Dynamic Trajectories via Convex Optimization

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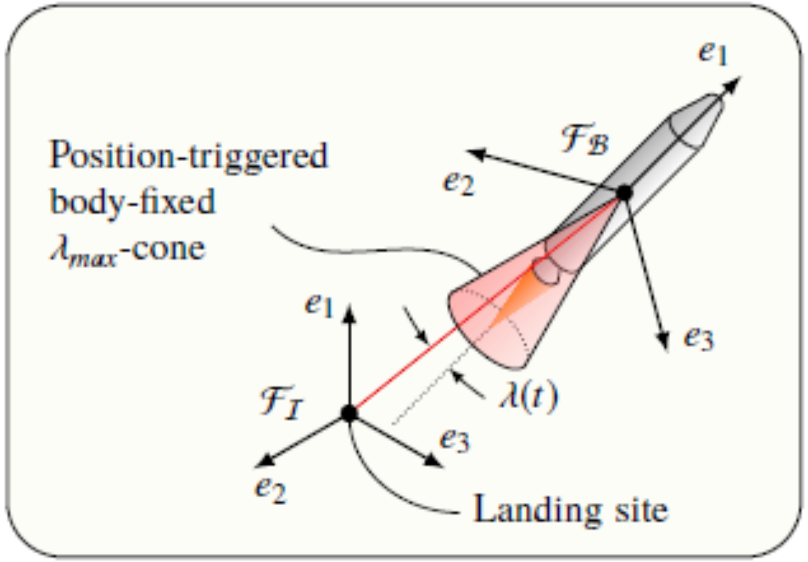
March, 26th 2020

Problem Overview

Physics-based optimization…



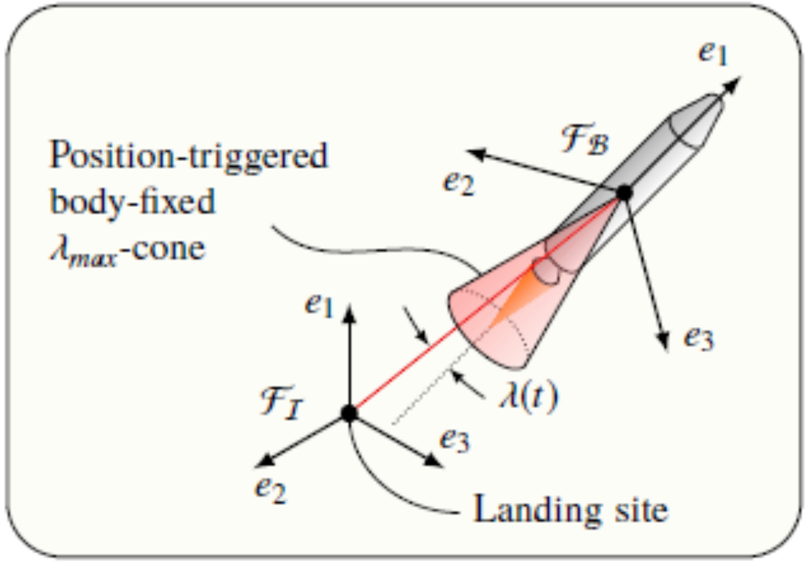
Szmuk, Reynolds, Acikmese. (2020).
Successive Convexification for Real-Time 6-DOF Powered Descent Guidance with State-Triggered Constraints. JGCD 2020.

…results in dynamically feasible solutions

Scharf, Acikmese, Dueri et al. (2014).
ADAPT Demonstrations of Onboard Large-Divert Guidance with a VTVL Rocket. IEEE Aerospace Conference 2014.

Problem Overview

Optimal trajectory generated while satisfying necessary constraints



Szmuk, Reynolds, Acikmese. (2020). Successive Convexification for Real-Time 6-DOF Powered Descent Guidance with State-Triggered Constraints. JGCD 2020.

Reynolds, Szmuk, Malyuta, Mesbahi, Acikmese (2020). Dual Quaternion Based Powered Descent Guidance with State-Triggered Constraints. JGCD 2020.

Szmuk, Acikmese. (2018). Successive Convexification for 6-DOF Mars Rocket Powered Landing with Free-Final-Time. AIAA GNC Conference 2018.

Problem 1. **Minimum-fuel rocket-landing problem**

Cost Function:

$$\underset{t_c,\, t_b,\, T_\mathcal{B}(t)}{\text{minimize}} \quad -m(t_f)$$

$$\text{s.t.} \quad t_c \in [0, t_{c,max}]$$

Boundary Conditions:

$$\begin{aligned} m(t_{ig}) &= m_{ig} & q_{\mathcal{B}\leftarrow\mathcal{I}}(t_f) &= q_{id} \\ r_\mathcal{I}(t_{ig}) &= p_{r,ig}(t_c) & r_\mathcal{I}(t_f) &= 0 \\ v_\mathcal{I}(t_{ig}) &= p_{v,ig}(t_c) & v_\mathcal{I}(t_f) &= -v_d e_1 \\ \omega_\mathcal{B}(t_{ig}) &= 0 & \omega_\mathcal{B}(t_f) &= 0 \end{aligned}$$

Dynamics:

$$\begin{aligned} \dot m(t) &= -\alpha_{\dot m} \|T_\mathcal{B}(t)\|_2 - \beta_{\dot m} \\ \dot r_\mathcal{I}(t) &= v_\mathcal{I}(t) \\ \dot v_\mathcal{I}(t) &= \frac{1}{m(t)} C_{\mathcal{I}\leftarrow\mathcal{B}}(t)\big(T_\mathcal{B}(t) + A_\mathcal{B}(t)\big) + g_\mathcal{I} \\ \dot q_{\mathcal{B}\leftarrow\mathcal{I}}(t) &= \frac{1}{2}\Omega(\omega_\mathcal{B}(t))\, q_{\mathcal{B}\leftarrow\mathcal{I}}(t) \\ J_\mathcal{B}\dot\omega_\mathcal{B}(t) &= r_{T,\mathcal{B}} \times T_\mathcal{B}(t) + r_{cp,\mathcal{B}} \times A_\mathcal{B}(t) - \omega_\mathcal{B}(t) \times J_\mathcal{B}\omega_\mathcal{B}(t) \end{aligned}$$

State Constraints:

$$\begin{aligned} m_{dry} &\le m(t) \\ \tan\gamma_{gs} \left\|H_\gamma r_\mathcal{I}(t)\right\|_2 &\le e_1 \cdot r_\mathcal{I}(t) \\ \cos\theta_{max} &\le 1 - 2\left\|H_\theta q_{\mathcal{B}\leftarrow\mathcal{I}}(t)\right\|_2 \\ \|\omega_\mathcal{B}(t)\|_2 &\le \omega_{max} \end{aligned}$$

Control Constraints:

$$\begin{aligned} 0 < T_{min} \le \|T_\mathcal{B}(t)\|_2 &\le T_{max} \\ \cos\delta_{max} \|T_\mathcal{B}(t)\|_2 &\le e_3 \cdot T_\mathcal{B}(t) \end{aligned}$$

State-Triggered Constraints:

$$h_\alpha\big(v_\mathcal{I}(t), q_{\mathcal{B}\leftarrow\mathcal{I}}(t)\big) \le 0$$

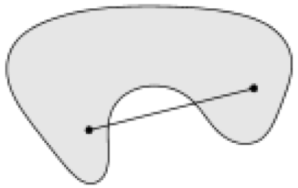
Optimal Guidance Problem Overview

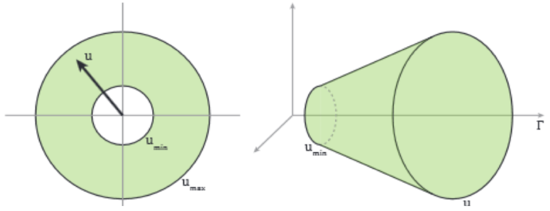
Why is it so fast?

- **Harness speed of convex optimization to solve complex, nonlinear problems**
  - **Take local convex approximation of nonlinear problem to achieve relevant run time**

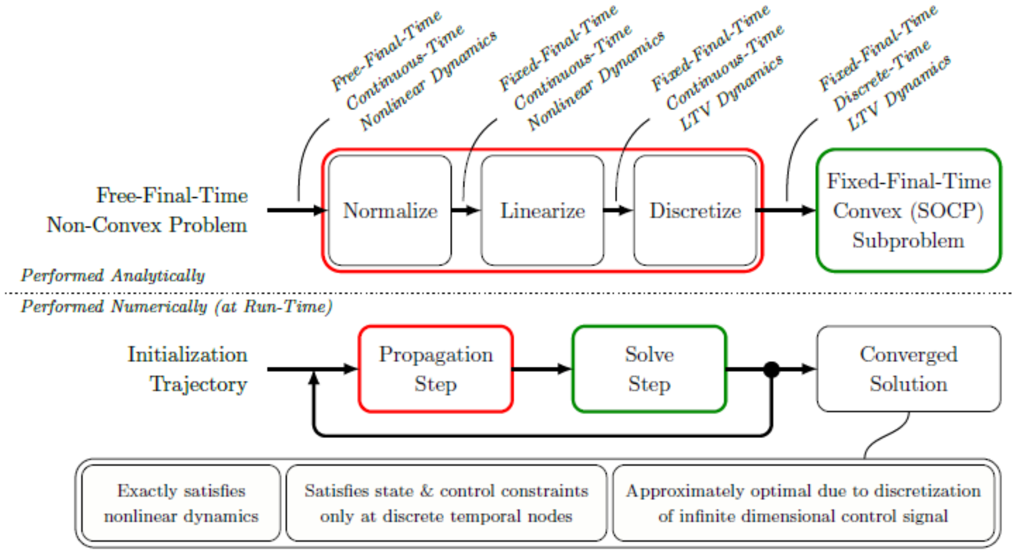
- **Convexify an optimal control problem (lossless convexification)**

- **Create sequence of convexified subproblems (successive convexification)**
  - **In practice, number of iterations shown to be small**





Szmuk, Pascucci, Dueri, Acikmese (2017).
Convexification and Real-Time On-Board Optimization for Agile Quad-Rotor Maneuvering and Obstacle Avoidance. IEEE IROS 2017.



Szmuk, Reynolds, Acikmese, Mesbahi (2019).
Successive Convexification for 6-DOF Powered Descent Guidance with Compound State-Triggered Constraints. AIAA SciTech 2019.

Optimal Guidance Problem Overview

Autonomous Controls Lab

How do we deal with conditional decision making?

