The Relational Spin-Closure Hypothesis

Mathematical baseline, open problems, and falsification criteria

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Abstract

This document formulates a hypothesis and a falsification programme. Its mathematical baseline is the companion paper *A Conditional Construction of Lorentzian Geometry from Euclidean Three-Space: Clifford Double-Cover Data, Gluing, and a Soldering Isomorphism* [1]. Starting from the positive-definite Euclidean space

$$V = \mathbb{R}^3,$$

that paper constructs

$$\mathcal{S} = \mathbb{R} \oplus V, \qquad N(a, v) = a^2 - \langle v, v\rangle, \qquad \mathrm{Cl}_3 = \mathrm{Cl}(V, q),$$

the subgroup G_{Spin} defined in Paper I by the Clifford norm-one condition, a proper orthochronous Lorentz group $G_{\mathrm{Lor}} \subset O(B)$, and a central two-fold covering

$$1 \longrightarrow \{\pm 1\} \longrightarrow G_{\mathrm{Spin}} \xrightarrow{\rho} G_{\mathrm{Lor}} \longrightarrow 1.$$

It also constructs the orthogonal Lie algebra

$$\mathfrak{so}(B) \cong \Lambda^2 \mathcal{S}, \qquad \dim \mathfrak{so}(B) = 6.$$

Global Lorentzian geometry requires additional data. A gluing datum yields a smooth four-manifold only under explicit topological and smoothness hypotheses. Independently, a G_{Spin}-valued Čech 1-cocycle $\widetilde{g}_{ij}$ projects to $g_{ij} = \rho(\widetilde{g}_{ij})$. Local soldering data determine a vector-bundle isomorphism

$$s : E_g \xrightarrow{\sim} TM$$

and hence the Lorentzian metric

$$g_x(X, Y) = B\left(s_x^{-1} X, s_x^{-1} Y\right).$$

Neither the global gluing datum nor the soldering isomorphism follows from the local Euclidean input alone.

The explicit periodic one-parameter family studied in Paper I is used here as a deliberately symmetric reference family: its members are gauge equivalent under G_{Spin}-valued Čech 0-cochains and corresponding soldering data induce the same Lorentzian metric. The hypothesis therefore does not treat that family as an attempted construction of curvature. The local spin-factor, Clifford and Lorentz structures are to remain fixed. A later finite transport model may instead use one global scalar λ to scale an edge-dependent generator assignment,

$$U_{xy}(\lambda) = \exp(\lambda K_{xy}), \qquad K_{yx} = -K_{xy}, \qquad K_{xy} \in \mathfrak{s},$$

without introducing independent local parameters. This is a comparison family for nontrivial discrete holonomy; it is not a dynamical law.

Each local spin factor contains the distinguished unit $e_x = (1, 0)$ with $N(e_x) = 1$. The future unit hyperboloid $N(u) = 1$ therefore supplies an intrinsic dimensionless mass-shell normalization. No dimensionful mass or energy scale is thereby generated. If a later scale $m > 0$ is independently supplied or derived, the standard dimensionless variables are

$$\varepsilon = E/(mc^2), \qquad \boldsymbol{\pi} = \mathbf{p}/(mc), \qquad \varepsilon^2 - \|\boldsymbol{\pi}\|^2 = 1.$$

The same local Lorentz-boost subgroup is to be compared in three settings: special-relativistic kinematics, a future comparison with Levi–Civita parallel transport and timelike geodesics, and the composition of kinematic and transport contributions. The distinguishing geometric information is not a deformation of the local Minkowski model but the path dependence, holonomy and, in a continuum formulation, curvature of the transport law.

Subsequent machine-checked work has now constructed the (-1)-eigenspace $\mathfrak{s} \subset \mathrm{Cl}_3$ of Clifford conjugation, the bracket-preserving real-linear isomorphism $D_B : \mathfrak{s} \xrightarrow{\sim} \mathfrak{so}(B)$, and, for every $X \in \mathfrak{s}$, the algebraic one-parameter subgroup $t \mapsto \exp(tX) \in G_{\mathrm{Spin}}$. It proves that the derivative at $t = 0$ of the projected action on $\mathcal{S}$ is exactly $D_B(X)$. For every unit vector $v \in V$, it also proves the exact grade-one rapidity relation

$$\rho\Big(\exp\Big(\frac{\eta}{2}\,\iota(v)\Big)\Big)\, e = (\cosh\eta, \sinh\eta\, v), \qquad N(\cosh\eta, \sinh\eta\, v) = 1,$$

including the factor $\eta = 2t$ between the Clifford exponential parameter and Lorentz rapidity. Thus the local Clifford exponential, the standard Lorentz boost and the future unit mass shell are now linked by a formally verified identity. Subsequent machine-checked work has also constructed compatible smooth Lie-group structures on G_{Spin} and G_{Lor}, identified their tangent Lie algebras with $\mathfrak{s}$ and $\mathfrak{so}(B)$, proved that the Lie bracket on the Spin side is the Clifford commutator, proved smoothness of ρ, and established the genuine differential identity $d\rho_e = D_B$ under the canonical tangent-space identifications. The next open mathematical layer is the standard local connection-one-form formalism and its functorial passage from the Spin group to the Lorentz group. Parallel transport, holonomy, curvature and torsion remain subsequent structures.

The Relational Spin-Closure Hypothesis begins only after these distinctions. It proposes that a future classical configuration space carries a gauge-equivariant constraint whose solution set couples state and geometric variables nontrivially. The finite-dimensional research programme is deliberately staged: first test algebraic transport compatibility and frustration on a graph; then introduce a controlled driven dynamics with fixed link variables; only afterwards promote the transport variables themselves to dynamical degrees of freedom and test nonlinear back-reaction. A continuum limit, an effective geometric description, and large-scale gravitational phenomenology come later. Rotation curves, gravitational lensing, dark-matter phenomenology and MOND-type scaling are therefore late falsification benchmarks, not ingredients of the defining equations. Quantization and particle interpretations are later still.

Contents

1 Status convention

The labels used below have the following meanings:

Paper I	proved or constructed in Paper I;
Formalized	proved in the accompanying machine-checked formalization;
Standard	standard external mathematics;
Open	open mathematical problem;
Hypothesis	hypothesis introduced here;
Benchmark	later mathematical or empirical benchmark.

No statement labelled Open, Hypothesis or Benchmark is asserted as a theorem of Paper I.

2 Mathematical results used from Paper I

2.1 Local algebraic construction

The primitive local datum is

$$V = \mathbb{R}^3, \qquad \langle u, v \rangle > 0 \quad (u \neq 0). \tag{2.1}$$

No manifold, tangent bundle, Lorentzian metric, orientation, principal bundle or connection is part of this datum.

Paper I constructs the real spin factor

$$\mathcal{S} = \mathbb{R} \oplus V, \qquad N(a, v) = a^2 - \langle v, v \rangle, \tag{2.2}$$

with polarized bilinear form B of signature $(1, 3)$, together with the real Clifford algebra

$$\mathrm{Cl}_3 = \mathrm{Cl}(V, q), \qquad q(v) = \langle v, v \rangle. \tag{2.3}$$

The Clifford norm-one group G_{Spin} acts on the paravector model and yields

$$1 \longrightarrow \{\pm 1\} \longrightarrow G_{\mathrm{Spin}} \xrightarrow{\rho} G_{\mathrm{Lor}} \longrightarrow 1, \tag{2.4}$$

where $G_{\mathrm{Lor}} \subset O(B)$ is the proper orthochronous Lorentz group defined by the determinant and future-cone conditions used in Paper I. The homomorphism ρ is continuous and admits local continuous sections. No smooth Lie-group structure on G_{Spin} is asserted there.

The B-skew-adjoint endomorphisms form

$$\mathfrak{so}(B) = \{A \in \mathrm{End}_{\mathbb{R}}(\mathcal{S}) : B(Ax, y) + B(x, Ay) = 0\}, \tag{2.5}$$

and Paper I proves a linear isomorphism

$$\Lambda^2 \mathcal{S} \xrightarrow{\sim} \mathfrak{so}(B), \qquad \dim_{\mathbb{R}} \mathfrak{so}(B) = 6. \tag{2.6}$$

The identification

$$\mathrm{Lie}(G_{\mathrm{Spin}}) \cong \mathfrak{so}(B) \tag{2.7}$$

is *not* a result of Paper I.

2.2 Global quotient and tangent transitions

Let $\{D_i\}_{i\in I}$ be local four-dimensional pieces. A gluing datum specifies overlap domains and homeomorphisms

$$\phi_{ij} : W_{ij} \longrightarrow W_{ji} \tag{2.8}$$

satisfying the stated identity, inverse and cocycle conditions. Under the closed-gluing-relation and countability hypotheses of Paper I, the quotient is Hausdorff and second countable. If the transition maps are smooth, the resulting atlas defines a smooth four-manifold M.

The local pieces alone do not determine a unique quotient:

$$\{D_i\}_{i\in I} \not\Longrightarrow \text{a unique } M. \tag{2.9}$$

The tangent transition cocycle is

$$T_{ij}(y) = D\phi_{ij}(y). \tag{2.10}$$

Independently, let

$$\widetilde{g}_{ij} : U_i \cap U_j \longrightarrow G_{\mathrm{Spin}} \tag{2.11}$$

be a G_{Spin}-valued Čech 1-cocycle, and set

$$g_{ij} := \rho \circ \widetilde{g}_{ij} : U_i \cap U_j \longrightarrow G_{\mathrm{Lor}}. \tag{2.12}$$

In general,

$$D\phi_{ij} \neq g_{ij}. \tag{2.13}$$

Thus the tangent and Lorentz-valued transition cocycles are distinct data before an additional compatibility condition is imposed.

