Split Complex Lorentzian Dynamics

Part XIV: Spatial Closure, Central Path Defects, and the Migration of Difficulty toward Dynamics

A formally verified continuation of the dependency audit

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Abstract

Part XIII established a universal one-axis carrier type. Every minimal left carrier has the same $2+2$ axis-relative decomposition into two rank-one lines over the exact central plane

$$Z = \mathrm{Center}(W) = \mathrm{span}_{\mathbb{R}}\{1, S\}, \qquad S^2 = -1,$$

the same half-angle reference action, and the same rigid relative full-angle law. At the same time the complete continuous internal-lift freedom

$$U_{\alpha,\beta}(\theta) = e^{\alpha\theta}\big(\cos(\beta\theta)1 + \sin(\beta\theta)S\big)U_{\mathrm{ref}}(\theta)$$

remained common to both sectors and therefore invisible to their relative action. The decisive unresolved question was whether this apparently harmless one-axis freedom remained harmless once independently reconstructed spatial directions were forced to coexist on one minimal carrier.

Part XIV performs that test without importing a full spatial rotation theory. A second old spatial axis is reconstructed independently. Its longitudinal/transverse decomposition, old transformation family, unique multiplicative extension, internal implementation plane, half-parameter law, reduced carrier slices and relative action are all derived without conjugating or relabelling the first-axis formulas. Only after this derivation is complete are the two axis systems compared. They agree in their reduced relative law, but their $2+2$ decompositions are maximally transverse: for nonparallel basis axes every pair of opposite-or-equal sign slices has zero intersection.

Mixed composition then exposes the next layer of structure. The reference implementers do not commute; their commutator produces the third noncentral spatial generator. The three derived infinitesimal generators

$$G_A = -\tfrac{1}{2}R, \qquad G_B = \tfrac{1}{2}Q, \qquad G_C = -\tfrac{1}{2}P$$

close with no further generator. The smallest product-closed implementer algebra is exactly the four-real-dimensional space $\mathrm{span}_{\mathbb{R}}\{1, P, Q, R\}$. From the three basis cases an intrinsic linear map $J : \mathbb{R}^3 \to W$ is reconstructed and shown to satisfy

$$J(n)^2 = -h(n,n)1, \quad J(n)J(m) + J(m)J(n) = -2h(n,m)1,$$

$$J(n)J(m) - J(m)J(n) = -2J(n \times m),$$

where the antisymmetric spatial operation is defined only after the product relation has been obtained. Arbitrary-axis reference implementations close inside this already available algebra, and the same carrier $2+2$ law holds for every unit spatial direction. Thus the spatial noncentral problem closes without new data.

The price of that success appears one level deeper. Reference words implementing the same algebra automorphism can differ internally by an exact central sign, and the reference defect set is exactly $\{+1, -1\}$. More generally, arbitrary full implementations of the same automorphism may differ by any invertible element of Z. Most importantly, there is no globally multiplicative assignment of internal implementers to the implementable automorphisms. Two commuting half-turn automorphisms necessarily have anticommuting internal implementers. An explicit closed mixed path induces the identity automorphism while its internal product is -1. Spatial automorphisms close, reference implementations close, yet globally coherent internal multiplication is obstructed and path dependence remains explicit.

This changes the Conservation-of-Difficulty diagnosis. The remaining obstruction is no longer a missing spatial generator, a missing associative closure, or a missing minimal carrier. Those structures have repaired themselves successively. The unresolved data now sit in the continuous, differentiable and path-dependent relation between internal implementations. The formal audit itself identifies this frontier: arbitrary multi-axis residual factors were classified only pointwise, while continuity/differentiability across axes, global section-defect classification, nonlinear rate alternatives and optional quadratic compatibility remain open. In the heuristic language developed alongside the experiment, the one-axis parameters α and β correspond respectively to a common hole-density scaling and a common band-transport rate. Part XIV proves that static spatial closure alone does not determine them. The next task should therefore test whether continuous multi-axis coherence converts these previously invisible common freedoms into genuine relational state data, or whether still another minimal datum is forced.

The Aristotle run ended in a time-budget timeout only after the Task-XIV mathematics, main audit and README had been completed. The supplied artifact contains all Task-XIV modules, a complete 641-line audit, a recorded successful full build and the principal axiom report; the stale cumulative summary remains at Task XIII. The timeout therefore affects the final summary/documentation layer, not the mathematical Task-XIV result reported here.

Keywords: multi-axis reconstruction; minimal carrier; half-angle; spatial closure; central defect; path dependence; coherent-lift obstruction; residual freedom; formal verification; Lean 4; conservation of difficulty.

Contents

1 The frontier after Part XIII

Part XIII ended at an unusually sharp boundary. The first half-angle signal, originally obtained on the full eight-dimensional carrier, had survived every subsequent reduction. It survived the classification of actual four-real-dimensional minimal carriers, survived restriction to arbitrary members of the carrier family, survived the complete continuous two-parameter internal-lift freedom, and survived carrier equivalence. What had become rigid was not a preferred normalized implementer, but the relative action between the two axis sectors [5, 6, 7].

For one selected spatial direction A, the two carrier slices obey

$$K_{A,+}(L) \oplus K_{A,-}(L) = L, \qquad \dim_{\mathbb{R}} K_{A,+}(L) = \dim_{\mathbb{R}} K_{A,-}(L) = 2, \tag{1}$$

and each is one line over Z. The reference factors are

$$c_+(\theta) = \cos(\theta/2)1 - \sin(\theta/2)S, \qquad c_-(\theta) = \cos(\theta/2)1 + \sin(\theta/2)S, \tag{2}$$

with rigid relative element

$$\mathrm{Rel}(\theta) = \cos\theta\, 1 - \sin\theta\, S. \tag{3}$$

Yet every continuous full lift remains

$$U_{\alpha,\beta}(\theta) = z_{\alpha,\beta}(\theta) U_{\mathrm{ref}}(\theta), \tag{4}$$

where

$$z_{\alpha,\beta}(\theta) = e^{\alpha\theta}\big(\cos(\beta\theta)1 + \sin(\beta\theta)S\big). \tag{5}$$

The entire residual factor is common to both sectors, hence invisible to equation (3).

There were two logically different ways this could have ended. The first was benign: perhaps a second axis would remove the residual freedom or reduce it to a conventional sign. The second was diagnostic: perhaps one-axis kinematics simply lacked the comparisons required to expose the remaining degrees of freedom. In that case the next obstruction should appear only when independent directions are composed.

