$T_r : L \to L', \qquad T_r(\psi) = \psi r, \tag{25}$$

is a bijective intertwiner for the left W-action. It was not necessary to prove that r is globally invertible in W.

Associativity already implies

$$T_r(x\psi) = xT_r(\psi) \tag{26}$$

for every $x \in W$. Task XIII pushes this simple identity through the entire reduced one-axis structure.

Theorem 6 (Universal carrier intertwining)**.** *For every pair of minimal carriers L, L' there exists a restricted right-multiplication equivalence T_r such that*

$$T_r(K_+(L)) = K_+(L'), \qquad T_r(K_-(L)) = K_-(L'), \tag{27}$$

$$T_r(U_{\text{ref}}(\theta)\psi) = U_{\text{ref}}(\theta)T_r(\psi), \tag{28}$$

$$T_r(U(\theta)\psi) = U(\theta)T_r(\psi) \tag{29}$$

for every full lift U, every θ, and every $\psi \in L$.

The preservation of the plus/minus slices is derived from the defining A-eigenrelations and associativity, not from equal dimensions. Thus all minimal carriers carry precisely the same one-axis transformation structure up to the already reconstructed notion of carrier equivalence.

Status 1 (Universal axial carrier type)**.** There is one intrinsic one-axis carrier type. The infinitely many literal minimal carriers are equivalent realizations of that type, and their axial slices and all full-lift actions are matched by the intrinsic carrier intertwiners.

This settles transformation-type dependence but not representative selection.

10 Internal action and carrier-family action are different

Every full lift acts on a chosen minimal carrier by left multiplication and preserves it. The algebra automorphism Φ_θ, however, acts on subspaces by image:

$$\phi_\theta(L) := \Phi_\theta[L]. \tag{30}$$

Task XIII proves that ϕ_θ maps minimal carriers to minimal carriers and forms a one-parameter action on the full minimal-carrier family.

For a full lift U, the two levels are connected by

$$\phi_\theta(L) = L\,U(-\theta), \tag{31}$$

where the right-hand side is the image of L under right multiplication. This formula makes the distinction explicit:

- $\psi \mapsto U(\theta)\psi$ is an internal action preserving every minimal carrier;
- $L \mapsto \phi_\theta(L)$ is a family action that can move a minimal carrier to a different carrier.

An explicit nonaxial carrier is moved at $\theta = \pi/2$.

Methodological principle 4 (Transformation is not selection)**.** A carrier can possess the same internal one-axis law as every other carrier while occupying a different orbit under the automorphism action on the carrier family. These are different classification problems.

11 The full minimal-carrier family and the sphere normal form

Part XII established infinitude and one equivalence class but left the global parameter space unresolved. Part XIII closes this gap.

The key instrumental map is the coordinate reflection

$$\mathrm{dg}(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (x_0, x_1, x_2, x_3, -x_4, -x_5, -x_6, -x_7). \tag{32}$$

It fixes the old scalar/spatial coordinates $1, A, B, C$ and reverses the mixed directions P, Q, R, S. The formal development attaches no metric, conjugation or physical meaning to this map. It is used only as an algebraic device to construct a convenient generator inside an arbitrary minimal carrier.

For a suitable self-product generator e of a minimal carrier, the unit coordinate of $\mathrm{dg}(e)e$ becomes a sum of coordinate squares and is positive, while its P, Q, R, S coordinates vanish. The inherited null condition then forces the remaining three normalized spatial coefficients to satisfy a unit-sphere equation.

Define

$$e(n) := \frac{1}{2}(1 + n_1 A + n_2 B + n_3 C), \qquad n_1^2 + n_2^2 + n_3^2 = 1. \tag{33}$$

Theorem 7 (Exact carrier parametrization)**.** *Every minimal left carrier equals $L_{e(n)}$ for a unique real unit vector $n = (n_1, n_2, n_3)$. Conversely every unit vector defines a minimal carrier. Hence*

$$\{\textit{minimal left carriers}\} \longleftrightarrow \{n \in \mathbb{R}^3 : \|n\|^2 = 1\} \tag{34}$$

is a bijection in the inherited coordinates.

