Split Complex Lorentzian Dynamics

Part XVI: Joint Regularity, a Discrete Global Representative Obstruction, and the Local-Repair Frontier

A formally verified continuation of the dependency audit

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Abstract

Part XV ended with a deliberately sharpened question. For every fixed unit spatial direction n, continuity in the one-parameter variable had reduced an arbitrary central residual to two real rates, and a multi-axis family could still carry arbitrary direction-dependent functions $\alpha(n)$ and $\beta(n)$ because the formal regularity predicate imposed no continuity in n. Under the stronger requirement that equal visible automorphisms receive equal internal representatives, however, three necessary constraints had already appeared:

$$\alpha(n) = 0, \qquad \beta(n) \in \mathbb{Z} + \frac{1}{2}, \qquad \beta(-n) = -\beta(n).$$

Part XV therefore left a conditional diagnostic rather than a conclusion: if genuine joint direction–parameter regularity forced β to be continuous, then continuity, half-odd discreteness and antipodal oddness appeared mutually incompatible. The frontier was no longer another algebraic generator or another carrier. It was the existence of a globally coherent internal representative.

Part XVI reports the formal hardening performed in Task 16. First, the exact axis–parameter coincidence relation is classified. For unit directions n, m, two inherited visible automorphisms agree if and only if their already reconstructed reference implementers agree up to the real central sign ± 1. Equivalently, the half-angle scalar and vector coordinates agree with one common sign. Parameter periodicity by 2π, axis reversal, the complete identity locus $2\pi\mathbb{Z}$, same-axis periodicity, generic axis determination up to reversal, and the zero-angle directional degeneracy are all derived as consequences of this classification rather than inserted as a presentation of a known rotation group.

Second, Task 16 introduces the genuinely stronger predicate that $(n, \theta) \mapsto U(n, \theta)$ be continuous jointly on the unit-direction space times $\mathbb{R}$, while retaining the one-axis lift laws. It then proves, rather than assumes, that both recovered central rates are continuous functions of direction. Conversely every continuous pair α, β produces a jointly regular family. Thus the Task 15 parametrization survives unchanged, but its arbitrary functions are narrowed exactly to continuous functions. The technically nontrivial step is recovery of the angular rate β: fixed-θ continuity would leave integer branch ambiguity, so the proof uses the whole parameter interval, compactness through a tube lemma, a trigonometric lower bound, and the intermediate value property. The relevant Mathlib declarations have been checked against the pinned source revision.

Third, the conditional Part XV no-go becomes a theorem. The inherited exact-transformation conditions force β into the discrete set $\mathbb{Z} + \frac{1}{2}$ and make it antipodally odd; joint regularity makes it continuous. Task 16 constructs an intrinsic continuous path of unit directions from n to $-n$. Along that path a continuous half-odd-valued function cannot change its integer label, so its endpoint values are equal. Antipodal oddness makes them opposite. Hence $\beta(n) = 0$, contradicting $0 \notin \mathbb{Z} + \frac{1}{2}$. Therefore no global jointly regular family can be an exact function of the visible transformation.

The obstruction does not destroy the internal lift. If equality of representatives is weakened only to equality up to the explicit central sign, a global jointly regular family exists and is unique: it is precisely the inherited reference family. Its defect on closed visible coincidences is exactly $\{+1, -1\}$, with the nontrivial sign realized. A negative control proves that this discreteness is not a generic consequence of joint regularity: without the sign normalization a jointly regular family can have a closed-relation defect of modulus $e^{2\pi}$. Hence the continuous central residual and the inherited discrete sign remain logically distinct.

Finally, exact equality can be restored locally. On the proper direction domains

$$\mathrm{Dom}(v) = \{n : h(n,n) = 1,\ h(n,v) > 0\},$$

which cover all unit directions and contain no antipodal pair, Task 16 constructs the families

$$U_k^{\mathrm{loc}}(n,\theta) = z_{0,k+1/2}(\theta)\, U_n^{\mathrm{ref}}(\theta), \qquad k \in \mathbb{Z},$$

and proves exact transformation-valuedness on each such domain. Every admissible local family has pointwise $\alpha = 0$ and $\beta \in \mathbb{Z} + \frac{1}{2}$; every constant label is realized. Two constructed local choices differ by a unique central unit independent of direction, and these transition factors satisfy the expected finite chain law directly by associativity.

The last statement requires a precise editorial qualification. The Task 16 audit headline says "minimal local repair classified", but the same audit explicitly records that domainwise constancy of the integer label has not been proved, that the chosen domains have not been shown minimal or maximal, and that only finite transition chains are treated. Part XVI therefore records the stronger global results as closed while downgrading the local conclusion to the theorem actually established: *local existence plus pointwise local-rate classification, with explicit transition data; full domainwise exhaustion and minimality remain open.* This is the new Conservation-of-Difficulty boundary. Task 17 should not branch into new physics. It should harden precisely this local frontier: prove the geometry and path connectedness of the domains, derive constancy and uniqueness of the integer label on connected domains, characterize the largest class of exact domains supported by the coincidence theorem, upgrade the overlap formulas to arbitrary indexed covers without invoking infinite products, and audit stronger regularity without importing conventional bundle or covering-space structure. Only after that repair is genuinely exhausted should the experiment widen again toward state dynamics.

Keywords: joint direction–parameter regularity; exact axis–angle coincidence; global representative obstruction; central sign defect; half-angle; local exact lift; transition data; formal verification; Lean 4; conservation of difficulty.

Contents

1 The question left hanging by Part XV

Part XV did not end with a missing formula. It ended with a mismatch between two levels of regularity. For each fixed unit axis n, the complete continuous lift class had already been proved to be

$$U(n,\theta) = z_{\alpha(n),\beta(n)}(\theta)\, U_n^{\mathrm{ref}}(\theta), \tag{1}$$

where

$$z_{\alpha,\beta}(\theta) = e^{\alpha\theta}(\cos(\beta\theta)1 + \sin(\beta\theta)S), \qquad S^2 = -1, \tag{2}$$

and the exact centre is

$$Z = \mathrm{Center}(W) = \mathrm{span}_{\mathbb{R}}\{1, S\}. \tag{3}$$

The one-axis group law and continuity had removed every arbitrary function of θ. What remained was one pair of real numbers for each direction, hence two functions α, β on the unit-direction set [10, 3].

