Split Complex Lorentzian Dynamics

Part XVII: Local Exhaustion, Non-Hemispherical Admissibility, and the Reconstruction Mismatch

When the local problem closes and the global difficulty changes type

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Abstract

Part XVI ended by refusing to promote a suggestive local picture into a theorem. Task 16 had proved the global jointly regular exact representative impossible, had identified the unique global sign-relaxed reference family, and had constructed exact local representatives on the inherited domains

$$\mathrm{Dom}(v) = \{n : h(n,n) = 1,\ h(n,v) > 0\},$$

but it had only pointwise classified the local rate and had not proved that one integer label exhausts a whole domain. It also had not shown that these domains were optimal, nor that overlap formulas for explicit models exhausted arbitrary local representatives. Task 17 was therefore posed as a hardening task rather than a new physical branch.

The hardening succeeds, but it does not merely confirm the expected picture. First, every nonempty inherited domain is proved intrinsically open, path-connected and antipodal-free. Joint regularity plus local exactness then forces the second central rate to be constant on the whole domain, and every locally exact family is proved equal, for every direction in the domain and every real parameter, to exactly one model U_k^{loc}, $k \in \mathbb{Z}$. Thus the local family-level gap left by Part XVI is genuinely closed.

Second, the geometry of exact domains turns out to be substantially larger than the inherited hemispherical class. Task 17 defines admissibility for an arbitrary set of unit directions and proves an exact functional criterion: a domain is admissible if and only if there exists a rate function continuous on the full unit-direction space, half-odd on the domain, and odd on each antipodal pair contained in the domain. Antipodal-freeness is sufficient, but—counter to the first attempted proof—it is not necessary. The disconnected two-point set $\{n, -n\}$ is admissible with opposite rates $+\frac{1}{2}$ and $-\frac{1}{2}$. At the same time, on every preconnected admissible domain the half-odd rate is forced to be constant. Hence a connected admissible domain cannot contain an antipodal pair. Combining two Task 17 theorems yields an immediate paper-level corollary not packaged as a named Lean endpoint: among preconnected sets of unit directions, admissibility is equivalent to antipodal-freeness. This corollary should be formalized explicitly in Task 18.

Third, the inherited domains are neither exhaustive nor maximal. Aristotle constructs an explicit open, path-connected, antipodal-free three-leg domain Y that is admissible yet contained in no $\mathrm{Dom}(v)$, and proves that every nonempty $\mathrm{Dom}(v)$ can be strictly enlarged while retaining admissibility. The phrase "minimal local repair" from the previous stage is therefore not merely unproved; as an optimality claim for the inherited domains it is false.

Fourth, arbitrary local exact representatives are compared without assuming the model formula. Their relative factor is derived to be central and unique; on a preconnected overlap it is direction-independent, equals a one-parameter central element with an integer rate equal to the difference of the two local labels, is multiplicative in the transformation parameter, and composes along arbitrary finite chains. This closes the finite overlap problem at the level actually asked in Task 17.

The most important result, however, is the one that requires the most careful interpretation. Task 17 exhibits a covering system of local exact representatives for which every local family is literally the same global family U_0^{loc}, every overlap factor is the unit, and yet no global *sign-relaxed* family can restrict to those local representatives by literal equality. The theorem is correct, but it is not an ordinary failure of gluing: the local data already glue globally, namely to U_0^{loc}. What fails is membership of that glued family in a separately required global class. The unique jointly regular sign-relaxed family is U^{ref}, whose recovered second rate is zero, whereas exactness on any nonempty local domain forces the rate to be half-odd. Thus literal equality between an exact local representative and the global sign-relaxed representative is already incompatible at the normalization level. This is distinct from the earlier global-exact no-go, where continuity, half-oddness and reversal oddness become incompatible only when the whole direction space is considered.

This distinction changes the Conservation-of-Difficulty frontier. The remaining question is no longer whether local exact representatives exist, nor whether their ordinary overlaps are coherent. Both are closed. The next question is which explicit central conversion data relate the half-odd local exact class to the zero-rate global sign-relaxed class, whether the integer overlap factors are differences of those conversion data, and whether a global sign-relaxed representative can be reconstructed after allowing such explicit local conversion rather than demanding literal equality. Task 18 is formulated around that question, together with three hardening obligations exposed by the audit itself: formalizing the connected-domain criterion, classifying maximal admissible connected domains, and determining whether the claimed neutrality of C^1 regularity really holds for arbitrary admissible sets rather than only for the subclasses formally tested.

Keywords: local exact representative; joint regularity; half-odd rate; admissible direction domain; antipodal relation; non-hemispherical domain; central overlap factor; reconstruction; normalization mismatch; formal verification; Lean 4; conservation of difficulty.

Contents

1 The boundary inherited from Part XVI

Part XVI closed the global question and deliberately left the local one open [6, 1]. The inherited jointly regular family has the form

$$U(n,\theta) = z_{\alpha(n),\beta(n)}(\theta)\, U_n(\theta), \tag{1}$$

where U_n is the already reconstructed reference implementer and joint regularity forces α and β to be continuous functions of direction. Exact transformation-valuedness at one direction forces

$$\alpha(n) = 0, \qquad \beta(n) \in \mathbb{Z} + \frac{1}{2}. \tag{2}$$

Globally, exact transformation-valuedness additionally forces reversal oddness,

$$\beta(-n) = -\beta(n), \tag{3}$$

and the coexistence of continuity, (2) and (3) yields the Task 16 no-go.

The local repair of Task 16 used

$$\mathrm{Dom}(v) = \{n :\ h(n,n) = 1,\ h(n,v) > 0\} \tag{4}$$

and the explicit families

$$U_k^{\mathrm{loc}}(n,\theta) = z_{0,k+1/2}(\theta)\, U_n(\theta), \qquad k \in \mathbb{Z}. \tag{5}$$

Task 16 proved that every U_k^{loc} is exact on every $\mathrm{Dom}(v)$ and that every arbitrary locally exact family has the pointwise rate restrictions (2). What it did *not* prove was constancy of k throughout the domain, family-level exhaustion, or any optimality of the domains.

