Endogenous Production Network and Knowledge Diffusion in Supplier-Customer Relationships

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Motivation

Network dynamics

- Extensive margin of production network is driven by business cycle (Lim, 2018; Martin et al., 2024)
- There is also a general declining trend in network dynamics after 2000.

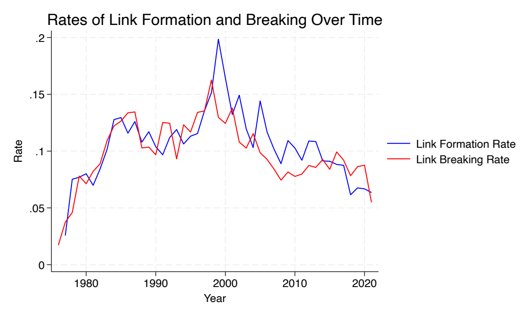


Figure 1: Rate of production link breaking and forming 1977-2021, Compustat

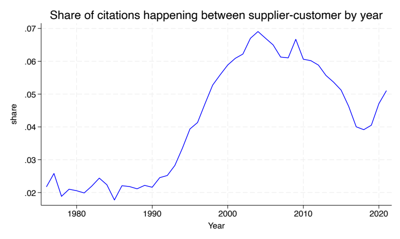
Possible mechanism

- Allow increasing trend simply for search cost in Lim (2018)
- Larger contracting frictions between firms (Boehm et al., 2024)
- Declining firm dynamics (Decker et al., 2016) $\Rightarrow$ less available new and better supplier
- My hypothesis:

  Increasing relationship-specific knowledge diffusion.

Citation trend

- Increasing share of supplier-customer citations out of total citations.
- Increasing share of patents that cites its supplier or customer.



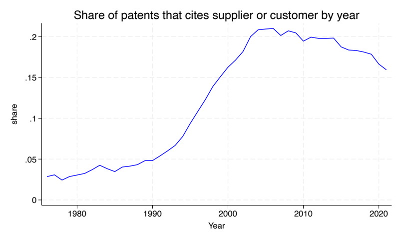
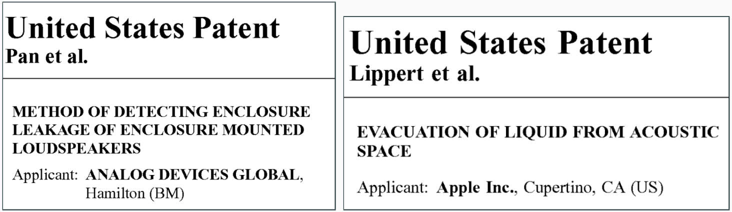


Figure 2: Yearly trend on the share of citation between supplier-customer (left), and the share of patents that cites supplier or customer (right). The share is computed out of all patents and citations in OECD patent citation statistics whose applicants got mapped to a firm PERMCO using Kogan et al. (2017)

An introductory example


United States Patent
Pan et al.

METHOD OF DETECTING ENCLOSURE LEAKAGE OF ENCLOSURE MOUNTED LOUDSPEAKERS

Applicant: **ANALOG DEVICES GLOBAL**, Hamilton (BM)

United States Patent
Lippert et al.

EVACUATION OF LIQUID FROM ACOUSTIC SPACE

Applicant: **Apple Inc.**, Cupertino, CA (US)

Figure 3: Apple (right panel, granted in 2019) cites its supplier Analog Devices (left panel, granted in 2016)

Analog Devices, Inc. designs, manufactures, and markets integrated circuits used in analog and digital signal processing. (Source: Bloomberg)

Auxiliary research question

- Impact of relationship-specific knowledge diffusion on growth patterns

Production linkage	Diffusion rage	Reference	Impact on Growth
$\times$	economy-wide	Kortum (1997)	mean growth $\uparrow$
$\times$	within-sector	Akcigit and Ates (2023)	growth dispersion $\downarrow$

- Impact on network centrality

Data

Data Source

- Production Network
  - WRDS Supply Chain with IDs (Compustat Segment)
  - Concordance mapping GVKEYs to PERMCOs.
- Patents
  - OECD Citation Statistics
  - Mapping from US patent numbers to PERMCOs, compiled by Kogan et al. (2017)