- State-Triggered Constraints (STCs) model logical implications
  - Constraints enforced when state-dependent criterion are met
  - Extension to optimization-based trajectory generation methods
- Integrate conditional logic into a continuous optimization framework
  - Embed a subset of discrete decision constraints into continuous problem
  - Formulated using continuous variables of the optimization problem
  - Nonlinear function models implications that trigger constraints
- Composed of *trigger function* and *constraint function*
  - *g(z)* – trigger function
  - *c(z)* – constraint function
  - *z* – optimization variable
- *Constraint function* is conditionally enforced based on the value of the *trigger function*
  - If the trigger function is non-negative, then optimization variable is not subject to the constraint condition
  - If trigger function becomes negative, then constraint is enforced

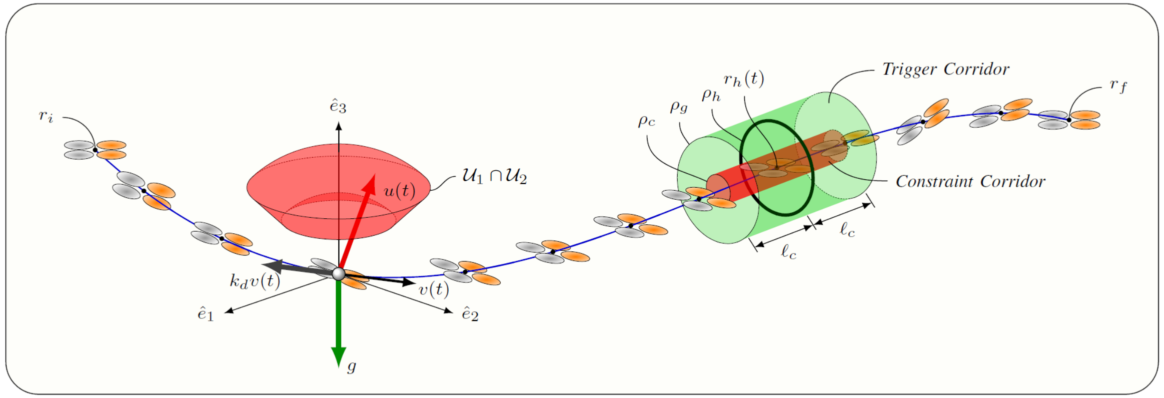
$$g(z) < 0 \quad \Rightarrow \quad c(z) \leq 0$$

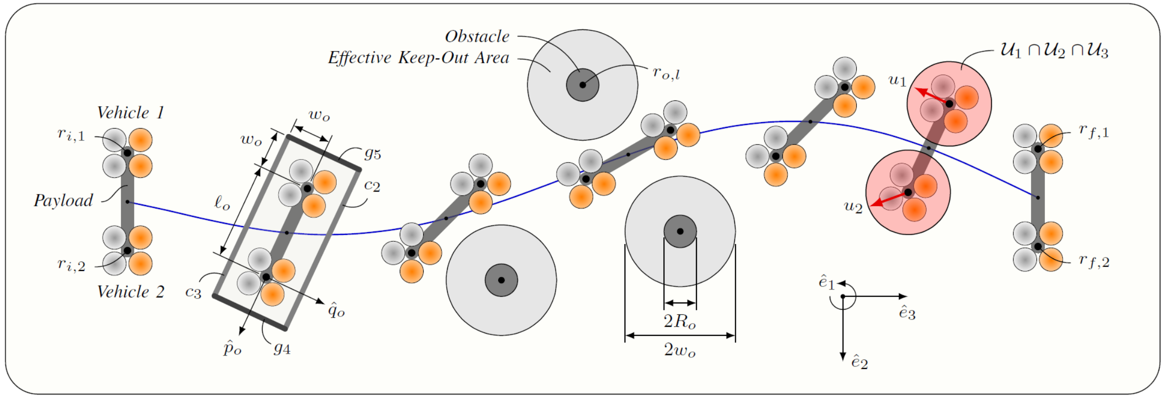
Triggers:	*Constraints:*
• speed	• angle of attack
• distance	• heating level
• battery voltage	• pointing attitude constraint

Szmuk, Malyuta, Reynolds, Mceowen, Acikmese. (2019). Real-Time Quad-Rotor Path Planning Using Convex Optimization and Compound State-Triggered Constraints. IEEE IROS 2019.

Optimal Guidance Problem Overview

Example 1: quadrotor with STCs





Szmuk, Malyuta, Reynolds, Mceowen, Acikmese. (2019).
Real-Time Quad-Rotor Path Planning Using Convex Optimization and Compound State-Triggered Constraints. IEEE IROS 2019.

Problem 1: ***Non-Convex Formulation of Scenario 1***

$$\underset{u}{\text{minimize}} \int_0^{t_f} \|u(t)\|_2 dt$$

subject to:

$$r(0) = r_i,\ r(t_f) = r_f,$$
$$v(0) = v(t_f) = 0_{3\times1},$$
$$u(0) = u(t_f) = mg\hat{e}_1,$$
$$\dot{x}(t) = Ax(t) + Bu(t) + Ew,\ u(t) \in \mathcal{U}_1 \cap \mathcal{U}_2,$$
$$\|v(t)\|_2 \le v_{max},\ h_1\big(r(t), r_h(t)\big) \le 0.$$

Objective: Min. fuel

Boundary conditions

Dynamics & control

State and safety

Problem 2: ***Non-Convex Formulation of Scenario 2***

$$\underset{u}{\text{minimize}} \int_0^{t_f} \big(\|u_1(t)\|_2 + \|u_2(t)\|_2\big) dt$$

subject to:

$$r_1(0) = r_{i,1},\ r_1(t_f) = r_{f,1},$$
$$r_2(0) = r_{i,2},\ r_2(t_f) = r_{f,2},$$
$$v_1(0) = v_2(0) = v_1(t_f) = v_2(t_f) = 0_{3\times1},$$
$$u_1(0) = u_2(0) = u_1(t_f) = u_2(t_f) = mg\hat{e}_1,$$

$$\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}\tilde{u}(t) + \tilde{E}\tilde{w},$$
$$u_1(t), u_2(t) \in \mathcal{U}_1 \cap \mathcal{U}_2 \cap \mathcal{U}_3,$$

$$h_2\big(r_1(t), r_2(t), r_{o,l}\big) = 0,\quad \forall j \in \mathcal{N}_o,$$
$$\|v_1(t)\|_2 \le v_{max},\ \|v_2(t)\|_2 \le v_{max},$$
$$\|r_1(t) - r_2(t)\|_2 = \ell_o.$$

Objective: Min. fuel

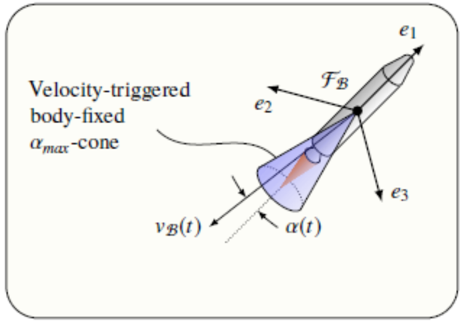
Boundary conditions

Dynamics, state and control

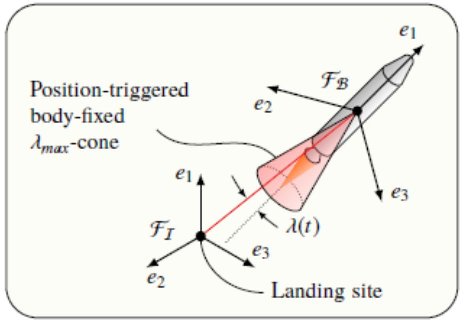
Safety constraints

Optimal Guidance Problem Overview

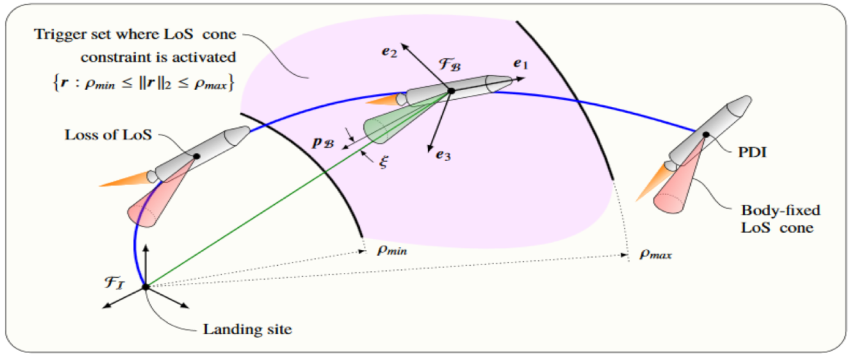
Example 2: rocket landing problem



(a) Angle of Attack State-Triggered Constraint



(b) Field of View State-Triggered Constraint



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Successive Convexification for Real-Time 6-DOF Powered Descent Guidance with State-Triggered Constraints. JGCD 2020.

Reynolds, Szmuk, Malyuta, Mesbahi, Acikmese (2020).
Dual Quaternion Based Powered Descent Guidance with State-Triggered Constraints. JGCD 2020.

Reynolds, Malyuta, Mesbahi, Acikmese. (2020).
A Real-Time Algorithm for Non-Convex Powered Descent Guidance. AIAA SciTech 2020.

Problem 1. **Minimum-fuel rocket-landing problem**

Cost Function:

$$\underset{t_c,\, t_b,\, T_{\mathcal{B}}(t)}{\text{minimize}} \quad -m(t_f)$$

$$\text{s.t.} \quad t_c \in [0, t_{c,max}]$$

Boundary Conditions:

$$m(t_{ig}) = m_{ig} \qquad q_{\mathcal{B}\leftarrow\mathcal{I}}(t_f) = q_{id}$$
$$r_{\mathcal{I}}(t_{ig}) = p_{r,ig}(t_c) \qquad r_{\mathcal{I}}(t_f) = 0$$
$$v_{\mathcal{I}}(t_{ig}) = p_{v,ig}(t_c) \qquad v_{\mathcal{I}}(t_f) = -v_d e_1$$
$$\omega_{\mathcal{B}}(t_{ig}) = 0 \qquad \omega_{\mathcal{B}}(t_f) = 0$$

Dynamics:

$$\dot{m}(t) = -\alpha_{\dot{m}} \|T_{\mathcal{B}}(t)\|_2 - \beta_{\dot{m}}$$
$$\dot{r}_{\mathcal{I}}(t) = v_{\mathcal{I}}(t)$$
$$\dot{v}_{\mathcal{I}}(t) = \frac{1}{m(t)} C_{\mathcal{I}\leftarrow\mathcal{B}}(t)\big(T_{\mathcal{B}}(t) + A_{\mathcal{B}}(t)\big) + g_{\mathcal{I}}$$
$$\dot{q}_{\mathcal{B}\leftarrow\mathcal{I}}(t) = \frac{1}{2}\Omega(\omega_{\mathcal{B}}(t))\, q_{\mathcal{B}\leftarrow\mathcal{I}}(t)$$
$$J_{\mathcal{B}}\dot{\omega}_{\mathcal{B}}(t) = r_{T,\mathcal{B}} \times T_{\mathcal{B}}(t) + r_{cp,\mathcal{B}} \times A_{\mathcal{B}}(t) - \omega_{\mathcal{B}}(t) \times J_{\mathcal{B}}\omega_{\mathcal{B}}(t)$$