Task 17 was designed to close exactly those gaps. Its source order is

`SafeBase` → `DomainGeometry` → `RateRigidity` → `LocalExhaustion` → `AdmissibleDomains` →
`OverlapTransitions` → `Reconstruction` → `RegularityAudit` → `Identification`.

The final identification module is comparison-only and is imported only by the Task 17 top module [2].

Methodological principle 1 (What Part XVII records)**.** The paper follows the theorem boundary rather than the intended narrative. A result that refutes an intended intermediate claim is part of the result, not noise to be removed. Conversely, a suggestive interpretation of a proved theorem is not promoted to a proved theorem merely because it fits the expected conventional picture.

2 Verification boundary and the "REVIEW SUGGESTED" status

The Aristotle user interface surfaced the completed Task 17 run with the workflow label `REVIEW SUGGESTED`. The supplied archive itself does not describe the formal development as incomplete. Its summary begins with "Task 17 is complete", and its audit reports a successful full build and focused build. Because the platform semantics of the workflow label are not part of the formal archive, this paper treats `REVIEW SUGGESTED` as provenance metadata, not as a mathematical status.

The supplied archive is run

`1fe8d6fc-ec55-42db-a7f4-350971665d0a`

and has SHA-256

`450f847ec72b1469140e7c36ac9cd058fc563965fc9dc057269a0423ba2923c6`.

It records Lean `leanprover/lean4:v4.28.0` and mathlib `v4.28.0` at commit `8f9d9cff6bd728b17a24e163c9402775d9e6a365` [8]. The archive audit reports

- `lake build`: successful, 8213 jobs;
- `lake build RequestProject.NullSectorTask17.Task17`: successful;
- no warnings in Task 17 modules;
- no `sorry`, `admit`, custom `axiom`, `@[implemented_by]`, `native_decide`, or `bv_decide`;
- all 43 principal endpoints reporting exactly `[propext, Classical.choice, Quot.sound]`.

A local static scan of the supplied Task 17 source independently confirms the absence of those prohibited proof escapes. The present document environment does not contain the pinned Lean/Lake toolchain, so the 8213-job build was not independently rerun here. The build result is therefore an archive-reported verification claim, while the source inventory, hashes and static scan were independently rechecked.

Proof boundary 1 (Editorial review is still mathematically useful)**.** *A green formal build certifies the formal statements that were actually encoded. It does not certify that prose summaries have selected the strongest correct theorem, that two differently named obstructions have not been conflated, or that a claimed equivalence has been proved for the full advertised class. Part XVII therefore reviews the theorem statements and their quantifiers separately from the audit prose.*

3 The inherited domains are now genuinely understood

Task 17 first closes the geometric gap in $\mathrm{Dom}(v)$. Define

$$\|w\| = \sqrt{h(w,w)}, \qquad \widehat{w} = \|w\|^{-1}w,$$

and for two domain points n_0, n_1,

$$q(t) = (1-t)n_0 + tn_1, \qquad \gamma(t) = \widehat{q(t)}, \qquad t \in [0,1]. \tag{6}$$

Because both endpoint pairings with v are strictly positive,

$$h(q(t), v) = (1-t)h(n_0, v) + th(n_1, v) > 0. \tag{7}$$

Hence $q(t) \neq 0$, normalization is defined, and the normalized path remains in $\mathrm{Dom}(v)$.

Theorem 1 (Task 17 domain geometry)**.** *For the inherited domains* $\mathrm{Dom}(v)$*:*

(i) $\mathrm{Dom}(v) \neq \varnothing$ *if and only if* $v \neq 0$*;*

(ii) $\widehat{v} \in \mathrm{Dom}(v)$ *if and only if* $v \neq 0$*;*

(iii) $\mathrm{Dom}(cv) = \mathrm{Dom}(v)$ *for* $c > 0$*;*

(iv) $\mathrm{Dom}(v)$ *is open in the inherited unit-direction space;*

(v) $\mathrm{Dom}(v)$ *contains no pair* $n, -n$*;*

(vi) every nonempty $\mathrm{Dom}(v)$ *is path-connected by the explicit path* (6)*, hence connected and preconnected.*

This matters because the local rate is not merely discrete; it is a continuous map into a discrete affine lattice. Once connectedness is no longer assumed but proved, the domainwise rigidity step becomes legitimate.

4 The local family classification is closed

Task 17 deliberately avoids the opaque shortcut "continuous map into a discrete set is locally constant". Instead it proves the relevant half-odd rigidity using the intermediate-value property on the actual inherited domain. If two values are distinct,

$$k + \frac{1}{2} \quad \text{and} \quad \ell + \frac{1}{2},$$

then some integer value such as $k + 1$ lies strictly between them. A continuous image of a preconnected set must contain the intermediate value, contradicting half-oddness.

Theorem 2 (Domainwise rate rigidity)**.** *Let $v \neq 0$, let U be jointly regular, and suppose U is exactly transformation-valued on* $\mathrm{Dom}(v)$*. Then there is a unique integer k such that for every $n \in \mathrm{Dom}(v)$,*

$$\alpha_U(n) = 0, \qquad \beta_U(n) = k + \frac{1}{2}. \tag{8}$$

The result is then lifted from rates to families.

Theorem 3 (Complete local exhaustion on $\mathrm{Dom}(v)$)**.** *For $v \neq 0$ and a jointly regular family U,*

$$U \textit{ exact on } \mathrm{Dom}(v) \iff \exists!\, k \in \mathbb{Z} : \; U(n, \theta) = U_k^{\mathrm{loc}}(n, \theta) \tag{9}$$

for every $n \in \mathrm{Dom}(v)$ and every $\theta \in \mathbb{R}$.

This closes the principal gap explicitly recorded in Part XVI. The phrase "local family indexed by one integer" is now theorem-level on every nonempty inherited domain.

Status 1 (Closed local question)**. LOCAL EXACT EXISTENCE: PROVED.**
DOMAINWISE RATE CONSTANCY ON $\mathrm{Dom}(v)$: PROVED.
FAMILY-LEVEL EXHAUSTION ON $\mathrm{Dom}(v)$: PROVED.
UNIQUENESS OF THE INTEGER LABEL: PROVED.