2.3 Soldering isomorphism and induced Lorentzian metric

Let $E_g \to M$ be the Lorentz vector bundle defined by the cocycle g_{ij}. Local soldering data are smooth maps

$$A_i : D_i \longrightarrow \mathrm{GL}(\mathcal{S}) \tag{2.14}$$

satisfying

$$A_j(\phi_{ij}(y)) = D\phi_{ij}(y)\, A_i(y)\, g_{ij}(x_y). \tag{2.15}$$

They determine a vector-bundle isomorphism

$$s : E_g \xrightarrow{\sim} TM. \tag{2.16}$$

The inverse

$$s^{-1} : TM \xrightarrow{\sim} E_g \tag{2.17}$$

has the conventional direction of an E_g-valued solder form.

The induced metric

$$g_x(X, Y) := B\Big(s_x^{-1}X, s_x^{-1}Y\Big) \tag{2.18}$$

is a smooth Lorentzian metric on M.

The logical dependence is therefore

$$\boxed{(V, \langle\ ,\ \rangle) \Longrightarrow (\mathcal{S}, B, \mathrm{Cl}_3, G_{\mathrm{Spin}}, \rho, \mathfrak{so}(B))} \tag{2.19}$$

locally, but globally

$$\boxed{\text{gluing datum} + \widetilde{g} + \text{soldering data} \Longrightarrow (M, TM, g).} \tag{2.20}$$

Neither additional term on the left of (2.20) is derived solely from (2.19).

2.4 Lifting obstruction on the chosen cover

For a Lorentz-valued Čech 1-cocycle g_{ij}, assume continuous lifts

$$h_{ij} : U_i \cap U_j \longrightarrow G_{\mathrm{Spin}}, \qquad \rho(h_{ij}) = g_{ij} \tag{2.21}$$

have been chosen on the double overlaps. On triple overlaps define

$$\epsilon_{ijk} := h_{jk} h_{ij} h_{ik}^{-1} \in \{\pm 1\}. \tag{2.22}$$

This defines the $\mathbb{Z}/2$-valued Čech 2-cocycle class used in Paper I. The paper does not identify this class with the classical characteristic class $w_2(TM)$. Likewise the determinant condition obtained from (2.15) is not identified there with $w_1(TM)$.

In particular, the following identifications are not part of the mathematical baseline:

$$[\epsilon] = w_2(TM), \qquad [\det D\phi] = w_1(TM). \tag{2.23}$$

2.5 Examples and nonuniqueness

Paper I establishes the following limitations:

$$\text{topological gluing} \not\Longrightarrow \text{smooth gluing}, \tag{2.24a}$$
$$\text{pointwise fibre isomorphisms} \not\Longrightarrow \text{continuous soldering}, \tag{2.24b}$$
$$\text{orientation-reversing two-chart gluing} \Longrightarrow \text{no soldering data}, \tag{2.24c}$$
$$\text{combinatorial labels} \not\Longrightarrow \text{existence of soldering data}. \tag{2.24d}$$

On the chosen periodic cover, distinct lifts may have the same Lorentz projection; this is a statement about the chosen cover and the specified equivalence relation, not a computation of $H^1(M; \mathbb{Z}/2)$.

2.6 One-parameter Čech 1-cocycle family

For the explicit periodic family of Paper I, let $\tilde{g}^{(\lambda)}$ denote the G_{Spin}-valued Čech 1-cocycle and let

$$\mathcal{E}_\lambda := \{\text{local soldering data for } \tilde{g}^{(\lambda)}\}. \tag{2.25}$$

Then

$$\forall \lambda \in \mathbb{R}, \qquad \mathcal{E}_\lambda \neq \varnothing. \tag{2.26}$$

Moreover, for all $\lambda_1, \lambda_2 \in \mathbb{R}$, there are continuous G_{Spin}-valued Čech 0-cochains u_i such that

$$\tilde{g}_{ij}^{(\lambda_2)} = u_i \, \tilde{g}_{ij}^{(\lambda_1)} \, u_j^{-1}, \tag{2.27}$$

and there is an explicit bijection

$$\mathcal{T}_{\lambda_1,\lambda_2} : \mathcal{E}_{\lambda_1} \xrightarrow{\sim} \mathcal{E}_{\lambda_2} \tag{2.28}$$

with

$$g_{\mathcal{T}_{\lambda_1,\lambda_2}(A)} = g_A. \tag{2.29}$$

No statement quantified over arbitrary covers, arbitrary cocycle families or arbitrary one-parameter subgroups follows from (2.26)–(2.29).

The selected one-parameter subgroup used in Paper I is explicitly a Lorentz-boost subgroup,

$$G(\lambda) = \cosh\left(\frac{\lambda}{2}\right) + \sinh\left(\frac{\lambda}{2}\right) \gamma_0,$$

whose Lorentz projection acts on the distinguished timelike–spacelike plane by

$$\begin{pmatrix} \cosh\lambda & \sinh\lambda \\ \sinh\lambda & \cosh\lambda \end{pmatrix}.$$

The direction γ_0 is selected, not canonical. In the present hypothesis this local one-parameter Lorentz-boost subgroup is not assigned exclusively to either special-relativistic momentum or gravitation. The intended distinction lies in the transport structure: special-relativistic kinematics may coexist with path-independent transport and trivial loop products, whereas a future connection-theoretic gravitational comparison would require path-dependent transport with nontrivial holonomy or curvature after those objects have been constructed.

3 The Relational Spin-Closure Hypothesis

The constructions above determine the present mathematical boundary. The hypothesis begins beyond that boundary.

3.1 Configuration space and gauge symmetry

The future classical variables have not yet been fully constructed. Accordingly, their final product structure is not assumed. Let

$$\widetilde{\mathcal{Q}} \tag{3.1}$$

denote a future space of admissible classical configurations extending the geometric structures already constructed, and let $\mathcal{G}$ be its gauge group. If a quotient is mathematically well behaved, write

$$\mathcal{Q} := \widetilde{\mathcal{Q}}/\mathcal{G}. \tag{3.2}$$

No manifold structure on $\mathcal{Q}$ is assumed at this stage.

Hypothesis 3.1 (Relational Spin-Closure Hypothesis)**.** *There exist a classical configuration space $\widetilde{\mathcal{Q}}$, a gauge group $\mathcal{G}$, a target space $\mathcal{Y}$, and a map*

$$\mathfrak{C} : \widetilde{\mathcal{Q}} \longrightarrow \mathcal{Y} \tag{3.3}$$

such that:

(i) $\mathfrak{C}$ is gauge equivariant and $0 \in \mathcal{Y}$ is fixed by the gauge action;

(ii) the physical solution set

$$\mathcal{Z} := \mathfrak{C}^{-1}(0)/\mathcal{G} \tag{3.4}$$

is nonempty and contains nontrivial gauge classes;

(iii) the geometric variables entering $\widetilde{\mathcal{Q}}$ are solved jointly with the state variables rather than prescribed as fixed external functions;

(iv) the local Euclidean/Clifford/Lorentz construction of Paper I is not altered in order to obtain a desired macroscopic equation.

The notation $\mathfrak{C}$ names the proposed equation only. It is not identified with torsion, curvature, a Čech cocycle, a connection, or any previously constructed object.

3.2 Symmetric reference configuration

The flat and homogeneous family studied in Paper I is an intentional reference configuration. It is not evidence that every admissible configuration must be flat. The local algebraic model is held fixed:

$$(V_x, \langle\ ,\ \rangle_x, \mathcal{S}_x, B_x, \mathrm{Cl}_x) \cong (\mathbb{R}^3, \langle\ ,\ \rangle, \mathcal{S}, B, \mathrm{Cl}_3)$$

for every local copy under consideration. No gravitational degree of freedom is introduced by deforming this local Clifford/Lorentz model.

The previously studied one-parameter family is homogeneous at the level of the transition data: the same parameter changes the transition cocycle coherently, and

$$\widetilde{g}^{(\lambda_1)} \sim_{\mathcal{G}} \widetilde{g}^{(\lambda_2)}, \qquad g_{\lambda_1} = g_{\lambda_2}.$$

Its gauge equivalence therefore characterizes that particular family; it is not a theorem that all inhomogeneous or path-dependent transport assignments are gauge equivalent.

Later extensions are required to preserve the local Clifford/Spin and Lorentz structures while allowing transport data between local frames to vary subject to explicitly stated compatibility and evolution equations. In particular, a non-flat geometry must not be obtained merely by prescribing

$$s = s(x), \qquad \lambda = \lambda(x), \qquad \omega = \omega(x)$$

as independent external functions and then computing the resulting curvature. Such fields may enter only after they have been constructed or constrained by the mathematical variables of the model.

For a finite graph model with identical local spin-factor/Clifford structures and edge transport variables

$$U_{xy} \in G_{\mathrm{Spin}}, \qquad U_{yx} = U_{xy}^{-1},$$

the reference configuration is characterized by trivial ordered products around every closed path,

$$H_C = 1 \qquad \text{for every closed path } C.$$

A nontrivial conjugacy class

$$[H_C] \neq [1]$$

is therefore a gauge-invariant indication of path dependence in the graph transport data.

In a continuum theory based on a principal connection ω, the corresponding standard notions must be distinguished carefully. The curvature two-form is

$$F_\omega = \mathrm{d}\omega + \frac{1}{2}[\omega \wedge \omega].$$

The condition $F_\omega = 0$ means that the connection is flat, whereas $F_\omega \neq 0$ is local curvature of that connection. Nontrivial holonomy can nevertheless occur for a flat connection on a topologically nontrivial base. Consequently, a nontrivial graph loop product is not identified either with continuum curvature or with continuum holonomy until an explicit comparison theorem has been proved.

The mathematical question is therefore whether the coupled compatibility and evolution equations for several locally identical spin-factor/Clifford structures admit or require gauge-inequivalent transport data while preserving the local Lorentzian model.