State Constraints:

$$m_{dry} \le m(t)$$
$$\tan\gamma_{gs} \|H_\gamma r_{\mathcal{I}}(t)\|_2 \le e_1 \cdot r_{\mathcal{I}}(t)$$
$$\cos\theta_{max} \le 1 - 2\,\|H_\theta q_{\mathcal{B}\leftarrow\mathcal{I}}(t)\|_2$$
$$\|\omega_{\mathcal{B}}(t)\|_2 \le \omega_{max}$$

Control Constraints:

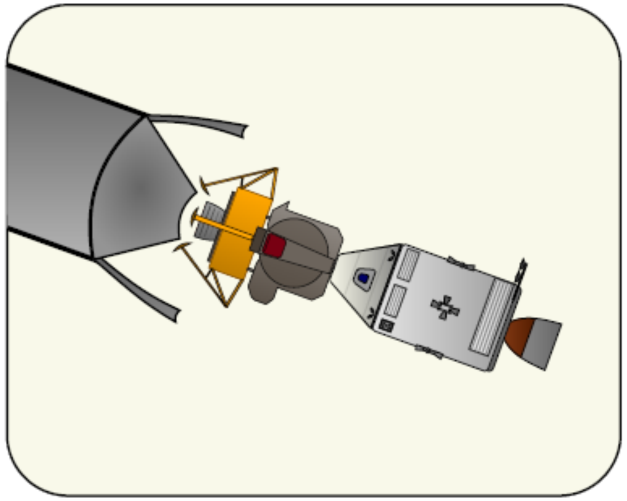
$$0 < T_{min} \le \|T_{\mathcal{B}}(t)\|_2 \le T_{max}$$
$$\cos\delta_{max} \|T_{\mathcal{B}}(t)\|_2 \le e_3 \cdot T_{\mathcal{B}}(t)$$

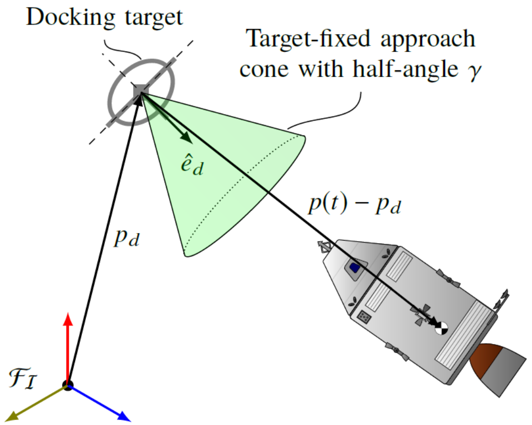
State-Triggered Constraints:

$$h_\alpha(v_{\mathcal{I}}(t), q_{\mathcal{B}\leftarrow\mathcal{I}}(t)) \le 0$$

Optimal Guidance Problem Overview

Example 3: spacecraft rendezvous





$$
\begin{aligned}
\underset{\sigma_i(t)}{\text{minimize}} \quad & J_f \\
\text{subject to :} \quad & \dot{p}(t) = v(t), \\
& \dot{v}(t) = \frac{1}{m} \sum_{i=1}^{M} q(t) \otimes f_i(t) \otimes q(t)^*, \\
& \dot{q}(t) = \frac{1}{2} q(t) \otimes \omega(t), \\
& \dot{\omega}(t) = J^{-1} \left[\sum_{i=1}^{M} r_i \times f_i(t) - \omega(t) \times (J\omega(t)) \right], \\
& 0 \le \Delta t_k^i \le \Delta t_{\max} \text{ for all } i = 1, \ldots, M \text{ and } k \in \mathbb{Z}_{\ge 0}, \\
& \Delta t_k^i < \Delta t_{\min} \Rightarrow \Delta t_k^i = 0 \text{ for all } i = 1, \ldots, M \text{ and } k \in \mathbb{Z}_{\ge 0}, \\
& \|p(t) - p_f\|_2 < r_a \Rightarrow e_1^{\mathsf{T}} [q_f]_{\otimes} q(t)^* \ge \cos(\Delta\theta_{\max}/2), \\
& \|p(t) - p_f\|_2 < r_a \Rightarrow \sigma_i(t) = 0 \text{ for all } i \in \mathcal{M}, \\
& \|p(t) - p_d\|_2 \cos(\gamma) \le (p(t) - p_d)^{\mathsf{T}} \hat{e}_d, \\
& p(0) = p_0,\ v(0) = v_0,\ q(0) = q_0,\ \omega(0) = \omega_0, \\
& p(t_f) = p_f,\ v(t_f) = v_f,\ q(t_f) = q_f,\ \omega(t_f) = \omega_f.
\end{aligned}
$$

Objective: min. fuel
min Σ (thruster-fire duration)

Dynamics

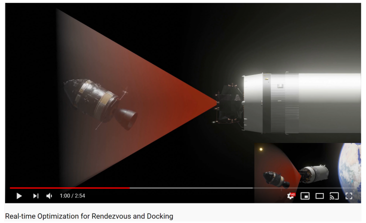
Thruster pulse duration

Safety constraints

Boundary conditions

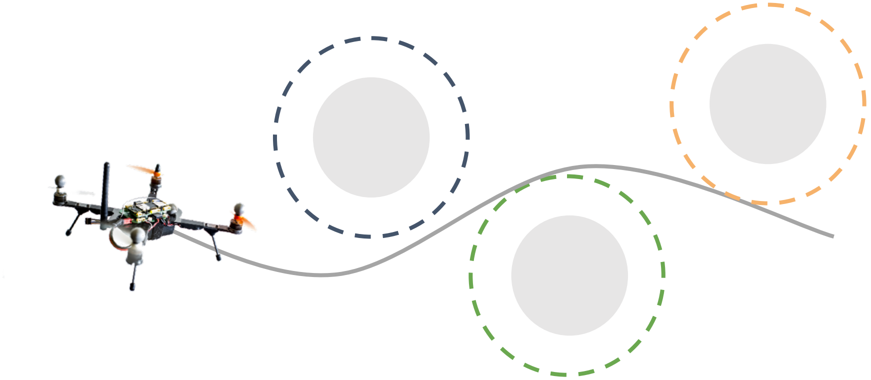
Won: Best AIAA 2020 GNC Graduate Paper at SciTech
Video Link: https://www.youtube.com/watch?v=vU1nBL2cg04

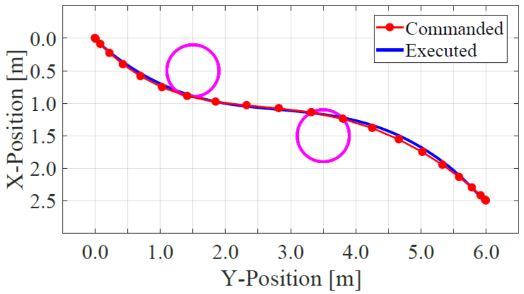
Malyuta, Reynolds, Szmuk, Acikmese, Mesbahi. (2020). Fast Trajectory Optimization via Successive Convexification for Spacecraft Rendezvous with Integer Constraints. AIAA SciTech 2020.



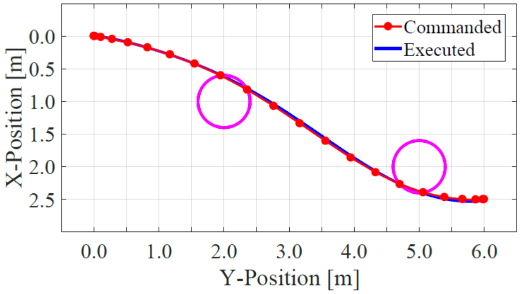
Optimal Guidance Problem Overview

Example 4: quadrotor obstacle avoidance





Szmuk, Pascucci, Dueri, Acikmese (2017). Convexification and Real-Time On-Board Optimization for Agile Quad-Rotor Maneuvering and Obstacle Avoidance. IEEE IROS 2017.



Dueri, Mao, Mian, Ding, Acikmese (2017). Trajectory Optimization with Inter-sample Obstacle Avoidance via Successive Convexification. IEEE CDC 2017.

Problem 2:

$$\underset{\mathbf{u}^k(t),\Gamma^k(t)}{\text{minimize}} \quad w\int_0^{t_f} \left(\Gamma^k(t)\right)^2 dt + \sum_{j\in\mathbb{J}} \nu_j$$

subject to:

$$\mathbf{r}^k(0) = \mathbf{r}_i \qquad \mathbf{v}^k(0) = \mathbf{v}_0 \qquad \mathbf{u}^k(0) = g\mathbf{e}_3$$
$$\mathbf{r}^k(t_f) = \mathbf{r}_f \qquad \mathbf{v}^k(t_f) = \mathbf{v}_f \qquad \mathbf{u}^k(t_f) = g\mathbf{e}_3$$

$$\dot{\mathbf{r}}^k(t) = \mathbf{v}^k(t)$$
$$\dot{\mathbf{v}}^k(t) = \mathbf{u}^k(t) - g\mathbf{e}_3$$

$$\|\mathbf{u}^k(t)\|_2 \le \Gamma^k(t)$$
$$0 < u_{min} \le \Gamma^k(t) \le u_{max}$$
$$\Gamma^k(t)\cos(\theta_{max}) \le \mathbf{e}_3^T\mathbf{u}^k(t)$$

$$x_{min} \le \mathbf{e}_1^T\mathbf{x}^k(t) \le x_{max}$$
$$y_{min} \le \mathbf{e}_2^T\mathbf{x}^k(t) \le y_{max}$$
$$z_{min} \le \mathbf{e}_3^T\mathbf{x}^k(t) \le z_{max}$$

For all $j \in \mathbb{J}$ and for $t \in [0, t_f]$:

$$\nu_j \ge 0$$
$$H_j \succeq 0$$
$$\Delta\mathbf{r}^{k,j}(t) \triangleq (\mathbf{r}^{k-1}(t) - \mathbf{p}_j)$$
$$\delta\mathbf{r}^k(t) \triangleq \mathbf{r}^k(t) - \mathbf{r}^{k-1}(t)$$
$$\xi^{k,j}(t) \triangleq \|H_j\Delta\mathbf{r}^{k,j}(t)\|_2$$
$$\zeta^{k,j}(t) \triangleq \frac{H_j^T H_j \Delta\mathbf{r}^{k,j}(t)}{\|H_j\Delta\mathbf{r}^{k,j}(t)\|_2}$$

$$\xi^{k,j} + \left[\zeta^{k,j}(t)\right]^T \delta\mathbf{r}^k(t) \ge R_j - \nu_j$$