The difficulty does not remain here. It moves immediately to the geometry of the domain class itself.

5 Admissibility is larger than the inherited hemispherical class

Task 17 now drops the special shape $\mathrm{Dom}(v)$. For an arbitrary set D of directions it defines

$$\mathrm{Adm}(D) \iff (\forall n \in D,\; h(n, n) = 1) \;\wedge\; \exists U : \; U \text{ jointly regular and exact on } D. \tag{10}$$

The exact theorem is functional rather than purely geometric.

Theorem 4 (Task 17 admissibility criterion)**.** *Assume every point of D is a unit direction. Then D is admissible if and only if there exists a real-valued function b, continuous on the full unit-direction space, such that*

$$b(n) \in \mathbb{Z} + \frac{1}{2} \qquad \textit{for every } n \in D, \tag{11}$$

$$b(-n) = -b(n) \qquad \textit{whenever } n, -n \in D. \tag{12}$$

The corresponding exact representative is

$$U_b(n, \theta) = z_{0,b(n)}(\theta) U_n(\theta). \tag{13}$$

This theorem should be read exactly as stated. It is a necessary-and-sufficient criterion, but it classifies domains through the existence of a globally continuous auxiliary rate. It is not yet a closed geometric description of every possible shape of D.

5.1 First counterintuitive result: an antipodal pair is admissible

The first attempted proof in Task 17 tried to show that the presence of n and $-n$ already obstructs local exactness. The branch equations forced only

$$b(-n) = -b(n),$$

which is perfectly compatible with half-oddness if the two points are not linked by connectedness. Aristotle therefore replaced the intended theorem by a counterexample.

Theorem 5 (Antipodal-pair admissibility)**.** *For every unit direction n, the disconnected two-point set*

$$D = \{n, -n\}$$

is admissible. One explicit globally continuous rate is

$$b(m) = \frac{1}{2}h(m, n), \tag{14}$$

which gives

$$b(n) = \frac{1}{2}, \qquad b(-n) = -\frac{1}{2}.$$

This result is not a contradiction with the global no-go. The global obstruction uses more than the existence of the antipodal pair: it uses continuity along the connected unit-direction space while remaining half-odd at every point. The disconnected pair allows the two opposite half-odd values without requiring a half-odd interpolation between them.

5.2 Antipodal-freeness is nevertheless sufficient

Task 17 proves a broad converse.

Theorem 6 (Antipodal-free sufficiency)**.** *Every antipodal-free set D of unit directions is admissible. In fact every constant-label model U_k^{loc} is exact on D.*

Thus the inherited domains are only one convenient family inside a much larger exact-domain class.

6 Connectedness resolves the apparent antipodal paradox

The two facts just established look contradictory only if connectedness is left implicit. Task 17 separately proves that the recovered half-odd rate is constant on every preconnected admissible domain.

Theorem 7 (Task 17 preconnected rigidity)**.** *If D is a preconnected set of unit directions, U is jointly regular and exact on D, then*

$$\beta_U(n) = \beta_U(m)$$

for all $n, m \in D$.

Combining theorem 7 with the reversal condition in theorem 4 yields a consequence that the Task 17 audit lists as open in a stronger-sounding form but does not package as a named endpoint.

Corollary 1 (Paper-level connected-domain criterion)**.** *Let D consist of unit directions and assume its inherited subspace is preconnected. Then*

$$\mathrm{Adm}(D) \iff D \text{ contains no antipodal pair.} \tag{15}$$

Proof. If D is antipodal-free, admissibility is exactly theorem 6. Conversely, let D be admissible and suppose $n, -n \in D$. Preconnected rigidity gives $\beta(-n) = \beta(n)$, while exactness gives $\beta(-n) = -\beta(n)$. Hence $\beta(n) = 0$. But local exactness also forces $\beta(n) \in \mathbb{Z} + \frac{1}{2}$, impossible. $\square$

Proof boundary 2 (Formal status of corollary 1)**.** *Every ingredient of the proof is a Task 17 theorem, but the equivalence itself is not exposed as a named Lean theorem in the supplied archive. It is therefore a paper-level corollary of formally verified endpoints, not an independently checked Task 17 endpoint. Task 18 should formalize it explicitly.*

This already sharpens one item in the Task 17 unresolved-obligation list. What remains open is not whether connected admissibility has *some* geometric criterion; the criterion *antipodal-free* follows immediately. What remains nontrivial is the geometry of maximal, canonical, or otherwise distinguished connected antipodal-free domains.

7 The inherited domains are neither exhaustive nor maximal

Task 17 constructs an explicit domain Y from three open linear wedges. The three preferred equatorial directions are

$$p_1 = (1,0,0), \qquad p_2 = (0,1,0), \qquad p_3 = \left(-\frac{3}{5}, -\frac{4}{5}, 0\right),$$

with corresponding transverse directions

$$m_1 = (0,1,0), \qquad m_2 = (-1,0,0), \qquad m_3 = \left(\frac{4}{5}, -\frac{3}{5}, 0\right).$$

A leg is defined by two strict linear inequalities of the form

$$|h(n,m)| < \frac{1}{100} h(n,p) + n_3, \tag{16}$$

and Y is the union of the three unit-direction legs.

Theorem 8 (Non-hemispherical admissible domain)**.** *The domain Y is*

(i) antipodal-free;

(ii) admissible;

(iii) open in the unit-direction space;

(iv) path-connected;

(v) contained in no inherited domain $\mathrm{Dom}(v)$*.*

The non-containment is certified by four explicit members whose positive pairing with a hypothetical common v would violate a positive linear combination relation. In particular, normalized versions of

$$(1,0,-10^{-3}), \quad (0,1,-10^{-3}), \quad \left(-\frac{3}{5}, -\frac{4}{5}, -10^{-3}\right), \quad (0,0,1)$$

all lie in Y, while

$$3(1,0,-10^{-3}) + 4(0,1,-10^{-3}) + 5\left(-\frac{3}{5}, -\frac{4}{5}, -10^{-3}\right) = -\frac{12}{1000}(0,0,1). \tag{17}$$

No single v can pair positively with all four.