Methodological caveat 1 (What the sphere theorem does and does not say)**.** *The theorem is an exact global parametrization. It does not prove that the carrier family has been intrinsically characterized as a canonical geometric sphere independently of the inherited coordinates and the instrumental reflection equation* (32)*. No metric-state-space or Bloch-sphere interpretation is part of the reconstruction theorem.*

This distinction matters because a successful coordinate normal form can be stronger than an existence theorem without yet being a canonical geometric characterization.

12 The exact axial fixed locus

In the sphere coordinates equation (33), the action of Φ_θ becomes transparent. The axial coordinate n_1 remains fixed while the transverse pair rotates:

$$(n_1, n_2, n_3) \mapsto (n_1, \cos\theta\, n_2 - \sin\theta\, n_3, \sin\theta\, n_2 + \cos\theta\, n_3). \tag{35}$$

This is not introduced as a target; it follows from the inherited action of Φ_θ on A, B, C and the exact carrier normal form.

Part XIII carefully separates fixity at one parameter from fixity under the entire family.

Theorem 8 (Single-angle fixity). *For a unit vector n,*

$$\phi_\theta(L_{e(n)}) = L_{e(n)} \iff (\cos\theta = 1 \textit{ and } \sin\theta = 0) \quad \textit{or} \quad n_2 = n_3 = 0. \tag{36}$$

At exceptional full-turn angles the automorphism is the identity and every carrier is fixed. This enlarged single-angle fixed set must not be confused with global axial invariance.

Theorem 9 (Exact global fixed locus). *Let L be minimal. Then*

$$(\forall\theta,\ \phi_\theta(L) = L) \iff L = L_{e_+} \textit{ or } L = L_{e_-}, \tag{37}$$

where

$$e_+ = \frac{1}{2}(1 + A), \qquad e_- = \frac{1}{2}(1 - A). \tag{38}$$

The two carriers are distinct.

Thus the complete carrier family is infinite—indeed sphere-parametrized—but global invariance under the chosen axial automorphism leaves exactly two poles.

Status 2 (Axial fixed-family verdict). The one-axis data reduce the full minimal-carrier family to exactly two globally Φ-invariant members. They do not choose one of those two.

The word "select" must therefore be used carefully. What is mathematically derived is a two-element fixed locus under a specified symmetry family. No physical mechanism choosing one pole has been reconstructed.

13 Universal transformation, residual selection freedom

The preceding results allow two questions that were previously entangled to be answered separately.

First, does a chosen minimal carrier have a special one-axis transformation law? No. The law is universal. Every minimal carrier has the same Z-line decomposition, the same reference half-angle factors, the same full-lift common residual freedom, the same rigid relative factor and the same intertwining class.

Second, does the one-axis automorphism make every minimal carrier equivalent as a point of the carrier family? No. Under the family action, generic carriers move while exactly two are globally fixed.

Moreover the two fixed carriers are not distinguished by their internal one-axis action. An explicit moved equatorial carrier carries the same restricted reference factors as the fixed axial carriers. Thus fixed-locus status and internal transformation type are logically distinct.

The residual selection problem after Part XIII is therefore sharp:

$$\text{infinite carrier family} \xrightarrow{\text{global axial invariance}} \{L_{e_+}, L_{e_-}\}, \tag{39}$$

with no inherited datum in Tasks VIII–XIII choosing between the final two.

14 No normalization is derived from carrier structure

A separate negative control asks whether the now much richer carrier structure could retroactively eliminate the Task-XI lift freedom. The answer is no.

For every $\alpha, \beta \in \mathbb{R}$, the continuous full lift equation (17):

- implements the same inherited algebra automorphism;
- preserves every minimal carrier;

- preserves both carrier slices;
- acts by central factors on those slices;
- is intertwined by all carrier equivalences.

Therefore none of the following forces $\alpha = \beta = 0$: minimality, the $2+2$ decomposition, central stability, carrier equivalence, slice invariance or Φ-implementation.

Status 3 (No normalization derived)**.** Carrier structure leaves the complete continuous (α, β) plane free. Distinct parameter pairs define genuinely distinct internal lifts even though they induce the same algebra automorphism and the same relative sector action.

Only the separately labelled quadratic-compatibility condition inherited from Part XI collapses the freedom to the reference lift. That condition remains additional; it is not promoted to an axiom of a physical state space.