Objective: Min. fuel

Boundary conditions

Dynamics

Control constraints

State constraints

Safety constraints

Video: Aggressive Obstacle Avoidance

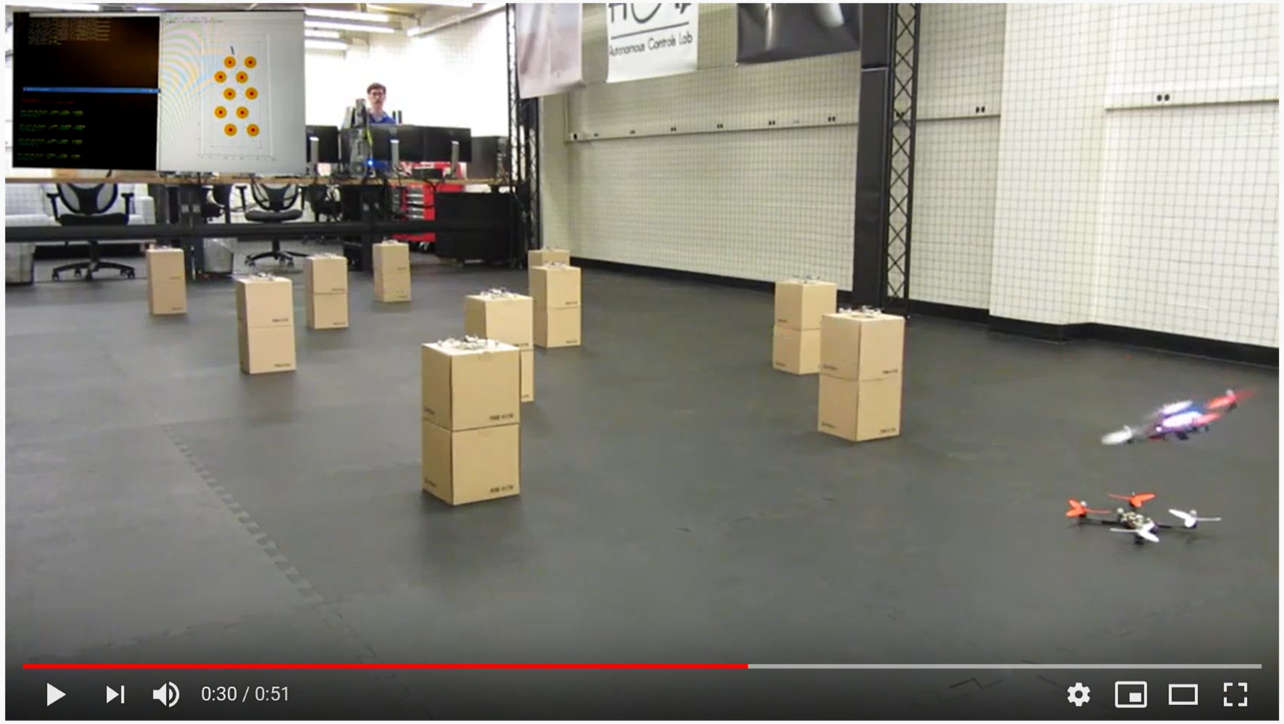
- **Optimality based on sufficiently high-fidelity dynamic model allows us to push aggressive performance**

- **Dynamic feasibility allows us to exploit most of the systems operating envelope**

- **Links:**
  - [Aggressive Obstacle Avoidance: https://www.youtube.com/watch?v=EK-X3kiTnn8](https://www.youtube.com/watch?v=EK-X3kiTnn8)

  - [Mobile Obstacle Avoidance: https://www.youtube.com/watch?v=0fP5kBx_rzE](https://www.youtube.com/watch?v=0fP5kBx_rzE)

  - [Obstacle Avoidance: https://www.youtube.com/watch?v=lmZDigf91Ss](https://www.youtube.com/watch?v=lmZDigf91Ss)

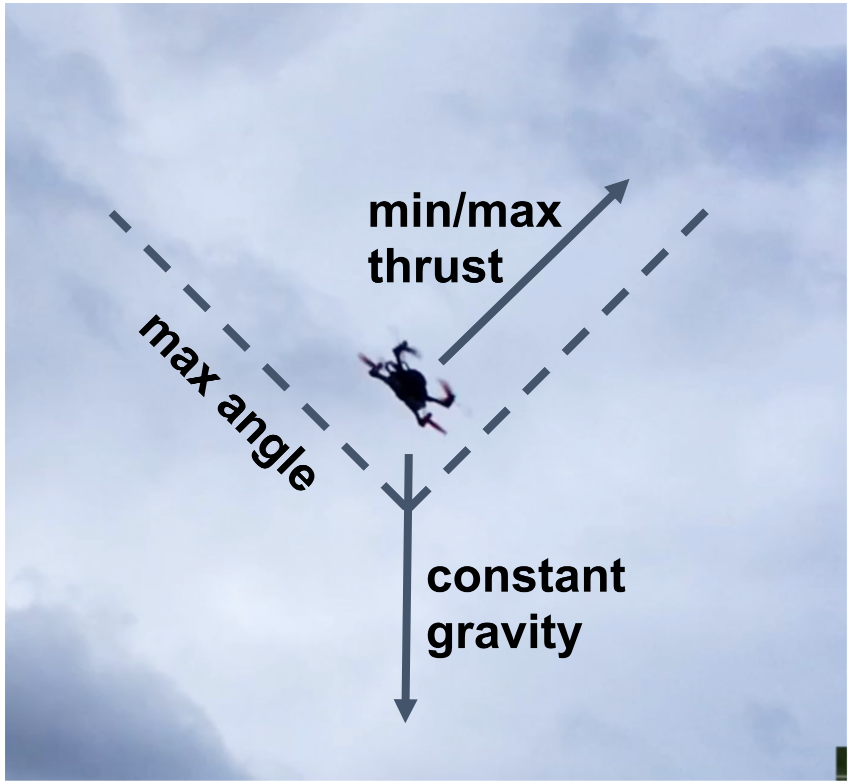


Real-Time Obstacle Avoidance

Tablet Interface Overview & Progress

Optimization Interface

Physics-based vehicle dynamics



Problem 1: ***Non-Convex Formulation***

$$\underset{\mathbf{T}(t)}{\text{minimize}} \quad \int_0^{t_f} \|\mathbf{T}(t)\|_2 dt$$

subject to:

$$\mathbf{r}(0) = \mathbf{r}_i \qquad \mathbf{v}(0) = \mathbf{v}_i \qquad \mathbf{T}(0) = \mathbf{T}_i$$
$$\mathbf{r}(t_f) = \mathbf{r}_f \qquad \mathbf{v}(t_f) = \mathbf{v}_f \qquad \mathbf{T}(t_f) = \mathbf{T}_f$$

$$\dot{\mathbf{r}}(t) = \mathbf{v}(t)$$
$$\dot{\mathbf{v}}(t) = \frac{1}{m}\mathbf{T}(t) - g\mathbf{e}_1$$

$$0 < T_{min} \leq \|\mathbf{T}(t)\|_2 \leq T_{max} \tag{1}$$
$$\|\mathbf{T}(t)\|_2 \cos(\theta_{max}) \leq \mathbf{e}_1^T \mathbf{T}(t) \tag{2}$$

$$\|H_j(t)\big(\mathbf{r}(t) - \mathbf{r}_j(t)\big)\|_2 \geq 1 \quad \forall j \in \mathbb{J}$$

Szmuk, Pascucci, Acikmese, B. (2018).
Real-Time Quad-Rotor Path Planning for Mobile Obstacle Avoidance Using Convex Optimization. IROS 2018

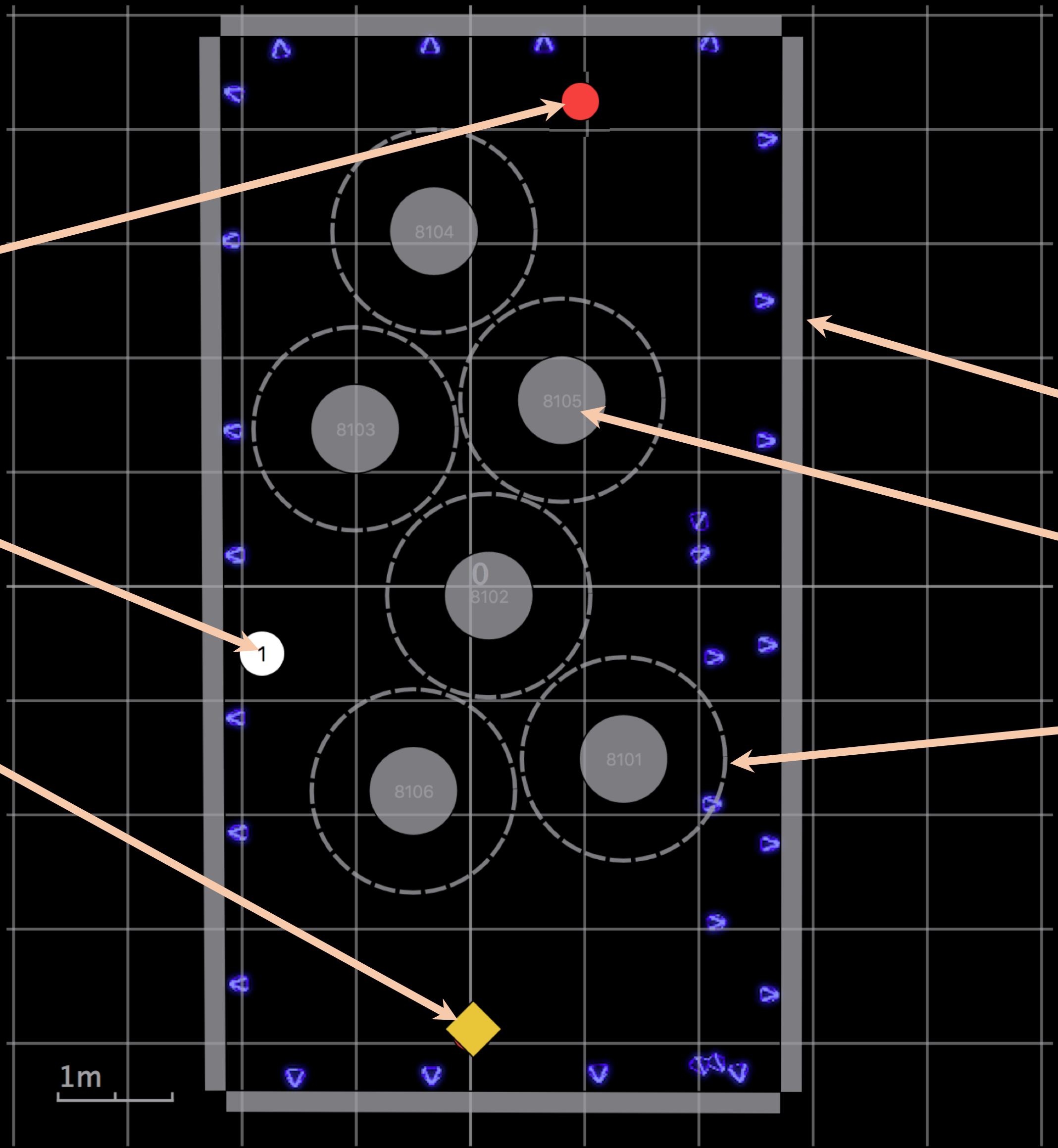
Interface: Task Specification

Task:

Goal

Waypoints

Vehicle (minimum fuel)



Map:

Safety Zone (hard)

Live obstacles

Clearance (soft)

Interface: Constraint Types

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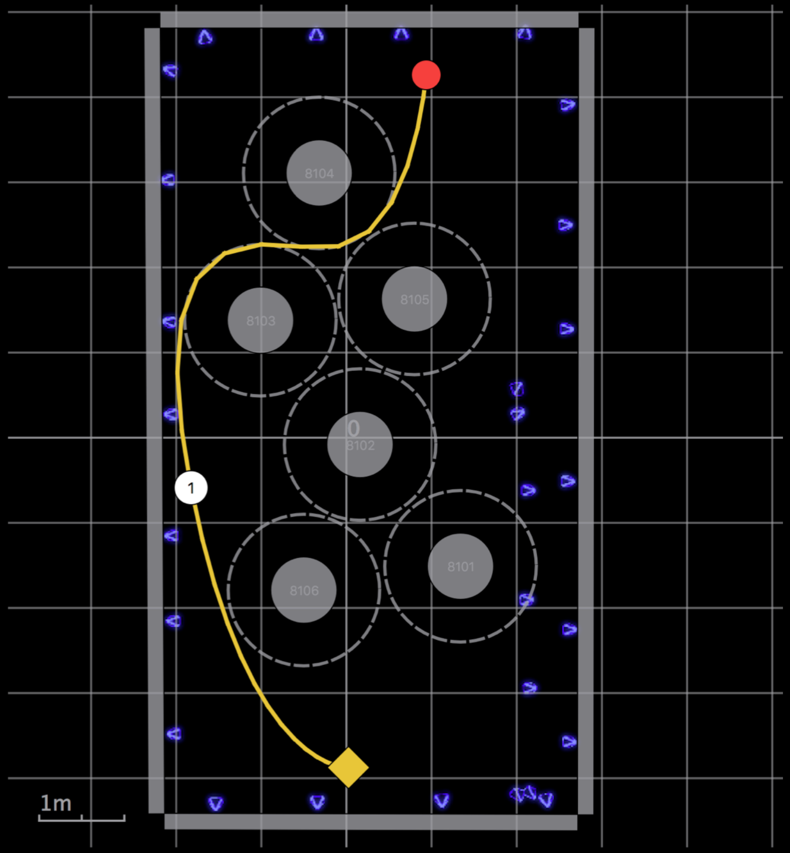
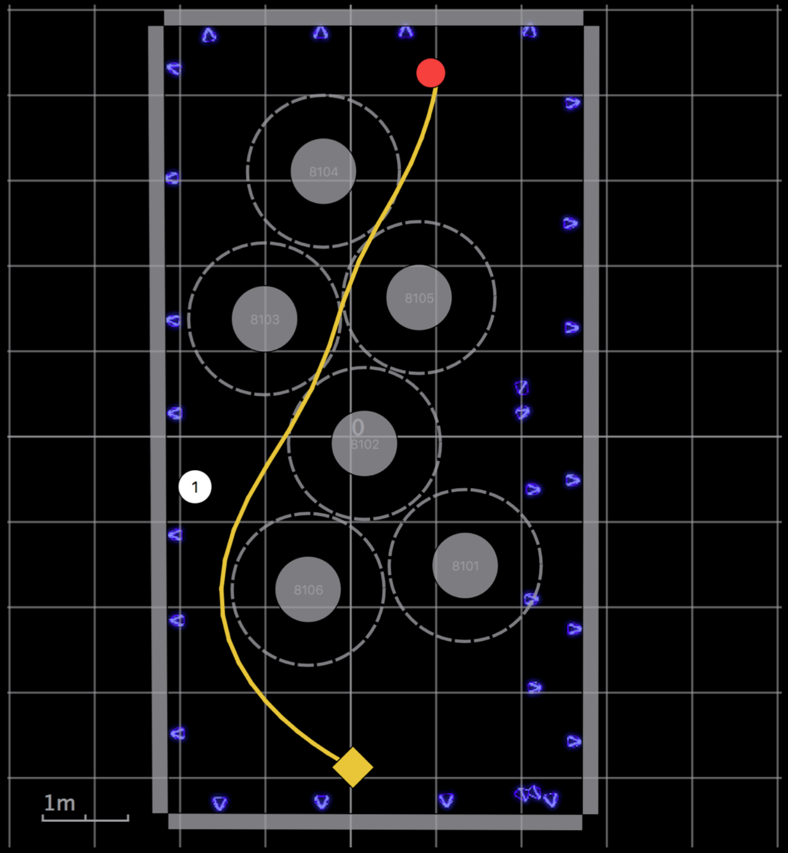
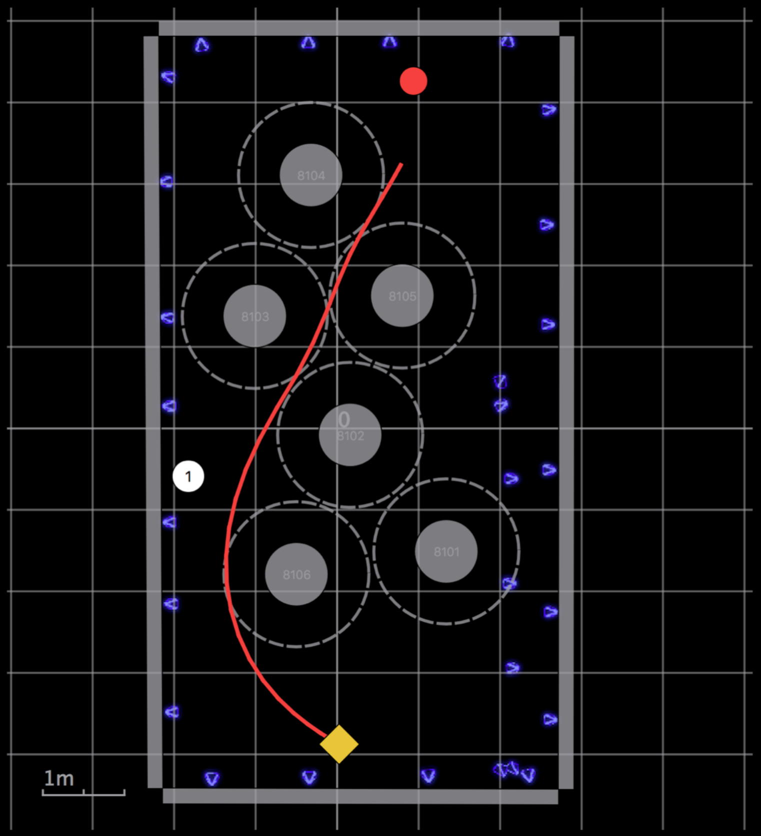
Soft and hard constraints



