Design of non-linear controller of Rotary Inverted Pendulum

Ayan Sengupta
Roll No-16307R005

April 14, 2019

1 Introduction

The Rotary Inverted Pendulum is a highly nonlinear system. A rotary inverted pendulum system (see Figure 1), has a rotating arm, which is driven by a motor with a pendulum mounted on its rim. The pendulum moves as an inverted pendulum in a plane perpendicular to the rotating arm. The controller needs

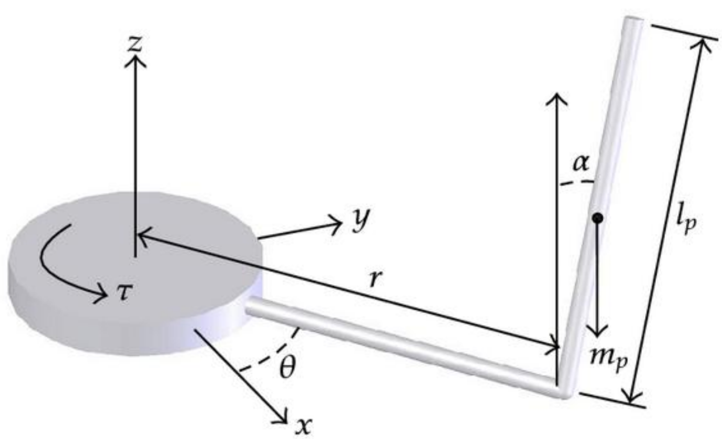

Figure 1: Rotary Inverted Pendulum

to stabilize two angles namely, θ-the angle of the motor joint and α-the angle of the pendulum. Now, θ has to be stabilized to be within a particular bound while α has to be made 180 °.

2 Dynamics of the System

In this section the dynamics of the Pendulum is given. The system parameters were taken from the simulink model provided.

Table 1: Values used for system model

Parameter	**Description**	**Value**
J_{eq}	Equivalent moment of inertia about motor shaft .	1.23×10^{-4}kg-m^2
m	Mass of the pendulum assembly	0.027kg
r	Length of arm pivot to pendulum pivot	0.08260m
J_m	Motor shaft moment of inertia	0.00011kg-m^2
L	Total length of pendulum.	0.191m
B_{eq}	Arm viscous damping	0.0015 $\frac{N-m}{rad/s}$
g	gravitational constant	9.81 m/s^2
K_m	Motor back-electromotive force constant	0.02797 $\frac{V}{rad/s}$
K_t	Motor torque constant	0.02797N-m
R$_m$	Motor armature resistance.	3.30 Ω

Using the values of Table(1) following constants were calculated.

$$\left.\begin{aligned} a &= J_{eq} + J_m + mr^2 \\ b &= mLr \\ c &= \frac{4mL^2}{3} \\ d &= mgL \\ G &= \frac{K_t K_m + B_{eq} R_m}{R_m} \end{aligned}\right\} \tag{1}$$

The nonlinear state equation of the system is given below:

$$\dot{x_1} = x_2 = \dot{\alpha} \tag{2}$$
$$\dot{x_2} = F_1(X) + G_1(X)u \tag{3}$$
$$\dot{x_3} = x_4 = \dot{\theta} \tag{4}$$
$$\dot{x_4} = F_2(X) + G_2(X)u \tag{5}$$

where F_1, F_2, G_1, G_2 are:

$$F_1(X) = \frac{-0.5b^2 sin(2x_1)x_2^2 - Gbx_4 cos(x_1) + adsin(x_1)}{ac - b^2 cos^2 x_1} \tag{6}$$

$$G_1(X) = \frac{K_t bcos(x_1)}{R_m(ac - b^2 cos^2 x_1)} \tag{7}$$

$$F_2(X) = \frac{cF_1(X) - dsin(x_1)}{bcos(x_1)} \tag{8}$$

$$G_2(X) = \frac{K_t c}{R_m(ac - b^2 cos^2 x_1)} \tag{9}$$

$$T = \frac{(u - K_m \dot{\theta})K_t}{R_m} \tag{10}$$

3 Sliding Mode Controller

The sliding mode controller for the inverted pendulum is designed as described in [1]. Here two sliding surface are considered for sliding mode control. They are defined below:

$$s_1 = x_2 + \lambda_1 x_1 \tag{11}$$

$$s_2 = x_4 + \lambda_3 x_3 \tag{12}$$

The Lyapunov function is taken as:

$$V = |s_1| + \lambda_2 |s_2| \tag{12}$$

To guarantee the stability of the feedback system, the control signal u is taken such that