3.3 One-parameter transport families and constrained dynamics

A one-parameter family used for comparison may be written

$$U_{xy}(\lambda) = \exp(\lambda K_{xy}), \qquad K_{xy} \in \mathfrak{s}, \qquad K_{yx} = -K_{xy}, \tag{3.5}$$

with one global amplitude $\lambda \in \mathbb{R}$ and a fixed edge-dependent generator assignment K_{xy}. Hence

$$U_{xy}(0) = 1, \qquad \frac{\mathrm{d}}{\mathrm{d}\lambda} U_{xy}(\lambda)\Big|_{\lambda=0} = K_{xy}.$$

The scalar λ scales a prescribed family of transport variables; it does not create independent local degrees of freedom. Equation (3.5) is therefore a comparison family, not an explanation of dynamics.

Intrinsic unit normalization and the dual role of Lorentz boosts

Every local spin factor has the distinguished unit

$$e_x = (1, 0), \qquad N(e_x) = 1. \tag{3.6}$$

Define the future unit hyperboloid

$$\mathcal{H}_x^+ := \{u = (\varepsilon, \boldsymbol{\pi}) \in \mathcal{S}_x : N(u) = 1,\ \varepsilon > 0\}. \tag{3.7}$$

The equation $N(u) = 1$ is an intrinsic dimensionless normalization. It is not a dimensional energy law and does not generate a mass scale. If a positive mass scale m is later independently supplied or dynamically derived, the standard dimensionless comparison is

$$\varepsilon = \frac{E}{mc^2}, \qquad \boldsymbol{\pi} = \frac{\mathbf{p}}{mc}, \qquad \varepsilon^2 - \|\boldsymbol{\pi}\|^2 = 1. \tag{3.8}$$

Thus the local rest state is represented by $u = e_x$, for which the dimensionless rest-energy ratio under this later physical identification is $E_{\text{rest}}/(mc^2) = 1$. Equation (3.8) is a later physical interpretation of the already present Lorentzian unit normalization, not a derivation of m, E, or $\mathbf{p}$.

For a unit spatial direction n, let $L_n(\eta)$ denote the standard proper orthochronous Lorentz boost of rapidity η in direction n. Its action on the rest vector is

$$L_n(\eta)e_x = (\cosh\eta, \sinh\eta\, n), \qquad N(L_n(\eta)e_x) = 1. \tag{3.9}$$

Hence, for an oriented one-dimensional boost subgroup,

$$\varepsilon = \cosh\eta, \qquad \pi_\parallel = \sinh\eta, \qquad \eta = \operatorname{arcosh}\varepsilon = \operatorname{arsinh}\pi_\parallel. \tag{3.10}$$

This gives an inverse kinematic reconstruction: a normalized state on $\mathcal{H}_x^+$ determines the rapidity of the corresponding one-dimensional Lorentz boost, subject to the stated orientation convention.

The accompanying machine-checked formalization has proved, for $X \in \mathfrak{s}$, that $\exp(tX) \in G_{\text{Spin}}$ is an algebraic one-parameter subgroup and that the derivative at $t = 0$ of its projected action is $D_B(X)$. For a unit vector $v \in V$, it also proves

$$\exp(t\,\iota(v)) = \cosh t\, 1 + \sinh t\, \iota(v). \tag{3.11}$$

The exact comparison with the standard boost normalization has now also been formally verified:

$$\rho\Big(\exp\Big(\frac{\eta}{2}\,\iota(v)\Big)\Big)\, e_x = L_v(\eta)e_x = (\cosh\eta, \sinh\eta\, v), \qquad \langle v, v\rangle = 1. \tag{3.12}$$

The projected state satisfies

$$N\left(\rho\left(\exp\left(\frac{\eta}{2}\,\iota(v)\right)\right)e_x\right) = 1,$$

and the Clifford exponential parameter t and Lorentz rapidity η obey the exact relation

$$\eta = 2t. \tag{3.13}$$

For the oriented one-dimensional subgroup the inverse relations in (3.10) are formally verified with the standard domain qualifications; in particular $\operatorname{arsinh}(\sinh\eta) = \eta$ and $\operatorname{arcosh}(\cosh\eta) = |\eta|$, with $\operatorname{arcosh}(\cosh\eta) = \eta$ for $\eta \geq 0$. The tangent vector of the explicit boost curve is Lorentz-orthogonal to the state, so the curve remains tangent to the unit hyperboloid. These statements are finite-dimensional Lorentz-kinematic results. A compatible smooth Lie-group structure on G_{Spin} is now available independently, but no connection or dynamical interpretation is used in these identities.

The same local Lorentz-boost subgroup then has two distinct standard roles in the comparison programme:

Comparison	Mathematical form	Interpretation
Special-relativistic kinematics	$u = L_n(\eta_{\mathrm{kin}})e_x$, trivial discrete holonomy	relative velocity, rapidity and momentum
General-relativistic free fall	$u = \dot\gamma$, $\nabla^g_u u = 0$ for the Levi–Civita connection	boost component of orthonormal-frame parallel transport
Combined comparison	$L_{\mathrm{obs}} = L_{\mathrm{tr}} L_{\mathrm{kin}}$	composition of kinematic and transport contributions

The second and third rows are comparison targets for a future connection theory, not present theorems of the hypothesis.

At the finite level, for an oriented closed path C, define the ordered product

$$H_C(\lambda) = \prod_{e\in C} U_e(\lambda). \tag{3.14}$$

The condition $H_C = 1$ for every closed path is the path-independent reference case for this finite model. A nontrivial conjugacy class $[H_C] \neq [1]$ is a gauge-invariant obstruction to such path independence. A nonzero special-relativistic rapidity therefore does not by itself imply curvature or nontrivial holonomy: one may have $\eta_{\mathrm{kin}} \neq 0$ while all loop products remain trivial.

For comparison with general relativity, a future connection constructed by the model must first be related to the Levi–Civita connection ∇^g of the induced Lorentzian metric g, or any torsion and non-metricity must be stated explicitly. Standard timelike free fall in general relativity is then described by a geodesic γ with four-velocity $u = \dot\gamma$ satisfying

$$\nabla^g_u u = 0. \tag{3.15}$$

In a local orthonormal frame, the Levi–Civita connection has the standard component form

$$\frac{\mathrm{d}u^a}{\mathrm{d}\tau} + \omega^a{}_{b\mu}\,\dot x^\mu u^b = 0. \tag{3.16}$$

In a one-generator boost reduction $u(\tau) = L_n(\eta(\tau))e_x$, the boost component would reduce, after fixing the sign and generator conventions, to an equation of the form

$$\frac{\mathrm{d}\eta}{\mathrm{d}\tau} = -\omega_K(\dot\gamma). \tag{3.17}$$

Equations (3.15)–(3.17) are standard comparison equations. They are not yet derived from the hypothesis, and no identification of a future G_{Spin}-connection with the Levi–Civita connection is assumed.

Composition of kinematic and transport contributions

Write a local comparison between an initially boosted state and a transported frame as

$$L_{\mathrm{obs}} = L_{\mathrm{tr}} L_{\mathrm{kin}}. \tag{3.18}$$

For collinear boosts along the same unit direction n,

$$L_{\mathrm{kin}} = L_n(\eta_{\mathrm{kin}}), \qquad L_{\mathrm{tr}} = L_n(\eta_{\mathrm{tr}}), \qquad L_{\mathrm{tr}} L_{\mathrm{kin}} = L_n(\eta_{\mathrm{kin}} + \eta_{\mathrm{tr}}), \tag{3.19}$$

so that $\eta_{\mathrm{obs}} = \eta_{\mathrm{kin}} + \eta_{\mathrm{tr}}$. Consequently,

$$\eta_{\mathrm{tr}} = \operatorname{arcosh}(\varepsilon_{\mathrm{obs}}) - \eta_{\mathrm{kin}} \tag{3.20}$$

under the same oriented one-dimensional convention. This is the simplest inverse reconstruction relevant to the combined comparison: once the kinematic rapidity is specified, the normalized observed state fixes the collinear transport rapidity required by the model.

For noncollinear $K_1, K_2 \in \mathfrak{s}$, set $A_i := D_B K_i \in \mathfrak{so}(B)$. If $[A_1, A_2] \neq 0$, scalar rapidity addition is invalid. On the Lorentz-algebra side the Baker–Campbell–Hausdorff expansion gives

$$\log(e^{\xi A_2} e^{\eta A_1}) = \eta A_1 + \xi A_2 + \frac{1}{2}\eta\xi[A_2, A_1] + \cdots . \tag{3.21}$$

The commutator term contains the additional Lorentz-algebra component determined by the chosen generators; for two noncollinear boost generators this includes the familiar rotational contribution. Such terms are consequences of the noncommutative composition law, not additional variables introduced by definition.

If a differentiable trajectory satisfies the invariant normalization $N(u(\tau)) = 1$, then differentiation gives

$$B(u, \dot{u}) = 0, \qquad \dot{u} \in T_u \mathcal{H}_x^+. \tag{3.22}$$

Thus any future dynamical mechanism compatible with the local normalization must move the normalized state tangentially to the unit hyperboloid or modify the transport relation between such states; it must not change the defining Lorentzian normalization of the local spin factor.

A one-parameter group-commutator comparison family may use $K_1, K_2 \in \mathfrak{s}$:

$$H(\lambda) = e^{\lambda K_1} e^{\lambda K_2} e^{-\lambda K_1} e^{-\lambda K_2}. \tag{3.23}$$

Formally near $\lambda = 0$,

$$H(0) = 1, \qquad H(\lambda) = 1 + \lambda^2 [K_1, K_2] + O(\lambda^3). \tag{3.24}$$

The first nontrivial term is therefore produced by the noncommutativity of the transport generators while the local spin-factor/Clifford structures remain unchanged. This is a prescribed family of group elements; it does not define an evolution equation.

The dynamical hypothesis requires more than a prescribed one-parameter family. Suppose a future formulation provides a $\mathcal{G}$-equivariant evolution vector field $\mathcal{V}$ on the constraint surface $\mathfrak{C}^{-1}(0)$, so that an admissible trajectory satisfies

$$\mathfrak{C}(q) = 0, \qquad \dot{q} = \mathcal{V}(q). \tag{3.25}$$

At a configuration q, evolution tangent to the gauge orbit,

$$\mathcal{V}(q) \in T_q(\mathcal{G} \cdot q), \tag{3.26}$$

represents no change of the corresponding gauge-equivalence class. Accordingly, a configuration is stationary modulo gauge when (3.26) holds.