The formal Task XV notion of a regular multi-axis family required only

$$\forall n: \quad \theta \longmapsto U(n,\theta) \text{ is a continuous one-axis lift.}$$

It imposed no topology across n. Therefore the theorem that arbitrary functions $\alpha(n), \beta(n)$ occur was exact for its definition, but could not settle genuine continuous $3+1$ coherence.

A second result had already narrowed the possibilities. If the internal value was required to depend only on the represented visible transformation, then Task XV proved the necessary relations

$$\alpha(n) = 0, \qquad \beta(n) \in \mathbb{Z} + \frac{1}{2}, \qquad \beta(-n) = -\beta(n). \tag{4}$$

No sufficiency theorem was available because the full coincidence relation

$$\Phi_n(\theta) = \Phi_m(\varphi) \tag{5}$$

had not yet been classified. Part XV nevertheless identified the critical conditional argument: continuity of β on the connected direction space would appear incompatible with equation (4). It therefore ended with the question

$$\boxed{\text{Can the internal central representative be chosen jointly and globally coherently?}} \tag{6}$$

[10].

Methodological principle 1 (A conditional diagnostic is a task specification, not a theorem)**.** When an obstruction follows only after adding an unproved regularity implication, the next task must prove the implication and the obstruction in the same dependency chain. It is not legitimate to import the expected topological answer merely because the conditional argument resembles a known construction.

Task 16 is exactly this hardening step.

2 Inherited structure, source order, and the two firewalls

The ambient carrier and all noncentral spatial structure remain inherited. Task 16 begins with

$$W = \mathrm{span}_{\mathbb{R}}\{1, A, B, C, P, Q, R, S\}, \qquad Z = \mathrm{span}_{\mathbb{R}}\{1, S\}, \qquad S^2 = -1, \tag{7}$$

together with the inherited real spatial carrier, its positive form h, the linear generator map J, the unit-axis predicate, the reference implementers $U_n(\theta)$, and the visible automorphism family $\Phi_n(\theta)$ [2, 3].

For a unit direction n the inherited reference implementation is

$$U_n(\theta) = \cos(\theta/2)1 - \sin(\theta/2)J(n). \tag{8}$$

This formula is not a Task 16 ansatz. The half-parameter was already forced for the first selected axis in Task 10 by solving an unknown internal-lift problem under the one-parameter law and conjugation requirement; Task 14 then reconstructed the same half-angle law independently for a second axis and extended the generator assignment to arbitrary spatial directions [8, 1, 9, 2].

Proof boundary 1 (Historical dependency of the half-angle)**.** *Task 16 does* not *derive the half-angle from bare Minkowski geometry. It inherits it from the already verified Task 10–14 dependency chain and asks a new question: whether those locally derived half-angle representatives can be made globally coherent over all directions. Consequently the $2\pi/4\pi$ result later in this paper is a consequence of the cumulative reconstruction, not of Task 16 in isolation.*

The Task 16 source order is

$$\begin{gathered}\texttt{SafeBase} \to \texttt{ExactCoincidences} \to \texttt{JointRegularity} \\ \to \texttt{TransformationValuedClosure} \to \texttt{RegularDefect} \\ \to \texttt{LocalRepair} \to \texttt{RateAudit} \\ \to \texttt{OptionCAudit} \to \texttt{Identification} \to \texttt{Task16}.\end{gathered}$$

The first seven modules form the reconstruction and hardening chain. `OptionCAudit` and `Identification` are comparison-only layers imported only at the top. The formal audit mechanically checks that language involving spin groups, quaternions, Pauli matrices, projective representations, coverings, bundles, connections, holonomy and physical particle notions does not enter the reconstruction layer [4].

Methodological principle 2 (Recognizable output must remain downstream)**.** The stronger the derived structure begins to resemble standard spinorial mathematics, the more important the source-order firewall becomes. Recognition is permitted only after the intrinsic coincidence, regularity, obstruction and local-repair theorems have been proved.

3 Exact axis–parameter coincidences

The first hardening obligation is purely algebraic. Task 16 defines

$$\mathrm{SignRelated}(n, m, \theta, \varphi) \iff \exists \varepsilon \in \{+1, -1\} : \quad U_n(\theta) = \varepsilon U_m(\varphi). \tag{9}$$

For unit n, m it then proves the exact equivalence

Theorem 1 (Exact coincidence classification)**.** *For unit spatial directions n, m and all real parameters θ, φ,*

$$\Phi_n(\theta) = \Phi_m(\varphi) \quad \iff \quad \mathrm{SignRelated}(n, m, \theta, \varphi). \tag{10}$$

The coordinate version is

$$\begin{aligned}\Phi_n(\theta) = \Phi_m(\varphi) \quad \iff \quad & \Big[\cos(\theta/2) = \cos(\varphi/2),\ \sin(\theta/2)n = \sin(\varphi/2)m\Big] \\ & \vee \Big[\cos(\theta/2) = -\cos(\varphi/2),\ \sin(\theta/2)n = -\sin(\varphi/2)m\Big].\end{aligned} \tag{11}$$

The same sign controls both the scalar half-angle coordinate and the spatial half-angle coordinate. No unclassified central factor remains: if two reference implementations differ centrally while implementing the same visible transformation, the central factor has vanishing S component and real part whose square is one.

The exceptional cases are then derived from theorem 1:

$$\Phi_n(\theta + 2\pi) = \Phi_n(\theta), \tag{12}$$
$$\Phi_{-n}(\theta) = \Phi_n(-\theta), \tag{13}$$
$$\Phi_n(\theta) = \mathrm{id} \quad \Longleftrightarrow \quad \theta = 2\pi k \text{ for some } k \in \mathbb{Z}, \tag{14}$$
$$\Phi_n(\theta) = \Phi_n(\varphi) \quad \Longleftrightarrow \quad \theta - \varphi = 2\pi k \text{ for some } k \in \mathbb{Z}. \tag{15}$$

Away from the degenerate case $\sin(\theta/2) = 0$, equality of visible transformations forces

$$n = m \quad \text{or} \quad n = -m. \tag{16}$$

At the identity parameters $2\pi k$, by contrast, the direction is completely undetermined.

Status 1 (Work Package 1)**.** The axis–parameter coincidence problem required for the global existence question is closed. The arbitrary spatial word problem is not solved and is not needed for this theorem.

This result changes the character of the next step. Transformation-valuedness is no longer a vague quotient condition: every equality that must be respected is now explicitly visible in the inherited coordinates.