The $2\pi/4\pi$ audit remains consistent with this conclusion. The algebra automorphism satisfies $\Phi_{2\pi} = \mathrm{id}$, whereas the generic carrier lift acquires its residual central factor. Familiar values such as $U(2\pi) = -1$ and $U(4\pi) = 1$ characterize the reference choice only after stronger conditions are imposed; they cannot be used as hidden normalization data.

15 Late conventional identification

Only after the reconstruction and negative controls are complete does the final identification module permit the standard complex-matrix language. Under the already established algebra comparison $W \cong M_2(\mathbb{C})$, a minimal carrier becomes a minimal left ideal, the central plane becomes the usual complex-scalar algebra, and the two carrier slices carry the factors

$$e^{-i\theta/2}, \qquad e^{+i\theta/2}. \tag{40}$$

The exact carrier-family normal form corresponds to the familiar family of rank-one projections, and the reconstructed one-axis action agrees algebraically with the standard axial spinorial action [2].

This comparison is useful precisely because it occurs late. It confirms that the intrinsic theorems land on familiar mathematics without allowing that familiar mathematics to choose the carrier, the slices, the relative factor or the fixed locus.

Methodological caveat 2 (No physical spin theorem)**.** *The permitted conclusion is an algebraic equivalence of the independently reconstructed one-axis carrier action with the standard axial spinorial action. Part XIII derives no fermion, no particle, no Hilbert space, no positive inner product, no projective state rule, no probability interpretation, no boost representation, no field equation and no physical time evolution.*

16 Formal verification and proof boundary

The supplied Aristotle artifact reports the following pinned environment:

Lean toolchain	`leanprover/lean4:v4.28.0`
Mathlib revision	`8f9d9cff6bd728b17a24e163c9402775d9e6a365`
Task-XIII archive SHA-256	`e8c053f1af782e79270ab71b92bb64de` `9a00d6cb3bf9f5aa637d6109c5eb13b2`

The Task-XIII source order is

$$\begin{gathered}\texttt{SafeBase} \to \texttt{MinusSectorHardening} \to \texttt{CarrierSlices}\\ \to \texttt{SliceCentralStructure} \to \texttt{ReferenceRestriction}\\ \to \texttt{ReferenceSectorAction} \to \texttt{FullLiftRestriction}\\ \to \texttt{RelativeSectorAction} \to \texttt{InfinitesimalRestriction}\\ \to \texttt{CarrierIntertwiners} \to \texttt{AutomorphismCarrierAction}\\ \to \texttt{FixedCarrierLocus} \to \texttt{SelectionAudit}\\ \to \texttt{Identification} \to \texttt{Task13}.\end{gathered}$$

The ordering is substantive. The minus-sector gap is closed first; the arbitrary carrier and slices are then fixed; only afterward are the reference and full lifts imported; the carrier-family geometry and fixed locus come later; conventional identification is last.

The supplied audit reports

```
lake build RequestProject.NullSectorTask13.Task13
[8116/8116] Built RequestProject.NullSectorTask13.Task13

lake build
Build completed successfully (8161 jobs).
```

A mechanical scan reports no `sorry`, `admit`, custom `axiom` or `@[implemented_by]` in Task XIII. Twenty-three exposed principal theorems were subjected to `#print axioms`; every one reports exactly

$$[\texttt{propext}, \texttt{Classical.choice}, \texttt{Quot.sound}]. \tag{41}$$

Proof boundary 1 (Verification boundary for this paper)**.** *The archive, source tree, theorem statements, audit, import order and relevant proof modules were inspected during preparation of this paper. The pinned Lean/Mathlib build was not independently rerun in the paper-generation environment. Build counts and axiom lists above are therefore reported from the supplied Aristotle audit, not from a second local build.*

The precise status is

$$\boxed{\text{TASK XIII: PASS.}} \tag{42}$$

No in-scope proof obligation required by the one-axis classification remains open.

17 Conservation of Difficulty after Part XIII

The main value of Part XIII is not that a known half-angle formula appears. Its value is the localization of which freedoms survive and which relations are forced when the formula is reconstructed on the correct carrier.