The hypothesis requires the existence of admissible configurations for which gauge-invariant transport data differ from the reference configuration and for which the evolution is not tangent to the gauge orbit,

$$\mathcal{V}(q) \notin T_q(\mathcal{G} \cdot q). \tag{3.27}$$

The vector field $\mathcal{V}$ and its dependence on the transport and geometric variables must be determined by the coupled equations of the model; they may not be supplied by prescribing the time dependence of λ, U_{xy}, ω, or the metric.

In the heuristic discussion below, the phrase *non-equilibrium* refers only to this dynamical-systems notion of a nonstationary gauge-equivalence class. No thermodynamic interpretation, entropy functional, statistical ensemble, or irreversible dynamics is assumed.

Remark 3.2 (Heuristic picture). One may picture each local copy as carrying the same Lorentzian unit hyperboloid and the same local spin-factor/Clifford structure. Special-relativistic motion changes the normalized state by a Lorentz boost. A gravitational comparison, if a later connection theory realizes it, would instead involve parallel transport between local orthonormal frames and the possible path dependence of that transport. The hypothesis is that the coupled equations can admit nonstationary gauge-equivalence classes while leaving the local Minkowski model unchanged. This picture is explanatory only; the mathematical content is expressed by Lorentz transformations, transport variables, gauge transformations, the constraint surface, and the evolution vector field defined above.

3.4 Linearized coupling between state and geometric variables

Suppose a future differentiable formulation admits a local splitting

$$T_q\widetilde{\mathcal{Q}} = X_\Phi \oplus X_G, \tag{3.28}$$

where X_Φ denotes variations of the state variables and X_G denotes variations of the geometric variables. At a solution $q \in \mathfrak{C}^{-1}(0)$, the linearized constraint is

$$D\mathfrak{C}_q(\delta\Phi, \delta G) = 0. \tag{3.29}$$

A minimal form of linearized coupling is the existence of a tangent vector in the linearized solution space with nonzero components in both summands:

$$\ker D\mathfrak{C}_q \cap \big((X_\Phi \setminus \{0\}) \times (X_G \setminus \{0\})\big) \neq \varnothing. \tag{3.30}$$

This states only that some infinitesimal solution variations change both sets of variables; it does not assert a constitutive law or a dynamical equation.

A stronger local statement can be formulated on specified subspaces when the partial derivatives

$$C_\Phi := D_\Phi\mathfrak{C}_q, \qquad C_G := D_G\mathfrak{C}_q \tag{3.31}$$

are invertible there. Then the linearized constraint can be solved in either direction,

$$\delta G = -C_G^{-1} C_\Phi \delta\Phi, \qquad \delta\Phi = -C_\Phi^{-1} C_G \delta G. \tag{3.32}$$

This is an implicit-function-type regularity condition and is not part of the present mathematical baseline.

3.5 Classical formulation prior to quantization

The hypothesis is classical at its defining stage. No Hilbert-space state axiom, Born rule, canonical commutation or anticommutation relation, Fock-space construction, path integral, or quantum-statistical postulate is used to define $\mathfrak{C}$.

No clock scalar, lapse, preferred foliation or simultaneity convention belongs to the primitive data. If such an object is introduced later, it requires an independent definition and status.

A variational realization, if it exists, would be a stronger statement:

$$\mathcal{A} : \widetilde{\mathcal{Q}} \longrightarrow \mathbb{R}, \qquad D\mathcal{A} = 0 \Longrightarrow \mathfrak{C} = 0. \tag{3.33}$$

No particular action functional is assumed.

4 Current local Lie-theoretic results and the open connection problem

4.1 The (-1)-eigenspace of Clifford conjugation and the induced Lie-algebra map

The accompanying machine-checked formalization defines the (-1)-eigenspace of Clifford conjugation

$$\mathfrak{s} := \{X \in \mathrm{Cl}_3 : c(X) = -X\} = \ker(\mathrm{id} + c). \tag{4.1}$$

It proves

$$\mathfrak{s} = \mathrm{Cl}_3^{(1)} \oplus \mathrm{Cl}_3^{(2)}, \qquad \dim_{\mathbb{R}} \mathfrak{s} = 6, \qquad [\mathfrak{s}, \mathfrak{s}] \subseteq \mathfrak{s}. \tag{4.2}$$

For the paravector embedding $A : \mathcal{S} \to \mathrm{Cl}_3$, the associated real-linear map is characterized by

$$A(D_X x) = XA(x) + A(x)\widetilde{X}, \tag{4.3}$$

where $\widetilde{X}$ denotes reversion. The resulting map is a bracket-preserving real-linear isomorphism

$$D_B : \mathfrak{s} \xrightarrow{\sim} \mathfrak{so}(B). \tag{4.4}$$

4.2 Exponential curves and the explicit boost sector

For every $X \in \mathfrak{s}$, the machine-checked formalization proves

$$\gamma_X(t) := \exp(tX) \in G_{\mathrm{Spin}}, \tag{4.5}$$

with

$$\gamma_X(0) = 1, \qquad \gamma_X(t+s) = \gamma_X(t)\gamma_X(s), \qquad \gamma_X(-t) = \gamma_X(t)^{-1}. \tag{4.6}$$

It also proves

$$c(\exp(tX)) = \exp(-tX) \tag{4.7}$$

and the exact projected paravector action

$$A(\rho(\exp(tX))x) = \exp(tX)A(x)\exp(t\widetilde{X}). \tag{4.8}$$

The derivative at the origin of this explicit projected curve is

$$\left.\frac{\mathrm{d}}{\mathrm{d}t}\rho(\exp(tX))x\right|_{t=0} = D_B(X)x. \tag{4.9}$$

For a unit vector $v \in V$, the grade-one exponential is proved in closed form:

$$\langle v, v\rangle = 1 \quad \Longrightarrow \quad \exp(t\,\iota(v)) = \cosh t\,1 + \sinh t\,\iota(v). \tag{4.10}$$

Equation (3.12), the relation $\eta = 2t$, and preservation of the unit Lorentz norm for the projected rest state are formally verified. The selected boost family of Paper I is therefore the specialization of this grade-one Clifford exponential to the chosen unit spatial direction.

4.3 Smooth Lie groups and the differential of the Spin–Lorentz homomorphism

The smooth local Spin–Lorentz bridge is now machine-checked. The Clifford norm-one group is equipped with a smooth Lie-group structure compatible with its finite-dimensional Clifford-algebra realization. The corresponding Lorentz group is likewise equipped with a smooth Lie-group structure. At the identity the tangent Lie algebras are canonically identified as

$$T_1 G_{\mathrm{Spin}} \cong \mathfrak{s}, \qquad T_1 G_{\mathrm{Lor}} \cong \mathfrak{so}(B). \tag{4.11}$$

Under the first identification, the Lie bracket is the Clifford commutator,

$$[X, Y]_{\mathrm{Lie}(G_{\mathrm{Spin}})} \longleftrightarrow XY - YX. \tag{4.12}$$

The existing homomorphism

$$\rho : G_{\mathrm{Spin}} \to G_{\mathrm{Lor}}$$

is smooth, and its genuine differential at the identity satisfies

$$\boxed{d\rho_e = D_B} \tag{4.13}$$

under the tangent-space identifications (4.11). Thus

$$\mathrm{Lie}(G_{\mathrm{Spin}}) \xrightarrow[\sim]{d\rho_e} \mathrm{Lie}(G_{\mathrm{Lor}}) \cong \mathfrak{so}(B) \tag{4.14}$$

is a Lie-algebra isomorphism, and the algebraic map constructed before the smooth structure was available is exactly the differential of the smooth Spin–Lorentz homomorphism.

A regular-level-set description underlies the Spin-side tangent identification. For

$$\mathfrak{c}_+ := \{Y \in \mathrm{Cl}_3 : c(Y) = Y\}, \qquad \Phi(g) = g\, c(g),$$

one has $G_{\mathrm{Spin}} = \Phi^{-1}(1)$ and

$$D\Phi_g(H) = H\, c(g) + g\, c(H), \qquad D\Phi_1(H) = H + c(H), \qquad \ker D\Phi_1 = \mathfrak{s}. \tag{4.15}$$

For $Y \in \mathfrak{c}_+$, the explicit choice $H = \frac{1}{2} Y g$ gives a right inverse of $D\Phi_g$ along the norm-one level set. These facts justify the tangent-space description rather than merely postulating it.

4.4 Local Spin and Lorentz connection one-forms

The next open mathematical layer is the standard local connection-one-form formalism. Let $U \subseteq M$ be open. A local Spin connection form is a smooth Lie-algebra-valued one-form

$$\omega \in \Omega^1(U, \mathfrak{s}). \tag{4.16}$$

For a smooth map $g : U \to G_{\mathrm{Spin}}$, the left Maurer–Cartan pullback is

$$\theta_g := g^{-1} \mathrm{d}g \in \Omega^1(U, \mathfrak{s}). \tag{4.17}$$

With the convention used here, local Spin connection forms on an overlap are related by

$$\omega_j = \mathrm{Ad}_{g_{ij}^{-1}}\, \omega_i + g_{ij}^{-1} \mathrm{d}g_{ij}. \tag{4.18}$$

Because $d\rho_e = D_B$ is now established, the corresponding Lorentz-algebra-valued one-form is

$$\Omega_i := D_B \circ \omega_i \in \Omega^1(U_i, \mathfrak{so}(B)). \tag{4.19}$$

The standard naturality statements to be formalized next are

$$D_B\Big(\mathrm{Ad}^{\mathrm{Spin}}_g X\Big) = \mathrm{Ad}^{\mathrm{Lor}}_{\rho(g)} D_B(X), \tag{4.20}$$

and

$$D_B(g^{-1}\mathrm{d}g) = (\rho\circ g)^{-1}\mathrm{d}(\rho\circ g). \tag{4.21}$$

Together these imply, for $h_{ij} := \rho\circ g_{ij}$, the Lorentz gauge-transformation law

$$\Omega_j = \mathrm{Ad}_{h_{ij}^{-1}} \Omega_i + h_{ij}^{-1}\mathrm{d}h_{ij}. \tag{4.22}$$

Equations (4.18)–(4.22) are standard Lie-group connection formulas; their machine-checked realization is the current open formalization problem. No curvature, parallel transport or gravitational interpretation is assumed at this stage.