4 Joint direction–parameter regularity

Let

$$\mathbb{S} = \{n : h(n, n) = 1\} \tag{17}$$

with its inherited subspace topology. Task 16 defines a family $U(n, \theta)$ to be jointly regular when

1. for every unit n, $\theta \mapsto U(n, \theta)$ is an inherited axis lift;
2. the map
$$(n, \theta) \longmapsto U(n, \theta)$$
is continuous on $\mathbb{S} \times \mathbb{R}$.

Crucially, no regularity of α or β is placed in the definition.

The first recovered rate is comparatively direct. Task 16 constructs explicit central coordinate functions recA and recB and proves that, for the Task XV representation,

$$\mathrm{recA} = e^{\alpha\theta} \cos(\beta\theta), \qquad \mathrm{recB} = e^{\alpha\theta} \sin(\beta\theta). \tag{18}$$

Hence

$$\mathrm{recA}^2 + \mathrm{recB}^2 = e^{2\alpha\theta}, \tag{19}$$

and evaluating at a fixed nonzero parameter recovers α through a logarithm. Joint continuity of U therefore yields continuity of $n \mapsto \alpha(n)$.

The angular rate is harder because a single circle value cannot distinguish rates that differ by an integer number of turns. The proof therefore does not argue from continuity at one fixed θ. It establishes the analytic lemma

Theorem 2 (Joint circle recovery)**.** *If the map*

$$(x, \theta) \longmapsto (\cos(b(x)\theta), \sin(b(x)\theta))$$

is jointly continuous on the relevant product, then b is continuous.

The formal proof uses compactness of a parameter interval to obtain uniform control, then a trigonometric lower bound to exclude a nonzero jump of the rate. The artifact explicitly uses Mathlib's `generalized_tube_lemma`, `Real.mul_le_sin`, and the interval intermediate-value theorems at the pinned revision [13, 12, 14]. The primary Mathlib source labels `Real.mul_le_sin` as one half of Jordan's inequality.

The result is an exact equivalence.

Theorem 3 (Joint regularity classification)**.** *A family U is jointly regular if and only if there exist functions*

$$\alpha, \beta : \mathbb{S} \to \mathbb{R}$$

continuous in the direction and satisfying

$$U(n, \theta) = z_{\alpha(n),\beta(n)}(\theta)\, U_n(\theta) \tag{20}$$

for every unit n and every θ. Conversely every continuous pair α, β produces such a family.

Status 2 (Work Package 2)**.** Task XV's arbitrary directional functions do not survive unchanged. Joint regularity reduces them exactly to arbitrary *continuous* directional functions. No third function, branch datum or refinement of the one-axis α/β parametrization is required.

The Conservation-of-Difficulty lesson is subtle: stronger regularity has not yet selected the rates. It has made them sufficiently rigid for the discrete transformation identifications to act globally on them.

5 The global exact representative does not exist

The main Task 16 theorem is obtained by combining the Task XV necessary conditions equation (4) with the new continuity theorem. No conventional topology of a spin cover is imported.

Fix a unit direction n. The inherited spatial geometry supplies a unit vector m orthogonal to n. Task 16 constructs

$$\gamma(t) = \cos(t)n + \sin(t)m, \qquad 0 \le t \le \pi. \tag{21}$$

The formal development proves

$$\gamma(0) = n, \qquad \gamma(\pi) = -n, \qquad h(\gamma(t), \gamma(t)) = 1, \tag{22}$$

and continuity of γ.

Now suppose U were both jointly regular and exactly transformation-valued. Then

$$\beta(\gamma(t)) \in \mathbb{Z} + \frac{1}{2} \tag{23}$$

for every t. Task 16 proves directly, using the intermediate value theorem, that a continuous real function constrained to a fixed translate of $\mathbb{Z}$ cannot change its integer label along an interval. Therefore

$$\beta(-n) = \beta(n). \tag{24}$$

But exact transformation-valuedness also gives the inherited antipodal relation

$$\beta(-n) = -\beta(n). \tag{25}$$

Combining equations (24) and (25) yields

$$\beta(n) = 0, \tag{26}$$

contradicting equation (23).

Theorem 4 (Global exact-representative no-go)**.** *There exists no family U satisfying both joint direction–parameter regularity and exact transformation-valuedness:*

$$\boxed{\neg\exists U: \quad \mathsf{JointReg}(U) \wedge \mathsf{TransformationValued}(U).} \tag{27}$$

Task 16 also proves the explicit contraposition form: every jointly regular family has at least one pair of axis–parameter data representing the same visible transformation but carrying unequal internal representatives.

Methodological principle 3 (The missing compatibility is not another continuous equation)**.** Part XV left two continuous rate functions and asked whether stronger coherence would relate them. Task 16 shows that exact global coherence does not contribute one additional scalar relation. It makes the candidate class empty because continuous directional variation is incompatible with a discrete antipodal identification.

This is the point at which the earlier underdetermination changes type. The difficulty has moved from continuous freedom to a discrete global obstruction.

6 The strongest surviving global class is unique

The failure of exact global representatives does not mean that no coherent global family exists. Task 16 weakens only the final equality, not the algebraic carrier or the regularity requirement.

Definition 1 (Sign-relaxed transformation-valuedness)**.** A family U is sign-transformation-valued when equal visible transformations imply

$$U(n,\theta) = U(m,\varphi) \quad \text{or} \quad U(n,\theta) = -U(m,\varphi). \tag{28}$$

The sign remains an actual central carrier element; no quotient is taken.

The inherited reference family

$$U^{\mathrm{ref}}(n,\theta) = U_n(\theta) \tag{29}$$

is jointly regular and satisfies equation (28). More importantly, Task 16 proves uniqueness.

A full visible turn first forces the continuous modulus rate to disappear:

$$\alpha(n) = 0. \tag{30}$$

It confines $\beta(n)$ to a half-integer lattice. Axis reversal again makes β odd. Joint continuity and the same intrinsic path argument then force

$$\beta(n) = 0 \tag{31}$$

for every direction. Therefore the residual factor is identically one.

Theorem 5 (Unique sign-relaxed global family)**.** *If U is jointly regular and sign-transformation-valued, then*

$$U(n,\theta) = U^{\mathrm{ref}}(n,\theta) \tag{32}$$

for every unit n and every θ.

Thus the stronger regularity test does something that Part XV could not: it removes both continuous central rate functions completely once the minimal explicit sign ambiguity is admitted globally.

7 The central defect becomes exactly discrete – but only after normalization

For the unique surviving class, Task 16 proves that whenever

$$\Phi_n(\theta) = \Phi_m(\varphi),$$

there exists a central factor c with

$$U(n, \theta) = c\, U(m, \varphi), \qquad c \in \{+1, -1\} \subset Z^{\times}. \tag{33}$$

The negative sign genuinely occurs: the visible transformation is 2π-periodic while the reference implementation changes sign after one visible period.