Task-XII minimal-carrier family; one unresolved H_- hardening obligation

$\downarrow$

close H_- exactly: six-dimensional stabilizer and reducibility

$\downarrow$

arbitrary minimal carrier L: two Z-lines $K_+(L), K_-(L)$

$\downarrow$

re-derive reference action: opposite half-angle central factors

$\downarrow$

retain all full lifts: complete residual freedom is common

$\downarrow$

relative action rigid: $\mathrm{Rel}(\theta) \quad = \quad \cos\theta\, 1 \;-\; \sin\theta\, S$

$\downarrow$

carrier intertwiners: universal axial carrier type

$\downarrow$

global carrier parametrization and Φ action: exact two-point fixed locus

$\downarrow$

exported difficulty: choice between two axial carriers; (α, β) freedom; all questions beyond one axis

The outcomes can be summarized as follows.

Question	Result
Does the Part-X half-angle formula survive reduction?	Yes, re-derived on every minimal carrier.
Does it depend on the carrier representative?	No; all carriers have the same axial type.
Does the Task-XI residual lift freedom vanish?	No; the whole (α, β) freedom survives internally.
Does that freedom affect the relative plus/minus action?	No; it is entirely common and cancels exactly.
What is the relative finite transformation?	$\cos\theta\, 1 - \sin\theta\, S$.
What is the relative infinitesimal generator?	$-S$.
Are the two slices arbitrary real planes?	No; each is a rank-one line over the exact center.
Does a slice have a canonical nonzero generator?	No.
Are different minimal carriers different one-axis types?	No; intrinsic intertwiners match slices and every full lift.
How large is the carrier family?	Exactly parameterized by an explicit real unit two-sphere.
Is that sphere already a canonical physical state sphere?	No such intrinsic geometric or physical interpretation is proved.
What does global axial invariance select?	Exactly two carriers, generated by $(1 \pm A)/2$.
Does it choose between the two?	No.

Question	Result
Does carrier structure normalize the lift?	No.
What remains outside the present proof boundary?	Selection between the two axial carriers, residual lift normalization, discontinuous-lift classification, and compatibility across more than one spatial axis.

The conservation-of-difficulty picture is therefore unusually clean. Some freedoms have disappeared because they were never genuine after the correct quotient of questions: carrier dependence disappears from the transformation type, and lift dependence disappears from the relative sector action. Other freedoms remain exactly where theorems say they remain: the common internal lift factor and the two-point axial fixed-locus ambiguity.

18 Open mathematical obligations

The in-scope one-axis task is complete, but several deliberately exported questions remain.

1. **Two-point selection.** The whole axial automorphism family fixes exactly two minimal carriers. No inherited datum selects one of them.
2. **Residual internal freedom.** The continuous (α, β) plane survives every carrier-structural requirement. It is visible on the carrier, invisible to the algebra automorphism and invisible to the relative sector action.
3. **Discontinuous full lifts.** Task XI established that the algebraic residual class without continuity is larger than the closed-form continuous family. Task XIII's abstract relative theorems cover arbitrary full lifts, but no complete parameterization of the discontinuous class is supplied.
4. **Canonical geometry of the carrier family.** The sphere parametrization is exact in the inherited coordinates. A coordinate-free characterization of the family as a canonical geometric object is not established and was not required.
5. **Multiple axes.** Every theorem in Part XIII is relative to the single inherited axis A. Compatibility between distinct axis choices has not yet been tested.
6. **Physical structure.** No inner product, positivity condition, projectivization, probability rule, time evolution, mass relation or field equation has been reconstructed.

The fifth item is the important next structural boundary. A single-axis result can match a familiar axial representation while still failing to glue coherently across different axes. The missing information could be none, or it could reappear as an orientation/sign choice, a central cocycle-like freedom, a path dependence or a new normalization condition. That must be tested rather than assumed away.

19 Outlook: Task XIV should test multi-axis compatibility

Part XIII closes the one-axis carrier problem strongly enough that the next task can finally move laterally rather than deeper along the same axis. The appropriate question is not yet a physical spin question and not yet a request to construct the standard full rotation group. The neutral question is:

> Given two independently selected unit spatial directions inside the already reconstructed $3 + 1$ algebra, do the corresponding axis-relative carrier decompositions and internal implementations coexist and compose consistently on an arbitrary minimal carrier, and what additional structure—if any—is required to glue them?

A useful Task XIV should proceed in stages.

