

## 1. CONVENTIONS

Item	Convention	Examples
Top level structures	Lower case bold Greek	$\sigma$ , the world state $\mu$ , the machine state.
Functions on highly structured values	Upper case Greek	$\Upsilon$ , the Ethereum state transition function.
Most functions	Upper case letters, possibly subscripted	$C$ , the general cost function $C_{\text{SSTORE}}$ , the cost function for the $\text{SSTORE}$ operation.
Specialised functions	Typewriter	$\text{KEC}$ , the Keccak-256 hash $\text{KEC512}$ , the Keccak-512 hash function.
Tuple	Upper case letter	$T$ , a transaction.
Component of a Tuple	Subscripted upper-case letter. A capital subscript refers to a component that is a tuple.	$T_n$ , the transaction nonce $I_H$ , The header of the current block (a tuple).
Scalars, fixed size byte sequences/arrays	Usually a lower-case letter Sometimes Greek	$n$ , a transaction's nonce $\delta$ , the number of stack items required.
Arbitrary length sequences	Bold lower-case	$\mathbf{o}$ , output data of message call.
Sets	Double struck capitals	$\mathbb{P}_{256}$ , positive integers less than $2^{256}$ $\mathbb{B}_{32}$ , byte sequences of length 32.
Components or subsequences of sequences	Square brackets	$\mu_s[0]$ , the first item on the stack $\mu_m[0..31]$ the first 32 items in memory.
Modified (and utilisable) value	Prime mark	$g'$ gas remaining.
Intermediate values	Asterisk superscripts	$g^*$ gas to be refunded $g^{**}$ available gas remaining after code execution.
Element-wise transformations	Asterisk superscript on a function	$f^*((x_0, x_1, \dots)) \equiv (f(x_0), f(x_1), \dots)$ for any function $f$ .

## 2. SYMBOLS

Name	Description
<b>High level constructs</b>	
$\sigma$	The world-state, comprising all accounts' nonces, balances, storage and code.
$\sigma_t$	World-state at time $t$ .
$\mu$	Machine-state tuple, $(g, pc, \mathbf{m}, i, \mathbf{s})$ , which are gas, program counter, memory, memory size, stack.
$T$	An Ethereum transaction
$T_0, T_1, \dots$	Individual transactions within a block
$B$	A block: $B \equiv (\dots, (T_0, T_1, \dots))$
$\Upsilon$	The Ethereum state transition function: $\sigma_{t+1} \equiv \Upsilon(\sigma_t, T)$
$\Omega$	The block-finalisation state transition function (pays out the mining reward).
$\Pi$	The block-level state-accumulation function: $\Pi(\sigma, B) \equiv \Omega(B, \Upsilon(\sigma, T_0), T_1, \dots)$
<b>World state</b>	
$\sigma[a]$	The account state of account $a$ , being a tuple of (nonce, balance, storageRoot, codeHash).
$\sigma[a]_n$	The nonce of account $a$ .
$\sigma[a]_b$	The balance of account $a$ .
$\sigma[a]_s$	A 256-bit hash of the root node of a Merkle Patricia tree that encodes the storage contents of account $a$ . Note that $\text{TRIE}(L_T^*(\sigma[a]_s)) \equiv \sigma[a]_s$
$\sigma[a]_c$	The hash of the EVM code of account $a$ . Equal to $\text{KEC}(\mathbf{b})$ where $\mathbf{b}$ is the account's code.