The exact result must be distinguished from a false stronger statement. Joint regularity by itself does *not* discretize every central discrepancy. Task 16 constructs a jointly regular negative control whose full-turn discrepancy is

$$-e^{2\pi}, \tag{34}$$

which is neither $+1$ nor -1. Therefore

$$\boxed{\text{joint regularity} \not\Rightarrow \text{discrete central defect.}} \tag{35}$$

The discreteness is a consequence of the sign-relaxed global closure theorem, not a generic property of the centre.

For the normalized class the finite consistency relations remain exact. Same-axis compositions have the expected trivialized discrepancy, and the defect factors satisfy the chain relation inherited from associativity. No central element is identified with another by quotienting.

Methodological principle 4 (Do not merge two central phenomena)**.** The continuous Task XV residual lives in all of $Z^{\times}$ before closure. The ± 1 reference defect is discrete and inherited from the half-angle implementation. Task 16 proves that the first disappears under sign-relaxed global closure while the second remains. Their common location in the centre does not make them the same datum.

8 Local exact representatives

The global no-go singles out the antipodal identification. Task 16 therefore asks whether exact transformation-valuedness can be restored on direction regions that avoid antipodal pairs.

For a fixed spatial vector v, define

$$\mathrm{Dom}(v) = \{n : h(n, n) = 1 \text{ and } h(n, v) > 0\}. \tag{36}$$

These are proper direction domains. The formal development proves two elementary but decisive facts:

$$n \text{ unit} \quad \Longrightarrow \quad n \in \mathrm{Dom}(n), \tag{37}$$

$$n \in \mathrm{Dom}(v) \quad \Longrightarrow \quad -n \notin \mathrm{Dom}(v). \tag{38}$$

Hence the family of such domains covers every unit direction while excluding the exact identification that caused the global contradiction.

For each integer k, Task 16 constructs

$$U_k^{\mathrm{loc}}(n, \theta) = z_{0,k+1/2}(\theta)\, U_n(\theta). \tag{39}$$

The shift by $k + \frac{1}{2}$ is not inserted as a conventional spin label. It is the central rate that cancels the sign branch in the exact coincidence theorem once antipodal direction pairs are excluded from a common domain.

Theorem 6 (Local exact existence)**.** *For every $k \in \mathbb{Z}$ and every v, the family U_k^{loc} is jointly regular and is exactly transformation-valued on* $\mathrm{Dom}(v)$*.*

The same family is proved not to be globally transformation-valued, so local success does not evade the no-go by an accidental stronger theorem.

Task 16 also proves a local necessity statement. If U is jointly regular and exactly transformation-valued on $\mathrm{Dom}(v)$, then at every direction $n \in \mathrm{Dom}(v)$,

$$\alpha(n) = 0, \qquad \beta(n) = k_n + \frac{1}{2} \quad \text{for some } k_n \in \mathbb{Z}. \tag{40}$$

Every value in this pointwise class is realized by one of the constant-label families equation (39).

This is already a strong repair theorem. It says that restricting the direction domain removes the global impossibility and collapses the continuous central freedom to a discrete local remnant.

9 Overlap transitions and finite coherence

Two constructed local families with labels $k, l \in \mathbb{Z}$ satisfy the exact relation

$$U_k^{\mathrm{loc}}(n,\theta) = t_{kl}(\theta)\, U_l^{\mathrm{loc}}(n,\theta), \tag{41}$$

where

$$t_{kl}(\theta) = z_{0,k-l}(\theta). \tag{42}$$

Task 16 proves that $t_{kl}(\theta)$ lies in the exact centre, is invertible there, is independent of the direction n, and is uniquely determined by equation (41) whenever the axis is unit.

The transition factors obey

$$t_{kl}(\theta)t_{lm}(\theta) = t_{km}(\theta). \tag{43}$$

This is proved at the carrier level from the central exponential law and associativity. For arbitrary two regular families, their discrepancy on a common direction is at least central even before one specializes to the explicit constant-label families.

Status 3 (What is genuinely closed locally)**.** The following are theorem-level results: proper domains cover all directions; each contains no antipodal pair; exact local families exist; their continuous scaling rate vanishes; the remaining rate is pointwise half-odd; every constant integer label is realized; and the explicit constant-label families have unique direction-independent central overlap factors satisfying finite composition.

At this point the Task 16 audit uses the phrase “minimal local repair classified”. The theorem inventory itself forces a more careful reading.

10 The local classification gap exposed by the audit itself

The Task 16 audit explicitly records five unresolved obligations. The most important one changes the correct strength of the local verdict.

From equation (40) one knows only

$$\forall n \in \mathrm{Dom}(v)\ \exists k_n \in \mathbb{Z} : \beta(n) = k_n + \frac{1}{2}. \tag{44}$$

Task 16 also knows that β is continuous. What is *not* proved is that the same integer k works throughout the whole domain. Such a theorem requires a connectedness or path-connectedness argument internal to $\mathrm{Dom}(v)$.

Consequently the current Lean development does not yet prove the exhaustive family-level statement

$$\forall U \text{ admissible on } \mathrm{Dom}(v), \quad \exists! k \in \mathbb{Z} : \quad U|_{\mathrm{Dom}(v)} = U_k^{\mathrm{loc}}|_{\mathrm{Dom}(v)}. \tag{45}$$

It proves the pointwise rate condition and proves that each constant label gives an admissible family. Those are not logically identical statements.

A second overreach would concern the word "minimal". Task 16 proves that the domains equation (36) are sufficient and proper. It does not prove that they are minimal, maximal, canonical, or exhaustive among all direction domains on which exact closure is possible. In particular, the exact coincidence theorem suggests that the decisive property may be more abstractly related to exclusion of antipodal pairs, but this broader domain classification has not been formalized.

Third, equation (43) treats finite transition chains. The audit states correctly that arbitrary infinite indexed families of domains have not been treated. This should not be confused with a need to define an infinite product: an arbitrary-index cover can potentially be handled by pairwise and triple-index coherence, but that theorem has not yet been written.

Fourth, no stronger C^1 joint notion has been analyzed. Since continuity already suffices for the global obstruction, this omission does not weaken theorem 4; it is a robustness question for the local classification.

Fifth, the arbitrary spatial word problem remains outside the task. Task 16 classifies axis–parameter coincidences, not every relation among arbitrary products of spatial transformations.