1. Reconstruct a second-axis copy without importing the answer. Choose a second old unit spatial direction B' by inherited spatial data and repeat only the minimum definitions needed to obtain its two carrier slices and its axial automorphism/lift family. Do not assume that the A-axis formulas transform covariantly into the second-axis formulas unless that covariance is proved from the already reconstructed algebra.

2. Compare decompositions on one fixed minimal carrier. For arbitrary minimal L, classify the intersections and change-of-decomposition map between the A-relative and B'-relative $2 + 2$ splittings. Determine whether the exact center alone controls this change or whether new mixed products enter essentially.

3. Test composition of finite axial actions. Compose the independently derived lifts for two nonparallel axes in both orders. Determine exactly what changes under order reversal. Do not call the result a rotation group, spin group or quaternionic composition law before the intrinsic composition theorem is complete.

4. Audit central residual freedom under composition. Part XIII shows that (α, β) is irrelevant to the relative action for a single axis. Task XIV should test whether separate residual choices for two axes cancel harmlessly under mixed composition or generate a new compatibility condition. This is a critical negative control: one-axis invisibility does not imply multi-axis invisibility.

5. Test path independence and closure. If an element can be reached by different sequences of axis rotations, compare the induced carrier transformations. Classify any residual central discrepancy rather than normalizing it away. Only if the discrepancy vanishes or is intrinsically controlled may one claim a well-defined glued spatial action.

6. Preserve the carrier-family distinction. Continue to distinguish internal action on a fixed carrier from the induced action on the sphere-parametrized family of carriers. Determine how mixed-axis transformations act on the exact two-point fixed loci associated with each individual axis.

7. Late comparison only. If the multi-axis composition closes canonically, only then compare the resulting structure with the conventional double-cover description of spatial rotations. A successful comparison would establish an algebraic full-rotation correspondence on the reconstructed minimal carrier. It would still not supply an inner product, probability interpretation, particle ontology or dynamics.

The most informative possible outcomes of Task XIV are therefore not limited to success. If the one-axis structures fail to glue without a new orientation, sign, normalization or central-compatibility datum, that failure would locate the next missing dependency with greater precision than simply importing the standard full rotation group.

Methodological principle 5 (Task-XIV boundary)**.** Do not complete the known rotation theory by recognition. Take two independently reconstructed axis systems, compose them inside the existing algebra and let the compatibility calculation determine whether a full spatial structure is forced or whether new data are required.

20 Conclusion

Part XIII turns the first half-angle signal of Part X into a carrier-level theorem and, in doing so, separates several questions that conventional presentations often package together.

The inherited minus-sector asymmetry is first closed exactly. An arbitrary minimal carrier then splits into two real two-dimensional subspaces that are, more rigidly, one-dimensional lines over the already derived center. No generator of either line is canonical. The reference internal action is re-derived on those lines and yields mutually inverse half-angle central factors. Their relative transformation is the full-angle element $\cos\theta\, 1 - \sin\theta\, S$.

Retaining the entire Task-XI lift family reveals the decisive rigidity statement. The (α, β) freedom remains completely visible on each slice, but it is common to both and disappears from the relative action. Infinitesimally the same fact becomes $G_+ - G_- = -S$. Thus the robust one-axis datum is not a particular normalized implementer but the relative transformation between the two central-line sectors.

The result is also independent of which minimal carrier is chosen. The intrinsic right-multiplication equivalences between minimal carriers preserve the plus/minus decomposition and intertwine every full lift. All minimal carriers therefore realize one universal axial carrier type.

Representative selection remains a separate matter. Part XIII derives an exact sphere parametrization of the entire minimal-carrier family and computes the induced axial automorphism action. Global invariance under that family leaves exactly two carriers, the axial poles. The transformation law is nevertheless the same on fixed and moved carriers, so symmetry fixity does not create a distinct carrier type and does not choose between the two poles.

The final conventional comparison recognizes the one-axis carrier action as the standard axial spinorial action, but this is only a late algebraic identification. The one-axis proof does not yet establish a full spatial rotation structure, because compatibility between different axes has not been reconstructed. That is the appropriate next boundary. Task XIV should therefore test multi-axis gluing directly, preserving all residual central freedoms until composition either removes them, controls them or exposes them as genuinely new missing data.

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