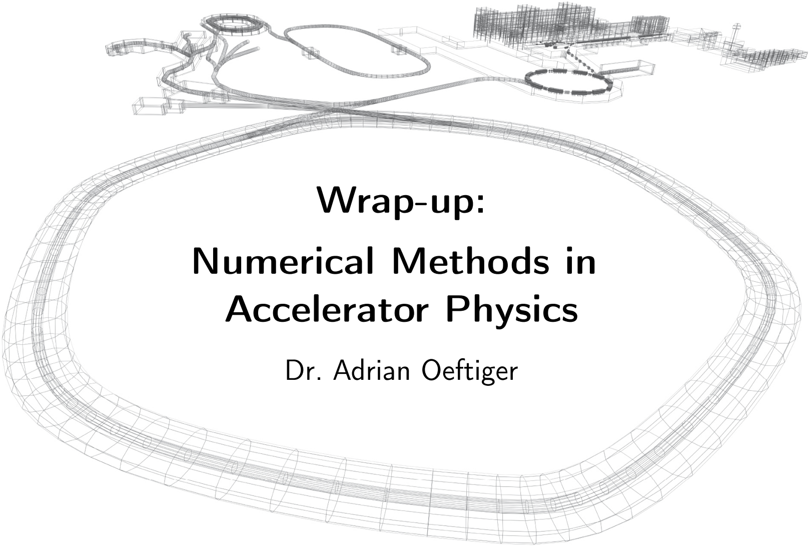

Wrap-up:

Numerical Methods in Accelerator Physics

Dr. Adrian Oeftiger

Overview

Learning objectives for lecture series:

- basic models of accelerator physics
- suitable methods for their implementation

Topics covered:

1. **reduced models**: steering, focusing, acceleration
2. **maps** of linear periodic systems + **stability analysis**
3. beam **transport models**: particles, beam distributions (self-consistent modelling)
4. **control-room** applications and **diagnostics**:
   - ⟶ tune reconstruction
   - ⟶ tomographic reconstruction of phase space
   - ⟶ closed-orbit control

Examination

- **30 min** oral exam
- format = conceptual discussion on models of accelerator physics and numerical implementations
- exam material = $\sum$ summary slides

⚠ you do <u>not</u> need to know how to write python code –
focus on the **ideas and concepts** we discussed!

Golden Thread

I. basic concepts: lectures 1-3
II. longitudinal dynamics: lectures 4-6
III. transverse dynamics: lectures 7-8
IV. applications: lectures 9-13

Golden Thread

I. **basic concepts**: lectures 1-3

- $\Longrightarrow$ using the simple pendulum as example
- ⟶ time scales in a synchrotron
  (transverse / longitudinal motion period, storage times)
- ⟶ phase space (system state), Hamiltonian (equations of motion)
- ⟶ discrete integrators: Euler, Euler-Cromer, leapfrog
- ⟶ statistical moments, emittance
- ⟶ non-linearities, Liouville theorem vs. filamentation (emittance growth)
- ⟶ discrete frequency analysis, NAFF algorithm (vs. FFT)
- ⟶ control of simulation error sources:
  1. discretisation error (symplecticity!)
  2. modelling error
  3. numerical artefacts
  4. (input error)
- ⟶ deterministic chaos,
  early indicators: (max.) Lyapunov exponent, frequency map analysis

Golden Thread

I. **basic concepts**: lectures 1-3

II. **longitudinal dynamics**: lectures 4-6

- ⟶ Lorentz force, electric longitudinal field E_z to accelerate
- ⟶ beam rigidity, paraxial approximation
- ⟶ momentum compaction, phase slippage, transition energy
- ⟶ phase focusing and stability (classical vs. relativistic regime)
- ⟶ longitudinal tracking equations (discrete one-turn map)
- ⟶ synchrotron Hamiltonian, rf bucket
- ⟶ Monte-Carlo sampling (random number generation)
- ⟶ equilibrium distributions (thermal PDF), small-amplitude approximation vs. nonlinear matching
- ⟶ emittance growth mechanisms (filamentation ↔ bucket non-linearity) from dipole and quadrupole moment oscillations

Golden Thread

I. **basic concepts**: lectures 1-3

II. **longitudinal dynamics**: lectures 4-6

III. **transverse dynamics**: lectures 7-8

- ⟶ magnetic fields for bending (steering) and focusing
- ⟶ multipole representation, dipole / quadrupole / sextupole magnets
- ⟶ Hill differential equation, quasi-harmonic oscillation
- ⟶ betatron transport matrices
- ⟶ FODO cell, alternate-gradient focusing
- ⟶ optics / Twiss functions, β-function as beam envelope, dispersion function
- ⟶ stability of periodic transport maps
- ⟶ betatron tune, chromaticity

Golden Thread

I. **basic concepts**: **lectures 1-3**

II. **longitudinal dynamics**: **lectures 4-6**

III. **transverse dynamics**: **lectures 7-8**

IV. **applications**:

- ⟶ longitudinal phase-space tomography **(lecture 9)**
  - Radon transform, sinogram, Fourier Slice Theorem
  - filtered back projection vs. algebraic reconstruction technique
- ⟶ closed orbit distortion **(lecture 10)**
  - local orbit correction (bumps)
  - global orbit correction (orbit response matrix, SVD)
- ⟶ machine learning **(lectures 11-12)**
  - ⟶ Bayesian optimisation (Gaussian processes, uncertainty modelling)
  - ⟶ reinforcement learning (discrete vs. continuous state/action spaces, Q-learning & actor-critic methods)
- ⟶ self-consistent modelling / collective effects **(lecture 13)**
  - categories of beam interactions (space charge, ...)
  - longitudinal space charge, line density derivative λ' model
  - microwave instability

You have gained solid fundamental knowledge on numerical modelling of periodic physics + have seen in action some dynamical examples from accelerator physics!

... and perhaps became a happy python user.

Well done! :))