Methodological principle 1 (Do not infer multi-axis coherence from one-axis success)**.** A complete one-parameter implementation theorem does not imply that independently reconstructed one-parameter systems admit a single globally multiplicative internal realization. Mixed composition must be tested before any full spatial interpretation is permitted.

Task XIV was designed precisely around that distinction. It did not ask Lean to construct a known full rotation group. It asked whether two independently reconstructed direction systems, acting on the same already reconstructed minimal carrier, close using inherited data alone and, if so, whether the internal implementations close without a residual discrepancy [1].

2 Inherited data and the two firewalls

The ambient algebra remains the Task-VIII real associative algebra

$$W = \mathrm{span}_{\mathbb{R}}\{1, A, B, C, P, Q, R, S\}, \qquad P = AB,\ Q = AC,\ R = BC,\ S = ABC. \tag{6}$$

Its exact center, inherited from Tasks IX–XI, is

$$Z = \mathrm{Center}(W) = \mathrm{span}_{\mathbb{R}}\{1, S\}, \qquad S^2 = -1. \tag{7}$$

The old spatial subspace carries the inherited positive form h, while the minimal left carriers and their exact sphere parametrization are inherited from Tasks XII–XIII.

Task XIV adds two especially important firewalls.

First, the reconstruction modules contain no imported full-rotation or spin-group language. No $SO(3)$, $SU(2)$, Spin group, quaternion, Pauli matrix, Euler-angle, Rodrigues formula, projective representation, group extension, or topological covering-space argument is allowed before the final comparison module. The internal generators are deliberately named G_A, G_B, G_C and J; the product-closed internal algebra is named $\mathcal{I}$; the emergent antisymmetric spatial operation is introduced only after the corresponding algebraic relation is proved.

Second, no physical target is available to the proof. The Task-XIV sources contain no electron, fermion, photon, spin observable, helicity, momentum, mass, energy, frequency, Hamiltonian, Hilbert space, probability rule or field equation. This matters because the result will later resemble standard spinorial mathematics very strongly. The firewall makes that resemblance an output rather than a construction guide.

3 Independent reconstruction of the second axis

The most important source-order requirement is that the second axis B be reconstructed from inherited old spatial data, not obtained by conjugating the existing A-axis answer.

Define the new old-space decomposition by

$$\mathrm{Long}_B := \mathrm{span}_{\mathbb{R}}\{B\}, \qquad \mathrm{Trans}_B := \{v \in \mathrm{Rest} : h(v, B) = 0\}. \tag{8}$$

The formal development proves

$$\mathrm{Long}_B \cap \mathrm{Trans}_B = \{0\}, \qquad \mathrm{Long}_B + \mathrm{Trans}_B = \mathrm{Rest}, \tag{9}$$

with dimensions one and two respectively. Only afterward is it derived that A and C span the transverse plane.

A one-parameter old transformation $\mathrm{rot}_B(\varphi)$ is then reconstructed. It fixes the temporal unit and B, preserves the old spatial subspace, the transverse plane and h, is nontrivial transversely, and satisfies the group law. The crucial next theorem is uniqueness: there exists exactly one unital multiplicative extension $\Phi_B(\varphi)$ to the full algebra W.

The resulting basis action is

$$\begin{aligned}
\Phi_B(\varphi)A &= \cos\varphi\, A - \sin\varphi\, C,\\
\Phi_B(\varphi)B &= B,\\
\Phi_B(\varphi)C &= \sin\varphi\, A + \cos\varphi\, C,\\
\Phi_B(\varphi)P &= \cos\varphi\, P + \sin\varphi\, R,\\
\Phi_B(\varphi)Q &= Q,\\
\Phi_B(\varphi)R &= -\sin\varphi\, P + \cos\varphi\, R,\\
\Phi_B(\varphi)S &= S.
\end{aligned} \tag{10}$$

This is not the first-axis table with labels silently exchanged. The fixed noncentral mixed direction is now Q, and this difference becomes essential when the two axes are later composed.

3.1 The second half-angle law is forced again

The internal implementation problem is posed without prescribing an implementation plane, a normalization, or a half-parameter. The formal result is that the smallest product-closed plane containing the reference implementation is exactly

$$\mathrm{span}_{\mathbb{R}}\{1, Q\}, \qquad Q^2 = -1. \tag{11}$$

The conjugation requirement then forces the half-parameter, after which the reference family may be written

$$U_B(\varphi) = \cos(\varphi/2)1 + \sin(\varphi/2)Q. \tag{12}$$

Its derivative at zero gives

$$G_B = \frac{1}{2}Q, \qquad G_B^2 = -\frac{1}{4}1. \tag{13}$$

Thus the second-axis half-angle structure is a genuinely independent derivation.

4 The second-axis carrier law and the first convergence

For a minimal carrier L define

$$K_{B,\pm}(L) = \{\psi \in L : B\psi = \pm\psi\}. \tag{14}$$

The Task-XIV proof repeats the carrier analysis for this new axis rather than invoking the first-axis dimension theorem. It obtains

$$L = K_{B,+}(L) \oplus K_{B,-}(L), \qquad \dim_{\mathbb{R}} K_{B,+} = \dim_{\mathbb{R}} K_{B,-} = 2, \tag{15}$$

and each slice is again a rank-one Z-line.

The exact reduced action is then

$$c_{B,+}(\varphi) = \cos(\varphi/2)1 - \sin(\varphi/2)S, \qquad c_{B,-}(\varphi) = \cos(\varphi/2)1 + \sin(\varphi/2)S. \tag{16}$$

The relative factor is

$$\mathrm{Rel}_B(\varphi) = \cos\varphi\, 1 - \sin\varphi\, S. \tag{17}$$

Only after these formulas have been proved does the comparison theorem show

$$c_{B,+} = c_{A,+}, \qquad c_{B,-} = c_{A,-}, \qquad \mathrm{Rel}_B = \mathrm{Rel}_A\,. \tag{18}$$

Status 1 (First multi-axis convergence)**.** The same reduced $\pm$ law has now been reconstructed independently from two different spatial axes and two different internal implementation planes. The equality is derived; it is not a covariance assumption.

This is the first positive signal of full spatial universality. It is immediately followed, however, by a result that prevents the two decompositions from being regarded as the same hidden split.