Name	Description
<b>Machine state</b>	
$\mu_g$	The gas available.
$\mu_{pc}$	The program counter.
$\mu_m$	The memory contents.
$\mu_i$	The number of memory words allocated.
$\mu_s$	The stack.
$\mu_s[n]$	Item at stack depth $n$ .
<b>Substate</b>	
$A$	A Transaction substate during execution: $A \equiv (A_s, A_l, A_t, A_r) \equiv (\mathbf{s}, \mathbf{l}, \mathbf{t}, r)$ .
$A_s$	The self-destruct set. These accounts will be discarded following the transaction's completion.
$A_l$	The log series.
$A_t$	The set of touched accounts. Empty ones are deleted at the end of the transaction.
$A_r$	The gas refund balance. Can partially offset execution costs.
$A^0$	The empty substate: $A^0 \equiv (\emptyset, (), \emptyset, 0)$ .
<b>Execution environment</b>	
$I$	Tuple of the following items provided to the execution environment.
$I_a$	The address of the account which owns the code that is executing.
$I_o$	The sender address of the transaction that originated this execution.
$I_p$	The price of gas in the transaction that originated this execution.
$I_d$	The byte array that is the input data to this execution; if the execution agent is a transaction, this would be the transaction data.
$I_s$	The address of the account which caused the code to be executing; if the execution agent is a transaction, this would be the transaction sender.
$I_v$	The value, in Wei, passed to this account as part of the same procedure as execution; if the execution agent is a transaction, this would be the transaction value.
$I_b$	The byte array that is the machine code to be executed.
$I_H$	The block header of the present block.
$I_e$	The depth of the present message-call or contract-creation (i.e. the number of CALLs or CREATEs being executed at present).
$I_w$	Flag for permission to make modifications to the state. See EIP-214, STATICCALL
<b>Execution</b>	
$\Xi$	The code execution function $(\sigma', g', A, \mathbf{o}) \equiv \Xi(\sigma, g, I)$ .
$\mathbf{o}$	The output data of a message call, $\mathbf{o} \equiv H(\mu, I)$ .
$\mathbf{i}$	At contract creation, the contract bytecode to be deployed.
$\mathbf{i}$	The initialisation EVM code for newly deployed contract (contract constructor).
$H(\mu, I)$	The normal halting function, usually the value provided by the RETURN or REVERT opcodes, or empty in the case of STOP.
$Z(\sigma, \mu, I)$	The exceptional halting function.
$w$	The current operation to be executed: $w \equiv I_b[\mu_{pc}]$ if $\mu_{pc} < \ I_b\ $ , and STOP otherwise.
<b>Blocks</b>	
$B$	A block: $B \equiv (B_H, B_T, B_U)$ .
$B_H$	The block's header.
$B_T$	The block's transactions.
$B_U$	Headers of ommer/uncle blocks of this block.
$B_R$	Transaction receipts.
$D(H)$	The difficulty of the block with header $H$ .
$P(H)$	The parent block of the block with header $H$ .
$V(H)$	The block header validity function.

Name	Description
Block header	
$H_p$	<b>parentHash</b> The Keccak 256-bit hash of the parent block's header, in its entirety.
$H_o$	<b>ommersHash</b> The Keccak 256-bit hash of the ommers list portion of this block.
$H_c$	<b>beneficiary</b> The 160-bit address to which all fees collected from the successful mining of this block be transferred.
$H_r$	<b>stateRoot</b> The Keccak 256-bit hash of the root node of the state trie, after all transactions are executed and finalisations applied.
$H_t$	<b>transactionsRoot</b> The Keccak 256-bit hash of the root node of the trie structure populated with each transaction in the transactions list portion of the block.
$H_e$	<b>receiptsRoot</b> The Keccak 256-bit hash of the root node of the trie structure populated with the receipts of each transaction in the transactions list portion of the block.
$H_b$	<b>logsBloom</b> The Bloom filter composed from indexable information (logger address and log topics) contained in each log entry from the receipt of each transaction in the transactions list.
$H_d$	<b>difficulty</b> A scalar value corresponding to the difficulty level of this block.
$H_i$	<b>number</b> A scalar value equal to the number of ancestor blocks. The genesis block has a number of zero.
$H_l$	<b>gasLimit</b> A scalar value equal to the current limit of gas expenditure per block.
$H_g$	<b>gasUsed</b> A scalar value equal to the total gas used in transactions in this block.
$H_s$	<b>timestamp</b> A scalar value equal to the reasonable output of Unix's time() at this block's inception.
$H_x$	<b>extraData</b> An arbitrary byte array containing data relevant to this block. This must be 32 bytes or fewer.
$H_m$	<b>mixHash</b> A 256-bit hash which proves combined with the nonce that a sufficient amount of computation has been carried out on this block.
$H_n$	<b>nonce</b> A 64-bit hash which proves combined with the mix-hash that a sufficient amount of computation has been carried out on this block.

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### Transactions

$T_n$	Transaction nonce.
$T_p$	Gas price for the transaction.
$T_g$	The maximum gas for a transaction.
$T_t$	The "to" address for the transaction.
$T_v$	The value to be transferred by the transaction.
$T_w, T_r, T_s$	The $v, r, s$ values of the transaction signature.
$T_i$	EVM-code for account initialisation (i.e. contract deployment).
$T_d$	Input data of a message call.
$S(T)$	Sender function—recovers the sender address from the transaction: $S(T) \equiv \mathcal{B}_{96..255}(\text{KEC}(\text{ECDSARECOVER}(h(T), T_w, T_r, T_s)))$ .