Overlapping of knowledge network and production network

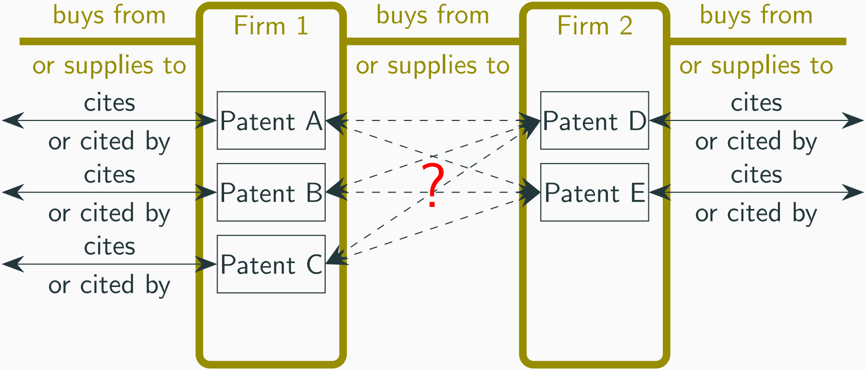


Figure 4: Citation behavior between supplier-customer pair

Overlapping of knowledge network and production network

Table 1: Citing behavior in production network

	Frequency	Percent
customer cites supplier	3,628	11.68%
supplier cites customer	3,513	11.31%
direction undistinguished	4,710	15.16%
# of supplier-customer pair	31,070	100.00%
customer cites supplier	3,628	28.46%
supplier cites customer	3,513	27.56%
direction undistinguished	4,710	36.95%
# of innovative supplier-customer pair	12,748	100.00%

Duration and mean sales of supply chain relationship

Table 2: Production linkage duration and mean sales on whether there is a citation

VARIABLES	(1) Duration	(2) Duration	(3) lgSales	(4) lgSales
has_citation	0.894***	-0.627***	0.0355	-0.0947*
	(0.193)	(0.189)	(0.0486)	(0.0501)
has_citation × cite_in_range		4.662***		0.331***
		(0.279)		(0.0591)
Supplier FE	Yes	Yes	Yes	Yes
Customer FE	Yes	Yes	Yes	Yes
Observations	10,566	10,566	7,928	7,928
R-squared	0.512	0.565	0.843	0.845

Standard errors clustered on supplier and customer level

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Simultaneity of link formation and patent citation

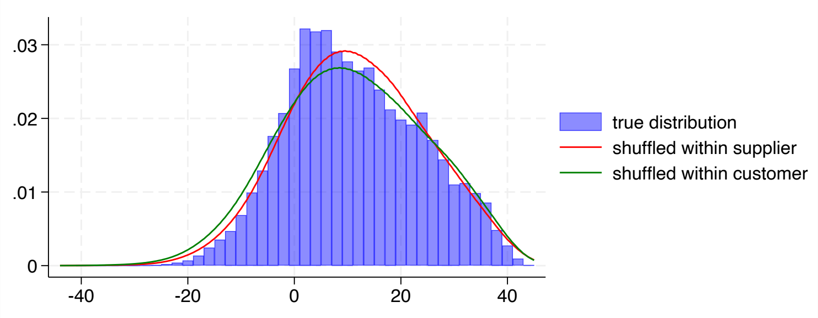


Figure 5: Histogram of patent filed, categorized by the year relative to production linkage formation

- Placebo exercise: shuffle link formation year within supplier or within customer

Model

Household

- Infinite horizon and log-utility of representative household

$$\max_{C_t} \sum_{t=0}^{\infty} \beta^t \log(C_t)$$
$$\text{s.t.} \qquad C_t + A_{t+1} = w_t L + (1 + r_t) A_t$$

- C_t is a CES aggregator over a continuum of varieties

$$C_t = \left(\int_0^1 c_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma-1}{\sigma}}$$

- HH problem gives the Euler Equation (Only care about the steady state) and the price index as the numeraire.