5 Two decompositions of one carrier are genuinely transverse

For every minimal carrier L and every pair of signs $\sigma, \tau \in \{+, -\}$, the basis-axis theorem gives

$$K_{A,\sigma}(L) \cap K_{B,\tau}(L) = \{0\}. \tag{19}$$

Thus the two $2+2$ decompositions share no nonzero vector. More generally, for unit axes n, m and signs encoded by $\sigma, \tau = \pm 1$, the generic intersection theorem shows that

$$K_{n,\sigma}(L) \cap K_{m,\tau}(L) = \{0\} \quad \text{unless} \quad h(n,m) = \sigma\tau. \tag{20}$$

The exceptional equality forces $n = \pm m$.

The mixed projectors

$$e_{n,+} = \frac{1}{2}(1+n), \qquad e_{n,-} = \frac{1}{2}(1-n) \tag{21}$$

are introduced only after their idempotency and complementary relations have been established. For the two basis axes, Task XIV derives sandwich identities such as

$$(e_{A,+}e_{B,+})e_{A,+} = \frac{1}{2}e_{A,+}, \qquad (e_{B,+}e_{A,+})e_{B,+} = \frac{1}{2}e_{B,+}. \tag{22}$$

The induced maps between the corresponding central lines are nonzero and invertible. No amplitude or probability interpretation is attached to the factor $1/2$.

The geometry is therefore already nontrivial at carrier level: different axes do not select common subspaces of one carrier. Instead, they provide different decompositions related by nontrivial mixed products.

6 Mixed finite composition generates the missing spatial direction

The next null test compares the two independently reconstructed reference implementations in both orders. The exact difference is

$$U_A(\theta)U_B(\varphi) - U_B(\varphi)U_A(\theta) = -2\sin\frac{\theta}{2}\sin\frac{\varphi}{2}\,P. \tag{23}$$

Thus the mixed product is order-dependent and its difference lies in the previously available noncentral direction P.

The automorphisms themselves also fail to commute generically. Task XIV proves a complete criterion for their commutation, rather than only one counterexample. In particular, generic mixed transformations are order-dependent, while the two half-turn automorphisms form an exceptional commuting pair that will later expose the internal central obstruction.

At the infinitesimal level the result becomes still sharper. The three generators are

$$G_A = -\frac{1}{2}R, \qquad G_B = \frac{1}{2}Q, \qquad G_C = -\frac{1}{2}P. \tag{24}$$

Their products obey

$$G_AG_B = -\frac{1}{4}P, \qquad G_BG_A = \frac{1}{4}P, \qquad G_A^2 = G_B^2 = G_C^2 = -\frac{1}{4}1, \tag{25}$$

and the commutators close exactly:

$$[G_A, G_B] = G_C, \qquad [G_B, G_C] = G_A, \qquad [G_C, G_A] = G_B. \tag{26}$$

No fourth noncentral generator appears.

Moreover the third direction is not introduced by recognition. Only after the commutator has been obtained is it compared with the inherited old spatial direction C, yielding

$$G_A = -\frac{1}{2}(SA), \qquad G_B = -\frac{1}{2}(SB), \qquad G_C = -\frac{1}{2}(SC). \tag{27}$$

Status 2 (Infinitesimal spatial closure)**.** Two independently reconstructed axes force the third noncentral direction, and the resulting three-generator system closes with no further generator.

This is a major Conservation-of-Difficulty result. The anticipated multi-axis stress test does produce new mixed structure, but that structure was already latent in the inherited eight-dimensional algebra. No new spatial carrier extension is required.

7 The exact implementer algebra

The smallest product-closed real subspace containing the two reference families is

$$\boxed{\mathcal{I} = \operatorname{span}_{\mathbb{R}}\{1, P, Q, R\}.} \tag{28}$$

Its real dimension is four, and Task XIV proves both product closure and minimality. In coordinates $x = (a, b, c, d)$ relative to $(1, P, Q, R)$, the product is

$$\begin{aligned}(a, b, c, d)(a', b', c', d') = (&aa' - bb' - cc' - dd',\\ &ab' + ba' - cd' + dc',\\ &ac' + ca' + bd' - db',\\ &ad' + da' - bc' + cb').\end{aligned} \tag{29}$$

The internal center of $\mathcal{I}$ consists exactly of real multiples of 1.

This last fact is crucial:

$$S \notin \mathcal{I}. \tag{30}$$

The exact center $Z = \operatorname{span}_{\mathbb{R}}\{1, S\}$ of the ambient algebra is therefore not part of the minimal non-central implementer algebra. Spatial implementation closes in a four-real-dimensional subalgebra, while the larger central residual freedom lives outside that subalgebra.

This separation foreshadows the final result of the task: the spatial problem will close, but the unresolved difficulty will survive in the ambient center.

8 The intrinsic axis-to-generator map

Only after the basis-axis cases and their mixed commutator have been established does Task XIV define the general axis-generator map. In inherited coordinates it is

$$J(n_1, n_2, n_3) = n_1 R - n_2 Q + n_3 P. \tag{31}$$

The development proves that this is a real-linear map $J : \mathbb{R}^3 \to W$. More importantly, the map satisfies the exact product laws

$$J(n)^2 = -h(n, n)1, \tag{32}$$

$$J(n)J(m) + J(m)J(n) = -2h(n, m)1, \tag{33}$$

and

$$J(n)J(m) - J(m)J(n) = -2J(n \times m). \tag{34}$$

The operation $n \times m$ is introduced only after the antisymmetric relation has emerged; it is not imported as a conventional cross product to obtain the answer. Combining the two parts gives

$$J(n)J(m) = -h(n, m)1 - J(n \times m). \tag{35}$$

Finally, and only late in the reconstruction, the already available central element S reveals the compact relation

$$\boxed{J(n) = Sn.} \tag{36}$$

This identity is an output of the basis reconstruction and linearity proof.

9 Arbitrary-axis closure and the universal relative law

For every unit old spatial direction n, define the reference implementation

$$U_n(\theta) = \cos(\theta/2)1 - \sin(\theta/2)J(n). \tag{37}$$

Task XIV proves its group law, invertibility and implementation of the corresponding algebra automorphism. Every minimal carrier is preserved by left multiplication, and every carrier again splits into two real two-dimensional, rank-one central lines

$$L = K_{n,+}(L) \oplus K_{n,-}(L). \tag{38}$$

The reduced factors are exactly the same $c_\pm$ as in equation (2). Hence

$$\boxed{\mathrm{Rel}_n(\theta) = \cos\theta\, 1 - \sin\theta\, S} \tag{39}$$

for every unit axis, every minimal carrier and every parameter.