Task 17 also proves that no nonempty $\mathrm{Dom}(v)$ is maximal among admissible domains: one can adjoin a suitable orthogonal unit direction and preserve antipodal-freeness.

Status 2 (The domain picture after Task 17)**.** The inherited $\mathrm{Dom}(v)$ are mathematically useful but not canonical by maximality or exhaustiveness. Exact connected domains can wrap around the unit-direction space more broadly than a single positive-half-space cut while still avoiding every antipodal pair.

This is the first genuine relocation of the Part XVI local picture. The local integer classification survives, but the geometry supporting it is much less rigid than the first repair suggested.

8 Arbitrary local representatives have rigid integer overlap factors

Task 16 computed transitions between explicit models. Task 17 derives the factor for arbitrary jointly regular families. Define

$$\mathrm{ovl}(U, V; n, \theta) := U(n, \theta)V(n, -\theta). \tag{18}$$

The inherited one-axis inverse law gives

$$U(n, \theta) = \mathrm{ovl}(U, V; n, \theta)V(n, \theta). \tag{19}$$

Theorem 9 (Task 17 overlap classification)**.** *For two jointly regular families U, V:*

(i) $\mathrm{ovl}(U, V; n, \theta)$ *is central and is the unique factor implementing* (19)*;*

(ii) if both families are locally exact at n, then there exists $j \in \mathbb{Z}$ such that

$$\mathrm{ovl}(U, V; n, \theta) = z_{0,j}(\theta), \tag{20}$$

where j is the difference of the two half-odd local labels;

(iii) the parameter law is multiplicative:

$$\mathrm{ovl}(\theta + \phi) = \mathrm{ovl}(\theta)\,\mathrm{ovl}(\phi), \qquad \mathrm{ovl}(0) = 1;$$

(iv) on a preconnected overlap where both families are exact, the factor is independent of direction;

(v) for three families,

$$\mathrm{ovl}(U, V)\,\mathrm{ovl}(V, T) = \mathrm{ovl}(U, T), \tag{21}$$

and the corresponding identity holds along arbitrary finite chains.

The arithmetic pattern is now exact:

$$\text{local labels} \in \mathbb{Z} + \frac{1}{2}, \qquad \text{relative labels} \in \mathbb{Z}. \tag{22}$$

The integer transition is not an independent new degree of freedom once the two local labels are fixed; it is their difference.

9 The Task-17 reconstruction counterexample

Package F asks whether compatible local exact representatives reconstruct a unique global sign-relaxed family. The result is stated as a refutation of existence and a proof of uniqueness if existence were to hold.

The key counterexample is intentionally extreme. Index the local system by directions v and take

$$F_v = U_0^{\mathrm{loc}} \tag{23}$$

for every index. Then:

1. every F_v is jointly regular;
2. every F_v is exact on $\mathrm{Dom}(v)$;
3. the $\mathrm{Dom}(v)$ cover every unit direction;
4. every pair of local families is not merely compatible on overlaps but literally equal everywhere;
5. every overlap factor is exactly the unit.

Yet Task 17 proves that there exists no jointly regular sign-relaxed family G whose restriction to every $\mathrm{Dom}(v)$ equals these local representatives.

At first sight this looks stronger than an ordinary obstruction to gluing. It is essential to state what the theorem does and does not say.

10 Trivial overlaps do glue — just not into the demanded class

Because every F_v in (23) is the same function, the local data already determine a global jointly regular family:

$$F(n, \theta) = U_0^{\mathrm{loc}}(n, \theta). \tag{24}$$

No topological gluing problem remains in this example. The obstruction appears only after imposing the additional global requirement

$$F \in \{\text{sign-relaxed transformation-valued families}\}.$$

Task 16 had already proved that this global class contains exactly one jointly regular family, U^{ref}. But

$$\beta_{U_0^{\mathrm{loc}}} = \frac{1}{2}, \qquad \beta_{U^{\mathrm{ref}}} = 0. \tag{25}$$

Thus the literal equality demanded by the reconstruction statement compares objects in two different normalization classes.

Proposition 1 (Two distinct obstructions)**.** *The cumulative Task 16–17 results expose two logically different impossibilities.*

(A) ***Global exact-representative obstruction.*** *A global jointly regular exact family would require a continuous rate that is half-odd everywhere and reversal-odd. Connectedness of the full direction space makes those requirements inconsistent.*

(B) ***Literal local-to-sign-relaxed reconstruction obstruction.*** *A local exact representative has a half-odd rate at every direction where exactness is imposed, whereas the unique global sign-relaxed family has zero central rate. Requiring literal equality of the two therefore conflicts with the rate normalization already on a nonempty local set.*

The second obstruction is stronger locally but weaker conceptually: it does not say that the local data fail to form a global jointly regular function. In the counterexample they demonstrably do. It says that the global function obtained by literal gluing is not the separately selected sign-relaxed representative.

Methodological caveat 1 (The phrase "missing datum is exactly reversal comparison")**.** *Task 17 correctly proves that no globally continuous rate can be simultaneously half-odd everywhere and odd under reversal. That theorem diagnoses the global* exact *no-go. The reconstruction counterexample, however, additionally contains the pointwise normalization mismatch* (25)*. Therefore the prose statement that reversal comparison is the sole missing datum for literal reconstruction into the sign-relaxed class is too compressed. Task 18 should separate the two questions formally.*

This distinction is the central new Conservation-of-Difficulty event of Part XVII.

11 A stronger immediate consequence: the reference family is not locally exact anywhere

Task 17 exposes the theorem that U^{ref} is exact on no nonempty inherited $\mathrm{Dom}(v)$. The pointwise rate theorem suggests a stronger statement.

Corollary 2 (Paper-level strengthening)**.** *For any nonempty set D of unit directions, the inherited reference family U^{ref} cannot be exactly transformation-valued on D.*

Proof. Choose $n \in D$. Exactness at n forces the recovered second rate to belong to $\mathbb{Z} + \frac{1}{2}$. The reference family has recovered rate zero. Since $0 \notin \mathbb{Z} + \frac{1}{2}$, contradiction. □

As with corollary 1, this is an immediate mathematical consequence of Task 17 endpoints but is not exposed as a named theorem in the archive. It should be formalized in Task 18 because it sharply identifies the literal-restriction obstruction before any global cover is mentioned.