6.0 seconds

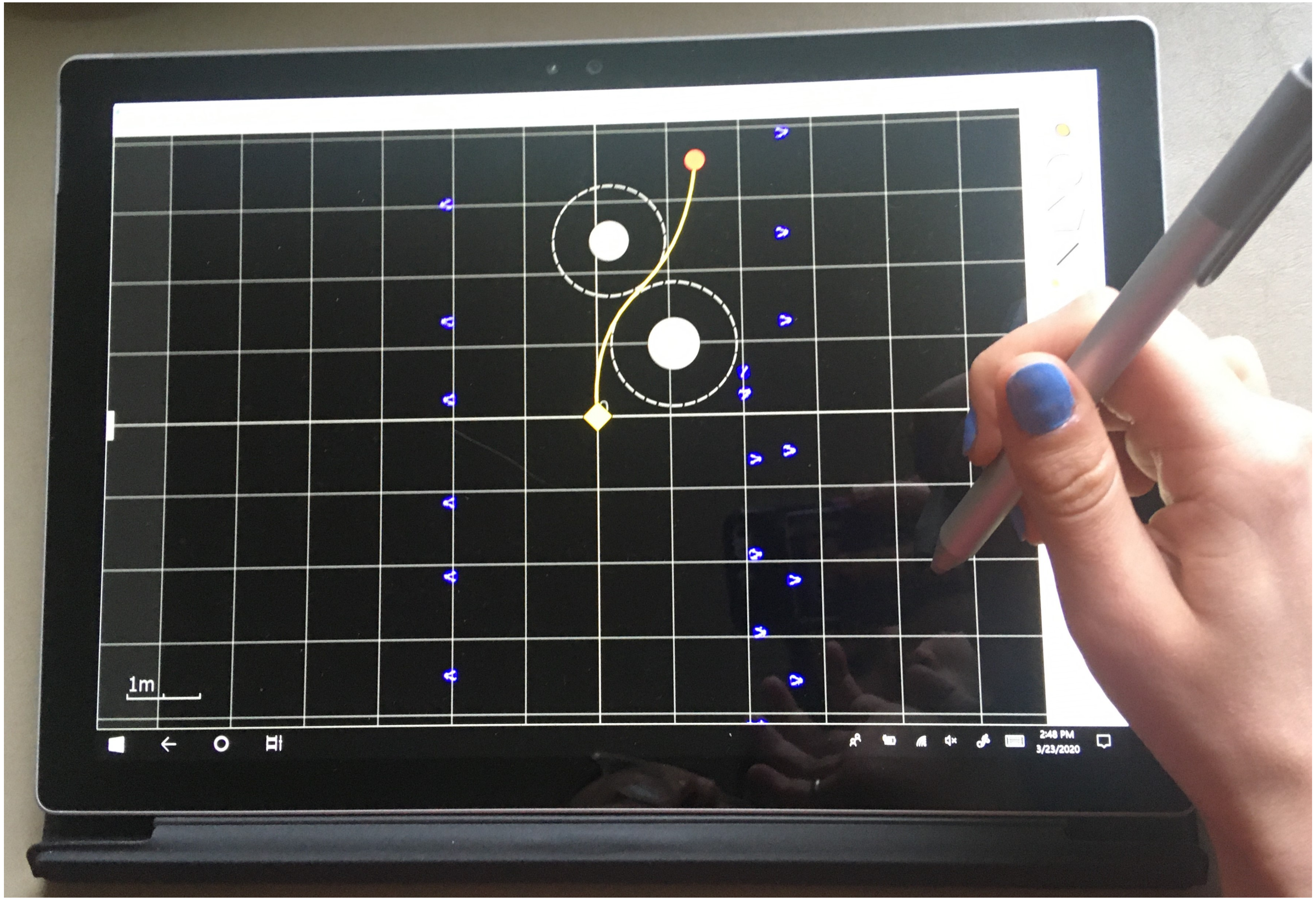


3.0 seconds



2.5 seconds

Interface: Handheld Tablet



Interface: Field Operations

Interface: Field Operations

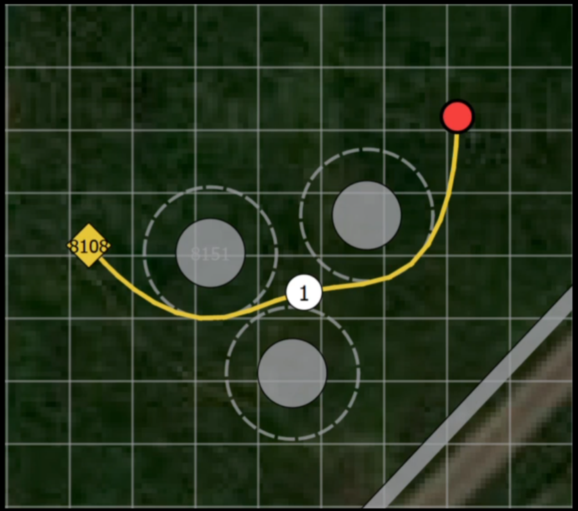

File
K 25
dK 1
n_recalcs 14
a_min 5.00
a_max 12.00
theta_max 0.79
q_max 0.00
max_iter 10
delta_i 100.00
lambda 100.00
alpha 2.00
dL_tol 0.10
rho_0 -0.10
rho_1 0.25
rho_2 0.90
rirelax 1000.00
rfrelax 10.00
Optimization Problem Parameters
clearance 2.80
Obstacle Soft Clearance
Grid Scale
5m
8108
Set Final Point Mode
Create Circle Obstacle Mode
Create Polygon Constraint Mode
Create Plane Constraint Mode
Create Waypoint Mode
Eraser Tool Mode
Flip Constraint Mode
Duplicate Selected Obstacle
Duplicate
Trajectory remains feasible!
User Feedback
Final time
7.00s
Set Final Time
0.11x
Set Zoom
Stage
Unstage
Stage / Unstage Trajectory
Exec
Send Staged Trajectory to Drone

Interface: Field Operations

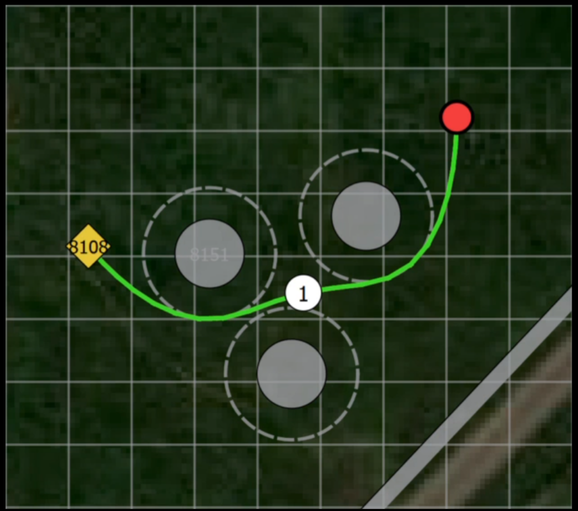
ACL
Autonomous Controls Lab

Nominal → Staged → Executed → Task completed

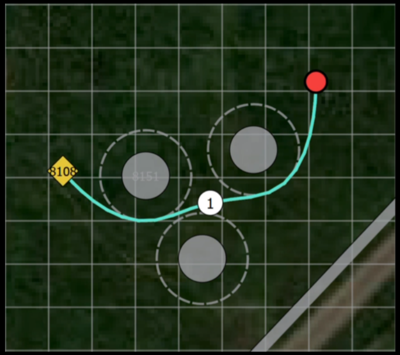
Nominal

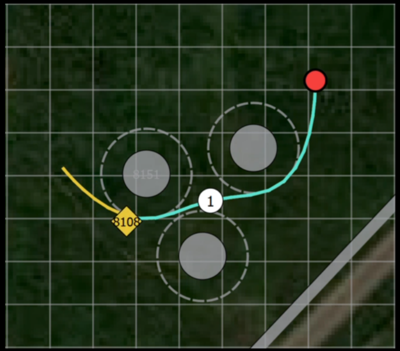


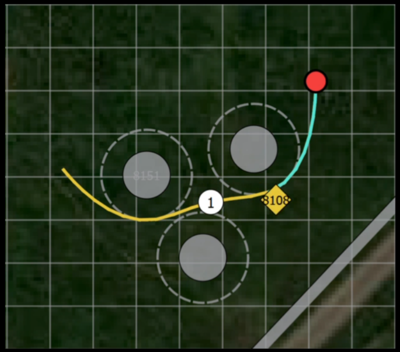
Staged



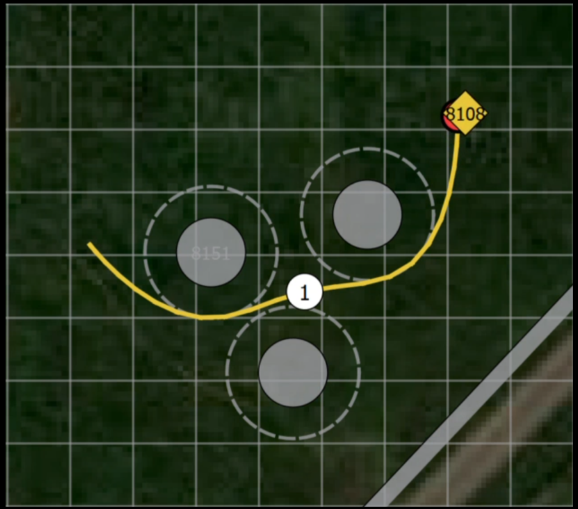
Executed

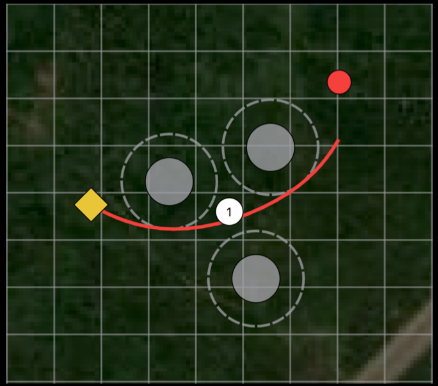






Task completed

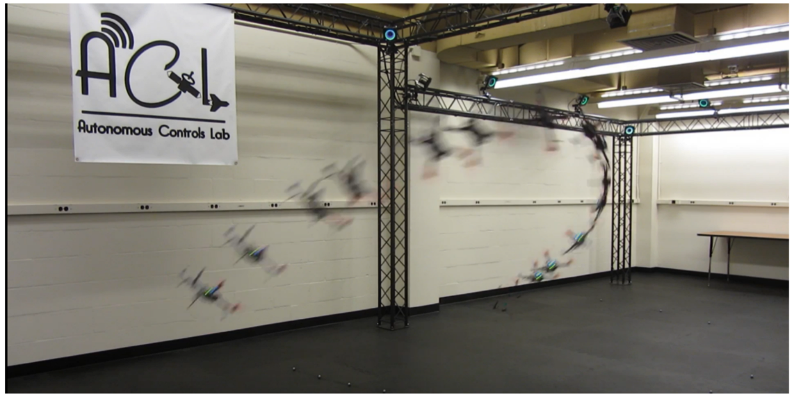




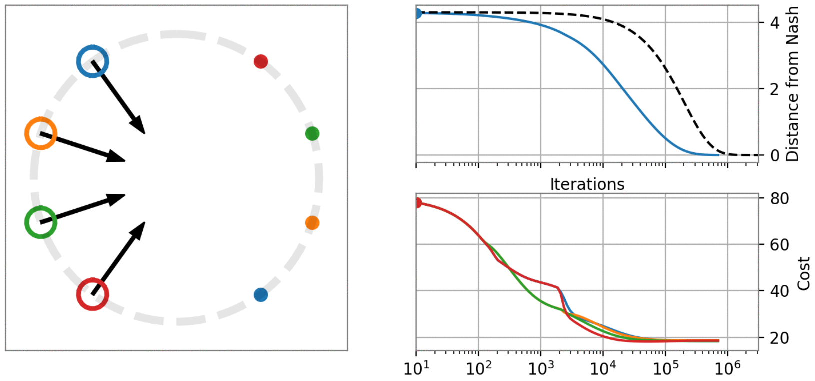
Constraint violated

Current Work

Higher Fidelity Dynamics

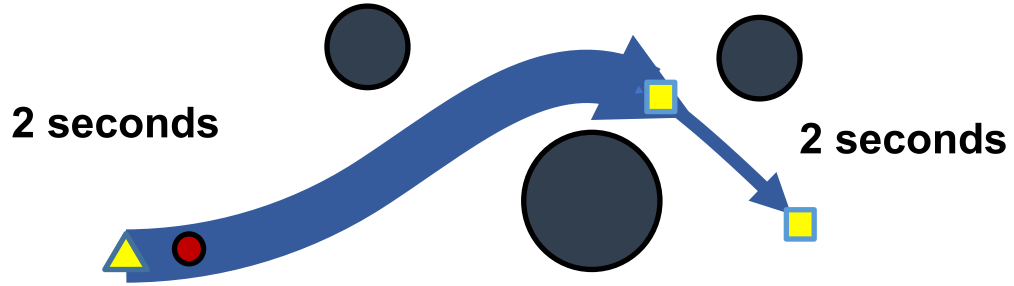


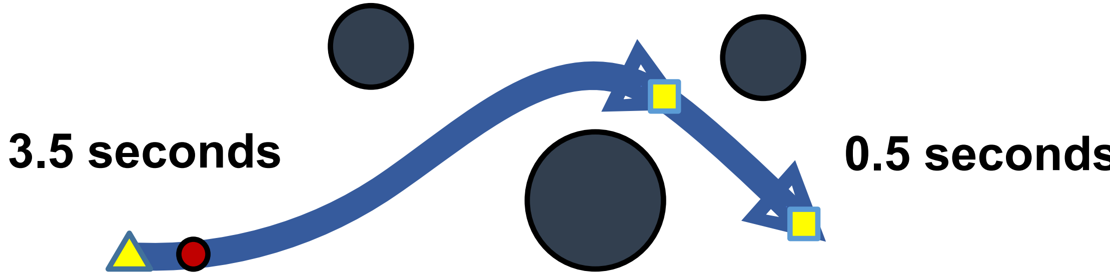
Game Theory Methods for Collision Avoidance