Finite products also close. Task XIV derives the generic product shape

$$U_n(\theta)U_m(\varphi) = \Big(\cos\frac{\theta}{2}\cos\frac{\varphi}{2} - \sin\frac{\theta}{2}\sin\frac{\varphi}{2}\,h(n,m)\Big)1 \\ - J\Big(\cos\frac{\theta}{2}\sin\frac{\varphi}{2}\,m + \sin\frac{\theta}{2}\cos\frac{\varphi}{2}\,n + \sin\frac{\theta}{2}\sin\frac{\varphi}{2}\,(n\times m)\Big). \quad (40)$$

An intrinsic quadratic quantity on $\mathcal{I}$ is proved multiplicative, every reference implementation has unit value, and every unit element of $\mathcal{I}$ is shown to admit an axis/parameter representation. Thus the product of two arbitrary-axis reference implementations is again a single reference implementation.

Status 3 (Spatial reference closure)**.** The complete reference spatial implementation closes without any additional noncentral degree of freedom.

This positive result is exactly what makes the next negative result significant. There is no longer any room to blame a missing spatial generator.

10 The first exact defect: reference words

Consider finite words of reference implementations. Their internal product implements the composed algebra automorphism. Task XIV proves a general rigidity theorem: any two internal implementations of the same algebra automorphism differ by an element of the exact center Z.

For reference words this central discrepancy is constrained much more strongly. It belongs simultaneously to Z and to $\mathcal{I}$. Since

$$Z \cap \mathcal{I} = \mathbb{R}\,1, \quad (41)$$

and the intrinsic implementer norm is one, the only possible reference defects are

$$\boxed{\{+1, -1\}.} \quad (42)$$

Task XIV proves not merely containment but exactness: both values occur.

One realization is already supplied by a full turn about one axis:

$$U_n(2\pi) = -1, \qquad \Phi_n(2\pi) = \mathrm{id}. \quad (43)$$

The algebra automorphism has returned to identity while the internal implementation has not.

The essential methodological point is that the sign is not quotiented away. It remains an explicit algebraic discrepancy requiring explanation.

11 The full residual defect is larger than a sign

The reference result is not the full internal-lift classification. For any axis and parameter, an arbitrary implementation has the form

$$\widetilde{U}_n(\theta) = z\,U_n(\theta), \qquad z \in Z^\times, \quad (44)$$

where $Z^\times$ denotes the invertible elements of the exact center. Conversely every such central rescaling remains an implementation of the same algebra automorphism.

Independent residuals on two axes simply combine:

$$(zU_n(\theta))(z'U_m(\varphi)) = (zz')(U_n(\theta)U_m(\varphi)). \quad (45)$$

No cross-axis compatibility condition is generated at this point. The residual remains common to the two $\pm$ sectors and therefore invisible to the relative law, but it remains absolutely visible on the internal carrier action.

The full defect set is therefore

$$\boxed{Z^{\times},} \tag{46}$$

not merely $\{\pm 1\}$. Task XIV explicitly realizes, for example, the defect $2 \cdot 1$ between two implementations of the same automorphism.

Status 4 (Common central freedom)**.** One-axis residual freedom remains invisible to the relative $\pm$ law after spatial gluing, but it does not disappear. Pointwise multi-axis composition preserves it as an absolute central discrepancy, and no relation between independent residual choices is forced by the static spatial algebra alone.

This is the precise place where Part XIV refuses a tempting simplification. The familiar ± 1 defect belongs to the normalized reference words. The actual unrestricted implementation problem still carries the complete invertible central freedom inherited from Part XI.

12 The coherent-lift obstruction

The central question can now be posed sharply. Does there exist a single assignment U that chooses one internal implementer for every implementable algebra automorphism and satisfies exact multiplication,

$$U(g \circ h) = U(g)U(h)? \tag{47}$$

Task XIV proves that the answer is no.

The proof is elementary enough to expose the obstruction directly. The two half-turn automorphisms about the independently reconstructed A and B axes commute:

$$\Phi_A(\pi)\Phi_B(\pi) = \Phi_B(\pi)\Phi_A(\pi). \tag{48}$$

Yet every pair u, v of internal implementers of these two automorphisms, including arbitrary central residuals, satisfies

$$uv = -vu. \tag{49}$$

An exactly multiplicative assignment would therefore require both $uv = vu$ and $uv = -vu$, forcing $uv = 0$, which is impossible for implementers.

Theorem 1 (Coherent-lift obstruction)**.** *There is no globally multiplicative assignment of internal implementers to the implementable spatial algebra automorphisms.*

This result cannot be repaired by selecting a different normalization of the same static data. The theorem already quantifies over arbitrary central residuals. Any exact repair must therefore add structure beyond a pointwise choice of implementer.

13 An explicit closed path that does not close internally

The obstruction is not merely existential. Task XIV supplies a concrete closed mixed word

$$(A, \pi), (B, \pi), (A, \pi), (B, \pi). \tag{50}$$

At the algebra-automorphism level,

$$\mathrm{wordAut}(\mathrm{closedPath}) = \mathrm{id}, \tag{51}$$

while internally

$$\mathrm{wordVal}(\mathrm{closedPath}) = -1. \tag{52}$$

The empty word and the closed mixed path therefore induce the same algebra automorphism but different internal actions.

Status 5 (Path dependence)**.** The spatial automorphism returns exactly to identity while the internal implementation retains a nontrivial central memory of the path.

Part XIV carefully keeps three closure statements separate:

Level	Question	Status
A	Are implementable algebra automorphisms closed under composition?	CLOSED
B	Do normalized reference implementations close under composition?	CLOSED
C	Do arbitrary full implementations close without residual central data?	CLOSED ONLY UP TO CENTRAL DEFECT

A positive result at one level is never substituted for another.

14 Carrier universality survives, carrier fixity does not

The mixed-axis difficulty does not come from choosing the wrong minimal carrier. The Task-XII right-multiplication equivalences between minimal carriers intertwine every mixed word and every arbitrary full implementation. They also match the $\pm$ slices for every unit spatial direction.

Status 6 (Universal multi-axis carrier type)**.** All minimal carriers realize the same exact multi-axis internal transformation structure up to the already reconstructed carrier equivalence.

The carrier-family action, however, becomes more restrictive. The Part-XIII sphere parametrization

$$e(p) = \frac{1}{2}(1 + p_1 A + p_2 B + p_3 C), \qquad h(p,p) = 1, \tag{53}$$

is retained only as an already proved coordinate theorem. The arbitrary-axis action on p is then derived from the internal algebra, yielding

$$\mathrm{rotv}(n,\theta,p) = \cos\theta\, p + \sin\theta\,(n \times p) + (1-\cos\theta) h(n,p) n. \tag{54}$$

For a unit axis n, a minimal carrier is fixed by the entire n-family if and only if its sphere coordinate is $p = n$ or $p = -n$.