A group homomorphism cannot be applied directly to a Lie-algebra-valued one-form. In particular,

$$\rho\circ\omega \tag{4.23}$$

is not a definition of a Lorentz connection. The mathematically correct infinitesimal map is the now-established differential $d\rho_e = D_B$, as in (4.19).

4.5 Curvature, torsion and later connection geometry

Only after the local connection-one-form layer is in place should curvature, parallel transport and holonomy be introduced. For a principal G_{Spin}-connection ω, standard differential geometry gives

$$F_\omega = \mathrm{d}\omega + \frac{1}{2}[\omega\wedge\omega]. \tag{4.24}$$

With a solder form $e : TM \to E_g$, torsion is

$$T = D_\omega e. \tag{4.25}$$

These are standard definitions [8, 3]; neither curvature nor torsion has yet been constructed in the present formalization. A nonzero connection one-form is not by itself curvature and is not by itself a gravitational field. The intended flat reference case remains $F_\omega = 0$, while any nonzero curvature attributed to the hypothesis must arise from the later coupled equations rather than being prescribed as input.

5 Discrete connection on a graph

A discrete connection on a graph may be studied before the continuum connection theory is complete, provided that no continuum-limit identification is assumed without proof.

Let

$$\Gamma = (\mathcal{V},\mathcal{E}) \tag{5.1}$$

be an oriented graph. Assign

$$U_{xy}\in G_{\mathrm{Spin}}, \qquad U_{yx} = U_{xy}^{-1} \tag{5.2}$$

to each oriented edge; these group elements play the role of link variables. A vertex gauge transformation $h_x \in G_{\mathrm{Spin}}$ acts by

$$U_{xy} \longmapsto h_y U_{xy} h_x^{-1}. \tag{5.3}$$

For a closed path $C = (x_0, \dots, x_n = x_0)$, define the ordered product (discrete holonomy)

$$H_C := U_{x_{n-1}x_n} \cdots U_{x_0 x_1}. \tag{5.4}$$

Then

$$H_C \longmapsto h_{x_0} H_C h_{x_0}^{-1}, \tag{5.5}$$

so its conjugacy class $[H_C]$ is gauge invariant, as expected for based holonomy under a change of local frame.

The reference configuration has trivial discrete holonomy,

$$H_C = 1 \qquad \text{for every closed path } C. \tag{5.6}$$

Nontrivial discrete holonomy is detected by

$$[H_C] \neq [1] \tag{5.7}$$

for at least one closed path. This condition leaves the local Clifford/Lorentz structure unchanged and concerns only the discrete connection.

No identification

$$[H_C] = [\mathrm{Hol}_\omega(C)] \tag{5.8}$$

is asserted before a continuum limit and an explicit comparison theorem have been constructed.

5.1 Parallel sections and frustrated compatibility conditions

At each vertex let

$$u_x \in \mathcal{H}_x^+ := \{u \in \mathcal{S}_x : N(u) = 1,\ u \text{ future-directed}\}, \tag{5.9}$$

and let

$$L_{xy} := \rho(U_{xy}) \in G_{\mathrm{Lor}}. \tag{5.10}$$

The edge compatibility condition is

$$u_y = L_{xy} u_x. \tag{5.11}$$

For a connected graph and a fixed base vertex x_0, a globally compatible family satisfying (5.11) exists precisely when there is a future unit vector at x_0 fixed by the discrete holonomy group generated by all based loop products. In standard terminology, the compatibility conditions are *frustrated* when they cannot all be satisfied simultaneously. Equivalently, in the present setting the holonomy group has no common fixed point on the future unit hyperboloid,

$$\mathrm{Fix}_{\mathcal{H}_{x_0}^+}\big(\mathrm{Hol}_{x_0}^\Gamma(U)\big) = \varnothing. \tag{5.12}$$

Nontrivial holonomy by itself is not sufficient for (5.12): a nontrivial subgroup may still fix a timelike unit vector. This distinction is essential. Equation (5.12) should also not be interpreted as the reason that no preferred inertial frame exists. The absence of a physically preferred inertial frame must follow from Lorentz covariance and gauge covariance of the theory; a dynamical system can exhibit motion while still possessing a distinguished congruence. The finite statement here is narrower: for the specified discrete connection there is no globally parallel admissible state.

For future-directed unit states define the relative rapidity $r_{xy} \geq 0$ by

$$B(u_y, L_{xy} u_x) = \cosh r_{xy}. \tag{5.13}$$

For positive edge weights w_{xy}, a useful nonnegative compatibility functional is

$$\mathcal{J}(u,U) := \sum_{\{x,y\}\in\mathcal{E}} w_{xy}(\cosh r_{xy} - 1) \geq 0. \tag{5.14}$$

It vanishes exactly when every weighted edge satisfies (5.11). The quantity $\mathcal{J}$ is introduced only as a mismatch functional. It is not identified with physical energy, mass, action, entropy, or stress–energy.

For small relative rapidities,

$$\mathcal{J}(u,U) = \frac{1}{2}\sum_{\{x,y\}\in\mathcal{E}} w_{xy} r_{xy}^2 + O\left(\sum_{\{x,y\}\in\mathcal{E}} w_{xy} r_{xy}^4\right). \tag{5.15}$$

Thus many individually small relative motions can produce a finite collective value of the compatibility functional. No interpretation of that collective value as additional gravitating mass is made at this stage.

5.2 Controlled driven dynamics with fixed transport data

The first dynamical model should keep the discrete connection fixed. Let

$$U_{xy}(\lambda) = \exp(\lambda K_{xy}), \qquad K_{yx} = -K_{xy}, \tag{5.16}$$

with one global control parameter λ and a fixed edge-dependent generator assignment. The state variables alone are then allowed to evolve on the product of future unit hyperboloids,

$$\dot{u}_x = F_x(u; U(\lambda)), \qquad B(u_x, F_x) = 0, \tag{5.17}$$

where the second condition preserves $N(u_x) = 1$. A suitable model must also be gauge covariant under the vertex action induced by ρ.

The controlled test begins from the symmetric initial state

$$u_x(0) = e_x. \tag{5.18}$$

At $\lambda = 0$, the equations must recover a stationary gauge class. For $\lambda \neq 0$, the research question is whether the equations themselves generate nonzero relative rapidities from (5.17), rather than having velocities inserted as initial or external data. Because $U(\lambda)$ is held fixed, this stage describes a driven finite system, not self-consistent back-reaction.

Frustration of the compatibility conditions does not by itself imply motion. A frustrated system can in principle possess stationary constrained states. The dynamical claim therefore requires an explicit evolution law and a proof or numerical demonstration that the relevant gauge-equivalence class is nonstationary according to (3.27).

5.3 Dynamical transport and nonlinear back-reaction

With the smooth Lie-group structure of G_{Spin} and its Lie algebra now established, the link variables can in principle be treated as dynamical group-valued degrees of freedom. The hypothesis nevertheless postpones this step until the simpler fixed-transport dynamics has been tested. In a local trivialization, a later coupled system may have the schematic standard form

$$\dot{u}_x = F_x(u,U), \qquad \dot{U}_{xy}U_{xy}^{-1} = G_{xy}(u,U), \tag{5.19}$$

with G_{xy} valued in the Lie algebra and with equations chosen so that $U_{yx} = U_{xy}^{-1}$, $N(u_x) = 1$, the constraints, and gauge covariance are preserved. Equation (5.19) is a structural target, not a presently derived law.

This step introduces genuine back-reaction:

$$\{U_{xy}\} \longrightarrow \{u_x\} \longrightarrow \{U_{xy}\}.$$

The resulting system is generically nonlinear for three independent reasons already visible in the mathematical structure: the hyperbolic dependence of relative rapidity, the noncommutative composition law of the Lorentz/Spin group, and the mutual dependence of state and transport variables. Noncollinear boost components can generate rotational Lie-algebra components through commutators; whether such components persist in self-consistent solutions is to be determined by the equations, not imposed as an additional degree of freedom.

A self-consistent stationary state is a fixed point only after quotienting by gauge transformations. The medium-term dynamical objective is therefore to determine whether the coupled finite system admits stable or recurrent collective states, whether some frustrated configurations remain nonstationary modulo gauge, and whether nontrivial relative motion survives after the external control parameter is returned to its reference value. Persistence after removal of the external driving would distinguish autonomous collective dynamics from a merely driven response.

6 Falsification criteria and later benchmarks

6.1 Mathematical falsification criteria

The following implications are excluded by the current mathematics:

$$V = \mathbb{R}^3 \not\Longrightarrow M, \tag{6.1a}$$

$$(B, \widetilde{g}) \not\Longrightarrow \exists\, s : E_g \xrightarrow{\sim} TM, \tag{6.1b}$$

$$\text{bare group homomorphism} \not\Longrightarrow d\rho_e, \tag{6.1c}$$

$$\prod_{e \in C} U_e \not\equiv \mathrm{Hol}_\omega(C) \quad \text{without a comparison theorem.} \tag{6.1d}$$

The present Spin–Lorentz homomorphism is now separately proved smooth, and its differential is computed as $d\rho_e = D_B$; the excluded implication concerns a group homomorphism without that additional smooth structure and proof.

