① LP ② NLP. ③ IP. ④ GD/GA

Gradient Decent (Min). Gradient Ascent (Max)

= Steppest descent (ascent).

max $f(x,y) = 2xy + 2y - x^2 - 2y^2$. (A concave, by Hessian matrix).

$\frac{\partial f}{\partial x} = 2y - 2x = 0 \Rightarrow y = x$.

$\frac{\partial f}{\partial y} = 2x + 2 - 4y = 0$. $2x + 2 - 4y = 0$, $-2x = -2$.

$x = 1, y = 1$. $(1,1)$ is global max.

The idea of gradient ascent:

$\vec{x}_{t+1} = \vec{x}_t + \alpha \nabla f(\vec{x}_t)$, $t = 0, 1, 2, \ldots$

$\vec{x}_1 = \vec{x}_0 + \alpha \cdot \nabla f(\vec{x}_0)$ first point is added by initial a directional small step.

direction is the ∇.

step size $= \alpha$.

initial point can be acheived arbitrarily :

let $\vec{x}_0 = (0, 0)$.

$$\nabla f = \begin{bmatrix} 2y - 2x \\ 2x + 2 - 4y \end{bmatrix}.$$

$$\bar{x}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \alpha \cdot \begin{pmatrix} 2\cdot 0 - 2\cdot 0 \\ 2\cdot 0 + 2 - 4\cdot 0 \end{pmatrix}$$

$$\bar{x}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \alpha \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix}.$$

$$\bar{x}_1 = \begin{pmatrix} 0 \\ 2\alpha \end{pmatrix}.$$ we have to determind α.

we have to put $\begin{cases} x = 0 \\ y = 2\alpha \end{cases}$ in to original function,

$$2xy + 2y - x^2 - 2y^2.$$

$$f(x,y) = 2\cdot 0\cdot 2\alpha + 2\cdot 2\alpha - 0 - 2\cdot 4\alpha^2$$

$$= 0 + 4\alpha - 8\alpha^2.$$

$$= 4\alpha - 8\alpha^2$$

we have to maximize this.(f).

$\frac{\partial f}{\partial \alpha} = 4 - 16\alpha = 0$. $\alpha = \frac{1}{4}$. $\therefore$ $\alpha = \frac{1}{4}$ maximizes the α.

$\therefore$ $\vec{x}_1 = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$

$\vec{x}_{t+1} = \vec{x}_t + \alpha \cdot \nabla f(\vec{x}_t)$

$\vec{x}_2 = x_1 + \alpha \cdot \nabla f(\vec{x}_1)$

$\nabla f = \begin{bmatrix} 2y - 2x \\ 2x + \cancel{2} - 4y \end{bmatrix}$.

$\nabla f \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 1 - 0 \\ 0 + 2 - 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$\vec{x}_2 = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} + \alpha \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\vec{x}_2 = \begin{pmatrix} \alpha \\ 0.5 \end{pmatrix}$.

$\boxed{2xy + 2y - x^2 - 2y^2}$.

$$f(\bar{x}_2) = \cancel{2}\alpha \cdot \frac{1}{\cancel{2}} + \cancel{2} \cdot \frac{1}{\cancel{2}} - \alpha^2 - \cancel{2} \cdot \frac{1}{\cancel{4}}$$

$$= \alpha + 1 - \alpha^2 - \frac{1}{2}$$

$$= -\alpha^2 + \alpha + \frac{1}{2}$$

$$\frac{\partial f}{\partial \alpha} = -2\alpha + 1 = 0 \qquad -2\alpha = -1$$

$$\alpha = \frac{1}{2} = 0.5 \qquad \therefore x_2 = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

$$\bar{x}_3 = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} + \alpha \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\nabla f(\bar{x}_2) = \begin{pmatrix} 2 \times \frac{1}{2} - 2 \times \frac{1}{2} \\ \cancel{2 \times \frac{1}{2}} + 2 - \cancel{4 \times \frac{1}{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\bar{x}_3 = \begin{pmatrix} 0.5 \\ 0.5 + \alpha \end{pmatrix}$$

$$1 + \cancel{2 - 2}$$

$$2xy + 2y - x^2 - 2y^2.$$

$$f(\tilde{x}_3) = 2 \times 0.5 \times (0.5+\alpha) + 2(0.5+\alpha) - \frac{1}{4} - 2\times(0.5+\alpha)^2$$

$$= 3(0.5+\alpha) - \frac{1}{4} - 2\times(0.5+\alpha)^2.$$

$$= \frac{6}{4} + 3\alpha - \frac{1}{4} - 2\times\left(\frac{1}{4} + \alpha + \alpha^2\right).$$

$$= \frac{5}{4} + 3\alpha - \frac{2}{4} - 2\alpha - 2\alpha^2.$$

$$= -2\alpha^2 + \alpha + \frac{3}{4}.$$

$$\frac{\partial f}{\partial \alpha} = -4\alpha + 1. \qquad -4\alpha + 1 = 0 \Rightarrow \alpha = \frac{1}{4},$$

$\therefore \bar{X}_3 = \begin{pmatrix} \frac{1}{2} \\ \frac{3}{4} \end{pmatrix}$, ...