The two-point fixed locus of Part XIII therefore generalizes axis by axis. But for nonparallel n, m there is no common fixed minimal carrier:

$$\boxed{\text{no common fixed carrier for two nonparallel axes.}} \tag{55}$$

A second axis does not select one of the two one-axis poles. It destroys the common fixed locus entirely.

This result is conceptually important for the later heuristic interpretation. The carrier *type* is universal, but a literal carrier representative is not a globally fixed state under arbitrary spatial changes. Identity of carrier type and constancy of carrier state have become mathematically distinct notions.

15 Rate, isotropy and orientation

Task XIV also audits several freedoms that could otherwise be confused with the central residual.

Axis-locally, a nonzero rescaling of the transformation parameter remains possible. Thus the normalization of a single axis generator is rate-dependent. However, once the generator assignment is required to be the derived linear map on the full spatial subspace, spatial isotropy forces the A, B and diagonal directions to share one common normalization. This is a genuine reduction of one-axis freedom by $3+1$ coherence.

The result is deliberately kept separate from the Task-XI central parameters (α, β). The spatial generator rate changes the algebra automorphism itself; central residual factors change only an internal implementation of the same automorphism.

Orientation behaves similarly. Reversing the sign of the central generator S reverses all axis generators simultaneously. Individual sign flips correspond to reversing the associated axis label, and the three commutator signs obey one product relation. No independent sign datum is silently absorbed.

The pattern is already suggestive:

$$\text{one-axis freedom} \quad \xrightarrow{\text{multi-axis linear coherence}} \quad \text{common spatial rate,} \tag{56}$$

while

$$\text{one-axis central freedom} \quad \xrightarrow{\text{pointwise multi-axis closure}} \quad \text{still free in } Z^{\times}. \tag{57}$$

The unresolved question is whether imposing *continuous and differentiable* multi-axis coherence will finally constrain the second line.

16 Conservation of Difficulty: the frontier moves again

The most important result of Part XIV is not a single multiplication table. It is the location to which the unresolved difficulty has migrated.

Earlier parts repeatedly encountered structures that the current carrier could not support. The $3+1$ sector laws could not close associatively on the original old carrier; an extension was forced. The three-direction closure produced an eight-dimensional associative algebra. A real-valued multiplicative quadratic scalar then failed; a central two-dimensional scalar target was forced. The first axial reduction exposed half-angle behavior, but the regular carrier was too large; minimal carriers had to be classified. The one-axis carrier result left residual internal freedom and a two-point fixed locus. Each time the theory either repaired the obstruction by forcing a new algebraic layer or precisely exported the remaining freedom [4, 5, 6, 7].

Task XIV tests the strongest obvious remaining kinematical suspicion: perhaps the one-axis result fails because spatial directions have not yet been glued. That suspicion is now largely eliminated. The mixed commutator forces the third direction, but the required generator is already present. The minimal implementer algebra closes. The generic axis map closes. Arbitrary-axis reference implementations close. The relative carrier law is universal for all axes. No new noncentral datum is required.

Yet the internal implementation problem becomes more, not less, explicit. Reference words retain an exact sign memory. Arbitrary full lifts retain the entire invertible center. A global multiplicative choice is impossible. Closed spatial paths can return the algebra to identity while leaving the internal carrier transformed by a central residual.

Methodological principle 2 (Migration of difficulty)**.** When spatial closure succeeds but internal path coherence fails, the next missing dependency should not be sought as another spatial generator. It should be sought in the law that relates internal implementations across parameters, axes and paths.

This is the sense in which the difficulty now appears to move toward the dynamical side. No physical time evolution has yet been derived, so the word "dynamics" must remain provisional. What has changed is the mathematical type of the open data: they are rates, continuous parameter dependence, section choices, path composition and residual internal transformations rather than static carrier dimensions or missing algebra generators.

16.1 Why the current theory cannot repair this defect by itself

The phrase "cannot repair" has a precise scope. The current inherited algebra can classify the defect but cannot eliminate it while preserving all already proved requirements.

1. Adding another noncentral spatial generator is unnecessary: the three-generator system and $\mathcal{I}$ already close.
2. Choosing another minimal carrier does not help: the multi-axis carrier type is universal.
3. Choosing another normalization does not help: the coherent-lift obstruction quantifies over arbitrary central residuals.
4. Quotienting the central defect away is forbidden by the audit and would erase exactly the information the task was designed to expose.
5. The missing datum, if any, must therefore control how central residuals are related along continuous multi-axis transformations or along different composition paths.

The search space has narrowed substantially. The next object is not allowed to be an arbitrary repair. It must explain why the static spatial algebra closes perfectly while internal implementation retains path-sensitive central information.

17 Formal verification, timeout, and provenance

The supplied Aristotle artifact reports the following pinned environment:

Lean toolchain	`leanprover/lean4:v4.28.0`
Mathlib revision	`8f9d9cff6bd728b17a24e163c9402775d9e6a365`
Task-XIV archive SHA-256	`89e8aa8f45f27138039006b3a3e3cbc` `903b9a3d3aa28c0954ab02adab9cc2936`
Task-XIV source files	22 reconstruction/comparison modules plus `Task14.lean`

The reported source order runs from the independent second-axis old structure through second-axis implementation and carrier slices, then through two-axis comparison, mixed finite and infinitesimal composition, implementer closure, generic axes, word defects, full-lift defects, coherence audit, carrier equivalences, carrier-family action, fixed-locus and selection audits. The conventional identification module comes last and is imported only by the top Task-XIV module [1].

The supplied audit records

```
lake build
Build completed successfully.
```

A mechanical scan reported in the artifact finds no `sorry`, `admit`, custom `axiom`, `@[implemented_by]` or `native_decide` in the Task-XIV layer. The principal exposed theorems were subjected to `#print axioms`; each reports exactly

$$[\texttt{propext}, \texttt{Classical.choice}, \texttt{Quot.sound}]. \tag{58}$$

The Aristotle dashboard run nevertheless ended with a time-budget timeout. The artifact makes the location of that timeout unusually clear. All Task-XIV modules are present; `TASK14_AUDIT.md` is complete, the README contains the Task-XIV documentation entry, the build and axiom reports are present, and the user-facing property list had no remaining `IN PROGRESS` item at handoff. The cumulative `ARISTOTLE_SUMMARY.md`, however, still ends with Task XIII and explicitly says there that it was not modified. The timeout is therefore confined to the final cumulative-summary/documentation step in the delivered artifact; it did not interrupt the Task-XIV mathematics or its main audit.