Criterion 6.1 (Nontriviality modulo gauge)**.** *A proposed constraint variable fails if every member of the tested family belongs to one gauge orbit and all observables used by the proposal are constant on that orbit.*

The one-parameter family (2.25)–(2.29) is an explicit example of such a negative result at the transition-cocycle level. It does not imply the same conclusion for connection-level families.

Criterion 6.2 (Independence from benchmark field equations)**.** *A candidate $\mathfrak{C}$ does not establish the hypothesis if it is defined by inserting the Einstein equation, a prescribed Schwarzschild field, a geodesic equation, Maxwell/Yang–Mills equations, an externally prescribed redshift/lapse profile, or a desired particle mass as a primitive constitutive law.*

Criterion 6.3 (No prescribed geometric field as explanatory input)**.** *A proposed non-flat geometric configuration does not establish the hypothesis if it is obtained merely by prescribing a position-dependent soldering field, scalar field, connection, or curvature and then computing the desired geometry from that prescription. A global scalar λ may parameterize a fixed edge-dependent transport assignment K_{xy}, but such a family is only a comparison family. Any physical departure from the symmetric reference configuration must instead be determined by the transport variables together with the coupled constraint and evolution equations.*

Criterion 6.4 (No prescribed time dependence as a dynamical explanation). *A nonzero comparison parameter or a nontrivial loop observable is not by itself a dynamical explanation. The hypothesis requires an evolution equation on the admissible configuration space for which some configuration with nontrivial gauge-invariant transport data has evolution not tangent to its gauge orbit. If the same behavior is obtained solely by prescribing the time dependence of* λ, U_{xy}, ω, *or the metric, no intrinsic dynamical mechanism has been established.*

Criterion 6.5 (Origin of dimensionful scales). *Every dimensionful quantity must have an explicit mathematical source: boundary or initial data, a dimensionful parameter, a normalization of a classical solution, or a later quantum/renormalization mechanism after quantization has independently been justified.*

Criterion 6.6 (Frustration is not an evolution law). *The absence of a globally compatible parallel state, or equivalently the absence of a common fixed point of the discrete holonomy group on the admissible state space, does not by itself establish dynamics. A dynamical claim requires an independently specified gauge-covariant evolution law and nonstationarity modulo gauge.*

Criterion 6.7 (No mass interpretation from the finite mismatch functional). *The functional* $\mathcal{J}$ *in* (5.14) *is not a mass density, energy density, action, or stress–energy tensor by definition. Such an interpretation requires a later variational or continuum construction that derives the corresponding source of the effective geometry.*

6.2 Late large-scale gravitational benchmarks

Large-scale galactic phenomenology enters only after the finite coupled dynamics, a controlled continuum or coarse-graining limit, and an effective geometric description have been constructed. The required logical chain is

$$\begin{gathered}\text{finite coupled dynamics}\\ \Downarrow\\ \text{continuum/coarse-grained variables}\\ \Downarrow\\ \text{effective source or geometric field}\\ \Downarrow\\ \text{Lorentzian metric and geodesic observables.}\end{gathered}$$

Only at the final stage may the theory be compared with circular-speed profiles, gravitational lensing, and other observations usually used to infer dark matter. A successful account must reproduce the relevant observables from the same derived solution; fitting a rotation curve alone is insufficient.

The small-rapidity expansion (5.15) makes it mathematically possible for a large number of weak relative motions to contribute to a macroscopic collective quantity. It does not show that this quantity gravitates. To obtain phenomenology conventionally attributed to dark matter, the continuum theory would have to derive an effective stress–energy tensor, an equivalent geometric source, or another covariantly defined quantity whose contribution changes both massive-particle dynamics and null-geodesic lensing in the required way.

A still more restrictive late benchmark is MOND-type low-acceleration scaling. In the deep-MOND regime the standard phenomenological relation is approximately [10]

$$a \simeq \sqrt{a_0 a_N}, \tag{6.2}$$

and for asymptotically circular motion this implies the baryonic Tully–Fisher scaling [11]

$$v_f^4 \simeq G M_b a_0. \tag{6.3}$$

Neither equation may be inserted into the microscopic or continuum dynamics as a defining law. If such scaling is to support the present hypothesis, it must emerge from the derived collective dynamics. In particular, the acceleration scale a_0 is dimensionful and cannot arise from the dimensionless normalization $N(u) = 1$ by algebra alone; its mathematical origin must be identified independently.

A later axisymmetric numerical model may search for self-consistent stationary solutions and compare their predicted circular-speed profile $v_c(r)$ and lensing observables with data. Such a calculation belongs after the continuum geometric map has been established. Before that stage, graph simulations test collective dynamics and scaling only; they are not galaxy models.

6.3 Benchmark hierarchy

Required mathematical structure	Later comparison	Status
Classical state space and evolution law	Lorentz-covariant dynamics on a flat Lorentzian background	Benchmark
Finite discrete connection and compatibility constraints	trivial/nontrivial holonomy and frustrated transport constraints	Benchmark
Driven finite dynamics with fixed link variables	emergence of relative rapidities from the equations	Benchmark
Coupled finite dynamics with back-reaction	nonlinear self-consistent collective states	Benchmark
Continuum connection and effective geometry	free fall, curvature and weak-field gravitational phenomenology	Benchmark
Coarse-grained stationary solutions	circular-speed profiles and gravitational lensing	Benchmark
Derived low-acceleration scaling	dark-matter phenomenology and MOND/Baryonic-Tully–Fisher tests	Benchmark
Stable localized finite-energy solutions	energy, momentum and invariant mass	Benchmark
Reduced phase space / Hamiltonian or symplectic structure	possible quantization	Benchmark
Consistent quantized physical modes	particle/statistics/radiation interpretation	Benchmark
Independent internal non-Abelian gauge sector	confinement / QCD comparisons	Benchmark

No row may be used as an assumption to prove an earlier row.

7 Research order

The logical order of the programme is stated so that no later physical interpretation is used as an assumption for an earlier mathematical construction.

1. Established baseline from Paper I. The current geometric construction is

$$\begin{gathered}(V, \langle\ ,\ \rangle) \Longrightarrow (\mathcal{S}, B, \mathrm{Cl}_3, G_{\mathrm{Spin}}, \rho, \mathfrak{so}(B)),\\ \text{gluing datum} + \widetilde{g} + \text{soldering data} \Longrightarrow (M, TM, g).\end{gathered} \tag{7.1}$$

This stage supplies the fixed local Lorentz/Clifford structure and the conditional global Lorentzian geometry. It does not supply a connection or dynamics.

2. Formalized infinitesimal and exponential algebra. The machine-checked formalization has established

$$\begin{aligned}\mathfrak{s} = \{X : c(X) = -X\}, &\qquad D_B : \mathfrak{s} \xrightarrow{\sim} \mathfrak{so}(B),\\ \gamma_X(t) = \exp(tX) \in G_{\mathrm{Spin}}, &\qquad \frac{\mathrm{d}}{\mathrm{d}t}\rho(\gamma_X(t))x\Big|_{t=0} = D_B(X)x.\end{aligned} \tag{7.2}$$

For unit $v \in V$, the closed formula

$$\exp(t\,\iota(v)) = \cosh t\,1 + \sinh t\,\iota(v)$$

is also formalized.

3. Finite-dimensional rapidity and mass-shell bridge. The exact identity

$$\rho\left(\exp\left(\frac{\eta}{2}\,\iota(v)\right)\right)e = (\cosh\eta, \sinh\eta\, v), \qquad N(\cosh\eta, \sinh\eta\, v) = 1, \tag{7.3}$$

is now formally verified for every unit $v \in V$. It fixes the factor $\eta = 2t$ between the Spin exponential parameter and Lorentz rapidity and closes the local special-relativistic kinematic bridge without introducing gravitation.

4. Smooth Lie-group and infinitesimal representation. The smooth local Spin–Lorentz bridge is now formally verified:

$$\mathrm{Lie}(G_{\mathrm{Spin}}) \cong \mathfrak{s}, \qquad \mathrm{Lie}(G_{\mathrm{Lor}}) \cong \mathfrak{so}(B), \qquad d\rho_e = D_B. \tag{7.4}$$

Under the Spin-side tangent identification, the Lie bracket is the Clifford commutator. The homomorphism $\rho : G_{\mathrm{Spin}} \to G_{\mathrm{Lor}}$ is smooth and its differential is a Lie-algebra isomorphism. This closes the finite-dimensional differential-geometric bridge needed for connection theory.

5. Local connection-one-form layer. The next open mathematical step is to formalize smooth $\mathfrak{s}$-valued one-forms $\omega_i \in \Omega^1(U_i, \mathfrak{s})$, the Maurer–Cartan term $g^{-1}\mathrm{d}g$, and the standard gauge-transformation law

$$\omega_j = \mathrm{Ad}_{g_{ij}^{-1}}\,\omega_i + g_{ij}^{-1}\mathrm{d}g_{ij}. \tag{7.5}$$

Using the proved identity $d\rho_e = D_B$, define $\Omega_i = D_B \circ \omega_i$ and establish adjoint equivariance and Maurer–Cartan naturality so that

$$\Omega_j = \mathrm{Ad}_{h_{ij}^{-1}}\,\Omega_i + h_{ij}^{-1}\mathrm{d}h_{ij}, \qquad h_{ij} = \rho \circ g_{ij}. \tag{7.6}$$

This stage introduces neither curvature nor gravitational dynamics.

6. Algebraic finite-graph transport and frustration. Independently of the later continuum connection, one may study a finite graph with link variables $U_{xy} \in G_{\mathrm{Spin}}$, gauge transformations at vertices, discrete holonomy, and the compatibility conditions $u_y = \rho(U_{xy})u_x$. The first questions are algebraic: determine when globally compatible state assignments exist, classify the common fixed-point set of the discrete holonomy group, and identify frustrated compatibility systems by (5.12). The nonnegative functional $\mathcal{J}$ in (5.14) may be used diagnostically, but it is not an energy.

