https://physnotes.jp/foundations/b_al/ → a vector (or matrix?)

min

subject to $\vec{x} \in \mathcal{X}$ (向量)　　$\vec{x}$ is called decision variable.

where $f: \mathbb{R}^n \to \mathbb{R}$ → output

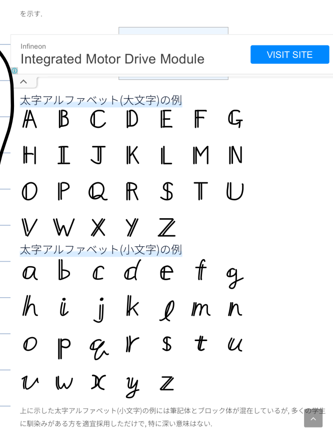
↓

input

$\mathcal{X} \subseteq \mathbb{R}^n$.

⇓　　⇓

矩阵　矩阵.



means that f maps each ordered pair (which contains n numbers as input) to a single number (as output).

$x \in \mathbb{R}$: x is simply one dimensional scalar. $x = -2$ or $x = 42$ ....

$\vec{x} \in \mathbb{R}^2$: $\vec{x}$ is a two dimensional vector, whose two components are both real numbers. ($\vec{x}$ is an ordered pair in the Cartesian plane, that has the form (x_1, x_2) where $x_1, x_2 \in \mathbb{R}$

∴ $\vec{x} = (-1, 7)$ or $\vec{x} = (\pi, 2.54)$.

$\vec{x} \in \vec{\mathcal{X}}$, $\vec{\mathcal{X}} \subseteq \mathbb{R}^n$.

⇓

$\vec{\mathcal{X}}$ is a subset of $\mathbb{R}^n$

model:　min $f(x)$

s.t. $\vec{x} \in \vec{\mathcal{X}}$

where $f: \mathbb{R}^n \to \mathbb{R}$ and $\vec{\mathcal{X}} \subseteq \mathbb{R}^n$.

① : vector of decesion variables $\vec{x}$: $[x_1, x_2 \cdots x_n]^T \in \mathbb{R}^n$.

② : the objective function $f(x)$

③ : constraint set or feasible region $\vec{\mathcal{X}}$.

$=$

$\Downarrow$

equalities
inequalities

$$\vec{x} = \{\vec{x} \in \mathbb{R}^n ; g_i(\vec{x}) = b_i, \text{ for } i=1,\cdots,m \text{ and } h_j(\vec{x}) \leq d_j, \text{ for } j=1,\cdots,p.$$

$\Downarrow$

is a set of equlities and inequalities functions.

$$\vec{x} \in \mathbb{R}^n ; g_i(\vec{x}) = b_i, \text{ for } i=1\cdots m.$$

means, for n dimensional vector $\vec{x}$,

there are m equality functions, each has the $\vec{x}$ and outputs a scalar number b.

in this case, the optimization problem can be written as,

$$\min_x f(\vec{x})$$
$$\text{s.t. } g_i(\vec{x}) = b_i, \quad i=1,2,\cdots m.$$
$$h_j(\vec{x}) \leq d_j, \quad j=1,2,\cdots p.$$

or in the more concise form

$$\min_x f(\vec{x})$$
$$\text{s.t. } \vec{g}(\vec{x}) = \vec{b}$$
$$\vec{h}(\vec{x}) = \vec{d}.$$

(vector must be a column, not a row)

$\Downarrow$ $\Uparrow$

means, $\vec{g}, \vec{b}, \vec{h}, \vec{d}$ is a one dimensional (vector)

(an array)

model: min $f(x)$

s.t. $\vec{x} \in \vec{\mathcal{X}}$

where $f: \mathbb{R}^n \to \mathbb{R}$ and $\vec{\mathcal{X}} \subseteq \mathbb{R}^n$.

a feasible point or feasible solution is a point in the constraint set $\vec{\mathcal{X}}$

is a matrix.

also a set of equalities and inequalities.

An optimal solution is a feasible point that attains the best possible objective value, that is a point

$\vec{x}^* \in \vec{\mathcal{X}}$, such that, $f(\vec{x}^*) \le f(\vec{x})$ for all $\vec{x} \in \vec{\mathcal{X}}$.

⇒ Case in the standard form, it's $\min f(\vec{x})$.

⇓

means, $f(\vec{x}^*)$ is smallest one, or equal to the smallest one.

$\vec{x}^*$ is a point lays in the feasible region ($\vec{\mathcal{X}}$), makes the objective function output is the minimum.

and. $f(\vec{x}^*)$ is the optimal value.

$\vec{x}^*$ is the optimal point (or optimal solution).

binding constraints: left LHS and RHS of the constraint is equal.
(or active constraints) $A\vec{x} = b$.

The problem is infeasible if $\vec{\mathcal{X}} = \emptyset$.

if $\vec{\mathcal{X}} = \emptyset$

$\Downarrow$ (null matrix)

constraint matrix is zero matrix

On the other hand

$$\boxed{\begin{aligned} \text{model: } & \min f(x) \\ & \text{s.t. } \vec{x} \in \vec{\mathcal{X}} \\ \text{where } & f: \mathbb{R}^n \to \mathbb{R} \text{ and } \vec{\mathcal{X}} \subseteq \mathbb{R}^n. \end{aligned}}$$

is unbounded if there exists $\vec{x}_k \in \vec{\mathcal{X}}$, $k = 1, 2, \cdots$ such that $f(\vec{x}_k) \to -\infty$.

means: if there is a point in the constraints set which makes the objective function value is $-\infty$, then the problem is unbounded or infeasiable.