Variable Way Point Times

(In collaboration w/ P.S. Lysandrou)





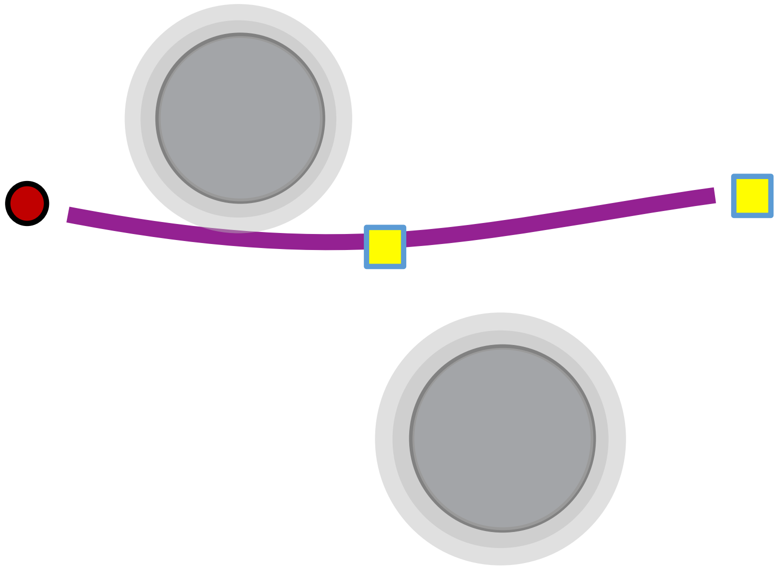
Future work:

- Turn potential targets into constraints for other trajectories to maintain feasibility

- Warm-start capabilities for replanning after task change

- User interface with tablet for quick online reassignment

Single Trajectory

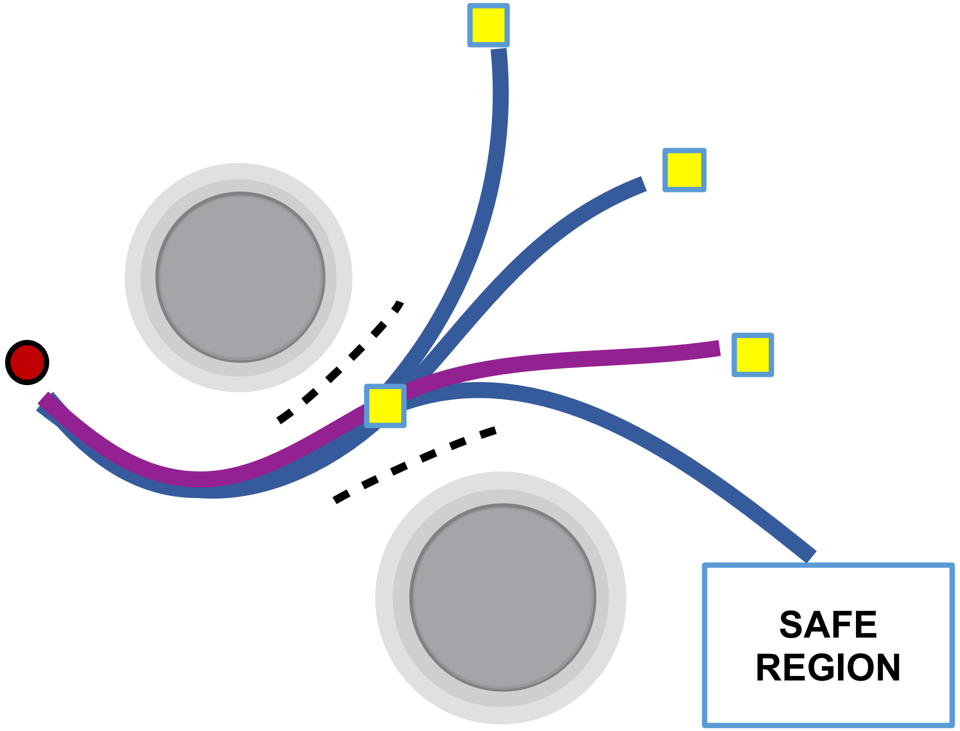


Trajectory Bundles

Single Trajectory

Trajectory Bundle

- Turn potential targets into constraints for other trajectories to maintain feasibility
- Warm-start capabilities for replanning after task change
- User interface with tablet for quick online reassignment



Trajectory Bundles

- Turn potential targets into constraints for other trajectories to maintain feasibility
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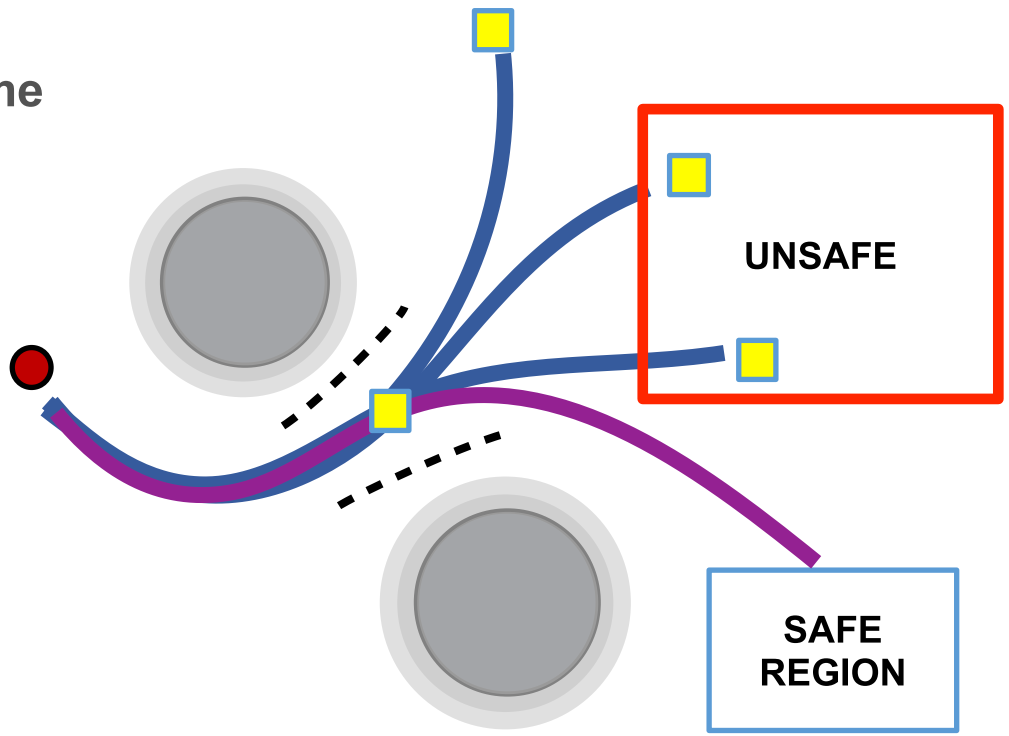
Single Trajectory

Trajectory Bundle

Real Time Update

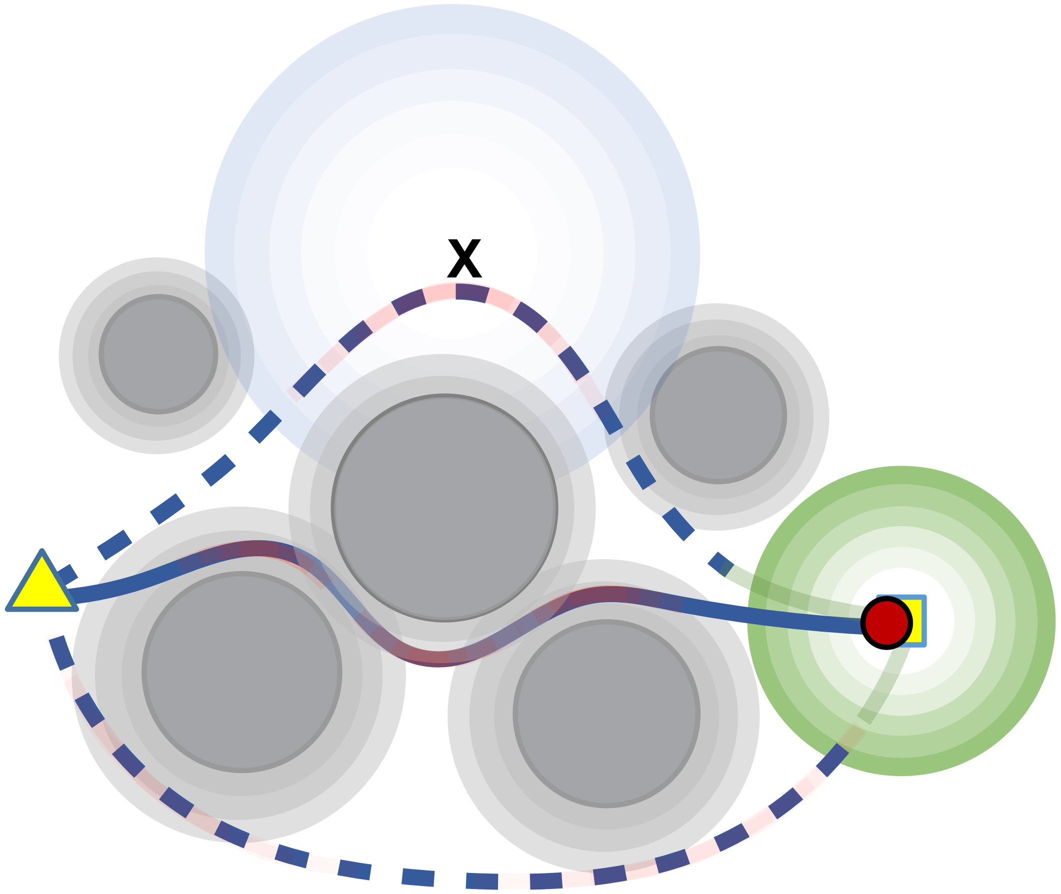


Multi-Objective Optimization

- Trade-offs in objective determine solution:

$$\begin{array}{ll} \min\limits_{x} & \overset{\text{Fuel}}{w_f F(x)} + \boxed{\overset{\text{Time}}{w_T T(x)} +} \overset{\text{Way Pts.}}{w_w W(x)} + \overset{\text{Perform}}{w_p P(x)} + \overset{\text{Boundary}}{w_b B(x)} \\ \text{s.t.} & x \in \mathcal{X} \end{array}$$