7. Controlled driven finite dynamics. With the transport assignment held fixed, use a family such as

$$U_{xy}(\lambda) = e^{\lambda K_{xy}} \tag{7.7}$$

as an externally controlled perturbation of the symmetric configuration. Evolve only the normalized states $u_x \in \mathcal{H}_x^+$. The reference case $\lambda = 0$ must be stationary modulo gauge. For $\lambda \neq 0$, the decisive test is whether relative rapidities are generated by the evolution equations from initially identical rest states rather than inserted as velocities or local parameters. This stage is intentionally a driven system.

8. Coupled finite dynamics and back-reaction. Only after the preceding test succeeds should the link variables become dynamical. The target is a gauge-covariant constrained system of the schematic form (5.19), preserving $N(u_x) = 1$ and the group-valued link constraints. The central questions are then whether state evolution modifies transport, whether the modified transport feeds back into the state evolution, whether noncommutative boost composition generates rotational components, and whether the resulting nonlinear system has nontrivial stationary, recurrent, or persistent states modulo gauge. Persistence after removal of the external driving distinguishes autonomous collective dynamics from a driven response.

9. Curvature, transport and continuum comparison. After the local connection-one-form layer is established, curvature, parallel transport and holonomy can be formalized on the continuum side. Only after a controlled scaling or continuum limit has also been defined should the finite transport variables be compared with those continuum objects

$$F_\omega = \mathrm{d}\omega + \frac{1}{2}[\omega \wedge \omega]. \tag{7.8}$$

The relation to the soldering isomorphism and the induced Lorentzian metric must then be established. A gravitational interpretation additionally requires a theorem relating the induced connection to the Levi–Civita connection of g, or an explicit statement of torsion and non-metricity. Discrete holonomy and continuum curvature remain distinct until this comparison is proved.

10. Coarse graining and macroscopic stationary solutions. If a continuum theory exists, derive rather than prescribe any effective source term, stress–energy tensor, constitutive relation, or modified geometric equation. Only then study stationary and axisymmetric solutions, circular geodesics, lensing, and collective rotational behaviour. The small-rapidity sum in (5.15) may motivate a scaling analysis, but it is not itself an effective mass density.

11. Galactic dark-matter and MOND benchmarks. Only after the preceding coarse-graining stage may one ask whether the derived macroscopic theory reproduces phenomena conventionally attributed to dark matter or yields MOND-type low-acceleration scaling such as (6.2) and the baryonic Tully–Fisher relation (6.3). The acceleration scale a_0, if it appears, must have an independently derived dimensionful origin. Neither a dark-matter halo profile nor a MOND interpolation law may be used as an input to earlier steps.

12. Localized excitations, quantization and additional interactions. Only after a successful classical continuum theory and its gravitational benchmarks should the programme ask whether it possesses stable localized finite-energy solutions, a reduced phase space, a consistent quantization, particle/statistics interpretations, or additional internal gauge sectors. Confinement, Standard-Model-like structure, and related strong-interaction questions are long-range comparison problems and are not assumptions of the gravitational programme.

The logical dependence is therefore

$$\boxed{\begin{array}{c}\text{Paper-I geometry}\\ \Downarrow\\ \text{Clifford–Lorentz algebra and rapidity bridge}\\ \Downarrow\\ \text{smooth Spin–Lorentz Lie-group bridge}\\ \Downarrow\\ \text{local connection-one-form layer}\\ \Downarrow\\ \text{finite discrete transport and frustration}\\ \Downarrow\\ \text{driven finite dynamics and finite back-reaction}\\ \Downarrow\\ \text{curvature, transport and continuum comparison}\\ \Downarrow\\ \text{coarse-grained macroscopic solutions}\\ \Downarrow\\ \text{galactic gravitational benchmarks}\\ \Downarrow\\ \text{later quantum and interaction tests.}\end{array}} \tag{7.9}$$

8 Object and status table

Object or statement	Mathematical form	Status
Euclidean input	$V = \mathbb{R}^3$ with positive-definite inner product	Paper I
Real spin factor	$\mathcal{S} = \mathbb{R} \oplus V$, $N(a, v) = a^2 - \langle v, v\rangle$	Paper I
Distinguished unit and unit hyperboloid	$e = (1, 0)$, $N(e) = 1$, $\mathcal{H}^+ = \{u : N(u) = 1,\ u_0 > 0\}$	Paper I
Dimensionless mass-shell interpretation	$\varepsilon = E/(mc^2)$, $\boldsymbol{\pi} = \mathbf{p}/(mc)$ if a scale m is later supplied	Benchmark
Real Clifford algebra	$\mathrm{Cl}_3 = \mathrm{Cl}(V, q)$	Paper I
Subgroup defined by the Clifford norm-one condition	G_{Spin} as defined in Paper I	Paper I
Central two-fold covering	$1 \to \{\pm 1\} \to G_{\mathrm{Spin}} \xrightarrow{\rho} G_{\mathrm{Lor}} \to 1$	Paper I
Orthogonal Lie algebra	$\mathfrak{so}(B) \cong \Lambda^2\mathcal{S}$, dimension 6	Paper I
(-1)-eigenspace of Clifford conjugation	$\mathfrak{s} = \{X : c(X) = -X\}$, a six-dimensional Lie algebra under the commutator	Formalized
Bracket-preserving linear isomorphism	$D_B : \mathfrak{s} \xrightarrow{\sim} \mathfrak{so}(B)$	Formalized
Algebraic one-parameter subgroup	$\gamma_X(t) = \exp(tX) \in G_{\mathrm{Spin}}$ for $X \in \mathfrak{s}$	Formalized
Derivative of projected exponential action	$\frac{\mathrm{d}}{\mathrm{d}t}\rho(\exp(tX))x\big\vert_0 = D_B(X)x$	Formalized
Explicit grade-one boost correspondence	$\rho(\exp((\eta/2)\iota(v)))e = (\cosh\eta, \sinh\eta\, v)$, $\lVert v\rVert = 1$	Formalized

Object or statement	Mathematical form	Status
Spin parameter / Lorentz rapidity relation	$\eta = 2t$ for the grade-one boost subgroup	Formalized
Unit mass-shell preservation	$N(\rho(\exp((\eta/2)\iota(v)))e) = 1$	Formalized
Smooth four-manifold	obtained from additional gluing data and smoothness hypotheses	Paper I
Tangent transition cocycle	$D\phi_{ij}$	Paper I
Group-valued Čech 1-cocycle	$\widetilde{g}_{ij} : U_i \cap U_j \to G_{\mathrm{Spin}}$	Paper I
Lorentz-valued cocycle	$g_{ij} = \rho(\widetilde{g}_{ij})$	Paper I
Soldering isomorphism	$s : E_g \xrightarrow{\sim} TM$, when local soldering data exist	Paper I
Lorentzian metric	$g_x(X, Y) = B(s_x^{-1}X, s_x^{-1}Y)$	Paper I
Čech lifting class on $\mathcal{U}$	$[\epsilon] \in \check{H}^2(\mathcal{U}; \mathbb{Z}/2)$ for chosen lifts	Paper I
Identification with $w_2(TM)$	not proved in Paper I	Open
Identification with $w_1(TM)$	not proved in Paper I	Open
Smooth Lie-group structure on G_{Spin}	smooth finite-dimensional Lie-group structure compatible with the Clifford realization	Formalized
Smooth Lie-group structure on G_{Lor}	smooth finite-dimensional Lie-group structure compatible with the Lorentz realization	Formalized
Smooth Spin–Lorentz homomorphism	$\rho : G_{\mathrm{Spin}} \to G_{\mathrm{Lor}}$ is smooth	Formalized
Tangent Lie algebra identification	$T_1 G_{\mathrm{Spin}} \cong \mathfrak{s}$, with Clifford commutator as Lie bracket	Formalized
Differential at the identity	$d\rho_e = D_B$ under the tangent-space identifications	Formalized
Local Spin/Lorentz connection one-forms	$\omega_i \in \Omega^1(U_i, \mathfrak{s})$, $\Omega_i = D_B \circ \omega_i$, standard gauge laws	Open
Curvature and torsion	F_ω, $T = D_\omega e$	Open
Discrete connection on a finite graph	$U_{xy} \in G_{\mathrm{Spin}}$, $U_{yx} = U_{xy}^{-1}$, vertex gauge action	Hypothesis
Discrete holonomy and compatible state assignment	H_C, $u_y = \rho(U_{xy})u_x$	Hypothesis
Frustrated compatibility conditions	no common fixed point of the discrete holonomy group on $\mathcal{H}^+$	Hypothesis
Compatibility functional	$\mathcal{J} = \sum w_{xy}(\cosh r_{xy} - 1)$, diagnostic only	Hypothesis
Driven finite state dynamics	fixed $U(\lambda)$, evolving $u_x \in \mathcal{H}_x^+$	Hypothesis
Coupled finite back-reaction	joint evolution of u_x and U_{xy}	Hypothesis
Continuum/coarse-graining map	finite coupled system to effective geometric variables	Open
Galactic phenomenology	rotation curves, lensing, dark-matter and MOND-type scaling	Benchmark
Classical configuration space	$(\widetilde{\mathcal{Q}}, \mathcal{G})$	Hypothesis
Gauge-equivariant constraint	$\mathfrak{C} : \widetilde{\mathcal{Q}} \to \mathcal{Y}$, $\mathfrak{C} = 0$	Hypothesis
Linearized state–geometry coupling	mixed state/geometric directions in $\ker D\mathfrak{C}$	Hypothesis
Quantum theory	conditional later extension	Benchmark

9 Relation to established mathematics

The algebraic terminology follows standard Clifford, Jordan-algebra, and spin-geometry usage [4, 5]. Smooth manifolds, vector bundles, principal bundles, gauge transformations, connections, parallel transport, holonomy, solder forms, curvature, torsion, geodesics, constrained dynamical systems, back-reaction, coarse graining, and stationary states modulo symmetry are used in their standard mathematical meanings [6, 7, 8, 3]. The word *frustrated* is used in its standard mathematical-physics sense: local compatibility conditions cannot all be satisfied simultaneously. No claim of novelty is attached to those structures or terms.