Proof boundary 2 (Corrected Part-XVI status)**.** *For this paper the defensible status is*

Global exact representative obstruction	***CLOSED***
Joint regularity classification	***CLOSED***
Sign-relaxed global class	***CLOSED AND UNIQUE***
Local exact existence	***CLOSED***
Local rate classification	***CLOSED POINTWISE***
Domainwise exhaustion by one integer label	***OPEN***
Minimal/maximal exact-domain classification	***OPEN***
Arbitrary-index overlap coherence	***OPEN****.*

Thus this paper does not repeat the stronger audit headline without qualification.

This is not a failure of Task 16's principal result. It is precisely the next conservation-of-difficulty location created by the success of the global theorem.

11 The complete rate audit

The sequence from Parts XV–XVI can now be summarized without conflating distinct requirements.

Requirement	α	β	spatial rate λ
Axiswise Task-XV regularity	arbitrary function of direction	arbitrary function of direction	independent coordinate
Joint Task-XVI regularity	arbitrary continuous function	arbitrary continuous function	implementation later forces 1
Exact global transformation-valuedness	no family exists		–
Sign-relaxed global closure	0	0	1
Exact closure on $\mathrm{Dom}(v)$	0	pointwise in $\mathbb{Z}+\frac{1}{2}$; every constant label realized	1

Task 16 separately proves that the spatial reparametrization rate is not being fixed by convention. If a rate-reparametrized family is required to implement the inherited $\Phi_n(\theta)$ with the same visible parameter, then

$$\lambda = 1. \tag{46}$$

At generator level, however, the decomposition

$$G_{\lambda,\alpha,\beta}(n) = \alpha 1 + \beta S - \frac{\lambda}{2} J(n) \tag{47}$$

remains injectively separated: equality of two such generators forces equality of λ, α and β separately.

Methodological principle 5 (A forced normalization is not a dynamical coupling)**.** The theorem $\lambda = 1$ fixes the parameter matching required to reproduce the inherited visible transformation. It does not create a functional relation between the spatial generator rate and the central rates. Their algebraic coordinates remain independently recoverable.

12 What became of the Option-C pattern

The heuristic comparison is deliberately late and contributes no theorem to the reconstruction. The earlier Task 1 Option-C dependency shape was schematically

$$\text{two variable quantities} + \text{insufficient compatibility information} \longrightarrow \text{underdetermination.} \tag{48}$$

Part XV, under axiswise regularity only, produced an even looser mathematical structure: two arbitrary directional functions and zero cross-direction relations.

Task XVI changes the answer qualitatively:

1. before closure, two continuous directional freedoms remain;
2. exact global closure does not add one scalar relation – it makes the class empty;
3. sign-relaxed global closure removes both continuous freedoms and leaves one unique family;
4. exact local closure restores existence but leaves a discrete half-odd remnant.

Therefore the principal Task 16 answer is not the old Option-C underdetermination. It is

$$\boxed{\text{continuous freedom} \longrightarrow \text{discrete global obstruction} \longrightarrow \text{local discrete repair.}} \tag{49}$$

In the alternatives used by the formal comparison layer, the principal global answer is C and the local repair has the D-type dependency shape. Alternative B holds only for the sign-relaxed global class.

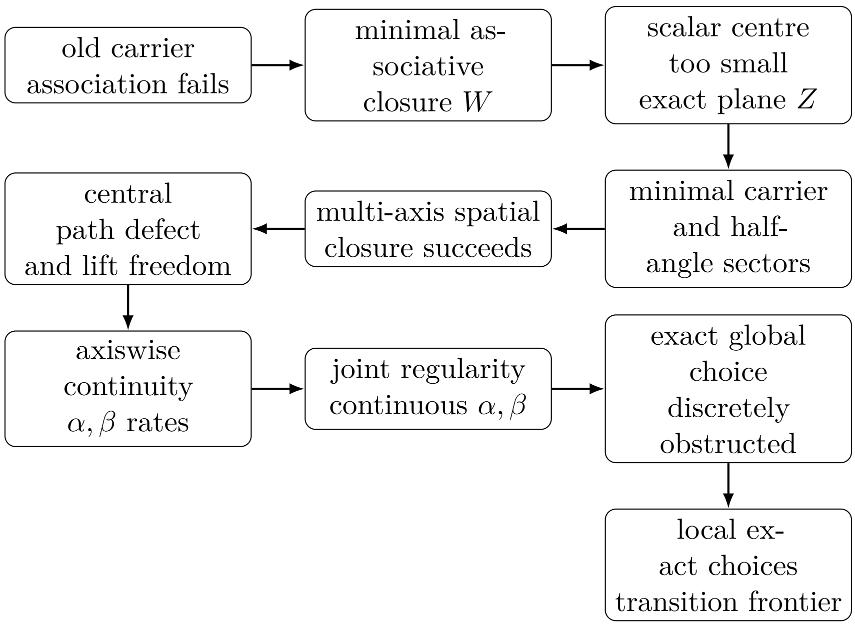
Only now may the old heuristic dictionary be recalled:

Formal datum	Heuristic reading only
$e^{\alpha\theta}$	common marker-density / local-scale change
$e^{\beta S\theta}$	common transport / internal progression rate
± 1 closed-relation residual	discrete internal memory after visible closure
local integer label	discrete local calibration class

None of these heuristic words occurs in the Lean statements or proofs. The mathematical result is stronger than the earlier analogy in one respect and weaker in another: stronger because a genuine discrete obstruction is proved; weaker because no physical meaning for the discrete label is derived.

13 Conservation of Difficulty: the obstruction has become global

The cumulative story from the associative-closure tasks to Task 16 now has a particularly clear migration pattern.



Each successful closure has made the next question narrower rather than eliminating it. Associativity forced a larger carrier; the full spatial algebra then closed; continuous one-axis lift freedom became two rates; joint regularity made those rates continuous; exact global coherence then failed for a discrete reason. The current frontier is therefore not "find the missing global function". The theorem says there is no such function in the exact class.

The mathematical difficulty now lies in describing how local exact representatives fit together and in proving the exact strength of that local classification. This is a more geometric and relational problem than Part XV's rate freedom, but it is still not physical dynamics. No time evolution, Hamiltonian, interaction law, field equation, probability rule, mass or charge has entered Task 16.

Methodological principle 6 (An obstruction is information)**.** When a stronger coherence requirement makes the global class empty, the correct response is not to weaken the proof until a global object reappears. The obstruction identifies which relation cannot be globally satisfied and therefore tells the next task where a local or relational datum must be audited.