$$\nabla f = \begin{bmatrix} 2y - 2x \\ 2x + \cancel{2} - 4y \end{bmatrix}.$$

$$\bar{X}_4 = \bar{X}_3 + \alpha \cdot \nabla f(\bar{X}_3)$$

$$\nabla f(\bar{X}_3) = \begin{pmatrix} \frac{3}{2} - 1 = \frac{1}{2} \\ \cancel{2 \times \frac{1}{2}} + 2 - \cancel{4 \times \frac{3}{4}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}.$$

$$\bar{X}_4 = \begin{pmatrix} \frac{1}{2} \\ \frac{3}{4} \end{pmatrix} + \alpha \cdot \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}$$

$$\bar{X}_4 = \begin{pmatrix} \frac{1}{2} + \alpha \cdot \frac{1}{2} \\ \frac{3}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(1+\alpha) \\ \frac{3}{4} \end{pmatrix}$$

$$\boxed{2xy + 2y - x^2 - 2y^2.}$$

$$f = \cancel{2 \times \frac{1}{2}}(1+\alpha) \cdot \frac{3}{4} + 2 \times \frac{3}{4} - \frac{1}{4}(1+\alpha)^2 - 2 \times \frac{9}{16}$$

$$= \frac{3}{4}(1+\alpha) + \frac{3}{2} - \frac{1}{4}(1+2\alpha+\alpha^2) - \frac{9}{8}$$

$$\frac{\partial f}{\partial \alpha} = \frac{3}{4} - \frac{1}{4}\left[2+2\alpha\right] = 0 .$$

$$\cancel{3} = \cancel{2} + 2\alpha .$$

$$1 = 2\alpha . \Rightarrow \alpha = \frac{1}{2} .$$

$$\therefore \bar{x}_4 = \begin{pmatrix} \frac{1}{2}(1+\frac{1}{2}) \\ \frac{3}{4} \end{pmatrix} = \begin{pmatrix} \frac{3}{4} \\ \frac{3}{4} \end{pmatrix}$$

$$2y - 2x = 0 .$$

$$-1$$

$$2x+2-4y = \frac{3}{2} + 2 - \cancel{4} \times \frac{3}{\cancel{4}}$$

$$= \frac{3}{2} - \frac{2}{2} = \frac{1}{2} ,$$

$$\therefore \bar{x}_5 = \begin{pmatrix} \frac{3}{4} \\ \frac{3}{4} \end{pmatrix} + \alpha \cdot \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$$

$$\bar{x}_5 = \begin{pmatrix} \frac{3}{4} \\ \frac{3}{4} + \frac{1}{2}\alpha \end{pmatrix}.$$

$$\boxed{2xy + 2y - x^2 - 2y^2.}$$

$$f = \cancel{2\times}\frac{3}{4}\times\left(\frac{3}{4}+\frac{1}{2}\alpha\right) + 2\times\left(\frac{3}{4}+\frac{1}{2}\alpha\right) - \frac{9}{16} - 2\times\left(\frac{3}{4}+\frac{1}{2}\alpha\right)^2$$

$$= 2\times\frac{3}{4}\times\frac{3}{4} + 2\times\frac{3}{4}\times\frac{1}{2}\alpha + 2\times\frac{3}{4} + \alpha - \frac{9}{16} - 2\times\left(\frac{9}{16} + \frac{3}{4}\alpha + \frac{1}{4}\alpha^2\right).$$

$$= \frac{3}{4}\alpha + \alpha - 2\times\frac{3}{4}\alpha - 2\times\frac{1}{4}\alpha^2 + \cdots$$

$$= \frac{1}{4}\alpha - \frac{1}{2}\alpha^2 + \cdots$$

$$\frac{\partial f}{\partial \alpha} = -\cancel{\frac{1}{2}\times 2}\,\alpha + \frac{1}{4} \qquad \alpha = \frac{1}{4}.$$

$$= -\alpha + \frac{1}{4}.$$

$$\overline{X}_5 = \begin{pmatrix} \frac{3}{4} \\ \frac{3}{4} + \frac{1}{2} \cdot \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{3}{4} \\ \frac{7}{8} \end{pmatrix}.$$

$$2y - 2x = 2 \times \frac{7}{8} - 2 \times \frac{3}{4}$$

$$= \frac{1}{4}.$$

$$\overline{X}_6 = \begin{pmatrix} \frac{3}{4} \\ \frac{7}{8} \end{pmatrix} + \alpha \cdot \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix}$$

$$2x + 2 - 4y = 2 \times \frac{3}{4} + 2 - 4 \times \frac{7}{8}.$$

$$= \frac{3+4}{2} - \frac{7}{2} = 0.$$

$$\overline{X}_6 = \begin{pmatrix} \frac{3}{4} + \frac{1}{4}\alpha \\ \frac{7}{8} \end{pmatrix}.$$

$$\frac{1}{4}(3+\alpha)$$

$$9 + 6\alpha + \alpha^2.$$

$$f(\alpha) = 2 \times \frac{1}{4}(3+\alpha) \cdot \frac{7}{8} + 2 \times \frac{7}{8} - \frac{1}{16}(3+\alpha)^2 - 2 \times \frac{56}{64}$$

$$\frac{7}{16}\alpha - \frac{1}{16} \times 6\alpha - \frac{1}{16} \cdot \alpha^2$$

$$\frac{\partial f}{\partial \alpha} = -\frac{1}{8}\alpha + \frac{1}{16} = 0 \Rightarrow \frac{1}{8}\alpha = \frac{1}{16}$$

$$2\alpha = 1, \quad \alpha = \frac{1}{2}.$$

in $$\bar{x}_f = \begin{pmatrix} \frac{3}{4} + \frac{1}{4} \times \frac{1}{2} \\ \frac{7}{8} \end{pmatrix} = \begin{pmatrix} \frac{7}{8} \\ \frac{7}{8} \end{pmatrix}.$$

...... skipped. final result = (1, 1).