$$\frac{1}{1+r_t} = \beta \frac{C_{t+1}^{-\phi}}{C_t^{-\phi}} = \beta \quad \text{and} \quad P_t = \left(\int_0^1 p_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \equiv 1$$

Firms

- Cobb-Douglas Production function

$$y = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z x^\alpha l^{1-\alpha}$$

  where x is the intermediate input and l is the labor input, z is a matching-specific productivity, dependent on supplier identity

- Oberfield (2018): "Firms are in a N-stable equilibrium"
  - productivity definition $q := w/MC$ and $q = q_x^\alpha z$.
  - to grow, firms keep drawing suppliers, i.e. (q_x, z) pairs, The arrival rate of drawing $z \geq x$ is $(x/\lambda)^{-\theta}$
  - $C_t = Q_t L^p$ where $Q_t = \left(\int_0^1 q_{i,t}^{\sigma-1} di\right)^{\frac{1}{\sigma-1}}$, and L^p is the mass of labor participating in production.
  - no entry or exit

Deviation from Oberfield (2018)

- I separate the updating of q_x and z, add a match-specific "learnability" b_x. In each period, firm chooses one of the two activities

  stay keep current supplier, draw a new z. The arrival rate of drawing $z \geq x$ is $(x/(\lambda b_x))^{-\theta}$

  switch draw new suppliers. The arrival rate of drawing $b_x \geq y$ is $(y/\mu)^{-\delta}$. μ is the search effort determined endogenously. After the identity of the supplier is revealed, they immediately draw an initial z from $H(z/b_x)$.

  The key deviation from Oberfield (2018): allowing firms to stay with current supplier and still have productivity growth, this is interpreted as "knowledge diffusion"

- Death shock $\mathrm{Bernoulli}(p)$ to stabilize firms' distribution

After some boring algebra...

- Reparameterize $b := q_x^\alpha b_x$ as the supplier quality ($=$ supplier's productivity$^\alpha$ $\times$ supplier's learnability).
- (q, b) summarizes firm's idiosyncratic state.
- Given the rate of knowledge diffusion λ, rate of switch μ, the current quality of the supplier b,
  - If the firm choose to stay, the best draw of q for that period is subject to $\mathrm{Fréchet}(\lambda^\theta b^\theta, \theta)$.
  - If the firm chooses to switch, the best firm value generated by the new draw is subject to $\mathrm{Fréchet}(\mu^\delta \mathcal{V}\mathcal{Q}, \frac{\delta}{\sigma-1})$

  where $\mathcal{Q} := \int q_i^{\alpha\delta} di$ is the $\alpha\delta$-th moment of productivity, and $\mathcal{V} := \int_0^\infty V(x,1)^{\frac{\delta}{\sigma-1}} dH(x)$. Assume that H has thin enough tails such that $\mathcal{V} < \infty$.

Bellman Equation

$$
\begin{aligned}
V(q,b) &= \pi(q,w,L_p) + (1-p)V_{survive}(q,b) \\
V_{survive}(q,b) &= pV_{switch}(q,b) + (1-p)\max\{V_{stay}(q,b), V_{switch}(q,b)\} \\
V_{stay}(q,b) &= \beta\mathbb{E}[\max\{V(q_{new},b), V(q,b)\}] \\
s.t. &\quad q_{new} \sim \text{Fréchet}(\lambda b^{\theta}, \theta) \\
V_{switch}(q,b) &= \max_{\ell_s}(\beta\mathbb{E}[\max\{v_{new}, V(q,b)\}] - w\ell_s) \\
s.t. &\quad v_{new} \sim \text{Fréchet}\left(\mu(\ell_s, S(q,b))\mathcal{V}\mathcal{Q}, \frac{\delta}{\sigma-1}\right)
\end{aligned}
$$

- $\mu(\ell_s, S(q,b))$ is the production of search effort from Klette and Kortum (2004). $S(q,b)$ is the search technology. The key assumption is that $\mu(\cdot,\cdot)$ is a homogeneous function.