### Transaction Receipt

$R$	A transaction receipt: $R \equiv (R_z, R_u, R_b, R_l)$
$R_z$	The status code of the transaction.
$R_u$	The cumulative gas used so far in the block.
$R_b$	The bloom filter composed from the information in the transaction logs.
$R_l$	The log entries created by the transaction, $(O_0, O_1, \dots)$ .
$O$	A log entry: $O \equiv (O_a, (O_{t0}, O_{t1}, \dots), O_d)$ .
$O_a$	The logger's address.
$O_t$	A 32-byte log topic.
$O_d$	The log data for this entry.
$\Upsilon^g$	The total gas used in this transaction.
$\Upsilon^l$	The logs created by this transaction.
$\Upsilon^z$	The status code of this transaction, $z$ .

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### Miscellaneous functions

$\ell(\mathbf{x})$	The last item in sequence $\mathbf{x}$ : $\ell(\mathbf{x}) \equiv \mathbf{x}[ \mathbf{x}  - 1]$
$L(n)$	The "all but one 64th" function: $L(n) \equiv n - \lfloor n/64 \rfloor$ .
$L_I((k, v))$	Representation of key-value pairs in the trie: $L_I((k, v)) \equiv (\text{KEC}(k), \text{RLP}(v))$
$L_R$	TODO

Name	Description
$L_S$	World-state collapse function. TODO: expand. Seems to have a different function in computing the message hash.
$L_T$	TODO
$M(s, f, l)$	Memory expansion function. $s$ is the current top of memory; $f$ is the start of writing; $l$ is the number of bytes to be written.
$\mathcal{B}$	Bit reference function such that $\mathcal{B}_j(\mathbf{x})$ equals the bit of index $j$ (indexed from 0) in the byte array $\mathbf{x}$
$\text{EMPTY}(\sigma, a)$	An account $a$ is <i>empty</i> when it has no code, zero nonce and zero balance, $\sigma[a]_c = \text{KEC}()$ $\wedge$ $\sigma[a]_n = 0 \wedge \sigma[a]_b = 0$ .
$\text{DEAD}(\sigma, a)$	An account $a$ is <i>dead</i> when its account state is non-existent or empty: $\emptyset \vee \text{EMPTY}(\sigma, a)$ .
$\text{TRIE}$	The root hash of the Merkle Patricia tree constructed from its arguments.
$\text{KEC}$	TODO
$\text{RLP}$	TODO
$\text{PoW}$	TODO

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### Operators and symbols

$\ \dots\ ,  \dots $	Length of a sequence. These seem to be used interchangeably, but I may have missed something.
$\wedge$	Logical “And”.
$\vee$	Logical “Or”.
$\emptyset$	The empty set.
$\cdot$	Concatenation, $(a, b, c, d) \cdot e \equiv (a, b, c, d, e)$ , or scalar multiplication depending on context.

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### Todo

$\mathbb{B}$	The set of all sequences of bytes.
$\mathbb{B}_n$	The set of all byte sequences of length $n$ bytes: $\mathbb{B}_n = \{B : B \in \mathbb{B} \wedge \ B\  = n\}$
$\mathbb{P}$	The set of positive integers [what’s wrong with $\mathbb{N}$ ??? Grrr...].
$\mathbb{P}_n$	The set of all positive integers smaller than $2^n$ : $\mathbb{P}_n = \{P : P \in \mathbb{P} \wedge P < 2^n\}$
$M_{3:2048}$	Specialised Bloom filter.
$\Lambda(\dots)$	Contract creation function.
$\Theta(\dots)$	“Message call”/contract execution function? Not very clearly defined anywhere, but used extensively.
$\Gamma(B)$	The “initiation state” of block $B$ . Usually $\sigma_i : \text{TRIE}(L_S(\sigma_i)) = P(B_H)_{H_r}$ .
$\Psi(B)$	A block transition function that maps an incomplete block $B$ to a complete block $B'$ (adds in mixHash, nonce, stateRoot).
$r(\dots)$	Calculates stateRoot? Used once but not defined.
<i>etc.</i>	

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