12 The natural next object is an explicit local conversion factor

The results themselves suggest a more precise reconstruction question. Instead of demanding that an exact local representative equal the global sign-relaxed reference literally, compare them by the same derived relative-factor operation already used for overlaps.

For a local exact family U, define formally at the paper level

$$B_U(n, \theta) := \mathrm{ovl}(U, U^{\mathrm{ref}}; n, \theta) = U(n, \theta) U^{\mathrm{ref}}(n, -\theta). \tag{26}$$

The existing Task 17 overlap theorem already implies that this factor is central and unique, and the recovered-rate formula gives, on every exact point,

$$B_U(n, \theta) = z_{0, \beta_U(n)}(\theta), \qquad \beta_U(n) \in \mathbb{Z} + \frac{1}{2}. \tag{27}$$

Thus

$$U(n, \theta) = B_U(n, \theta) U^{\mathrm{ref}}(n, \theta). \tag{28}$$

On a preconnected exact domain, $\beta_U = k + \frac{1}{2}$ is constant, so the entire local family differs from the global reference by one fixed half-odd central one-parameter factor.

For two local families U_i, U_j, one then expects the already proved integer overlap to factor as a difference of their two half-odd conversion factors:

$$\mathrm{ovl}(U_i, U_j) = B_{U_i} B_{U_j}^{-1}. \tag{29}$$

At the rate level this is merely

$$\left(k_i + \frac{1}{2}\right) - \left(k_j + \frac{1}{2}\right) = k_i - k_j \in \mathbb{Z},$$

but the carrier-level statement, its uniqueness, its domain hypotheses and its arbitrary-family form should be proved rather than inferred.

Open question 1 (The Task 18 reconstruction question)**.** *If local exact representatives are allowed to be related to a global sign-relaxed representative by explicit central conversion factors rather than literal equality, do all admissible local systems reconstruct the unique global reference family? If yes, are the integer overlap factors exactly the differences of the half-odd local-to-global conversion factors, and is the global exact no-go precisely the impossibility of choosing one such half-odd conversion continuously over the entire direction space while satisfying reversal?*

This formulation does not quotient away the central factor. It carries the factor explicitly, preserving the methodological rule that a sign or central discrepancy may not be declared physically or mathematically irrelevant merely because a conventional formulation often modds it out.

13 Why the domain result is more interesting than a hemisphere theorem

The original repair domains had the simple form of positive linear half-spaces intersected with the unit sphere. It would have been easy to mistake their success for evidence that such a hemispherical chart was intrinsic. Task 17 rules that out.

The domain Y of theorem 8 shows that a connected exact domain may extend around several azimuthal directions and still avoid every antipodal pair. Together with corollary 1, the local exactness condition on a connected domain is therefore insensitive to whether the domain is cut out by one positive linear functional. The important property is the absence of an antipodal pair, not hemispherical convexity.

This distinction relocates the domain problem:

Earlier question	Task 17 answer
Are $\mathrm{Dom}(v)$ sufficient?	Yes.
Are they connected?	Yes, intrinsically.
Do they exhaust connected exact domains?	No.
Are they maximal exact domains?	No.
Is antipodal-freeness sufficient?	Yes.
Is antipodal-freeness necessary in general?	No.
Is it necessary on connected domains?	Yes, by corollary 1.

The next geometric problem is therefore not to find another convenient patch. It is to determine the structure of maximal connected antipodal-free domains and whether there is any canonical selection principle available from the inherited algebra alone.

14 The stronger-regularity audit needs one scope correction

Task 17 introduces a stronger predicate requiring the two directional rate functions to be restrictions of C^1 functions on the ambient direction carrier. It proves:

1. every constant-label local model U_k^{loc} is C^1;
2. on every inherited $\mathrm{Dom}(v)$, the C^1 local classification is the same integer classification as the continuous one;
3. every antipodal-free domain admits a C^1 exact representative, namely a constant-label model;
4. the antipodal two-point domain also admits a C^1 exact representative through the linear rate (14);
5. overlap factors between C^1 local representatives have the same integer form as before.

These theorems are strong. They do *not*, however, expose an equivalence saying that every arbitrary domain satisfying the continuous admissibility criterion theorem 4 also possesses a C^1 admissible rate. The Task 17 status prose says that C^1 regularity "adds no restriction" to domains, but the principal endpoints only establish this for the inherited domains, all antipodal-free domains, the explicit antipodal pair, and the overlap classification.

Status 3 (Corrected C^1 boundary). **ON** $\mathrm{Dom}(v)$**: NO NEW RESTRICTION, PROVED.**
ON ANTIPODAL-FREE DOMAINS: C^1 **EXISTENCE, PROVED.**
ON THE EXPLICIT ANTIPODAL PAIR: C^1 **EXISTENCE, PROVED.**
FOR EVERY CONTINUOUSLY ADMISSIBLE ARBITRARY DOMAIN: EQUIVALENCE WITH C^1 **ADMISSIBILITY, NOT EXPOSED BY TASK 17.**

The general question may still have a positive answer, but it has not been proved by the endpoints cited for the global status claim. This is exactly the kind of quantifier boundary that a review should surface.

15 What "complete intrinsic classification" now means

The Task 17 audit concludes that local exact representability admits a complete intrinsic classification. That sentence is correct if "classification" means the exact functional criterion plus the family/rate classification. It should not be read as a completed geometric taxonomy of all admissible sets.

The precise state is:

1. **Arbitrary set, functional criterion: closed.** Admissibility is equivalent to existence of a globally continuous rate satisfying half-oddness on D and reversal oddness on antipodal pairs in D.

2. **Preconnected set, geometric criterion: essentially closed at paper level.** Admissibility is equivalent to antipodal-freeness, by corollary 1; a named Lean theorem remains to be added.