BGP equilibrium

- $V(q,b)$, $V_{stay}(q,b)$, $V_{switch}(q,b)$ are all homogeneous of degree $\sigma - 1$.
- Firm will stay if:

$$V_{stay}(q,b) \geq V_{switch}(q,b) \Rightarrow V_{stay}(q/b,1) \geq V_{switch}(q/b,1)$$

  Simply need to solve $V_{stay}(k,1) = V_{switch}(k,1)$

- It can be proven that such $k^* \in (0,\infty)$ exists and is unique.

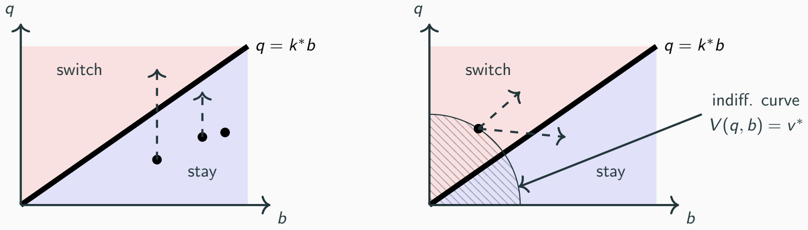


Figure 6: Evolution of firm productivity and supplier quality

Law of motion of measures

For demonstration purposes, consider only the case with no entry or exit now.

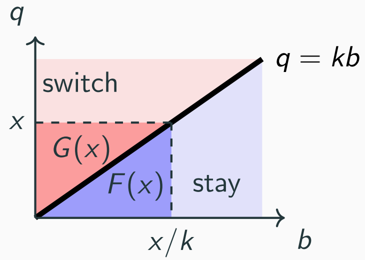


Figure 7: Definition of F and G

- LoM of $\{F, G\}_t$ summarized by a system of forward equations
- No closed form solution but the moments do!

Law of motion of moments

$$\begin{bmatrix} m^q_{t+1}(s) \\ m^b_{t+1}(s) \end{bmatrix} = \mathbf{A}(s) \begin{bmatrix} m^q_t(s) \\ m^b_t(s) \end{bmatrix}$$

$\mathbf{A}(s)$ an matrix, can be explicitly written as a function of s, λ, μ^*, V, k, $\mathcal{V}$, $\mathcal{Q}$

$m^q_t(s)$ the s-th moment of switching firms' productivity q at time t

$m^b_t(s)$ the s-th moment of staying firms' supplier quality b at time t

The backbone of this model is simply the **random growth model**, i.e. **Gibrat's Law**! The original 1-d random growth model $X_{t+1} = g_t X_t$ (g_t independent of X_t) has the same linear property of the moments: $\mathbb{E}X^s_{t+1} = \mathbb{E}g^s_t \cdot \mathbb{E}X^s_t$. Our model nests the random growth model by setting $\lambda = 0$.

Closing the model

Let the s-th moment of staying firms' productivity q under steady state be $\Phi(s)$

- The stationary technology frontier Q (i.e. $[M^q(\sigma-1)] + \Phi(\sigma-1)]^{\frac{1}{\sigma-1}}$) matches the Q in the bellman equation.
- The stationary $\alpha\delta$-th moment $\mathcal{Q}$ (i.e. $M^q(\alpha\delta) + \Phi(\alpha\delta)$) matches the $\mathcal{Q}$ in the bellman equation.
- Labor market clears: Labor used in production, and supplier search sums up to the total measure of labor.
- Free entry of entrants.

Distribution of linkage duration

Let $\{A, B, C, D\}$ be the four states defined by the following

- **A** staying
- **B** switching but linkage didn't break since last period (either just transitioned from A or the last supplier draw falls inside the indifference curve)
- **C** switching but linkage broke past period, including supplier exit
- **D** exit