Proof boundary 1 (Verification boundary for this paper)**.** *The supplied archive, Task-XIV source tree, theorem statements, audit, README, stale cumulative summary and relevant proof modules were inspected during preparation of this paper. The pinned Lean/Mathlib build was not independently rerun in the paper-generation environment. The successful build and axiom list are therefore reported from the supplied Aristotle audit.*

The correct status is

$$\boxed{\text{TASK XIV: MATHEMATICAL PASS; FINAL CUMULATIVE SUMMARY TIMED OUT.}} \tag{59}$$

18 Late conventional comparison

Only after the reconstruction and negative controls are complete does the final comparison module use conventional names. The derived product laws of P, Q, R agree, up to the documented sign convention, with the standard unit-quaternion relations. The bracket system equation (26) agrees with the standard three-dimensional spatial rotation Lie algebra. The reference implementation satisfies

$$U_n(2\pi) = -1, \qquad U_n(4\pi) = 1, \qquad \Phi_n(2\pi) = \mathrm{id}, \tag{60}$$

and the explicit mixed closed path has the characteristic nontrivial sign residual. In late conventional language, the independently reconstructed minimal-carrier action therefore agrees algebraically with the standard full spatial spinorial rotation structure [3, 2].

Two qualifications remain essential.

First, the unrestricted implementation freedom is larger than the normalized sign structure: arbitrary implementations carry all of $Z^\times$, not only $\{\pm 1\}$. No projective quotient has been imposed.

Second, this is still not a physical spin theorem. No particle, fermion, angular-momentum observable, Hilbert-space inner product, probability interpretation, boost representation, mass law or time evolution has been reconstructed.

19 Heuristic translation: from hole tapes to a dynamical synchronization boundary

The formal reconstruction above deliberately contains none of the original heuristic language. It is nevertheless useful, after the proof boundary has been fixed, to translate the result back into the picture that motivated the experiment.

The heuristic begins with two complementary "hole tapes" and a local object whose intrinsic tick is to remain invariant. Relative motion changes the two null-related marker structures reciprocally. In the more general Option-C version of the picture, two quantities were deliberately allowed to vary independently: the marker density and the transport rate of the tape. The synchronization condition then supplied too little information to determine both. That underdetermination was left unresolved rather than normalized away.

Part XI later found, independently of that picture, two continuous common internal parameters

$$z_{\alpha,\beta}(\theta) = e^{\alpha\theta}(\cos(\beta\theta)1 + \sin(\beta\theta)S). \tag{61}$$

The heuristic dictionary used in this research program is

Formal datum	Heuristic reading only
$e^{\alpha\theta}$	common change of hole density / local marker scale
$e^{\beta S\theta}$	common band-transport or internal progression rate
$\mp\frac{1}{2}S$	rigid relative half-angle contribution attached to the two axis sectors
$\mathrm{Rel}(\theta)$	relative synchronization law after the common freedom cancels
central path defect	internal memory left after the visible spatial transformation has closed

This dictionary is not part of the Lean theorem set.

The key Part-XIV result in this language is almost paradoxical. Adding all spatial directions does *not* remove the common density/transport freedom. The spatial bands can be glued consistently at the reference level, and the entire spatial generator system closes, yet the internal implementation retains central path memory and arbitrary common residual freedom.

Thus the old Option-C underdetermination has not been repaired by completing the spatial geometry. The missing relation has been pushed elsewhere.

19.1 Carrier identity versus carrier state

The carrier results sharpen the heuristic further. The multi-axis carrier type is universal, but no literal minimal carrier is fixed by two nonparallel full axis families. In heuristic language, the local object keeps its *type* while its internal state must change under changes of spatial relation.

This is exactly the distinction required if the eventual object is to be persistent across regimes: an electron-like object, if it is ever identified at a later stage, cannot become a different particle merely because its spatial relation or surrounding field changes. What must remain invariant is the defining carrier structure; what may change is the state carried by that structure. Part XIV establishes only the algebraic side of that distinction and does not identify the carrier physically.

19.2 Why the word "dynamics" is now less metaphorical

Before Task XIV, it was possible that the remaining (α, β) freedom was merely an artefact of looking along one axis. That possibility has now been tested and rejected at the pointwise spatial level. The audit's unresolved questions are instead explicitly about continuity, differentiability, global sections, composition defects and nonlinear rate assignments.

Those are precisely the mathematical ingredients needed to ask how an internal state changes continuously rather than merely which static transformations exist. Therefore the statement

$$\boxed{\text{the difficulty is migrating from kinematics toward dynamics}} \tag{62}$$

is now more than narrative, while still remaining short of a physical time-evolution theorem.

The heuristic expectation is that α and β should not be deleted. They should be subjected to the next compatibility layer and allowed either to become forced state variables, to collapse to a smaller common class, or to expose the need for one additional relational object. Any of those outcomes would be informative.

20 What has the search for the missing object actually narrowed to?

Task XIV eliminates several broad classes of candidates.

The missing object is not required to supply the third spatial direction; that direction is already forced by the mixed commutator. It is not required to close the spatial implementer algebra; $\mathcal{I}$ is already minimal and closed. It is not required to define arbitrary axes; J and U_n already do so. It is not required to select a special minimal carrier type; all minimal carriers realize the same multi-axis structure. It is not a sign convention; the orientation audit classifies the available sign choices.

What remains is more specific:

a law relating internal central residuals across continuous multi-axis change and composition paths

Whether that law is itself sufficient, or whether it forces a new carrier-level datum, is the next question.

This is a much narrower search than the one that existed after Part X. At that time almost every familiar component of a spinorial dynamics was still absent. After Part XIV the spatial noncentral structure is already complete enough to reproduce the conventional full spatial spinorial action in late comparison. The unresolved problem is concentrated in the internal common sector that the visible relative action cannot see.

That is exactly where the original heuristic placed its two underdetermined common quantities.

21 The six open obligations exported by Task XIV

The Aristotle audit closes the in-scope task but names six remaining mathematical obligations. They are unusually well aligned with the newly exposed frontier.

Open obligation	**Why it matters now**
Global classification of sections and their central composition defects	The coherent-lift theorem rules out exact multiplicativity, but does not classify all possible internal choices modulo central rescaling. This is the natural global form of the defect problem.
Word/relations classification	Mixed words are compared through their resulting automorphisms and internal products, but the complete relation structure of the generated automorphisms has not been reconstructed.
Continuity/differentiability of arbitrary lifts across axes	This is the central Task-XV target. Task XI classified continuous lifts on one axis; Task XIV classifies multi-axis residuals only pointwise.
Nonlinear alternatives to the linear rate assignment	Spatial isotropy forces a common rate under the already derived linear assignment. The remaining question is whether weaker assumptions permit genuinely nonlinear rate data.
Degenerate $n = \pm m$ intersection cases	The generic nonparallel theorem is complete away from the exceptional coincidence. The remaining carrier-level edge cases should be closed for a clean global classification.