3. **Inherited** $\mathrm{Dom}(v)$**, family classification: closed.** Exactly one integer label per jointly regular exact family.

4. **General disconnected admissible set: labels need not be constant.** The antipodal pair is the explicit witness.

5. **Maximal connected/open admissible domains: open.** The inherited domains are not maximal and the archive gives no classification of maximal ones.

This is stronger and more precise than either calling the domain problem "open" without qualification or calling it "fully geometrically classified".

16 Conservation of Difficulty: the obstruction changes category

The trajectory from Parts XV–XVII can now be written more sharply:

$$\begin{aligned}\text{arbitrary directional rates} &\longrightarrow \text{continuous directional rates}\\ &\longrightarrow \text{global exact no-go}\\ &\longrightarrow \text{local half-odd repair}\\ &\longrightarrow \text{one integer per connected exact domain}\\ &\longrightarrow \text{integer relative transitions}\\ &\longrightarrow \text{domains far larger than the original hemispheres}\\ &\longrightarrow \text{literal reconstruction mismatch with the global sign-relaxed class.}\end{aligned} \tag{30}$$

The surprising feature is that the overlap problem closes before the reconstruction problem does. Even the strongest possible overlap compatibility—all local representatives identical and all transition factors equal to the unit—does not produce the requested global sign-relaxed representative by literal restriction. The reason is not hidden in an unproved triple-overlap law. It lies in the fact that the local exact class and the global sign-relaxed class impose different central-rate normalizations.

Methodological principle 2 (Revised Conservation of Difficulty)**.** Difficulty has moved from *compatibility between local patches* to *compatibility between two representation requirements.* The next object to classify is therefore not another overlap factor between local exact families, but the explicit factor that converts a local exact family into the unique global sign-relaxed family.

This is a qualitatively different frontier. A simple patching metaphor is no longer sufficient.

17 Late conventional comparison

Only after the reconstruction chain is fixed may one compare the resulting pattern with familiar mathematics. The cumulative result has three conspicuous features:

1. the visible family is 2π-periodic while the inherited reference implementer changes sign after 2π and returns after 4π;
2. exact local representatives on connected admissible domains are indexed by the affine lattice $\mathbb{Z} + \frac{1}{2}$;
3. relative factors between two local exact representatives are indexed by the difference lattice $\mathbb{Z}$.

In conventional spin geometry, central signs and two-fold refinements of rotation data are characteristic structures [3, 4]. The Task 17 pattern can therefore be recognized as structurally reminiscent of local choices inside a two-fold refinement, with an affine family of local normalizations and integer differences between them.

That comparison must not be allowed to erase what Task 17 actually found. No conventional bundle, spin group, covering-space theorem, cohomology class or physical spin observable was imported into the reconstruction. More importantly, the Task 17 "no reconstruction" theorem is *not* yet a theorem that some conventional bundle fails to glue. The explicit local data in its counterexample already define a global function. The failure concerns the additional sign-relaxed transformation-valued requirement.

Methodological caveat 2 (No physical-state theorem)**.** *Part XVII does not derive fermionic statistics, a spin measurement law, a Hilbert-state space, a Born rule, a Hamiltonian, a field equation or a dynamical interaction. The terms "half-odd", "integer transition" and "two-fold" refer to formally reconstructed algebraic data. Any physical interpretation remains downstream of the present proof boundary.*

18 Formal verification and provenance

Task 17 contains ten Lean modules with 2048 total source lines in the supplied archive. The audit records 27 declarations in `SafeBase`, 20 in `DomainGeometry`, 5 in `RateRigidity`, 7 in `LocalExhaustion`, 37 in `AdmissibleDomains`, 13 in `OverlapTransitions`, 6 in `Reconstruction`, 8 in `RegularityAudit`, one comparison-only declaration in `Identification`, and 44 top-level declarations in `Task17`. The top-level theorem inventory exposes 43 principal endpoints with an axiom report.

The only standard-library material listed by the archive consists of elementary real analysis, arithmetic and point-set topology: continuity and continuous-on restrictions, preconnectedness and the intermediate-value property, paths and path-connectedness, square roots, elementary trigonometric identities, C^1 predicates, linear arithmetic and integer parity. No algebraic topology or conventional bundle/spin classification theorem appears in the reconstruction source. Lean and mathlib provide the formal environment [5, 7, 8].

Four failed attempts are especially important as engineering provenance because they changed the mathematics rather than merely the tactics:

1. a generic discrete-valued constancy argument was replaced by an explicit path plus intermediate-value proof;

2. the intended theorem "antipodal pair obstructs local exactness" was refuted and replaced by theorem 5;
3. a first non-hemispherical counterexample based on meridian arcs failed openness and was replaced by the open three-leg domain Y;
4. direction-independence of overlap factors was weakened from an unrestricted claim to the proved preconnected-overlap theorem after the disconnected antipodal example showed the stronger statement false.

These are not editorial blemishes. They are the exact points at which the formal search changed the conceptual picture.

19 Task XVIII: questions forced by the Task-XVII result

Task 18 should not widen into new physical dynamics. The new mathematics has created a sharper internal question than the one we expected to ask. The purpose of Task 18 should be to determine whether the reconstruction mismatch is merely an artifact of demanding literal equality, or whether a deeper obstruction survives after the correct explicit local-to-global conversion data are included.

A suitable neutral title is

Task XVIII – Local-to-Global Conversion Audit: Connected Admissibility, Central Bridge Factors, Relation Coverage, and Reconstruction after Explicit Renormalization.

The task should be organized around the following questions rather than around a predetermined conventional answer.

19.1 Question 1: formalize the connected-domain criterion

Task 17 already contains all ingredients of corollary 1. Task 18 should prove as a named endpoint, with exact hypotheses,

$$\mathrm{unit}(D) \wedge \mathrm{preconnected}(D) \implies (\mathrm{Adm}(D) \iff D \text{ antipodal-free}). \tag{31}$$

It should separately state the empty-domain edge case and distinguish preconnected, connected, path-connected and open hypotheses rather than collapsing them into one phrase.