In steady state, firms jump between these states in a Markov process, with the transition matrix $\mathcal{P}$ (endogenous). Need to find the distribution of a sequence like

$$\text{as an example: } \{A, A, A, B, B, (A, C, \text{or } D)\}$$

Distribution of linkage duration

$$
\mathcal{T} = \begin{array}{c|ccccc|c}
 & AA & AB & AX & BB & BX & X \\
\hline
AA & \mathcal{P}_{AA} & \mathcal{P}_{AB} & 1-\mathcal{P}_{AA}-\mathcal{P}_{AB} & & & \\
AB & & & & \mathcal{P}_{BB} & 1-\mathcal{P}_{BB} & \\
AX & & & & & & 1 \\
BB & & & & \mathcal{P}_{BB} & 1-\mathcal{P}_{BB} & \\
BX & & & & & & 1 \\
\hline
X & & & & & & 1
\end{array}
= \begin{bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{0} & 1 \end{bmatrix}
$$

This is what is called an absorbing markov, because a link will eventually break a.s.. Given that a production linkage always begins with A, the initial state is $\boldsymbol{\pi}_0 = [\mathcal{P}_{AA}, \mathcal{P}_{AB}, 1-\mathcal{P}_{AA}-\mathcal{P}_{AB}, 0, 0, 0]^T$. The periods ($N$) elapsed when reaching the absorbing state has pmf

$$
\Pr(N = n) = \boldsymbol{\pi}_0^T \mathbf{Q}^{n-1} \mathbf{R} \quad \text{(Mixed Geometric distribution)}
$$

True distribution of duration

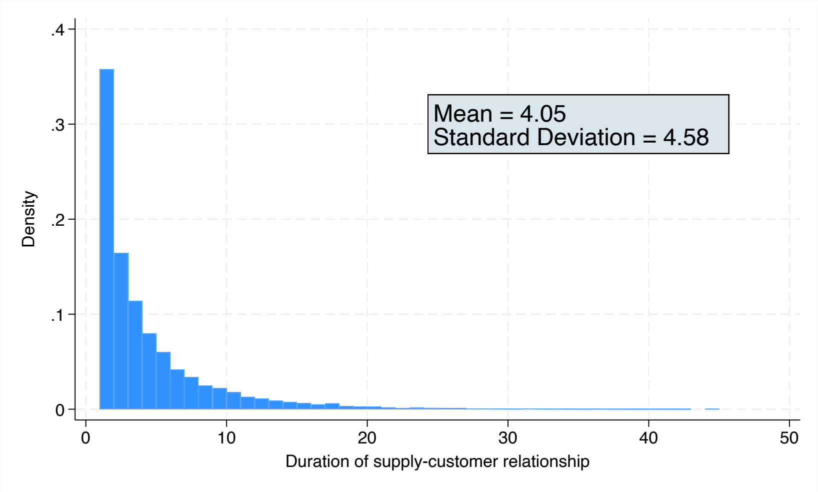


Figure 8: True distribution of duration

Simulation

var. name	meaning	value 1
λ	knowledge diffusion rate	0.000
α	Cobb-Douglas intermediate input share	0.2
β	discount rate	0.99
σ	CES elasticity of substitution	2.0
θ	tail exponent of new match-specific prod.	4.0
δ	tail exponent of learnability draw	4.0
p	death rate	0.1
a	unit search cost (incumbent)	4.5
η	elasticity of search cost (incumbent)	6.0
ψ	search efficiency (entrants)	1.0
H	initial distribution of match-specific prod.	Uniform(0,1)

Table 3: Exogenous variables

I simulate the model with $\lambda = 0.001, 0.002, ..., 0.01$.

Simluation cont.

var. name	meaning	sign of change
Q	technology frontier	$+$
L_p	labor in production	$+$
k	cut-off technology level	$+$
	labor hired for switching per firm value	$-$
	measure of entry	$-$
$\mathbb{E}[\text{Duration}]$	mean duration of a production linkage	$+$
	measure of staying firms	$+$

Table 4: Key endogenous variables

Main takeaways

- Firms choose NOT the "least-cost provider" as their supplier, but the supplier that generates the best firm value. They are happy to choose an expensive supplier but with high potential of knowledge diffusion. (Unlike Kortum (1997) or Oberfield (2018))
- High knowledge diffusion generates
  - High growth conditional on surviving (on par with Kortum (1997))
  - High growth dispersion conditional on surviving (complementing Akcigit and Ates (2023))
  - Declining share of firms that searches, "sticky" production network.

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