$$\dot{V} = -k \times sat(\frac{V}{\Phi})$$

where saturation function is defined as

$$sat(\frac{V}{\Phi}) = \frac{V}{\Phi} \qquad if \quad \Phi < |V| \tag{13}$$

$$sat(\frac{V}{\Phi}) = sign(V) \qquad otherwise \tag{14}$$

The final control signal is shown below:

$$u = \frac{-ksat(\frac{V}{\Phi}) - (\lambda_1 x_2 + F_1(X))sign(s_1) - \lambda_2(\lambda_3 x_4 + F_2(X))sign(s_2)}{G_1(X)sign(s_1) + \lambda_2 G_2(X)sign(s_2)} \tag{14}$$

4 Result

With this sliding mode controller with two sliding surfaces, there are five constants λ_1,λ_2,λ_3, k, Φ.The values of the constants are given in Table 2

Table 2: Values used for modeling

Constant	**Value**
λ_1	0.2
λ_2	0.9
λ_3	0.9
k	20
Φ	0.5

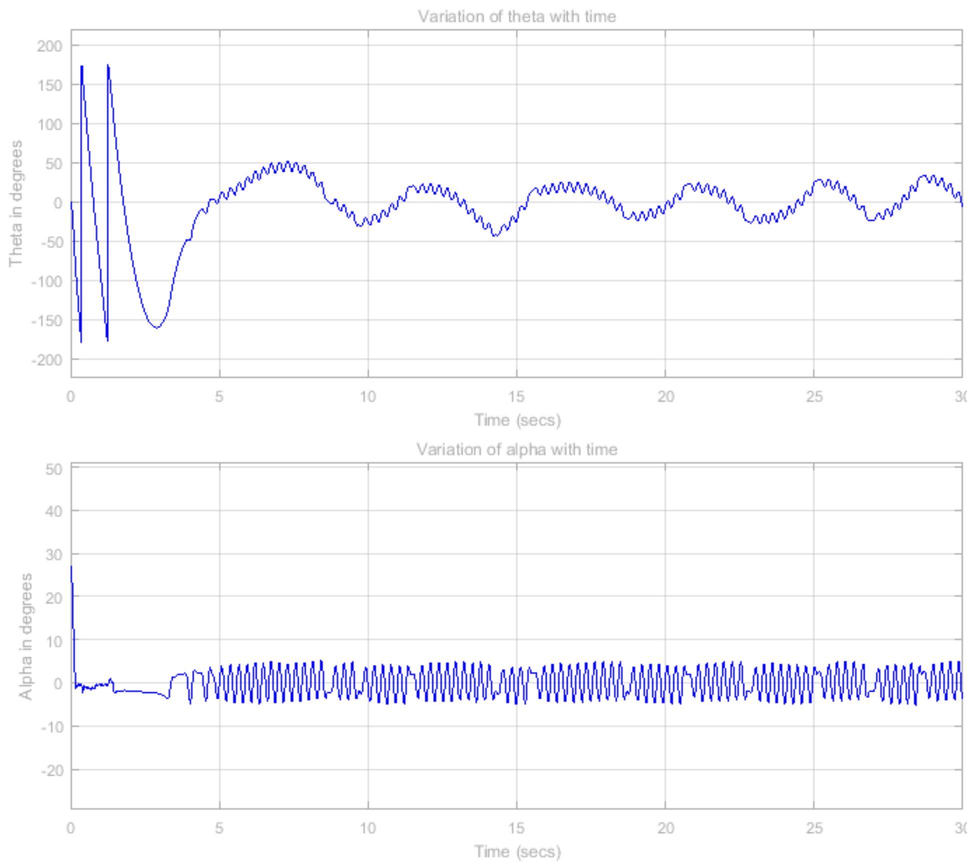

Figure 2: variation of θ and α with initial disturbance α=25°

The results achieved for the initial value of α being 27 ° is shown in Figures(2). The above figures show that the controller is able to stabilize both θ and α. We can see the prevailing chattering that is present in our output. The α has a maximum chattering of 5°. This controller is able to stablilize the inverted pendulum with an initial α in the range of 27°. Now we can reduce chattering to 3° by changing the constants, but that will reduce the maximum range of α for which this controller can stabilize the system.

References

[1] Khanesar, Mojtaba Ahmadieh, Mohammad Teshnehlab, and Mahdi Aliyari Shoorehdeli. ”Sliding mode control of rotary inverted pendulm.” Control & Automation, 2007. MED’07. Mediterranean Conference on. IEEE, 2007.