19.2 Question 2: what are the maximal connected exact domains?

Once (31) is formalized, the domain problem becomes a purely intrinsic geometry problem: classify maximal preconnected or maximal open connected antipodal-free subsets of the unit-direction space. Task 18 should determine:

- whether maximal domains exist in the chosen class;
- whether maximality by set inclusion is compatible with openness and connectedness simultaneously;
- what relation their boundary has to direction reversal;
- whether every inherited $\mathrm{Dom}(v)$ admits many inequivalent maximal extensions;
- whether any additional structure already reconstructed in the project selects one extension canonically.

No hemisphere theorem or standard classification should be inserted as an assumption.

19.3 Question 3: separate literal gluing from class-constrained reconstruction

Task 18 should define at least two reconstruction predicates:

(a) an unrestricted jointly regular global extension of local representatives by literal equality;

(b) a jointly regular global *sign-relaxed* representative related to local exact representatives by explicit central conversion factors.

The present Task 17 counterexample should then be reclassified under both predicates. In particular, the theorem should record formally that the family $F_v = U_0^{\mathrm{loc}}$ *does* have an unrestricted global extension, namely U_0^{loc}, even though it has no literal sign-relaxed reconstruction.

19.4 Question 4: derive the local-to-reference bridge

For an arbitrary jointly regular family U exact on an admissible set D, define the relative factor to the global reference intrinsically, for example by the already derived operation

$$B_U = \mathrm{ovl}(U, U^{\mathrm{ref}}).$$

Task 18 should prove or refute the following hierarchy without assuming the model form:

(i) B_U is central and unique;

(ii) on every exact point, its rate is half-odd;

(iii) on every preconnected exact domain, it is direction-independent and has one label $k + \frac{1}{2}$;

(iv) $U = B_U U^{\mathrm{ref}}$ on the domain;

(v) for two local exact representatives, the integer overlap factor is exactly the quotient/difference of their two bridge factors;

(vi) finite overlap composition follows from the bridge factorization and conversely recovers the already proved transition law.

The central factors must remain explicit; no quotienting by them is allowed.

19.5 Question 5: does bridge-normalized reconstruction always exist?

Given an arbitrary indexed family of admissible local exact representatives with a direction-space cover, ask whether the bridge-normalized local data all reduce to the same global sign-relaxed family U^{ref}. If so, prove existence and uniqueness. If not, identify the exact additional datum required and construct a counterexample.

This is the decisive Task 18 test. A positive result would mean that the Task 17 no-reconstruction theorem diagnoses only the incompatibility of *literal* normalization, while the explicit central bridge restores a coherent local-to-global relation. A negative result would reveal a genuinely new obstruction beyond the integer overlap laws.

19.6 Question 6: where exactly does the global exact no-go reappear?

If every local exact family can be converted to U^{ref}, Task 18 should identify the obstruction to replacing the family of local bridge factors by one global bridge factor. The expected candidate, to be tested rather than assumed, is the already proved impossibility of a continuous rate that is simultaneously half-odd everywhere and reversal-odd. The task should prove an equivalence if one exists:

$$\textit{global exact representative} \iff \textit{global admissible bridge to the sign-relaxed reference.}$$

If the equivalence fails, the missing implication must be isolated explicitly.

19.7 Question 7: ordinary overlaps versus coincidence relations

The Task 17 overlap factor compares local representatives at the *same direction* lying in an intersection of two direction domains. Global exactness, however, constrains pairs (n, θ) and (m, ϕ) whenever the visible transformations coincide, including relations between different directions.

Task 18 should therefore formalize an intrinsic "relation coverage" condition without importing conventional groupoid or bundle terminology: a family of direction domains is relation-complete if every relevant visible coincidence is witnessed inside at least one domain or is accompanied by explicit cross-domain comparison data. The task should determine:

- whether the inherited cover $\{\mathrm{Dom}(v)\}$ is relation-complete;
- which coincidence classes necessarily escape every connected admissible domain;
- whether antipodal comparison is the only missing relation after the exact coincidence theorem of Task 16;
- whether adding explicit cross-domain relation factors suffices to recover the global sign-relaxed defect law.

This work package should use the already proved Task 16 coincidence classification rather than reopen the arbitrary spatial word problem.

19.8 Question 8: disconnected domains and componentwise labels

Task 17 proves that connectedness forces one label and gives the antipodal two-point counterexample when connectedness is absent. Task 18 should classify the general disconnected case as far as the existing continuity assumptions allow. In particular:

- is the label constant on each connected component of D?
- can different components carry arbitrary half-odd labels subject only to accumulation constraints inherited from the globally continuous rate?
- what normal form do overlap factors have when an overlap has multiple components?
- can infinitely many components accumulate in a way that obstructs a globally continuous or C^1 extension?

This is where the exact functional criterion may contain more information than the simple antipodal-free test.

19.9 Question 9: audit the full C^1 domain class

Task 18 should correct the quantifier gap identified in section 14. Define a C^1-admissible arbitrary domain and decide whether

$$\mathrm{Adm}(D) \quad\Longleftrightarrow\quad \mathrm{Adm}_{C^1}(D) \tag{32}$$

holds for every unit-direction set D. A proof must construct or derive the required C^1 rate; a counterexample must satisfy the continuous criterion while excluding any C^1 realization. Only after this question is closed should higher smoothness or analyticity be considered.

19.10 Question 10: strengthen the negative controls

Task 18 should preserve the reconstruction firewall. In particular, it must not assume:

- a conventional two-fold covering structure;
- a bundle, cocycle, connection, holonomy or cohomology class;
- a preferred hemisphere or preferred axis;
- that bridge-normalized reconstruction succeeds;
- that maximal connected admissible domains have a known conventional classification;
- that all continuous admissible rates can be smoothed;
- any physical interpretation of the half-odd local labels or integer relative labels.

Conventional terminology may appear only after all reconstruction statements and counterexamples are fixed.