14 Formal verification and provenance

The supplied Task 16 Aristotle archive has SHA-256

`203d7a32c0c34b680a355ef43ce09db0acbc9a10f8d9ba34a940f68de0ba8d18`.

The cumulative summary identifies the run as

`71cdf7f1-99f8-4748-b32c-bc3d42779f3c`.

The formal environment is Lean `v4.28.0` with Mathlib revision

`8f9d9cff6bd728b17a24e163c9402775d9e6a365`.

The artifact records

```
lake build
Build completed successfully (8203 jobs).

lake build RequestProject.NullSectorTask16.Task16
Build completed successfully.
```

A mechanical Task 16 scan finds no `sorry`, `admit`, custom `axiom` declaration or `@[implemented_by]`. All 35 principal theorems exposed by `Task16.lean` report exactly

$$[\texttt{propext}, \texttt{Classical.choice}, \texttt{Quot.sound}]. \tag{50}$$

The formalization environment is described in [7, 11, 15].

The audit also identifies the specific imported Mathlib declarations used in the new proofs. The three technically central sources were independently checked at the pinned revision during preparation of this paper: `generalized_tube_lemma` in the compactness source, `Real.mul_le_sin` in the trigonometric bounds source, and `intermediate_value_Icc/Icc'` in the intermediate-value source [13, 12, 14].

Proof boundary 3 (Verification boundary for this paper)**.** *The supplied archive, Task 16 theorem sources, audit, README and cumulative summary were inspected, and the source-level dependencies and central theorem statements were cross-checked. The full Lean build was not independently rerun in the paper-generation environment because that runtime does not provide the project's Lake toolchain. The successful 8203-job build and the axiom outputs are therefore reported from the supplied Aristotle artifact rather than claimed as an independent rebuild.*

This distinction is relevant because the purpose of the paper is to report exactly what is machine-checked, exactly what is inferred at paper level, and exactly what remains open. The local-exhaustion correction in section 10 is part of the same provenance discipline.

15 Late conventional comparison: the two-fold pattern

Only after the reconstruction is fixed does Task 16's final comparison module use conventional language. It restates three already proved equations:

$$\Phi_n(\theta + 2\pi) = \Phi_n(\theta), \tag{51}$$

$$U_n(\theta + 2\pi) = -U_n(\theta), \tag{52}$$

$$U_n(\theta + 4\pi) = U_n(\theta). \tag{53}$$

Together with the global no-go, the unique sign-relaxed family and the local exact repair, this is the characteristic mathematical pattern of a two-fold refinement of the visible transformations. In conventional spin geometry, analogous $2\pi/4\pi$ and central-sign behavior is associated with spinorial lifting [5, 6].

The comparison must be stated at exactly the right strength.

Methodological caveat 1 (No physical spin-1/2 theorem)**.** *Task 16 does not derive a physical spin observable, fermionic statistics, a Hilbert-space probability rule, a spin measurement law, a Dirac field, or a complete Lorentz-spin representation. It also does not derive the half-angle from bare Lorentz geometry at this stage; that factor was inherited from the previous algebraic reconstruction. What is formally established is a spinorially characteristic internal* $2\pi/4\pi$ *and* ± 1 *structure, together with its global exact-representative obstruction, without importing a spin group,* $SU(2)$*,* $SO(3)$*, a covering-space theorem or bundle language into the reconstruction.*

This is already a substantial structural result. The conventional name is recognizable only after the dependency chain has produced the phenomenon on its own.

16 What Task XVI does not yet close

The global theorem is stronger than the Part XV diagnostic, but the experiment should resist converting that success into claims not present in the formal package.

1. **No exhaustive domainwise local classification yet.** Pointwise half-oddness is proved; constancy of the integer label on a connected domain is not.
2. **No minimality theorem for** $\mathrm{Dom}(v)$**.** These domains are sufficient, cover all directions and exclude antipodes. Their optimality among admissible domains is unproved.
3. **No arbitrary-index cover theorem.** Pairwise transitions and a three-label chain identity are proved. An arbitrary indexed local system is not formalized.
4. **No stronger differentiable directional audit.** Continuity suffices for the no-go; C^1 or smooth versions remain a robustness question.
5. **No arbitrary spatial word presentation.** Exact axis–parameter coincidences are classified, not all relations among arbitrary products.
6. **No new physical dynamics.** The surviving local integer labels and central transition factors are mathematical data. Their physical interpretation remains open.

The first two items are not peripheral. They determine whether the phrase "minimal local repair" is theorem-level or merely a plausible summary. They should therefore be resolved before a broader physical branch is opened.

17 Task XVII: hardening the local repair

Task 17 should be narrower than Task 16 in subject matter but more exhaustive in proof obligations. Its purpose is not to rediscover the global obstruction. That theorem is closed. Its purpose is to turn the current local existence result into an exact domainwise classification and to determine precisely which parts of the "minimal repair" language are justified.

A suitable neutral title is

Task XVII – Domainwise Exhaustion and Local-Repair Hardening: Path Connectedness, Exact-Domain Geometry, Transition Coherence, and Higher-Regularity Audit.

The following work packages are deliberately detailed because each one removes a specific ambiguity in the Task 16 audit.

17.1 Work Package 1: intrinsic geometry of the proper domains

For

$$\mathrm{Dom}(v) = \{n : h(n,n) = 1,\ h(n,v) > 0\},$$

separate the degenerate and nondegenerate cases. If $v = 0$, the domain is empty; if $v \neq 0$, normalize only when that normalization has been derived from the inherited positive form. Prove, for every nonempty domain used in the local repair:

1. openness in the inherited unit-direction topology;
2. nonemptiness under an explicit hypothesis on v;
3. absence of antipodal pairs;

4. path connectedness, preferably by an explicit path rather than by importing a conventional hemisphere theorem;
5. connectedness as a corollary;
6. stability of the defining strict positivity along the chosen path.

A robust candidate path between $n, m \in \mathrm{Dom}(v)$ is normalized linear interpolation. Let

$$q(t) = (1-t)n + tm, \qquad 0 \le t \le 1.$$

Since

$$h(q(t), v) = (1-t)h(n, v) + th(m, v) > 0,$$

$q(t)$ cannot vanish. One may therefore normalize $q(t)$ using the inherited positive norm and prove that the normalized path remains in $\mathrm{Dom}(v)$. This route is preferable to importing spherical-geodesic language because the positivity certificate is visible in the proof.