- Multi-objective optimization
  - Sampling based techniques
  - Hierarchical optimization
- User feedback
  - Clearly display tradeoffs on tablet
  - User changes weights/enforces constraints/selects trajectories on screen



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  - **Sampling based techniques**
  - **Hierarchical optimization**
- **User feedback**
  - **Clearly display tradeoffs on tablet**
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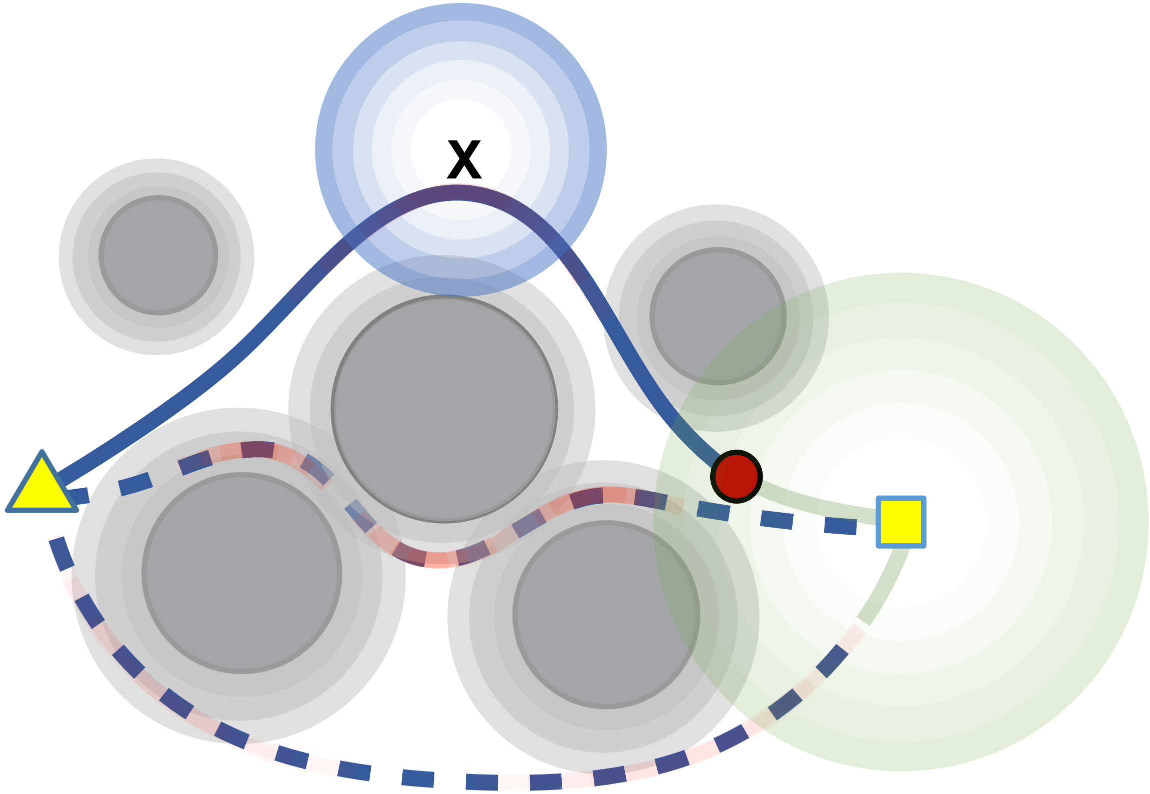

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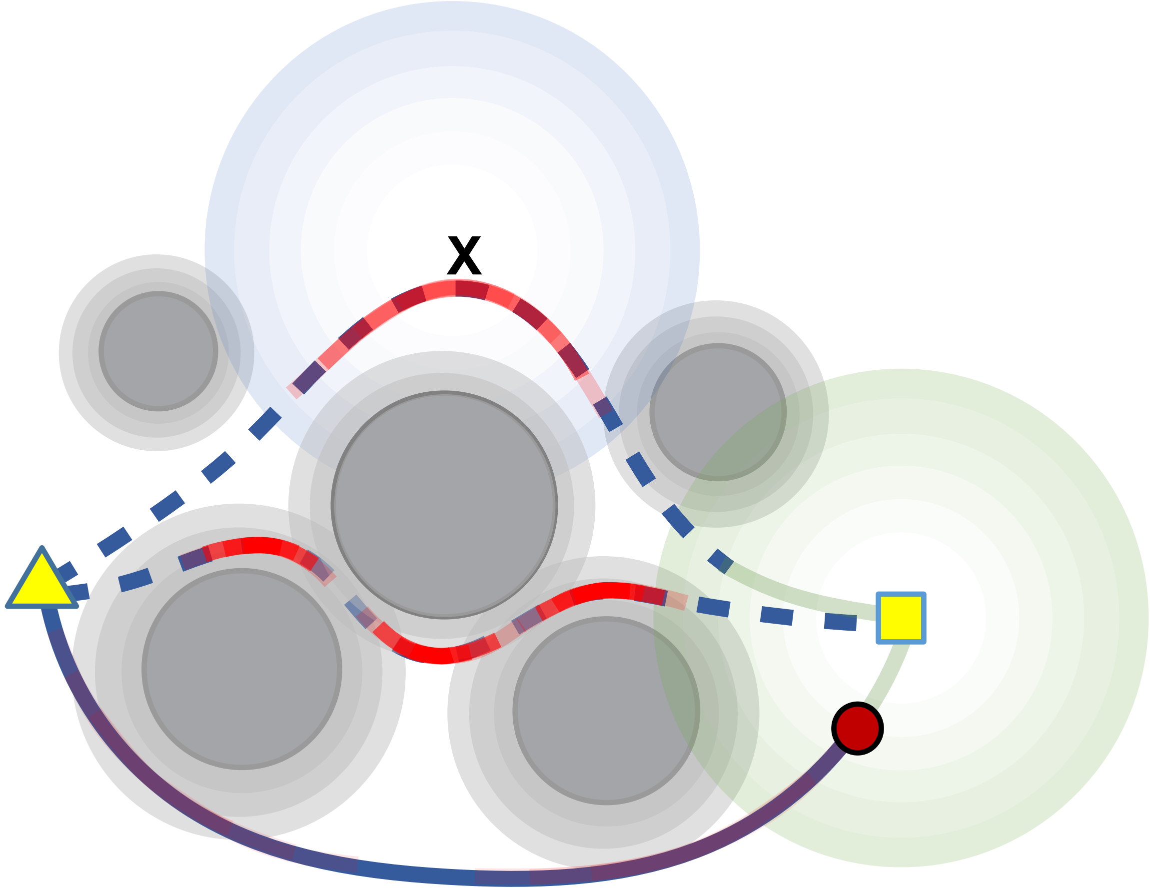

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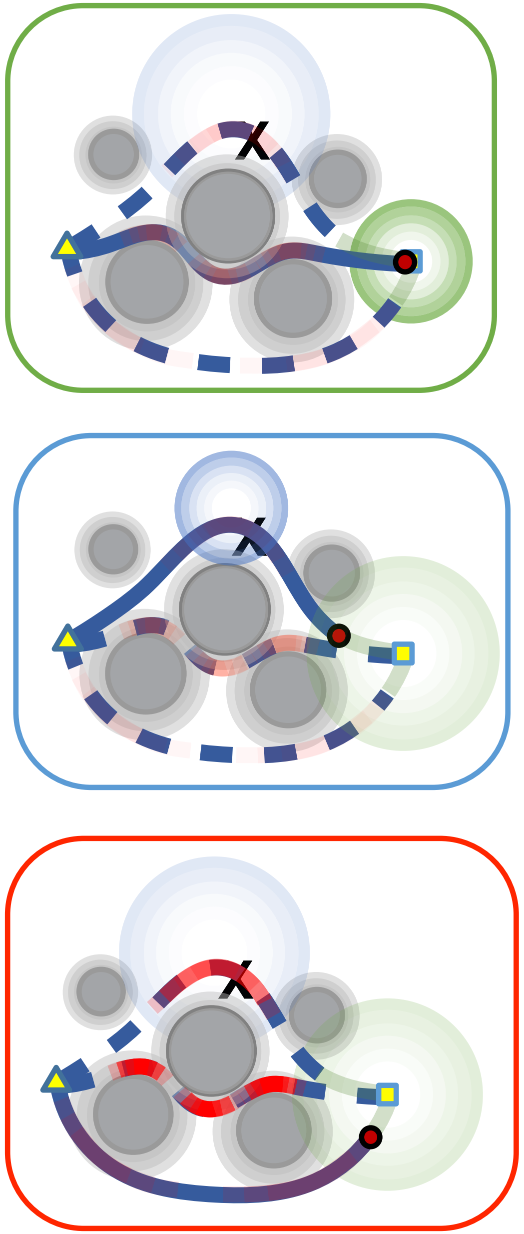
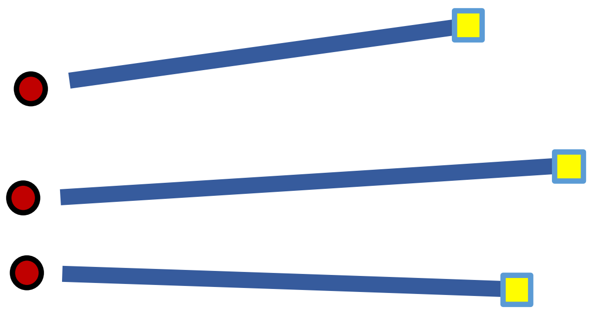
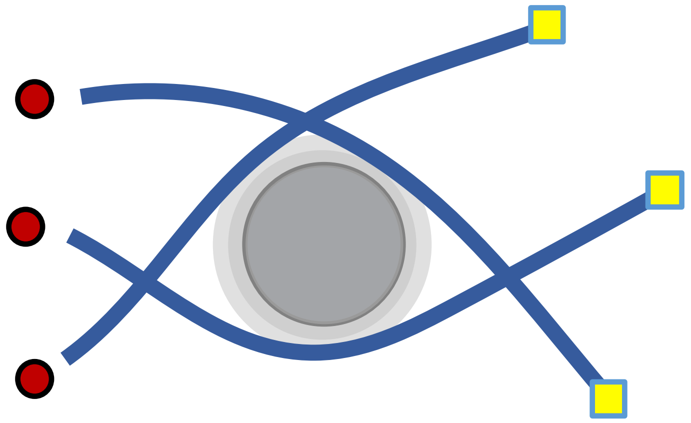


Multi-Agent Optimization

- Multi-agent SCVX
  - Primal-dual methods for decentralization of trajectory planning - ADMM



- Task matching using trajectories
  - Assignment Algorithms
    - *Optimal transport – shortest path*
    - *Extensions*
      - Obstacle/collision avoidance
      - User assignment



Thank you!

Q&A