19.11 Task-XVIII decision table

The final Task 18 report should distinguish at least the following endpoints:

Question	Required verdict form
Preconnected admissibility iff antipodal-free?	PROVED / REFUTED with exact hypotheses
Maximal connected/open admissible domains exist?	PROVED / CONDITIONAL / REFUTED
Classification or structural description of maximal domains?	PROVED / PARTIAL / OPEN
Task 17 trivial-overlap system has unrestricted global extension?	PROVED / REFUTED
Every local exact representative has a unique central bridge to U^{ref}?	PROVED / REFUTED
Integer overlap equals difference of half-odd bridges?	PROVED / REFUTED
Bridge-normalized reconstruction of arbitrary local systems?	PROVED / REFUTED / CONDITIONAL
Global exactness equivalent to existence of one global bridge?	PROVED / REFUTED / CONDITIONAL
Inherited domain cover relation-complete for visible coincidences?	PROVED / REFUTED
Disconnected-domain componentwise normal form?	PROVED / PARTIAL / OPEN
Continuous admissibility equivalent to C^1 admissibility for arbitrary D?	PROVED / REFUTED
Arbitrary spatial word problem	remain OPEN unless strictly required

The desired endpoint is not a predetermined positive theorem. If bridge-normalized reconstruction fails, the failure itself becomes the next Conservation-of-Difficulty datum. If it succeeds, then the apparent Task 17 reconstruction obstruction is reclassified precisely as a literal-normalization obstruction, and the true global difficulty is isolated in the nonexistence of one globally exact bridge.

20 Conservation-of-Difficulty ledger

Question	Part-XVII status
Are the inherited $\mathrm{Dom}(v)$ geometrically controlled?	Yes: nonempty iff $v \neq 0$, open, antipodal-free, path-connected, connected.
Is the local half-odd label constant on $\mathrm{Dom}(v)$?	Yes, proved from continuity plus intrinsic connectedness.
Does every locally exact family on $\mathrm{Dom}(v)$ equal one model U_k^{loc}?	Yes, for a unique integer k.
Is antipodal-freeness necessary for arbitrary admissibility?	No. The disconnected pair $\{n, -n\}$ is admissible with opposite labels.
Is antipodal-freeness sufficient?	Yes, for every set of unit directions.
What happens on preconnected domains?	Paper-level corollary: admissibility iff antipodal-free; named Lean endpoint still to be added.
Are the inherited $\mathrm{Dom}(v)$ exhaustive among connected open admissible domains?	No. Explicit Y is open, path-connected, admissible and lies in no $\mathrm{Dom}(v)$.
Are $\mathrm{Dom}(v)$ maximal admissible domains?	No. Every nonempty one has a strict admissible enlargement.
Are arbitrary local overlap factors classified?	Yes: central, unique, integer-labelled; direction-independent on preconnected overlaps; finite chain law proved.
Do trivial overlap factors guarantee a global jointly regular function?	In the Task 17 counterexample, yes trivially: all local functions already equal U_0^{loc}.
Do they guarantee a global sign-relaxed representative by literal equality?	No. This is Task 17's reconstruction refutation.
Why not?	The glued exact-local normalization has half-odd rate, while the unique global sign-relaxed family has zero rate.
Is this the same obstruction as the global exact no-go?	No. The global exact no-go additionally uses continuity plus reversal oddness over the full direction space.
Is the reference family locally exact on any nonempty domain?	Paper-level corollary: no; exactness at one point already forces a half-odd rate.
Does C^1 change the inherited-domain classification?	No, proved.
Does C^1 leave the entire arbitrary admissible-domain class unchanged?	Not established by the exposed Task 17 endpoints; Task 18 question.
What is the next frontier?	Explicit local-to-global central conversion, relation coverage, maximal connected admissible domains, and the full C^1 arbitrary-domain audit.
Has physical spin or quantum dynamics been derived?	No. The result remains a formally reconstructed internal representation structure.

21 Conclusion

Part XVI ended with a local classification gap. Task 17 closes that gap completely on the inherited domains. The domains are proved connected by an explicit intrinsic path; continuity then freezes the half-odd rate to one value; equality of rates freezes the family itself to exactly one integer-labelled model. The local repair is no longer merely an existence statement.

The formal search then changes the expected story. Antipodal pairs are not intrinsically forbidden from local exact domains: a disconnected pair supports opposite half-odd labels. What forbids antipodes is connectedness together with exactness. Antipodal-freeness is sufficient on every set and, on preconnected sets, becomes necessary as well. The inherited positive-half-space domains are therefore not the geometry itself. They are convenient connected examples inside a larger antipodal-free class. The explicit three-leg domain proves that connected exact regions can extend beyond every single inherited $\mathrm{Dom}(v)$, and every $\mathrm{Dom}(v)$ can be enlarged.

The overlap problem also closes more strongly than before. Arbitrary local exact representatives differ by unique central integer-labelled one-parameter factors, with direction-independence on preconnected overlaps and exact finite composition. If the Conservation-of-Difficulty principle were merely a metaphor for increasingly complicated patching, one might expect the next obstruction to live in a failed overlap law.

It does not.

Task 17's sharpest counterexample makes all local families identical and all overlap factors equal to the unit. Those data already form a global jointly regular family. What they do not form is the unique global sign-relaxed family. The contradiction therefore lies between representation requirements: local exactness requires a half-odd central rate, while global sign-relaxed coherence selects the zero-rate reference family. Literal restriction cannot satisfy both. This is distinct from the global exact no-go, whose contradiction appears only when the half-odd rate is required to remain continuous and reversal-odd over the entire connected direction space.

The new frontier is consequently more precise than "local data fail to glue". The right question is whether exact local representatives and the global sign-relaxed representative are related by explicit central conversion factors, whether the integer overlap factors are exactly differences of these half-odd conversion factors, and whether all local systems become globally coherent once that conversion is carried rather than suppressed. If so, the global obstruction will have been isolated to the impossibility of one global exact conversion. If not, Task 18 will expose a genuinely new layer of difficulty beyond ordinary overlap coherence.

The Conservation-of-Difficulty boundary after Part XVII is therefore

Local exact representatives and their finite overlap laws are classified. The remaining problem is to classify the explicit central bridge between the local exact normalization and the unique global sign-relaxed normalization, and to determine whether any obstruction survives after that bridge is included.

No physical interpretation is required to make this question sharp. The mathematics has supplied its own next experiment.

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