Open obligation	Why it matters now
Optional Task-IX $N_\pm$ compatibility	The old quadratic compatibility tests remain available but have deliberately not been promoted to inherited data. They can be applied later as an explicitly additional test, never as a hidden normalization.

These are not arbitrary leftovers. The first four all ask, in different forms, how the pointwise spatial algebra becomes a coherent continuously varying internal structure. That is why they should dominate the next task.

22 Outlook: Task XV should attack the continuous central defect

The next task should not move to boosts, mass, energy, electromagnetism, gravitation, a Hamiltonian or a field equation. It should first determine whether the internal central defect already contains a forced continuous law.

A suitable neutral title is:

> **Task XV – Continuous multi-axis internal coherence: classification of regular lift sections, central composition defects, and residual-rate compatibility.**

The target should remain blind to the heuristic interpretation of α as hole-density scaling and β as band-transport rate. Aristotle should receive only the formal open structure exported by Task XIV.

22.1 Stage 1: regular lifts over arbitrary axes

For every unit spatial direction n, start from the already reconstructed automorphism family $\Phi_n(\theta)$ and classify internal lifts $U(n,\theta)$ subject to continuity, then differentiability, jointly in the available parameters. Do not assume an axis-independent exponential residual. Derive whether the Task-XI form implies axis-dependent coefficients

$$\alpha = \alpha(n), \qquad \beta = \beta(n), \tag{63}$$

or whether a stronger common form is forced.

The first null test is therefore:

> Does regularity across spatial directions reduce the pointwise $Z^\times$ residual freedom, or does the complete central freedom survive continuously?

22.2 Stage 2: classify the composition defect of a regular section

Choose no quotient. For a regular internal section U, define the exact central discrepancy by the division-free relation

$$U(g)U(h) = c(g,h)U(g \circ h), \qquad c(g,h) \in Z^\times, \tag{64}$$

whenever the relevant inverses are available. Prove its algebraic identities from associativity rather than importing any conventional cohomological terminology. Classify which $c(g,h)$ can arise from regular sections and how it changes under a central redefinition of the section.

The key question is not whether the defect can be hidden by notation, but whether any regular choice can make it globally trivial. Task XIV says exact trivialization is impossible; Task XV should determine the strongest regular normal form that remains possible.

22.3 Stage 3: differentiate the defect

Once regularity is established, differentiate only after the finite composition law is fixed. Isolate the central infinitesimal data. Determine whether the common one-axis generator

$$\alpha 1 + \beta S \tag{65}$$

remains arbitrary for every direction or whether mixed infinitesimal compatibility constrains its axis dependence.

This is the precise place where the old single-axis residual parameters may cease to be removable conventions and become genuine state-change data. The task should report the possibilities separately:

Outcome	Meaning inside the formal program
unique regular residual	dynamics-like normalization is forced by multi-axis coherence
common constants remain	one global two-parameter freedom survives but no local variation is allowed
axis-dependent regular residual	a new continuous internal state datum survives
path-dependent residual law required	static implementer choice is insufficient; a relational law is forced
no globally regular section	the obstruction strengthens beyond exact multiplicativity

None of these outcomes should be labelled physically inside the reconstruction.

22.4 Stage 4: close the rate and exceptional-axis audits

The nonlinear rate alternatives and the degenerate $n = \pm m$ cases should be closed in the same task if feasible, because both can otherwise masquerade as exceptional dynamical freedoms. The key requirement is to keep the spatial automorphism rate separate from the central internal residual throughout.

22.5 Stage 5: optional additional compatibility only after classification

Only after the regular multi-axis residual class has been obtained should the Task-IX $N_{\pm}$ compatibility conditions be applied as an *additional* test. If they collapse the residual class, the paper must state explicitly that they are an extra compatibility condition. If regular coherence independently reproduces the same restriction, that would be a much stronger dependency result.

22.6 Stage 6: preserve the target firewall

The Task-XV reconstruction layer should forbid at least the following target language: physical spin dynamics, electron, fermion, mass, energy, frequency, Hamiltonian, wavefunction, gauge potential, connection, electromagnetism, gravity, cocycle, projective representation and standard spin-group machinery. Such names may appear only in a final comparison module after the intrinsic classification is complete.

Methodological principle 3 (Task-XV boundary)**.** Do not supply the missing dynamics by interpreting α and β. First classify every regular multi-axis implementation permitted by the inherited algebra, expose its exact central composition law, and let regularity decide whether the residual freedom is removable, global, local or path-dependent.

This is the shortest route that preserves the experiment. It attacks precisely the unresolved location exposed by Task XIV without feeding Aristotle the heuristic answer.

23 A cautious longer-range heuristic

The present paper stops before any physical identification, but the reason the Task-XV boundary matters can be stated heuristically.

The research picture now separates three layers:

$$\begin{aligned} &\text{invariant carrier type} \longrightarrow \text{internal state / synchronization data} \\ &\longrightarrow \text{causal interaction once local realizations are compared.} \end{aligned} \tag{66}$$

The formal Task-XIV result occupies the first two layers only. It shows that the same minimal carrier type supports all spatial directions, while internal implementations retain central path information that the visible spatial automorphism cannot erase.

The larger heuristic program asks whether later local comparison of such carriers could turn different kinds of synchronization data into different effective interaction sectors. In that picture an electron-like fermionic carrier would have to retain its defining identity in every allowed regime; acceleration, external fields and spatial reorientation would change its state or relational data, not the carrier type itself. None of this is yet proved. The importance of Task XV is that it may determine the first intrinsic law governing such state change without importing any interaction theory.

If that law is forced, the longstanding "missing object" question becomes narrower again. If it is not forced, the residual defect will state exactly what additional datum must be introduced next.

24 Conservation-of-Difficulty ledger

The complete Task-XIV audit answers thirty explicit questions. The following condensed ledger records the structural results most relevant to the Part-XIV argument.