17.2 Work Package 2: domainwise constancy of the discrete rate

Take an arbitrary jointly regular family U that is exactly transformation-valued on a fixed nonempty $\mathrm{Dom}(v)$. Reuse Task 16 to obtain

$$\alpha(n) = 0, \qquad \beta(n) \in \mathbb{Z} + \frac{1}{2}$$

for every n in the domain and continuity of β.

Then prove, using the explicit path from Work Package 1 and an intrinsic interval argument, that

$$\exists k \in \mathbb{Z}\ \forall n \in \mathrm{Dom}(v): \quad \beta(n) = k + \frac{1}{2}. \tag{54}$$

Do not take "a continuous map to a discrete set is constant" as an opaque imported slogan if a short interval proof using the already employed intermediate-value mechanism is available. The aim is to keep the critical dependency transparent.

The resulting theorem should upgrade `local_rates` from pointwise necessity to domainwise necessity.

17.3 Work Package 3: exhaustive classification and uniqueness on one domain

Combine equation (54) with the inherited regular-family parametrization. Prove the family-level theorem

$$\mathsf{JointReg}(U) \wedge \mathsf{TransformationValuedOn}(\mathrm{Dom}(v), U) \Longrightarrow \exists! k \in \mathbb{Z}: \quad U(n, \theta) = U_k^{\mathrm{loc}}(n, \theta) \tag{55}$$

for all $n \in \mathrm{Dom}(v)$ and all θ.

The uniqueness of k must be proved, not inferred from visual distinctness of the formulas. It should follow from uniqueness of the recovered β rate or from injectivity of the central exponential classification under the exact hypotheses already available.

This theorem is the missing statement that would justify saying "one integer label per connected proper domain" rather than merely "every point has a half-odd label and every constant label is realizable".

17.4 Work Package 4: characterize the admissible exact domains

The domains $\mathrm{Dom}(v)$ were sufficient, but Task 16 did not show that they are the only useful domains. Use the exact coincidence theorem rather than geometric intuition to study a general subset D of the unit directions.

At minimum test the following necessity/sufficiency boundary:

$$D \cap (-D) = \varnothing. \tag{56}$$

Questions to close are:

1. Is antipodal-freeness necessary for any exact transformation-valued jointly regular family on D?
2. Is antipodal-freeness sufficient for the explicit U_k^{loc} families to be exact on D?
3. If not, which additional exceptional coincidence from theorem 1 obstructs sufficiency?
4. Among open connected domains, what is the exact classification of those admitting an exact local family?
5. Are $\mathrm{Dom}(v)$ maximal within any natural class, or merely a convenient canonical cover?

Do not build a "minimal domain" theorem unless the order notion of minimality has first been defined. There are several inequivalent possibilities: minimal by inclusion, maximal admissible domain, minimal number of cover sets, or minimal amount of transition data. Task 17 should state explicitly which one is being tested.

17.5 Work Package 5: arbitrary-index transition coherence

Suppose a family of admissible domains D_i is indexed by an arbitrary type I, and each domain carries a classified local label k_i. On nonempty overlaps define

$$t_{ij}(\theta) = z_{0,k_i-k_j}(\theta). \tag{57}$$

Prove uniformly for all indices:

$$t_{ii}(\theta) = 1, \tag{58}$$

$$t_{ij}(\theta)^{-1} = t_{ji}(\theta), \tag{59}$$

$$t_{ij}(\theta)t_{jk}(\theta) = t_{ik}(\theta), \tag{60}$$

with uniqueness whenever the overlap contains an admissible direction.

This closes the audit's "only finite chains" caveat at the correct logical level. An arbitrary indexed cover does not require an infinite product. It requires universally quantified pairwise and triple-overlap laws. If a genuinely infinite composition is later desired, that would be a separate analytic problem and should not be smuggled into this hardening task.

17.6 Work Package 6: reconstruct local families from transition data

Test the converse direction. Given a family of local labels or transition factors satisfying the exact algebraic laws, determine whether the local representative family is uniquely reconstructed on each domain up to one base-domain label. Distinguish carefully among:

1. data that are forced by already chosen local families;
2. data sufficient to reconstruct such families;

3. data unique only after fixing one reference label;

4. data invisible to the visible automorphisms.

This is the proper place to determine whether "one integer per domain" and "one central transition per overlap" are genuinely equivalent descriptions. Task 16 proved one direction for its explicit families; Task 17 should prove the exact equivalence or state the remaining asymmetry.

17.7 Work Package 7: stronger regularity as a robustness test

Only after the continuous local classification is complete, introduce a stronger directional regularity notion if it can be formulated without changing the carrier. Possible targets are C^1 or smooth dependence on the unit-direction manifold inherited from the finite-dimensional real spatial space.

The question is not whether stronger regularity rescues a global exact representative – continuity already proves that impossible. The useful questions are:

1. does a C^1 jointly regular family force α and β to be C^1 on each domain?

2. are the classified local families automatically smooth in direction and parameter?

3. do transition factors acquire any additional restriction beyond their already derived direction independence?

4. does stronger regularity reduce the discrete local label class? A negative result is expected but must not be presumed.

If this work package becomes disproportionately expensive, it should be explicitly marked optional after the continuous domainwise exhaustion is closed. It must not delay the primary correction of the Task 16 local classification.

17.8 Work Package 8: hard negative controls and source-order audit

Retain the same reconstruction discipline. In particular prove or mechanically verify that the hardening does not depend on:

1. a preferred basis axis;

2. a preferred minimal carrier;

3. a preselected conventional hemisphere theorem if the path can be constructed intrinsically;

4. a spin group, $SU(2)$, $SO(3)$, covering-space, bundle, cocycle, connection or holonomy formalism;

5. quotienting the central sign;

6. an assumed global integer label;

7. an assumed connectedness theorem whose hypotheses have not been checked for the actual subtype $\mathrm{Dom}(v)$.

The comparison module may, again, name the standard structure only after the local exhaustion theorem and transition coherence are fixed.