The title "Relational Spin-Closure Hypothesis" is a name for the proposal, not the introduction of a new mathematical operation called "spin closure." The actual hypothesis is formulated only in terms of standard objects: configuration spaces, gauge group actions, equivariant constraint maps, evolution vector fields, discrete connections, holonomy, and, if later constructed, principal connections and curvature. Heuristic passages are explicitly marked as such and do not define additional mathematical structures.

The specific new proposal is restricted to the existence and physical adequacy of a future coupled classical system with the properties stated in Section 3 and the staged finite-to-continuum programme of Sections 5–7. General relativity, dark-matter phenomenology, MOND, gauge theory, soliton models, and non-Abelian strong-interaction theory are comparison targets only; none is used to define the preceding dynamics [9, 10, 11, 12, 13, 14, 15].

10 Conclusion

The current construction contains the chain

$$\begin{gathered}
V = \mathbb{R}^3 \Longrightarrow (\mathcal{S}, B, \mathrm{Cl}_3, G_{\mathrm{Spin}}, \rho, \mathfrak{so}(B)), \\
+ \text{ gluing datum} \Longrightarrow M, TM, \\
+ \widetilde{g}_{ij}, A_i \Longrightarrow s : E_g \xrightarrow{\sim} TM \Longrightarrow (TM, g), \\
\mathfrak{s} = \{X : c(X) = -X\} \xrightarrow{D_B} \mathfrak{so}(B), \qquad D_B \text{ an algebraic isomorphism}, \\
X \in \mathfrak{s} \Longrightarrow \gamma_X(t) = \exp(tX) \in G_{\mathrm{Spin}}, \qquad \frac{\mathrm{d}}{\mathrm{d}t}\rho(\gamma_X(t))x\bigg|_{t=0} = D_B(X)x.
\end{gathered} \tag{10.1}$$

For a unit vector v, the machine-checked formalization additionally proves

$$\exp(t\,\iota(v)) = \cosh t\,1 + \sinh t\,\iota(v). \tag{10.2}$$

The exact comparison with standard Lorentz rapidity is now also formally verified:

$$\rho\left(\exp\left(\frac{\eta}{2}\,\iota(v)\right)\right)e = (\cosh\eta, \sinh\eta\,v), \qquad N(\cosh\eta, \sinh\eta\,v) = 1. \tag{10.3}$$

The corresponding relation between the Clifford exponential parameter and Lorentz rapidity is exactly $\eta = 2t$. The smooth local Spin–Lorentz bridge is now also formally closed:

$$\mathrm{Lie}(G_{\mathrm{Spin}}) \cong \mathfrak{s}, \qquad \mathrm{Lie}(G_{\mathrm{Lor}}) \cong \mathfrak{so}(B), \qquad d\rho_e = D_B, \tag{10.4}$$

with the Spin-side Lie bracket represented by the Clifford commutator. The next structural problem is therefore no longer the existence of the infinitesimal representation, but the standard local connection-one-form layer: Spin-valued gauge transformations, the Maurer–Cartan term, and the induced Lorentz connection form $\Omega = D_B \circ \omega$. Curvature, parallel transport, holonomy and torsion remain subsequent structures.

The symmetric geometry studied so far is an intentional reference configuration. The local spin-factor, Clifford, and Lorentz structures are held fixed, and the graph model begins with trivial discrete holonomy. The hypothesis does not obtain gravitation by deforming the local Minkowski model or by freely prescribing a position-dependent soldering field, scalar field, connection, or curvature. Each local spin factor retains the normalization $e_x = (1, 0)$, $N(e_x) = 1$; the future unit hyperboloid is therefore a dimensionless mass-shell normalization, not a generated mass scale.

A single global scalar may parameterize the comparison family

$$U_{xy}(\lambda) = e^{\lambda K_{xy}},$$

with an edge-dependent generator assignment K_{xy}. The same local Lorentz-boost subgroup can then be compared with special-relativistic kinematics, with a future connection-theoretic description of orthonormal-frame transport, and with the composition of the two. Collinear rapidities add, whereas noncollinear boosts are governed by the noncommutative Lorentz composition law and its commutator terms. These are standard kinematic and group-theoretic facts; they do not constitute an evolution equation.

The dynamical hypothesis requires a coupled constrained evolution law on $\mathfrak{C}^{-1}(0)$. Some admissible configurations with nontrivial gauge-invariant transport data must have an evolution vector field that is not tangent to the gauge orbit, while the local Lorentzian normalization remains fixed. The time dependence of λ, U_{xy}, a connection, or the metric may not be prescribed merely to obtain this behavior.

The finite research programme therefore separates three logically different statements. First, a discrete connection may have frustrated state-compatibility conditions,

$$\mathrm{Fix}_{\mathcal{H}^+}\big(\mathrm{Hol}^{\Gamma}(U)\big) = \varnothing, \tag{10.5}$$

without this fact alone implying time evolution. Second, with $U_{xy}(\lambda)$ held fixed, a driven state equation may be tested for the generation of relative rapidities from initially identical rest states. Third, only a later coupled evolution of states and transport variables can test genuine back-reaction and autonomous collective dynamics.

For small relative rapidities the diagnostic functional obeys

$$\mathcal{J} = \frac{1}{2} \sum_{\{x,y\}} w_{xy} r_{xy}^2 + O(r^4),$$

so many weak relative motions may contribute collectively. This observation is not an identification with mass or stress–energy. Such an interpretation requires a continuum/coarse-graining construction that produces a covariant effective source or geometry.

Large-scale galactic comparisons are therefore deliberately late. Only after a continuum geometry has been derived may the theory be confronted with rotation curves and gravitational lensing, and only after a dimensionful acceleration scale has been obtained independently may MOND-type relations such as $a \simeq \sqrt{a_0 a_N}$ or $v_f^4 \simeq G M_b a_0$ be treated as predictions. They are falsification benchmarks, not defining equations.

In the graph model the distinction between trivial and nontrivial discrete holonomy is

$$H_C = 1 \quad \text{for every closed path } C, \qquad \text{versus} \qquad [H_C] \neq [1] \quad \text{for some } C. \tag{10.6}$$

For a future continuum principal connection, flatness is instead the local condition

$$F_\omega = 0, \tag{10.7}$$

and nonzero F_ω is curvature of that connection. Flatness does not exclude nontrivial global holonomy. Consequently, the discrete and continuum statements remain distinct until an explicit continuum-limit comparison has been proved.

The later hypothesis is the existence of

$$(\widetilde{\mathcal{Q}}, \mathcal{G}, \mathcal{Y}, \mathfrak{C}), \qquad \mathfrak{C} : \widetilde{\mathcal{Q}} \to \mathcal{Y}, \qquad \mathfrak{C}^{-1}(0)/\mathcal{G} \neq \varnothing, \tag{10.8}$$

with nontrivial gauge classes and nontrivial coupling between state and geometric variables in the linearized solution space. Failure to derive nontrivial gauge-invariant transport data, failure to construct such a classical constraint and evolution law, or failure of the later physical benchmarks falsifies the corresponding claim. Nothing in Paper I by itself establishes (10.8).

References

[1] J. Seeck (ed.), *A Conditional Construction of Lorentzian Geometry from Euclidean Three-Space: Clifford Double-Cover Data, Gluing, and a Soldering Isomorphism*, companion mathematical paper, 13 September 2026. `https://raw.githubusercontent.com/antaris82/The_Relational_Spin-Closure_Hypothesis/main/docs/mathematics/A_Conditional_Construction_of_Lorentzian_Geometry.pdf`.

[2] *The Relational Spin-Closure Hypothesis: machine-checked formalization.* `https://github.com/antaris82/The_Relational_Spin-Closure_Hypothesis`.

[3] M. Nakahara, *Geometry, Topology and Physics*, 2nd ed., Institute of Physics Publishing, Bristol and Philadelphia, 2003. ISBN-13: 978-0-7503-0606-5.

[4] C. Chevalley, *The Algebraic Theory of Spinors*, Columbia University Press, New York, 1954.

[5] H. B. Lawson Jr. and M.-L. Michelsohn, *Spin Geometry*, Princeton University Press, Princeton, 1989.

[6] J. M. Lee, *Introduction to Smooth Manifolds*, 2nd ed., Springer, 2013.

[7] D. Husemoller, *Fibre Bundles*, 3rd ed., Springer, 1994.

[8] S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry, Vol. I*, Interscience, New York, 1963.

[9] A. Einstein, *Die Grundlage der allgemeinen Relativitätstheorie*, Annalen der Physik 354(7) (1916), 769–822. DOI: 10.1002/andp.19163540702.

[10] M. Milgrom, *A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis*, The Astrophysical Journal 270 (1983), 365–370. DOI: 10.1086/161130.

[11] S. S. McGaugh, J. M. Schombert, G. D. Bothun, and W. J. G. de Blok, *The Baryonic Tully–Fisher Relation*, The Astrophysical Journal 533(2) (2000), L99–L102. DOI: 10.1086/312628.

[12] T. W. B. Kibble, *Lorentz Invariance and the Gravitational Field*, Journal of Mathematical Physics 2(2) (1961), 212–221. DOI: 10.1063/1.1703702.

[13] T. H. R. Skyrme, *A Non-Linear Field Theory*, Proceedings of the Royal Society A 260 (1961), 127–138. DOI: 10.1098/rspa.1961.0018.

[14] C. N. Yang and R. L. Mills, *Conservation of Isotopic Spin and Isotopic Gauge Invariance*, Physical Review 96 (1954), 191–195. DOI: 10.1103/PhysRev.96.191.

[15] K. G. Wilson, *Confinement of Quarks*, Physical Review D 10 (1974), 2445–2459. DOI: 10.1103/PhysRevD.10.2445.