Question	**Task-XIV answer**
Can a second axis be reconstructed without first-axis covariance?	Yes. Old decomposition, old family, unique algebra extension, implementation plane and half-angle law are derived independently.
Does the second axis reproduce the first reduced carrier law?	Yes, only after the independent derivation.
Do the two carrier decompositions share nonzero vectors?	No for nonparallel axes; all four basis-axis intersections vanish.
Do distinct-axis implementations commute?	No generically.
What does the mixed commutator generate?	The third noncentral direction already present in W.
Does the infinitesimal system close?	Yes, with three generators and no further noncentral datum.
What is the smallest product-closed implementer algebra?	$\mathrm{span}_{\mathbb{R}}\{1, P, Q, R\}$, real dimension four.
Is there an intrinsic generator map for arbitrary axes?	Yes; J is linear and obeys the exact symmetric and antisymmetric product laws.
Does every unit axis give the same carrier $2+2$ relative law?	Yes: universal all-axis relative law.

Question	Task-XIV answer
Do finite reference implementations close?	Yes.
Same automorphism, different internal implementation?	Always a central discrepancy.
Exact reference defect set?	$\{+1, -1\}$.
Exact arbitrary full-lift defect set?	All invertible elements of Z.
Does one-axis common residual freedom disappear in multi-axis closure?	No. It remains relatively invisible and absolutely visible.
Is a globally multiplicative internal section possible?	No; coherent lift is obstructed.
Can a closed spatial path leave a nontrivial internal residual?	Yes; an explicit path gives automorphism identity and internal value -1.
Is the multi-axis structure carrier-dependent?	No; universal multi-axis carrier type.
Do two nonparallel axes select a common fixed carrier?	No.
Does isotropy reduce the axis-rate freedom?	Yes, to one common normalization under the derived linear assignment.
Does spatial gluing require new noncentral data?	No for automorphisms and reference implementations; the unresolved defect is central and concerns coherent internal implementation.

The dramatic feature of this ledger is its asymmetry. Almost every spatial question closes positively. The decisive negative answers cluster exactly where one asks for a globally coherent internal realization.

25 Conclusion

Part XIV begins with the strongest possible challenge to the one-axis result: rebuild a second direction independently, place both direction systems on the same arbitrary minimal carrier, compose them in both orders, and refuse every conventional shortcut until the intrinsic algebra has answered the question.

The spatial answer is unexpectedly complete. The second-axis half-angle law is independently forced. The two carrier decompositions are genuinely transverse. Their mixed products generate the third spatial direction. The three infinitesimal generators close. The minimal implementer algebra is four-real-dimensional and product-closed. A linear arbitrary-axis generator map is reconstructed with exact symmetric and antisymmetric product laws. Every unit axis yields the same carrier $2 + 2$ structure and the same relative full-angle action. Arbitrary-axis reference implementations close without any new noncentral datum.

If Part XIV had stopped there, the Conservation-of-Difficulty program could have declared the spatial problem solved. Instead the closure exposes a deeper obstruction. Internal implementations of the same visible automorphism need not agree. Reference words retain an exact sign defect. Arbitrary full lifts retain the whole invertible center. Two commuting half-turn automorphisms have necessarily anticommuting internal implementations, so no globally multiplicative section exists. A closed mixed path returns the algebra automorphism to identity while leaving the internal value -1.

The theory has therefore repaired the spatial obstruction only to reveal one it cannot remove with the same static ingredients. The missing dependency is no longer plausibly another spatial direction or carrier extension. It lies in the continuous and path-dependent law relating internal implementations. The Aristotle audit independently identifies exactly this frontier through its unresolved regularity, section-defect and rate-classification questions.

In the original heuristic, the two common one-axis freedoms had already been interpreted as hole-density scaling and band-transport rate. Their cancellation from the relative one-axis law made them appear observationally invisible there. Part XIV shows that completing $3+1$ spatial kinematics still does not determine them. At the same time it produces unavoidable internal path memory. The old underdetermination has therefore survived every static repair and is now concentrated where a genuine state-change or synchronization law would have to live.

That is the correct tension at the end of Part XIV. The known spatial spinorial structure has emerged in late comparison without being supplied as a target, but the experiment still does not possess physical dynamics. Task XV should not import them. It should ask whether continuity, differentiability and mixed-axis coherence force a law for the central residuals themselves. If they do, the difficulty will have moved one step closer to a dynamical carrier. If they do not, the remaining defect will identify the next missing object with still greater precision.

References

[1] Aristotle (Harmonic). *Null-Sector Foundation – Task 14: Multi-axis Compatibility on the Minimal Carrier: Independent Axis Reconstruction, Mixed Composition, Central Defects, and Spatial Closure.* Cumulative formal Experiment-2 artifact; Lean 4.28.0; mathlib revision 8f9d9cff6bd728b17a24e163c9402775d9e6a365; supplied archive SHA-256 89e8aa8f45f27138039006b3a3e3cbc903b9a3d3aa28c0954ab02adab9cc2936; TASK14_AUDIT reports a successful full build. Sept. 2026.

[2] Jr. H. Blaine Lawson and Marie-Louise Michelsohn. *Spin Geometry.* Vol. 38. Princeton Mathematical Series. Princeton University Press, 1989.

[3] Pertti Lounesto. *Clifford Algebras and Spinors.* 2nd ed. Vol. 286. London Mathematical Society Lecture Note Series. Cambridge University Press, 2001. DOI: 10.1017/CBO9780511526022.

[4] Jan Seeck, OpenAI ChatGPT, and Aristotle. *Split Complex Lorentzian Dynamics – Part X: A Distinguished Axis, Transverse Channels, Half-Angle State Action, and the First Dynamics Boundary.* Companion paper of Experiment 2, generated from the audited Task-10 artifact. Sept. 2026.

[5] Jan Seeck, OpenAI ChatGPT, and Aristotle. *Split Complex Lorentzian Dynamics – Part XI: Exact Central Freedom, Full-Carrier Lift Rigidity, and the Separation of Observable and State Generators.* Companion paper of Experiment 2, generated from the audited Task-11 artifact. Sept. 2026.

[6] Jan Seeck, OpenAI ChatGPT, and Aristotle. *Split Complex Lorentzian Dynamics – Part XII: Minimal Left Carriers, Axis-Relative Half-Angle Splitting, and the Persistence of State-Selection Freedom.* Companion paper of Experiment 2, generated from the audited Task-12 artifact. Sept. 2026.

[7] Jan Seeck, OpenAI ChatGPT, and Aristotle. *Split Complex Lorentzian Dynamics – Part XIII: Universal Axial Carrier Type, Rigid Relative Sector Action, and the Exact Two-Point Fixed Locus.* Companion paper of Experiment 2, generated from the audited Task-13 artifact. Sept. 2026.