17.9 Target Task-XVII verdicts

Task 17 should end with a table that separates the possible outcomes rather than forcing one expected result. The decisive entries should be:

Question	Required final status
Is every nonempty $\mathrm{Dom}(v)$ path connected?	proved or explicit counterexample
Is β constant in its integer label on each such domain?	proved or exact obstruction
Does every admissible family equal a unique U_k^{loc} on the domain?	exhaustive theorem or exact residual freedom
What property of a general direction set D is necessary and sufficient for exact local closure?	classified as sharply as Task 16's coincidence theorem permits
Are $\mathrm{Dom}(v)$ minimal, maximal, canonical, or merely sufficient?	one precise notion, no ambiguous use of "minimal"
Do arbitrary-index transition data satisfy identity, inverse and triple-overlap laws?	proved without infinite-product sleight of hand
Are local representatives reconstructible from those transition data?	exact necessity/sufficiency statement
Does stronger regularity change the discrete local class?	theorem or explicitly deferred robustness question

A satisfactory Task 17 endpoint would justify replacing the current local status by something genuinely stronger, for example

LOCAL EXACT FAMILIES EXHAUSTIVELY CLASSIFIED ON THE SPECIFIED DOMAIN CLASS; TRANSITIONS COMPLETE.

But this sentence is a target, not a present result.

18 Why Task XVII should precede broader dynamics

The temptation after equations (51) to (53) is to move immediately toward familiar physical spin dynamics. The dependency audit argues for the opposite order.

Task 16 has shown that the internal representative cannot be made globally exact. Therefore any later state dynamics that uses these representatives must know whether its basic state object is global only up to sign, local with transition data, or something still less rigid. That is not a cosmetic mathematical choice. It determines the domain on which future dynamical variables would even be defined.

At the same time, Task 17 is sharply bounded. It should not open boosts, interactions, field equations, mass terms, gauge dynamics or probability theory. The current obstruction is already sufficiently localized that a broader task would mix new assumptions with an unresolved local-representation question.

Methodological principle 7 (Harden before interpreting)**.** A recognizable $2\pi/4\pi$ signature is not permission to import the physical theory that usually accompanies it. The correct next step is to close the exact mathematical object that the signature has forced: its domainwise representatives and their transition law.

Once that layer is closed, the Conservation-of-Difficulty method can legitimately ask the next broader question: whether any further state or dynamical law is forced by the local/global structure or remains independent input.

19 Conservation-of-Difficulty ledger

Question	Part-XVI status
Is the exact axis–parameter coincidence relation known?	Yes. Equal visible automorphisms are exactly reference implementers equal up to ± 1, with all periodic, antipodal and degenerate cases derived.
Does joint (n, θ) continuity force directional regularity of the Task-XV rates?	Yes. Both α and β are continuous; conversely every continuous pair occurs.
Does joint regularity eliminate central freedom by itself?	No. It leaves arbitrary continuous α, β.
Does a global jointly regular exact transformation-valued family exist?	No. The nonexistence theorem is closed.
What causes the no-go?	A discrete incompatibility: continuous half-odd β versus antipodal oddness along an intrinsic path from n to $-n$.
Is there a global weaker class?	Yes. Equality up to the explicit central sign is possible and selects exactly the inherited reference family.
What is its closed-relation defect?	Exactly $\{+1, -1\}$, with the nontrivial sign realized.
Is every jointly regular defect automatically ± 1?	No. A negative control realizes $-e^{2\pi}$.
Can exact equality be restored locally?	Yes, on every $\mathrm{Dom}(v)$ by every explicit U_k^{loc}.
What rates survive locally?	$\alpha = 0$ and pointwise $\beta \in \mathbb{Z} + \frac{1}{2}$; every constant label is realized.
Are the explicit overlap factors central and direction-independent?	Yes, uniquely, with the finite chain law $t_{kl} t_{lm} = t_{km}$.
Is one integer label proved constant over the entire domain for every admissible family?	No. This is the principal Task-XVII hardening obligation.
Are the domains $\mathrm{Dom}(v)$ proved minimal or maximal?	No. They are sufficient proper covering domains only.
Is arbitrary-index cover coherence proved?	No. Pairwise/finite-chain formulas are proved; a universally indexed local system is not yet formalized.
Has the Option-C underdetermination survived?	No. It is replaced globally by a discrete obstruction and locally by discrete repair data.
Has physical spin-1/2 been derived?	No. A spinorially characteristic two-fold internal structure has been derived; the physical interpretation remains outside the proved boundary.
What should be hardened next?	Domain geometry, label constancy, family-level exhaustion, admissible-domain classification, arbitrary-index transition coherence, and only then stronger regularity.

20 Conclusion

Part XV left the experiment at a carefully exposed gap. Axiswise continuity had reduced each lift to two central rates, but the formal definition allowed those rates to vary arbitrarily from direction to direction. A stronger transformation-valued audit had already forced the real scaling rate to zero, the angular central rate into $\mathbb{Z} + \frac{1}{2}$, and antipodal oddness, but it had not classified all visible coincidences or proved global existence or nonexistence. The next difficulty was therefore concentrated in one question: can the internal representative be chosen jointly and globally over all directions?

Task 16 answers that question decisively. The exact axis–parameter coincidence relation is classified. Joint regularity is classified and is shown to be equivalent to continuity of the two recovered directional rate functions. These two results turn Part XV's conditional diagnostic into a theorem: exact global representatives do not exist. The contradiction is discrete and intrinsic. A continuous half-odd rate cannot remain antipodally odd along the derived path connecting a direction to its reversal.

The failure is highly structured rather than destructive. If equality is weakened only by the explicit central sign, the global class becomes nonempty and in fact rigid: the inherited reference family is the unique jointly regular solution. Its visible period is 2π, its internal representative changes sign after one visible period and returns after 4π, and its closed-relation defect is exactly ± 1. Joint regularity alone does not explain this discreteness; the negative control preserves the distinction between the continuous central residual and the inherited sign.

Exact equality reappears on proper direction domains. There the continuous central freedom collapses to pointwise half-odd values, every constant integer label is realized, and explicit local choices are related by unique direction-independent central transition factors satisfying finite composition. The global difficulty has therefore not disappeared. It has been localized.

That final sentence must be read with the same discipline as the earlier parts of this series. Task 16 has not yet proved that every admissible family carries one constant integer label throughout a whole domain, nor that the chosen domains are minimal or maximal, nor that an arbitrary indexed cover has been exhausted. Those are now small enough to be attacked directly, and large enough that skipping them would blur the exact object produced by the global obstruction.

The Conservation-of-Difficulty frontier after Part XVI is therefore

> The global exact representative is impossible.
>
> What is the exact, exhaustive local replacement – its admissible domains, discrete labels, and transition law?

Task XVII should answer this question before the project widens toward physical state dynamics. If the hardening succeeds, the project will have moved from an internally derived half-angle action, through a global discrete obstruction, to a fully characterized local relational structure without importing its conventional name. Only then will the next dynamical question be posed on a mathematically closed foundation rather than on a suggestive analogy.

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